Exp # 1

Find area b/w $y = \sec^2 x$ & $y = \sin x$.

Sol:- Limit:- from 0 to $\pi/4$.

$y = y$.

$\sec^2 x = \sin x$.

$\frac{1}{\cos^2 x} = \sin x$.

$\sec^2 0 = 1$.

$\sin 0 = 0$.

For $\sec^2 x$

$A(0, 1)$

$x = \pi/4$.

$\frac{1}{(\cos \pi/4)^2} \Rightarrow 2$

$B(\pi/4, 2)$

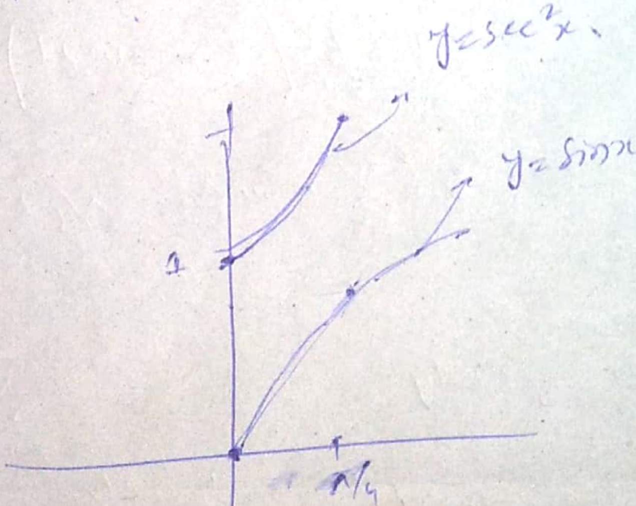
For $= \sin 0$.

$A(0, 0)$

For $\pi/4$.

$B(\pi/4, 0.7071.)$

Graph:-



$$\int_0^{\pi/4} (\sec^2 x - \sin x)' = \tan \pi/4 - 0 - \sin \pi/4$$

$= 1 - 0.7071.$

$= 0.2929$

A = $\sqrt{2}/2$ Ans

$$\int_0^{\pi} (1 - \cos^2 x)\, dx.$$

$$\int_0^{\pi} (\sin^2 x)\, dx = \int_0^{\pi} \frac{1 - \cos 2x}{2}\, dx$$

$$\left(\frac{1}{2}x - \frac{\sin 2x}{2}\right)\Big|_0^{\pi}$$

$$= \frac{\pi}{2} - \frac{0}{2} \quad \frac{\pi}{2} \text{ Ans}$$

$$\int_{-\pi/3}^{\pi/3} \cancel{f} \left(\frac{1}{2}\sec^2 \cancel{x}\, t + 4\sin^2 t\right)$$

$$= \left(\frac{1}{2}\tan t\right) - 4\int \left(\frac{1 - \cos 2t}{2}\right) dt.$$

$$= \cancel{\frac{4\pi}{2}} \; \frac{1}{2}\tan(\pi/3) - \frac{1}{2}\tan(-\pi/3)$$

$$- 4\left(\frac{1}{2}\right)\left[t - \sin 2t\right]$$

$$= -2\left(\pi/3 - \sin \pi/3 - \pi/3 + \overset{\sin}{\pi/3}\right.$$

Q3:-

$$\int_0^1 (y^2 - y^3)\, dy.$$

$$\left(\frac{y^3}{3} - \frac{y^4}{4}\right)\Big|_0^1 \Rightarrow \frac{1}{3} - \frac{1}{4} \Rightarrow \frac{1}{12}$$

Q4:-

$$\int \quad 12y^2 - 12y^3 = 2y^2 - 2y.$$

$$2(6y^2 - 6y^3 = y^2 - y.$$

$$6y^3 + y^2 - 6y^2 - y = 0.$$

$$6y^3 \cancel{+} - 5y^2 - y = 0.$$

Q10:- Find the area between $y = x^2$ & $y = 4$.

Sol:- Step # 1

Finding limit;

$y = y$

$x^2 = 4$

$x = \pm 2$.

So limit is -2 to 2.

For $x = -2$.

$\Rightarrow y = x^2$.

$\Rightarrow y = 4$

$A(-2, 4)$

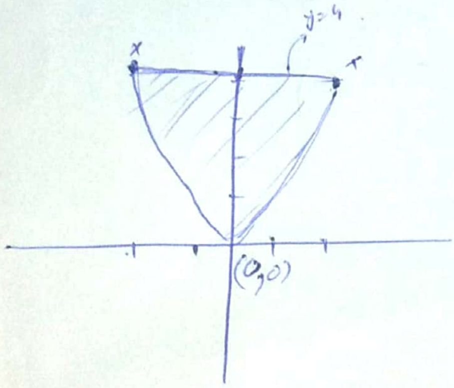
For $x = 2$.

$\Rightarrow y = x^2$.

$y = 4$

$B(2, 4)$.

Second step # Graph:



Step # 3: Origin:-

To complete rough graph put $x = 0$.

$y = 0 \Rightarrow C(0, 0)$.

Step # 4. Using formula:-

Since; $A = \int_{-2}^{2} (f(x) - g(x))\, dx$ ∵ Putting values

$A = \int^{2} (4 - x^2)\, dx$.

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$$\text{Area} = \int_{-2}^{2}(4-x^2)\,dx.$$

$$= 4\int_{-2}^{2} dx - \int_{-2}^{2} x^2\,dx$$

$$= \left(4x - \frac{x^3}{3}\right)\Big|_{-2}^{2}$$

$$= \left(4(2) - \frac{(2)^3}{3}\right) - \left(4(-2) - \frac{(-2)^3}{3}\right).$$

$$= \left(8 - 8/3\right) - \left(-8 + 8/3\right).$$

$$= \left(\frac{24-8}{3}\right) - \left(\frac{-24+8}{3}\right).$$

$$= \frac{16}{3} - \left(\frac{-16}{3}\right)$$

$$= \frac{16}{3} + 16/3 \Rightarrow 2\left(\frac{16}{3}\right).$$

$$\boxed{\text{Area} = \frac{32}{3} \text{ Ans}}$$

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Q3,

$$= \pi \int_{-3}^{3} (R(x))^2 \, dx \Rightarrow \pi \int_{-3}^{3} (\sqrt{9-x^2})^2 \, dx.$$

$$= \pi \left((9-x^2)\right) \Big|_{-3}^{3}.$$

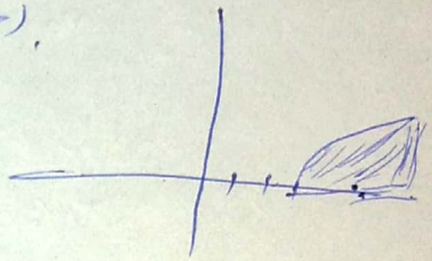
$$= \pi \left(9x - \frac{x^3}{3}\right) \Big|_{-3}^{3}.$$

$$= \left(9(3) - \frac{3^3}{3}\right) - \left(9(-3) - \frac{(-3)^3}{3}\right)$$

$$= \left(27 - \frac{27}{3}\right) - \left(-27 + 27/3\right)$$

$$= \left(\frac{51-27}{3}\right) - \left(\frac{-51+27}{3}\right) \Rightarrow$$

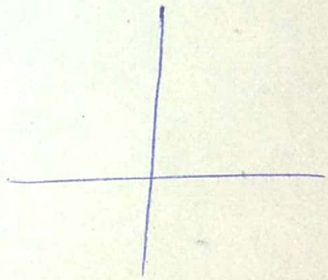
$$\boxed{V = 36\pi \text{ Ans}}$$



Q8:- $y = x - x^2$, $y = 0$

$x(x-1) = 0$

$x = 0$, $x = 1$



$$\pi \int_{0}^{1} \left((x - x^2)\right)^2 dx.$$

$$= \pi \int (x^2 + x^4 - 2x^3) \, dx \Rightarrow \pi \left(\frac{x^3}{3} + \frac{x^5}{5} - \frac{x^4}{2}\right) \Big|_{0}^{1}$$

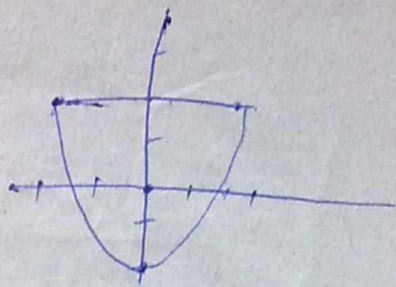
$$= \pi \left(\frac{1}{3} + \frac{1}{5} - \frac{1}{2}\right).$$

2 | 2, 3, 5
 | 1,

$$= \pi \left(\frac{10 + 6 - 15}{30}\right).$$

$$= \pi \, \frac{1}{30}$$

$x^2 - 4 = -x^2 - 2x$



$2x^2 + 2x - 4 = 0$

$x^2 + x - 2 = 0$

$x^2 + 2x - x - 2 = 0$

$x(x+2) - 1(x+2) = 0$

$x = -2, \quad x = 1$

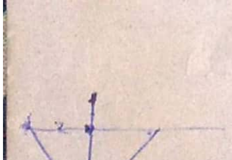
$$\int_{-3}^{-2} (x^2 - 4 + x^2 + 2x)\,dx + \int_{-2}^{1} (-x^2 - 2x - x^2 + 4)$$

$$= \int_{-3}^{-2} (2x^2 + 2x - 4)\,dx + \int_{-2}^{1} (-2x^2 - 2x + 4)\,dx$$

$$= \left(\frac{2x^3}{3} + x^2 - 4x\right)\Bigg|_{-3}^{-2} + \left(-\frac{2x^3}{3} - x^2 + 4x\right)\Bigg|_{-2}^{1}$$

$$= \left(\left(-\frac{16}{3} + 4 + 8\right) - \left(-\frac{54}{3} + 9 + 12\right)\right) + \left(\left(-\frac{2}{3} - 1 + 4\right) - \left(\frac{16}{3} - 4 - 8\right)\right)$$

$$= \left(-\frac{16}{3} + 12 + \frac{54}{3} - 21\right) + \left(-\frac{2}{3} + 3 - \frac{16}{3} + 12\right)$$



$$= \left(\frac{-16 + 36 + 54 - 63}{3}\right) + \left(\frac{-2 + 9 - 16 + 36}{3}\right)$$

$$= \frac{11}{3} + (9) = \frac{11 + 27}{3} = \frac{38}{3}$$

S13:- $y = x^2 - 2, \quad y = 2$

$x^2 - 2 = 2$

$x^2 = 4$

$x = \pm 2$

$(-2, 2), (2, 2)$

$$\int_{-2}^{2} (2 - x^2 + 2)$$

4 Am.

Q14:- $y = 2x - x^2$ & $y = -3$
2

$2x - x^2 = -3.$

$x^2 - 2x - 3 = 0. \Rightarrow x^2 - 3x + x - 3 = 0.$

$x(x-3) + 1(x-3)$

$y = 2(-1) - (-1)^2.$

$= -2 - 1.$

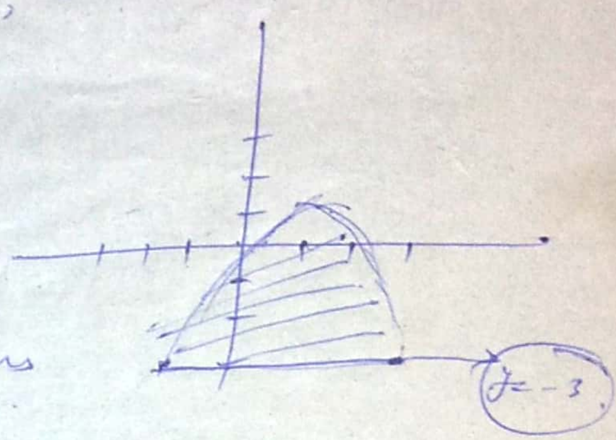
$= -3.$

$x = -1, \; x = 3.$

$x(2-x).$

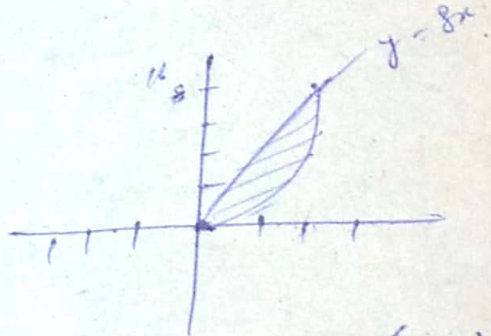
$A(-1,-3) \quad B(3,-3).$

origin
$\Rightarrow (0,0)$



$\int_{-1}^{3} (2x - x^2 + 3)\,dx.$ Ans
2.

Q15:- $y = x^4, \quad y = 8x.$
2



$x^4 = 8x.$

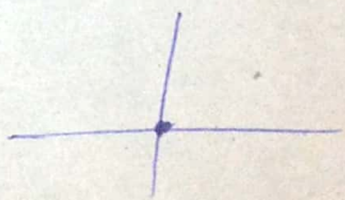
$x^3 = 8.$

$\boxed{x = 2}$

$A(0,0)$

$A(2, 16)$

$(0,0)$



$\int_0^2 (8x - x^4)\,dx.$

Q16 $y = x^2 - 2x, \quad y = x$
2

$x^2 - 2x - x = 0 \Rightarrow x^2 - 3x = 0.$

$x(x-3) = 0.$

$x = 0, \; x = 3$

$\int_0^3$

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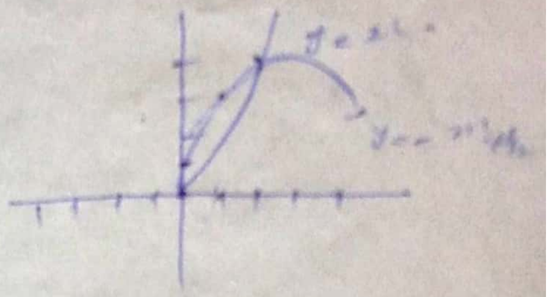
Q2. $y = x^2$ & $y = -x^2 + 4x$

$2x^2 - 4x = 0$.

$2x(x-2) = 0$.

$x = 0$, $x = 2$

$A(0,0)$, $B(2,4)$



$$\int_0^2 (-x^2 + 4x - x^2)\,dx$$

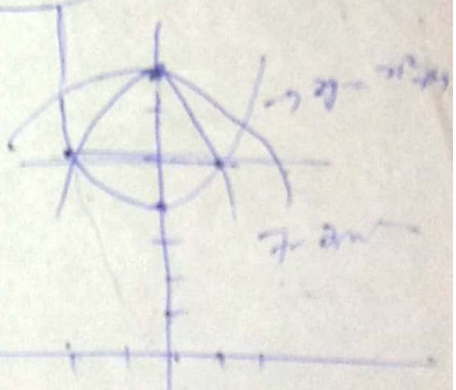
Q3:- $y = 7 - 2x^2$ & $y = x^2 + 4$

$x^2 + 4 = -2x^2 + 7$

$x^2 + 2x^2 + 4 - 7 = 0$.

$3x^2 = 3 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$.

$A(1,5)$, $B(-1,5)$



origin

$x = 0$
$y = 4$
$y = 7$

$$A = \int (7 - 3x^2 - x^2 - 4)\,dx$$

$$= \int_{-1}^{1} (3 - 3x^2)\,dx$$

$$(3x - x^3)\Big|_{-1}^{1} \Rightarrow (3-1) - (-3+1)$$

$= 2 - (-2)$

$= 4$ An

(19) $y = x^4 - 4x^2 + 4$ & $y = x^2$

$x^4 - 4x^2 + 4 = x^2$

$x^4 - 4x^2 - x^2 + 4 = 0$

$x^4 - 5x^2 + 4 = 0$

let $x^2 = y$

$y^2 - 5y + 4 = 0$

$y^2 - 4y - y + 4 = 0$

$y(y-4) - 1(y-4) = 0$

$y - 1 = 0$ or $y = 4$

$y = 1$

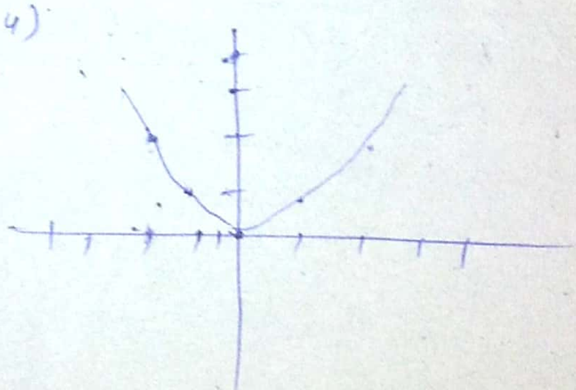
$x = \pm 1$ $x = \pm 2$

A $(-1,1)$, B$(1,1)$, C$(-2,4)$, $(2,4)$

if $x = 0$

$y = 4$

$(0,4)$

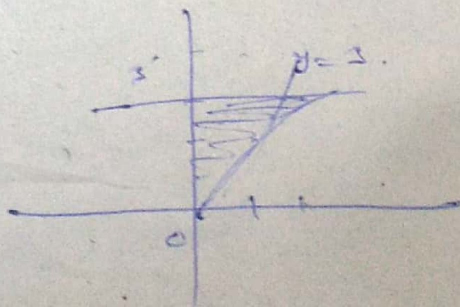


(20) $y = x\sqrt{a^2 - x^2}$, $a > 0$ & $y = 0$

Q23:- $x = 2y^2$. $x = 0$ & $y = 3$.

$$\int_0^3 2y^2\,dy.$$

Ans



Q:- Find $\frac{d^{73}}{dx^{73}} x \sin x = ?$

Sol:- We have to find $\frac{d^{73}}{dx^{73}} x \sin x = ?$

Since;

$$\frac{d}{dx} x \sin x = \sin x + x \cos x.$$

$$\frac{d^2}{dx^2}(\sin x + x\cos x) = \cos x + \cos x + x(-\sin x)$$

$$= 2\cos x - x\sin x.$$

$$\frac{d^3}{dx^3}(2\cos x - x\sin x) = -2\sin x - \sin x - x\cos x.$$

$$= -3\sin x - x\cos x.$$

$$\frac{d^4}{dx^4}(-3\sin x - x\cos x) = -3\cos x - (\cos x + x(-\sin x))$$

$$= -4\cos x + x\sin x.$$

$$\frac{d^5}{dx^5}(-4\cos x + x\sin x) = 4\sin x + \sin x + x\cos x.$$

$$= 5\sin x + x\cos x.$$

As first term repeated after 5 (each) derivati

so;

$$\boxed{\frac{d^{73}}{dx^{73}}(x \sin x) = -73\sin x - x\cos x.}$$

Ans.

Example #6

$u = u(y-z, z-x, x-y)$, prove That;

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0.$$

Sol:- Let $r = y - z$, $s = z - x$ & $t = x - y$.

From formula.

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial r} \cdot \frac{\partial r}{\partial x} + \frac{\partial u}{\partial s} \cdot \frac{\partial s}{\partial x} + \frac{\partial u}{\partial t} \cdot \frac{\partial t}{\partial x}.$$

$$= \frac{\partial u}{\partial r}(0) + \frac{\partial u}{\partial s}(-1) + \frac{\partial u}{\partial t} \cdot (+1).$$

$$= -\frac{\partial u}{\partial s} + \frac{\partial u}{\partial t} \longrightarrow (1).$$

$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial r} \cdot \frac{\partial r}{\partial y} + \frac{\partial u}{\partial s} \cdot \frac{\partial s}{\partial y} + \frac{\partial u}{\partial t} \cdot \frac{\partial t}{\partial y}.$$

$$= \frac{\partial u}{\partial r}(+1) + \frac{\partial u}{\partial s}(0) + \frac{\partial u}{\partial t}(-1).$$

$$= +\frac{\partial u}{\partial r} - \frac{\partial u}{\partial t} \longrightarrow (2).$$

$$\frac{\partial u}{\partial z} = \frac{\partial u}{\partial r} \cdot \frac{\partial r}{\partial z} + \frac{\partial u}{\partial s} \cdot \frac{\partial s}{\partial z} + \frac{\partial u}{\partial t} \cdot \frac{\partial t}{\partial z}.$$

$$= \frac{\partial u}{\partial r}(-1) + \frac{\partial u}{\partial s}(1) + \frac{\partial u}{\partial t}(0).$$

$$= -\frac{\partial u}{\partial r} + \frac{\partial u}{\partial s} \longrightarrow (3).$$

Add & (1), (2) & (3),

$$= -\cancel{\frac{\partial u}{\partial s}} + \cancel{\frac{\partial u}{\partial t}} - \cancel{\frac{\partial u}{\partial t}} + \cancel{\frac{\partial u}{\partial r}} - \cancel{\frac{\partial u}{\partial r}} + \cancel{\frac{\partial u}{\partial s}}$$

$= 0$ Ans.

Q8:- $\lim_{\substack{x \to 0 \\ y \to 0}} \dfrac{x^2 y^3}{x^2 + y^2}$, $x \neq 0, y \neq 0$.

Sol:-

$l_1 = \lim_{y \to 0} \left(\lim_{x \to 0} \dfrac{x^2 y^3}{x^2 + y^2} \right) = \lim_{y \to 0} (0) = 0.$

$l_2 = \lim_{x \to 0} \left(\lim_{y \to 0} \dfrac{x^2 y^3}{x^2 + y^2} \right) = \lim_{x \to 0} (0) = 0.$

l_3 : put $y = mx$.

$l_3 = \lim_{x \to 0} \left(\dfrac{x^2 (mx)^3}{x^2 + (mx)^2} \right).$

$= \lim_{x \to 0} \dfrac{mx^5}{x^2 + mx^2} \Rightarrow \lim_{x \to 0} \dfrac{\cancel{x^2}(mx^3)}{\cancel{x^2}(1+m)}.$

$= 0.$

l_4 = Put $y = mx^2$.

$l_4 : \lim_{x \to 0} \left(\dfrac{x^2 (mx^2)^3}{x^2 + (mx^2)^3} \right) \Rightarrow \lim_{x \to 0} \dfrac{m x^8}{x^2 + m^2 x^6}.$

$= \lim_{x \to 0} \dfrac{\cancel{x^2} \cdot m x^6}{\cancel{x^2}(1 + m^2 x^4)}$

$= \dfrac{0}{1+0} \Rightarrow 0.$

As $l_1 = l_2 = l_3 = l_4$ So limit exist

at $x = 0$.

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Q4:- $\lim\limits_{\substack{x\to 0\\ y\to 0}} \dfrac{xy}{y-x^2}$; $x\neq 0,\ y\neq 0$.

Sol:- $l_1 = \lim\limits_{y\to 0}\left(\lim\limits_{x\to 0}\left(\dfrac{xy}{y-x^2}\right)\right) = \lim\limits_{y\to 0}(0) = 0.$

$l_2 = \lim\limits_{x\to 0}\left(\lim\limits_{y\to 0}\left(\dfrac{xy}{y-x^2}\right)\right) = \lim\limits_{x\to 0}(0) = 0.$

l_3: $y = mx$.

$l_3 = \lim\limits_{x\to 0}\left(\dfrac{x(mx)}{mx-x^2}\right) = \lim\limits_{x\to 0}\left(\dfrac{mx^2}{mx-x^2}\right).$

$= \lim\limits_{x\to 0} \dfrac{\cancel{x^2}}{\cancel{x^2}}\left(\dfrac{m}{m/x-1}\right).$

$= \lim\limits_{x\to 0}\left(\dfrac{m}{-1}\right) = -m.$

l_4: $y = mx^2$. Put value of y.

$l_4 = \lim\limits_{x\to 0}\left(\dfrac{x(mx^2)}{mx^2-x^2}\right) \Rightarrow \lim\limits_{x\to 0} \dfrac{mx^3}{mx^2-x^2}.$

$= \lim\limits_{x\to 0} \dfrac{\cancel{x^2}(mx)}{\cancel{x^2}(m-1)} \Rightarrow \lim\limits_{x\to 0} \dfrac{mx}{m-1}$

$= 0.$

As $l_1 = l_2 \neq l_3 = l_3$. So limit does not exist.

Example # 16

$$z = e^{ax+by} \cdot f(ax-by). \text{ prove That;}$$

$$b\frac{\partial z}{\partial x} + a\frac{\partial z}{\partial y} = 2abz$$

Sol:-

$$\frac{\partial z}{\partial x} = e^{ax+by}(a).f(ax-by) + e^{ax+by} \cdot f'(ax-by)(a)$$

$$= a\left(e^{ax+by} f(ax-by)\right) + e^{ax+by} f'(ax-by)).$$

$$b\frac{\partial z}{\partial x} = abe^{ax+by}\left(f(ax-by) + f'(ax-by)\right).$$

$\hookrightarrow$ 1.

$$\frac{\partial z}{\partial y} = e^{ax+by}(b).f(ax-by) + e^{ax+by} f'(ax-by)(-b).$$

$$= e^{ax+by}\, b\left(f(ax-by) - f'(ax-by)\right).$$

$$a\frac{\partial z}{\partial y} = abe^{ax+by}\left(f(ax-by) - f'(ax-by)\right).$$

Now add.

$$b\frac{\partial z}{\partial x} + a\frac{\partial z}{\partial y} = abe^{ax+by}\left(f(ax-by) + f'(ax-by)\right)$$
$$+ abe^{ax+by}\left(f(ax-by) - f'(ax-by)\right)$$

$$= abe^{ax+by}\left[f(ax-by) + \cancel{f'(ax-by)} + f(ax-by) - \cancel{f'(ax-by)}\right]$$

$$= abe^{ax+by}\left(2f(ax-by)\right).$$

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Q. $f(x,y) = \begin{cases} x^3+y^3 & \text{when } x \neq 0,\ y \neq 0 \\ \cancel{0}\ 1 & \text{when } x=0,\ y=0 \end{cases}$ at origin

Sol:-

$l_1 = \lim_{y \to 0} \left(\lim_{x \to 0} (x^3+y^3) \right) = \lim_{y \to 0} (0) = 0$

$l_2 = \lim_{x \to 0} \left(\lim_{y \to 0} (x^3+y^3) \right) = \lim_{x \to 0} (0) = 0.$

~~$l_3 = \lim_{(x,y)\to(0,0)} (x^3+y^3) = 0.$~~

~~As $l_1 = l_2 = l_3$ so $f(x,y)$ is~~ continous

~~at $x=0.$~~

l_3: $y = mx.$

$\lim_{x \to 0} (x^3+(mx)^3) \Rightarrow \lim_{x \to 0} (x^3+mx^3) = 0.$

l_4: $y = mx^2.$

$\lim_{x \to 0} (x^3+(mx^2)^3) = \lim_{x \to 0} (x^3+mx^6)$

$= 0.$

$l_1 = l_2 = l_3 = l_4$ so limit exist at origin

Example # 11

$$f(x,y) = \begin{cases} \dfrac{x^2 + xy + x + y}{x+y} & (x,y) \neq (2,-2) \\ 4 & (x,y) = (2,-2) \end{cases}$$

Sol:

$$f(x,y) = \begin{cases} \dfrac{x^2 + xy + x + y}{x+y}, & (x,y) \neq (2,-2) \\ 4 & (x,y) = (2,-2). \end{cases}$$

$$l_1 = \lim_{(x,y)\to(2,-2)} \frac{x^2 + xy + x + y}{x+y}.$$

$$= \lim_{(x,y)\to(2,-2)} \frac{x(x+y) + 1(x+y)}{(x+y)}.$$

$$= \lim_{(x,y)\to(2,-2)} \frac{\cancel{(x+y)}(x+1)}{\cancel{(x+y)}}.$$

$$l_1 = \lim_{(x,y)\to(2,-2)} (x+1) = 2+1 = 3.$$

$$l_2 = 4.$$

As $l_1 \neq l_2$.

so limit does not exist.

Example #7

$$\lim_{\substack{x \to \infty \\ y \to 3}} \frac{2x-3}{x^3+4y^3}$$

Sol:-

$$\lim_{\substack{x \to \infty \\ y \to 3}} \frac{2x-3}{x^3+4y^3} \Rightarrow \lim_{\substack{x \to \infty \\ y \to 3}} \frac{x^3\left(2/x^2 - 3/x^3\right)}{x^3\left(1+4y^3/x^3\right)}$$

$$\lim_{y \to 3}\left(\lim_{x \to \infty} \frac{\left(2/x^2 - 3/x^3\right)}{1+4y^3/x^3}\right)$$

$$l_1 = \lim_{y \to 3}\left(\frac{0-0}{1+0}\right) \Rightarrow 0$$

Now;

$$\cancel{\lim_{\substack{x \to \infty \\ y \to 3}} x^3\left(2/x^2 - \right.}$$

$$\lim_{\substack{x \to \infty \\ y \to 3}} \frac{2x-3}{x^3+4y^3}$$

$$\lim_{x \to \infty}\left(\lim_{y \to 3} \frac{2(x)-3}{x^3+4(y)^3}\right)$$

$$= \lim_{x \to \infty}\left(\frac{2x-3}{x^3+4(3)^3}\right) \Rightarrow \lim_{x \to \infty}\left(\frac{2x-3}{x^3+(27)4}\right)$$

$$l_2 = \lim_{x \to \infty} \frac{x^3\left(2/x^2 - 3/x^3\right)}{x^3(1+108)} \Rightarrow \frac{0-0}{1+108} = 0$$

As $l_1 = l_2$ so limit exist at ∞ & 3.

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Example # 8

$$\lim_{\substack{x\to\infty \\ y\to 2}} \frac{xy+4}{x^2+2y^2}$$

Sol:- $l_1 = \lim_{y\to 2}\left(\lim_{x\to\infty} \frac{xy+4}{x^2+2y^2}\right)$

$$= \lim_{y\to 2}\left(\lim_{x\to\infty} \frac{x(y+4/x)}{x^2(1+2y^2/x^2)}\right)$$

$$= \lim_{y\to 2}\left(\frac{1}{\infty}\left(\frac{y+4/\infty}{1+2y^2/\infty^2}\right)\right)$$

$$= \lim_{y\to 2}(0) = 0.$$

$l_2 = \lim_{x\to\infty}\left(\lim_{y\to 2} \frac{xy+4}{x^2+2y^2}\right).$

$$= \lim_{x\to\infty}\left(\frac{2x+4}{x^2+8}\right).$$

$$= \lim_{x\to\infty}\left(\frac{x^2(2/x+4/x^2)}{x^2(1+8/x^2)}\right)$$

$$= \lim_{x\to\infty}\left(\frac{2/x+4/x^2}{1+8/x^2}\right) = \lim_{x\to 0}\frac{0}{1+\infty 0}$$

$$= 0$$

$l_1 = l_2$ so limit exist.