Q 43:- $\int \sqrt{x} \ln x \, dx.$

Sol:- As;

$= uv - \int v \, du.$

Let $u = \ln x.$

$\frac{du}{dx} = \frac{1}{x}.$

$du = \frac{1}{x} dx.$

$dv = \sqrt{x} \, dx.$

$\int dv = \frac{x^{1/2+1}}{1/2+1}.$

$v = \frac{2x^{3/2}}{3}.$

$= \frac{2}{3} x^{3/2} \ln x - \int \frac{2}{3} x^{3/2} \cdot \frac{1}{x} \cdot dx$

$= \frac{2}{3} x^{3/2} \ln x - \frac{2}{3} \int \frac{x^{3/2}}{x}$ $\quad \frac{3}{2} - 1$ $\quad \frac{1}{2}$

$\frac{2}{3} x^{3/2} \ln x - \frac{2}{3} \int \sqrt{x}$ $\quad x^{\frac{1}{2}+1}$

$\frac{2}{3} x^{3/2} \ln x - \frac{2}{3} x^{3/2} \left(\frac{2}{3}\right)$

$\frac{2}{3} x^{3/2} \ln x - \frac{4}{9} x^{3/2} + C$

Slope tangent, & normal.

Q35:- $2xy + \pi \sin y = 2\pi$ $\quad (1, \pi/2)$.

Sol:- $2\left(y + x\frac{dy}{dx}\right) + \pi \cos y \frac{dy}{dx} = 0.$

$$2y + 2x\frac{dy}{dx} + \pi \cos y \frac{dy}{dx} = 0.$$

$$\frac{dy}{dx}\left(2x + \pi^{\cos y}\right) = -2y.$$

$$\boxed{\frac{dy}{dx} = \frac{-2y}{2x + \pi \cos y}}$$

$$\frac{dy}{dx} = m = \frac{-2(\pi/2)}{2 + \pi \cos(\pi/2)}$$

$$m = \frac{-\pi}{2}.$$

Tangent line,

$$y - \pi/2 = -\pi/2(x - 1).$$

For normal line,

$$m_1 \cdot m_2 = -1 \Rightarrow m_2 = \frac{-1}{-\pi/2} \Rightarrow +\frac{2}{\pi}.$$

$$y - \frac{\pi}{2} = \frac{2}{\pi}(x - 1) \quad \text{Ans}$$

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$y)$

$s(\pi x - y)\frac{dy}{dx}$

$\pi\cos(\pi x - y)$

$s(\pi x - y)$

$)$.

Sol:- $y' = 2\cos(\pi x - y)\left(\pi - \frac{dy}{dx}\right)$

$$\frac{dy}{dx} = \pi 2\cos(\pi x - y) - 2\cos(\pi x - y)\frac{dy}{dx}$$

$$\frac{dy}{dx}\left(1 + 2\cos(\pi x - y)\right) = 2\pi\cos(\pi x - y).$$

$$\frac{dy}{dx} = \frac{2\pi\cos(\pi x - y)}{1 + 2\cos(\pi x - y)}.$$

$$\frac{dy}{dx} = m\Big|_{(1,0)} = \frac{2\pi(-1)}{1 + 2(-1)} \Rightarrow \frac{-2\pi}{-1} = 2\pi.$$

$m = 2\pi$

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Q38:-

$$x^2\cos^2 y - \sin y = 0 \qquad (0, \pi).$$

Sol:- $$\frac{d}{dx}\left(x^2\cos^2 y - \sin y\right) = 0.$$

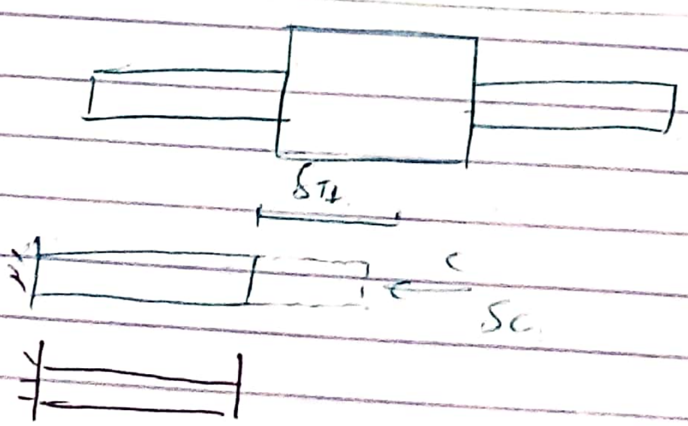
$$2x\cos^2 y + x^2 . 2\cos y . (-\sin y)\frac{dy}{dx} - \cos y\frac{dy}{dx} = 0.$$

$$\frac{dy}{dx}\left(2x^2\cos y(-\sin y) - \cos y\right) = -2x\cos^2 y.$$

$$\frac{dy}{dx} = \frac{\cancel{-}2x\cos^2 y}{\cancel{-}\left(2x^2\cos y\sin y - \cos y\right)}$$

$$\frac{dy}{dx} = \frac{2x\cos^2 y}{2x^2\cos y\sin y - \cos y}.$$

$$m = \frac{dy}{dx}\Big|_{(0,\pi)} = \frac{2(0)\cos(\pi)^2}{}$$



$t^2 (t^2+4)^{-1}$.

$$\frac{t^2}{t^2+4}$$

$$\cancel{\frac{t^2}{t^2(1+4/t^2)}}$$

$$\frac{\frac{4}{t^2+4}}{t^2}$$

Q16.→ $\int (\sec x + \tan x)^2 \, dx$.

$\sec^2 x + \tan^2 x + 2\sec x \tan x$.

$\sec^2 x + (\sec^2 \theta - 1) + 2\sec x \tan x$.

$2\sec^2 \theta - 1 + 2\sec x \tan x$.

$$\boxed{2\tan \theta - \theta + 2\sec\theta + c}$$ Ans →

Q $\frac{27}{2}$ $\int_{-4}^{4} |x| \, dx$.

$$|x| = \begin{cases} x = \cancel{x \geq 0}, & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

$$= \int_{-4}^{0} -x \, dx + \int_{0}^{4} x \, dx.$$

$$= \frac{-x^2}{2} + \frac{x^2}{2},$$

$$= + \frac{16}{2} + \frac{16}{2} \Rightarrow \frac{2(16)}{2} = 16 \text{ Ans}$$

$\int_{0}^{\pi/3} \sin^2 x \cos x \, dx$.

Let $\sin^2 x = u$.

$\frac{du}{dx} = 2\sin x \cos x$.

2

Q50:- $y = x^4 - 2x^3 + x^2 + 3$ $\quad 0 \le x \le 1$.

Sol:-

$y = 0$.

$(x^2)^2 - 2(x^2)(x)$

$x^4 - 2x^3 + x^2 + 3 = 0$.

$y = x^4 - 2x^3 + x^2 + 3$.

let $x^2 = y$.

$x^2 = y$.

$y^2 - 2y + y + 3 = 0$.

$y =$

$y^2 - 2y + y = -3$.

$y^2 - y - y +$

Q49:- $y = x^3 - 3x^2 + 2x$ $\quad 0 \le x \le 2$.

Sol:- $0 = x^3 - 3x^2 + 2x$.

$1 - 1/2$

$= \frac{1}{2}$

Sol:- let $u = x+5 \Rightarrow x = u-5$.

$$\frac{(u-5)^2}{\sqrt{u-5+5}}$$

$$= \int \frac{(u-5)^2}{\sqrt{u}} \Rightarrow \frac{u^2+25-10u}{u^{1/2}}.$$

$$-\frac{1}{2} - 1$$

$$\frac{-1-2}{2} =$$

$$= \int u^{3/2} + 25u^{-1/2} - 10u^{1/2}$$

$$\frac{20u^{1/2}}{1/2}$$

$1/2 + 1$

$$= \frac{2}{5}u^{5/2} + \frac{(25)(2)}{3\ u^{3/2}} - \frac{2(10)}{3}u^{3/2}$$

$$= \frac{2}{5}u^{5/2} + \frac{50}{3}u^{+1/2} - \frac{20}{3}u^{3/2}.$$

$U =$

Q18 :- $\int \sqrt{1+\sin 2x}\, dx.$

$\sqrt{1+2\sin x \cos x}$

$= \sqrt{1+2\sin x(1-\cos x^2)}$

$= \sqrt{1+2\sin x - 2\sin x . \cos^2 x}$

Q22:- $\int \sqrt{\sin x}\, \cos^3 x\, dx.$

$u\, \cancel{y} = \sin x$

$\frac{du}{dx}\, \cancel{\frac{dy}{dx}} = \cos x.$

$du = \cos x\, dx.$

$= \int \sqrt{u}.\ \cos^2 x . \cos x\, dx$

$= \int \sqrt{u}.\ \cos^2 x\, du.$

$= \int u^{1/2}(1-u^2)\, du.$

$= \int (u^{3/2} - u^{5/2})$

$x^{1/2}$

$\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}$

$\frac{1}{2}+2 = \frac{3}{2}$

$1-\sin^2 x.$

$1-(\sin^2 u)$

$1-u^2$

$$\int \frac{\sin(2t+1)\,dt}{\cos^2(2t+1)}$$

$$= \int \frac{\sin(2t+1)}{\cos(2t+1)} \cdot \frac{1}{\cos(2t+1)}\,dt.$$

$$\int \tan(2t+1) \cdot \sec(2t+1)\,dt.$$

$$\frac{\sec(2t+1)+c}{2} \quad \text{Ans.}$$

$$= \frac{1}{2\cos(2t+1)} + c \quad \text{Ans}$$

Q32 ::

$$\frac{\sec z \tan z}{\sqrt{\sec z}}.$$

$U = \sec z.$

$\frac{du}{dz} = \sec z . \tan z \; dz.$

$$= \int \frac{1}{\sqrt{u}}\,du$$

Q36 :- $\int \frac{\cos\sqrt{\theta}}{\sqrt{\theta}\sin^2\sqrt{\theta}}\,d\theta$

Let

$\therefore \; U = \sin^2\sqrt{\theta}.$

$\frac{du}{d\theta} = \cos\sqrt{\theta} . \frac{1}{2\sqrt{\theta}}.$

$2\,du = \cos\sqrt{\theta} . \frac{1}{\sqrt{\theta}}\,d\theta.$

$\frac{1}{2}\int \frac{1}{u^2}\,du.$

$= \frac{1}{2}(-1)\frac{1}{u}.$

$= -\frac{1}{2\sin\sqrt{\theta}} + c$

Q39:- $\frac{1}{x^2}\sqrt{2-\frac{1}{x}}\,dx$.

Sol:- let $u = 2 - 1/x$.

$du = +\frac{1}{x^2}\,dx$.

$u^{1/2}\,du$.

$2/3\,u^{3/2} + c \Rightarrow (2 - 1/x)^{3/2} + c$.

Q49:- $\int \sqrt{\frac{x^4}{x^3-1}}\,dx$

let $x^3 - 1 = u$

$\frac{du}{dx} = 3x^2$

$du = 3x^2\,dx$

$\frac{(x^4)^{1/2}}{(x^3-1)^{1/2}} \Rightarrow \frac{x^2}{(x^3-1)^{1/2}}$.

let

Q38: $\sqrt{\dfrac{x-1}{x^5}}$.

$= \dfrac{\sqrt{x-1}}{\sqrt{x^5}} \Rightarrow \dfrac{\sqrt{x-1}}{\sqrt{x}\cdot x^2}$.

$\dfrac{\sqrt{x-1}}{x^2\sqrt{x}}\,dx$.

$u = \sqrt{x}-1$

$= \int 2u^{1/2}\,du$.

$\dfrac{dy}{dx} = \dfrac{1}{2\sqrt{x}}$.

$du = \dfrac{1}{2\sqrt{x}}\,dx$.

$2du = \dfrac{1}{\sqrt{x}}\,dx$.

Q43:- $\int x(x-1)^{10}\,dx$

$u = x^2-1$.

$du = dx$.

$\int x(x-1)(x-1)^9$.

$x^2 - x$.

$du(2x-1) = dx$

$\int (x^2-x)(x-1)^9\,dx \Rightarrow$

Q43:-

$\int (u+1)(u)^{10}$.

$u = x-1$

$x = u+1$.

$\int (u^{11}+u^{10})\,du$.

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$$\int (x+1)^2 (1-x)^5 dx.$$

Let $u = 1-x$.

$\cancel{x = u+1}$.

$\cancel{(u+2)(}$

$x = 1-u$.

$$\int (1-u+1)^2 (1-(1-u))^5 du.$$

$$= \int (2-u)^2 (+u)^5 du.$$

$$\cancel{\int (2u)(-2u^5 + u^7) du.}$$

$$= \int (4 + u^2 - 4u)(+u)^5$$

$$= \int +4u^5 + u^7 - 4u^6.$$

$$= \frac{4u^6}{6} + \frac{u^8}{8} - \frac{4u^7}{7} + C$$

Ans

Q44. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}$

Sol:- Let $\sqrt{x} = u$.

$$du = \frac{1}{2\sqrt{x}} dx.$$

$$\int e^{u} = \frac{(2)}{} e^{u} + c.$$

$$= (2) e^{\sqrt{x}} + c.$$

Q46:-

$$\int \underset{\text{II}}{\sqrt{x}} \; \underset{\text{I}}{e^{\sqrt{x}}} \, dx$$

$u = \sqrt{x}$

$$\frac{du}{dx} = \frac{1}{2\sqrt{x}}.$$

$$du = \frac{1}{2\sqrt{x}} dx.$$

$v = e^{\sqrt{x}}.$

Q1.- $\int \cos \sqrt{x} \, dx.$

$\sqrt{x} = u.$

$$= \int \cos u \, . \, 2u \, dx.$$

$$du = \frac{1}{2\sqrt{x}}$$

$$= 2 \int u . \cos u \, du.$$

$2\sqrt{x} \, du = dx.$

continue ←

$$= 2 [u \sin u + \cos u]$$

$2u \, du = dx$

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Q48:- $\int_0^{\pi/2} x^3 \cos 2x \, dx.$

$\int_0^{\pi/2} x \cdot x^2 \cos 2x \, dx$

let

$u = x^2$

$\frac{du}{dx} = 2x$

$\frac{du}{2} = x dx$

$= \int_0^{\pi/2} u \cdot \cos$

$\frac{1}{2} \int u \cos$

x^3	+	$\cos 2x$
$3x^2$	−	$\frac{\sin 2x}{2}$
$6x$	+	$-\frac{\cos 2x}{4}$
6	−	$-\frac{\sin 2x}{8}$
0		$\frac{\cos 2x}{16}$

$\frac{x^3 \sin 2x}{2} + 3x^2 \frac{\cos 2x}{4} - 6x \frac{\sin 2x}{8} - 6 \frac{\cos 2x}{16}$

$= \frac{\pi^3}{8}(0) +$

~~Q64:- $y = 8$~~

Q65:- $y = \cos(\pi x/2)$ & $y = 1 - x^2$.

Sol:- $\cos(\pi x/2) = 1 - x^2$.

$x^2 = 1 - \cos(\pi x/2)$.

$x = \sqrt{1 - \cos(\pi x/2)}$.

Q63:-

$y = 2\sin x$, $y = \sin 2x$. $0 \le x \le \pi$

Q66:-

$y = \sin(\pi x/2)$ & $y = x$.

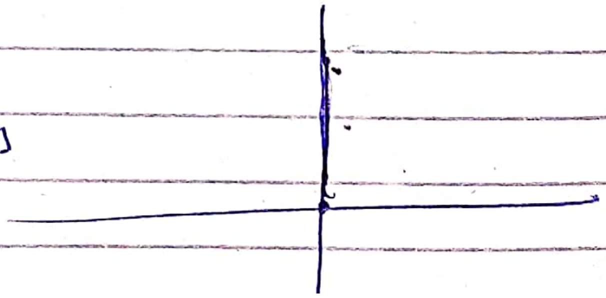
Sol:- $\sin \pi(x/2) = x$.

$\sin(\pi x/2) - x = 0$.

Q69:- $y = 3\sin y\sqrt{\cos y}$ & $x = 0$ $0 \le y \le \pi/2$.

Sol:-

Area = 6 Ans



d $x = \sqrt{\cos(\cancel{\pi}\, \pi y/4)}$ $-2 \leq y \leq 0$ $x = 0$.

$$= \int (R(y))^2 \, dy.$$

$$= \int_{-2}^{\cancel{0}} \left(\sqrt{\cos(\pi y/4)}\right)^2$$

$$\frac{\pi}{2} \left(\int_{-2}^{0} \left(\cos\left(\frac{\pi y}{4}\right)\right) dy. \right) \Rightarrow \sin\left(\frac{\pi y}{4}\right) \Big|_{-2}^{0}$$

$$= \pi\left(\sin 0 \cancel{2} + \sin \pi/2\right) = 1(\pi).$$

Q34:- $x = \sqrt{2y} / (y^2+1)$, $x = 0$, $y = 1$.

$$\frac{\sqrt{2y}}{y^2+1} = 0. \Rightarrow y = 0$$

$$(0, 1)$$

$$= \pi \int_0^{1} \left(\frac{\sqrt{2y}}{y^2+1}\right)^2 \cdot \frac{2y}{(y^2+1)^2}.$$

$$y^2+1 = u.$$

$$2y \frac{dy}{du} = 1.$$

$$\frac{dy}{du} = \frac{1}{2y}.$$

Q25:- $y = \sqrt{\cos x} \quad 0 \le x \le \pi/2, \; y = 0, \; x = 0$

Sol:-

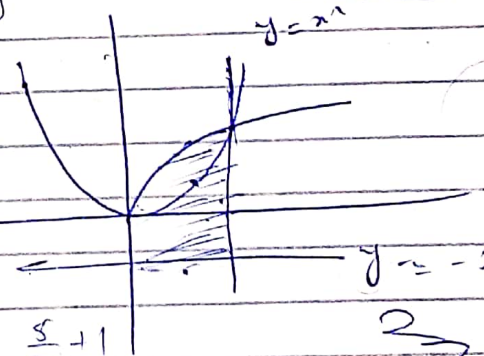
As we know that;

$$\int_0^{\pi/2} \left((\cos x)^{1/2} \right)^2 dx$$

$$\pi \int_0^{\pi/2} \cos x \, dx \Rightarrow \pi (\sin x) \Big|_0^{\pi/2}$$

$\pi(1)$ Ans.

$x = y$



$$\int_0^1 \pi (\sqrt{x} - x^2)^2 . dx$$

$2 + 1/2$

$$\pi \int_0^1 \left(x + x^4 - 2x^{5/2} \right) . dx$$

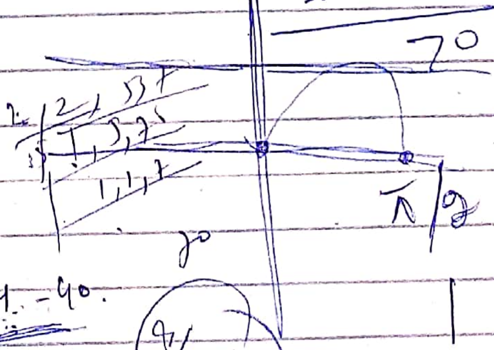
$\frac{5}{2} + 1$

$$\pi \left[\frac{x^2}{2} + \frac{x^5}{5} - 2 . x^{7/2} . \frac{2}{7} \right]_0^1$$

$\frac{1}{2} + \frac{1}{5} - \frac{4}{7}$

$\frac{35 + 7 - 20}{70}$

$$\pi \left[\frac{1}{2} + \frac{1}{5} - 2 . \left(\frac{2}{7} \right) \right]$$



$$\pi \left[\frac{22}{70} \right]$$

$\frac{35 + 14 - 40}{70}$

$\frac{9}{70}$

Q50:- $\int \frac{x}{(2x-1)^{2/3}}\,dx$

~~let $u = 2x-1$~~

~~$du = 2$~~

let $u = 2x-1$.

$\frac{u+1}{2} = x$.

$\int \frac{\frac{u+1}{2}}{u^{2/3}}$

$= \int \frac{\frac{u+1}{2}\,dx}{\left(\cancel{2}\left(\frac{u+1}{\cancel{2}}\right) - 1\right)^{2/3}} \Rightarrow \int \frac{(u+1)\,du}{2u^{2/3}}$

$\left(2\left(\frac{u+1}{2}\right) - 1\right)^{2/3}$

$\int \left(\frac{u}{2u} + \frac{1}{u}\right) du.$ $\quad u+1-1$

$= \frac{1}{2}u + \ln u + c.$ $\quad u^{2/3}$

$= \frac{1}{2}(2x-1) + \ln(2x-1) + c$ Ans

$1-\frac{2}{3}$ $\quad \frac{u+1}{2u^{2/3}}$ $\quad \frac{1}{2}\int \frac{u+1}{u^{2/3}}$

$= \frac{1}{2}\int u^{1/3} + u^{-2/3}$

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Q48:- $3x^5\sqrt{x^3+1}$

let

$u = x^3+1$

$3x^2 \cdot x^3 \sqrt{x^3+1}\,dx$

$\frac{du}{dx} = 3x^2$

$du = 3x^2\,dx$

$x^3 = u-1$

$= \int (u-1)\,u^{1/2}\,du$

$\int \left(u^{3/2} - u^{1/2}\right) du$ Ans

Q42 $y = x^2 - 2x$ & $y = x$

Sol:- $x^2 - 2x = x$

$\cancel{x^2 = 3x}$ $x^2 - 3x = 0$

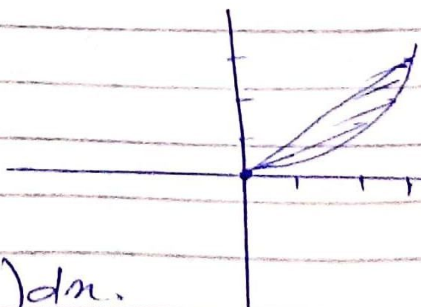
$\cancel{x^2 = 3}$ $x(x-3) = 0$

$x = 0$ & $x = 3$

For 0 $y = 0$

A(0,0)

B(3,3)



Origin:- $y = 0$

Area = $\int_0^3 \left(x - x^2 + 2x\right) dx.$

$$\int_0^3 (-x^2+3x)\,dx.$$

$$\cancel{= \left(\frac{-x^3}{3} - \frac{x^2}{2}\right)\Big|_0^3} \quad = \int_0^3 \left(-\frac{x^3}{3} + \frac{3x^2}{2}\right)$$

$$= -\frac{27}{3} - \frac{9}{2} \Rightarrow \frac{-54-27}{6} \Rightarrow$$

$$= -\frac{\cancel{27}}{\cancel{3}} + \frac{3(3)^2}{2} \Rightarrow -9 + \frac{27}{2}$$

$$\frac{-18+27}{2} \Rightarrow \frac{9}{2} \text{ Ans}$$

Q.45:- $y = x^2$, $y = -x^2+4x$

Sol:- $x^2 + x^2 = 4x.$

$2x^2 = 4x.$

$x = 2$

Q49:- $y = \sqrt{x4}$ $\quad 5y = x + 6.$

Q48:- $y = x^4 + x^3 + 16x + 4.$

&

$y = 6x^4 + 6x^2 + 8x + 4.$

Sol:-

$$\cancel{x^4} + x^3 + 16x + 4 = \cancel{x^4} + 6x^2 + 8x + 4$$

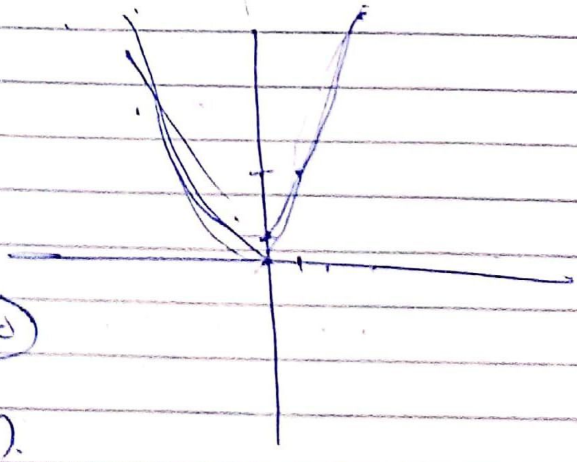
$$+x^3 - 6x^2 + 8x = 0.$$

$$x(x^2 - 6x + 8) = 0.$$

$x = 0$ $\quad x^2 - 2x - 4x + 8 = 0.$

$x = 2$, $x = 4$.

$A(0, 4)$, $B(2, 60)$, $C(4, 388)$.



origin

$y = 4$, $x = 0$

$A(0, 4)$.

$y = \sqrt{x},$ $\quad y =$

Q16.-

$$r - 2\sqrt{\theta} = \frac{3}{2}\theta^{2/3} + \frac{4}{3}\theta^{3/4}$$

$$\frac{2-3}{3} = -\frac{1}{3}$$

$$\frac{3}{4} - 1 \Rightarrow \frac{-1}{4}$$

Sol:- $\frac{dr}{d\theta} = ?$

$$\frac{dr}{d\theta} - 2\frac{1}{2\sqrt{\theta}} = \frac{3}{2}\left(\frac{2}{3}\right)\cdot\theta^{\frac{2}{3}-1} + \frac{4}{3}\left(\frac{3}{4}\right)\theta^{3/4-1}$$

$$\boxed{\frac{dr}{d\theta} = (\theta)^{-1/3} + \theta^{-1/4} + \frac{1}{\sqrt{\theta}}}$$

Q17 :- $\sin(r\theta) = 1/2$.

Sol:- $\frac{dr}{d\theta} = ?$

$$\cos(r\theta)\left(\theta\frac{dr}{d\theta} + r\right) = 0.$$

$$\theta\frac{dr}{d\theta} = -r \Rightarrow \boxed{\frac{dr}{d\theta} = \frac{-r}{\theta}}$$ Ans

Q18:- $\cos r + \cot\theta = r\theta$.

$\frac{dr}{d\theta} = ?$

Sol:- $$-\sin r.\frac{dr}{d\theta} - \csc^2\theta = \theta\frac{dr}{d\theta} + r.$$

$$\theta\frac{dr}{d\theta} - \sin r\frac{dr}{d\theta} = -r - \csc^2\theta.$$

$$\frac{dr}{d\theta}(\theta - \sin r) = -(r + \csc^2\theta)$$

$$\boxed{\frac{dr}{d\theta} = \frac{-(r + \csc^2\theta)}{\theta - \sin r}}$$ Ans

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Q.20:- $x^{2/3} + y^{2/3} = 1$. $\frac{d^2y}{dx^2} = ?$

$\frac{2}{3} x^{2/3-1} + \frac{2}{3} y^{2/3-1} \frac{dy}{dx} = 0$.

$\frac{-1-3}{3} = \frac{-4}{3}$.

$\frac{2}{3} x^{-1/3} + \frac{2}{3} y^{-1/3} y' = 0. \Rightarrow y' = -\frac{x^{-1/3}}{y^{-1/3}}$.

$$\frac{dy'}{dx} = -\frac{y^{-1/3} (-\frac{1}{3}) . x^{-1/3-1} - x^{-1/3} (-\frac{1}{3}) y^{-1/3-1} y'}{(y^{-1/3})^2}.$$

$$\boxed{y'' = \frac{\frac{1}{3} y^{-1/3} . x^{-4/3} + \frac{1}{3} x^{-1/3} . y^{-4/3} \left(-\frac{x^{-1/3}}{y^{-1/3}}\right)}{y^{2/3}}}.$$

Q33:- $6x^2 + 3xy + 2y^2 + 17y - 6 = 0$ $(-1, 0)$.

$6 - 6 = 0$

So the points on curve.

$\frac{dy}{dx} = ?$

$3x + 3y + 3xy' + 4y y' + 17y' = 0$.

$(3x + 4y + 17) y' = -3x - 3y$.

$y' = \frac{-3x - 3y}{3x + 4y + 17}$

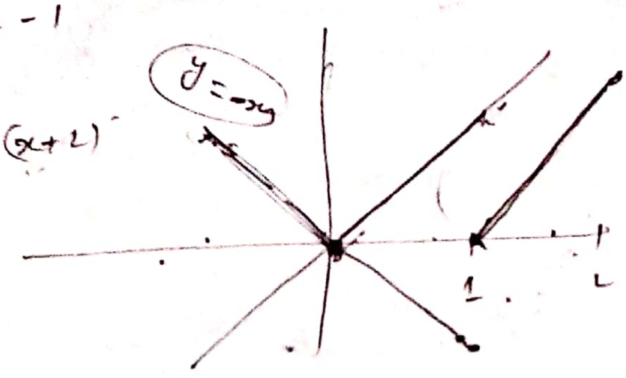
$y'|_{(-1,0)} = \frac{3}{-3 + 17} = \frac{3}{14}$ Ans

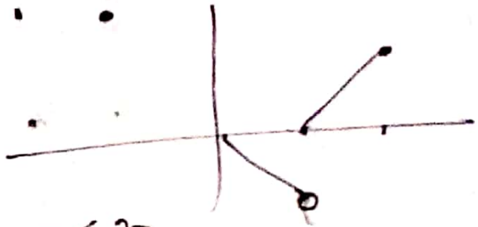
$y = \frac{3}{14}(x+1) \Rightarrow \frac{3}{14} x + \frac{3}{14}$ Ans.

$y = -x$

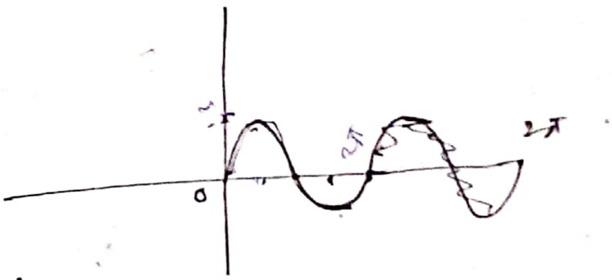
$y = x - 1$

$f(x+2) = (x+2)$

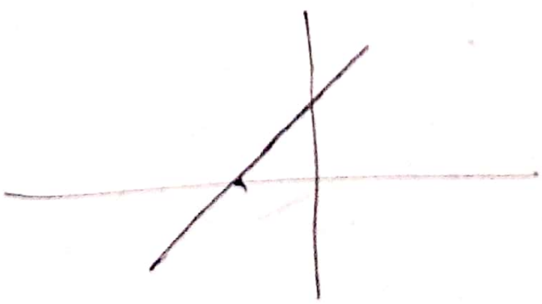




Q19
2. $y = 3\sin x, \quad 0 < x < 2\pi.$



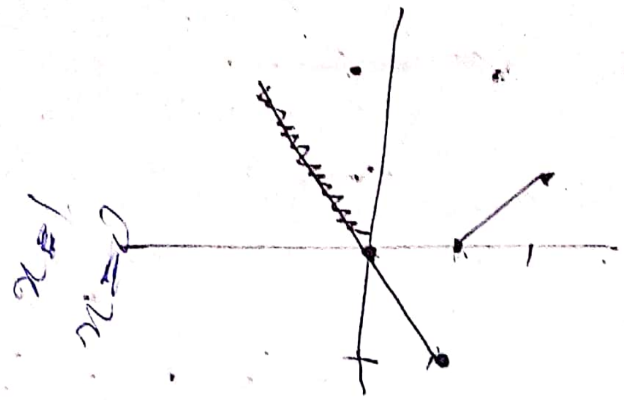
Q20:- $f(x) = \begin{cases} x+1, & -1 \le x < 0. \\ \cos x, & 0 < x \le \pi/2. \end{cases}$



Abs. max = no.

Ab min = 0.

Q 17 :- $g(x) = \begin{cases} -x & 0 \le x \le 1. \\ x-1 & , 1 \le x \le 2. \end{cases}$

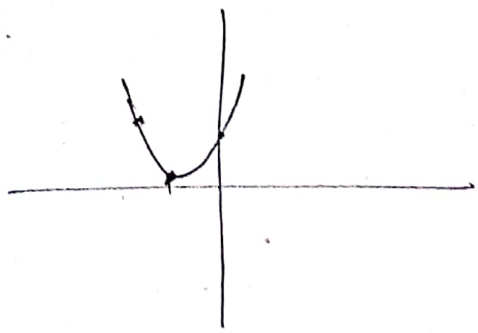


Q 23 :- $f(x) = x^2 - 1$, $-1 \le x \le 2$.

Sol :- $f(-1) = (-1)^2 - 1$

$f(-1) = 0$.

$f(2) = 3$.



$f'(x) = 2x$.

$f'(x) = x = 0$.

$f(0) = -1$

Absolute max at $2 = 3$.

Absolute min at 0, $= -1$.

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31. $f(\theta) = \sin\theta$, $-\pi/2 \le \theta \le \frac{5\pi}{6}$.

$f(-\pi/2) = -1$.

$f'(\theta) = \cos\theta$.

$\pi/2$.

$f\left(\frac{5\pi}{6}\right) = 0.5$.

$= 1$.

max at $\pi/2 = 1$.

min $-\pi/2 = -1$

Q49:- $f(x) = \sqrt{25-x^2}$, $-5 \le x \le 5$.

Q51:- $g(x) = \dfrac{x-2}{x^2-1}$ $0 \le x < 1$.

Sol:- $g'(x) = \dfrac{(x^2-1) - (x-2)(2x)}{(x^2-1)^2}$.

$= \dfrac{x^2-1-2x^2+2x}{(x^2-1)^2}$.

$g(x) = \dfrac{\cancel{x^2} - x^2 + 4x - 1}{(x^2-1)^2}$.

$= -\left(\dfrac{x^2 4x + 1}{(x^2-1)^2}\right)$.

$= x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \dfrac{+4 \pm \sqrt{16-4(1)(1)}}{2(1)}$ $\dfrac{4 \pm 2\sqrt{3}}{2}$

$= \dfrac{4 \pm \sqrt{12}}{2} \Rightarrow 2 \pm 2\sqrt{3}$

$\cancel{2\pm}$ $2+2\sqrt{3}$, $2-2\sqrt{3}$.

$2+\sqrt{3}$, $2-\sqrt{3}$

Q11 $y = x^3 - 3x + 3.$

$0, 3$

$y' = 3x^2 - 3.$

$y'' = 6x.$

$x = 0$ inflection point

$= 3 \Rightarrow (0,3)$ inflection point.

$3(x^2 - 1) = 0.$

$x = \pm 1.$

$y = \cancel{-1 + 3 + 3}$

$y'' = 6(-1) = -6 < 0$ ~~local min~~ local max

$y'' = 6(1) = 6 > 0$ ~~decreasing~~ local min.

Q14:- $y = 4x^2 - x^4 = x^2(x^2 - 2)$

$x = 0, -1, +1.$

Q17:- $y = x^4 - 2x^2$

$y' = 4x^3 - 4x \Rightarrow 4x(x^2 - 1).$

$y'' = 12x^2 - 4. \Rightarrow 4(3x^2 - 1) = 0.$

$x^2 = 1/3 \Rightarrow x = \pm\frac{1}{\sqrt{3}}$

$y = \left(-\frac{1}{\sqrt{3}}\right)^4 - 2\left(\frac{1}{\sqrt{3}}\right)^2$

$y = \frac{1}{9} - 2\frac{1}{3} \Rightarrow \frac{1-6}{9} \Rightarrow -5/9 = y$

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$$\left(\frac{1}{\sqrt{3}}\right)^4 - 2\left(\frac{1}{\sqrt{3}}\right)^2 = -5/9$$

Inflection points are,

$$\left(-\frac{1}{\sqrt{3}}, -5/9\right), \left(\frac{1}{\sqrt{3}}, \frac{-5}{9}\right).$$

Now;

$$y'' = 12\left(\frac{1}{\sqrt{3}}\right)^2 - 4.$$

$$= \frac{12}{3} - 4 \Rightarrow \frac{12-12}{3} = 0.$$

$y''(0) = -4. < 0$ local max.

$y''(-1) = 8 > 0$ local min

$y''(1) = 8 > 0$ local min

Q15. $y = \sqrt{3x} - 2\cos x \qquad 0 \le x \le 2\pi.$

$$y' = \frac{1}{2\sqrt{3x}}(3) + 2\sin x$$

$$= \frac{3 + 4\sqrt{3x}\,\sin x}{2\sqrt{3x}}$$

Critical point $= (0)$.

$$y'' = \frac{\cancel{2}\sqrt{3x}\,.\left(4(3)\sin x + 4\sqrt{3x}\,.\cos x\right) \,/\, \cancel{2}\sqrt{3x}}{(2\sqrt{3x})^2}$$

$$= \cancel{2\sqrt{3x}}\left(\frac{12\sin x + 8(3x)\cos x}{\cancel{2\sqrt{3x}}}\right.$$

$y = x^{x^x}$

let $y = x^x$

$\ln y = x \ln x$

$\frac{1}{y}\frac{dy}{dx} =$

$\lim_{h \to 0^+} = \frac{2(0+h) - (0+h)^3 - 1 + 1}{(0+h)}$

$\frac{2h - h^3}{h}$

$\frac{h(2-h^2)}{h} = 2$

$\lim_{x \to 0} x^{x^x}$

$\lim_{x \to 0} x^{x^x}$

let $y = x^x$

$\ln y = x \ln x$

$y = e^{x \ln x} \Rightarrow y = e^{\frac{\ln x}{1/x}}$

$y = e^{\frac{1/x}{-1/x^2}}$

$y = e^{x}$

$y = 1$

x^{x^1} $\quad x^x = 1$

1

$y = \ln(\ln(\ln(\ln(\ln(x))$

let $u = \ln x$

$\frac{du}{dx} = 1/x$

$v = \ln u$

$\frac{dv}{du} = \frac{1}{u} \Rightarrow \frac{1}{1/x} \Rightarrow x$

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$y = x^{x^{x^x}}$.

let $y = x^x$.

$\ln y = x \ln x$

$\frac{1}{y}\frac{dy}{dx} =$

$\lim_{h \to 0^+} = \frac{2(0+h) - (0+h)^3 - 1 + 1}{(\cancel{0} + h)}$

$\frac{2h - h^3}{h}$

$\frac{h(2-h^2)}{h} = 2$

$\lim_{x \to 0} x^{x^x}$.

$\lim_{x \to 0} x^{x^{x^x}}$.

let $y = x^x$.

$\ln y = x \ln x$.

$y = e^{x \ln x}$. $\Rightarrow y = e^{x \frac{\ln x}{x}}$.

$y = e^{\frac{1/x}{-1/x^2}}$.

$y = e^x$.

$y = 1$.

x^{x^1} $x^x = Z$

1

$y = \ln(\ln(\ln(\ln(\ln(x))))$.

let $u = \ln x$.

$\frac{du}{dx} = 1/x$.

& $v = \ln u$.

$\frac{dv}{du} = \frac{1}{u} \Rightarrow \frac{1}{1/x} \Rightarrow x$.

$w = \ln v$.

$$\frac{dw}{dv} = \frac{1}{w}.$$

$$= 1/x.$$

let $\cancel{x}^{y} = d \ln w$

$$\frac{dx y}{dw} = \frac{1}{w}$$

$$= x.$$

$$\frac{dy}{dx} = x \cdot (1/x)(x(1/2x))$$

$$\frac{dy}{dx} = \frac{1}{2} \text{ Ans}.$$

$$y = \ln(\ln(\ln(\ln(\ln(x))))).$$

let $u = \ln(x)$

$$\frac{du}{dx} = 1/x$$

$v = \ln u$

$$\frac{dv}{du} = \frac{1}{u}$$

$w = \ln v$.

$$\frac{dw}{dv} = \frac{1}{v}.$$

$p = \ln w$.

$$\frac{dp}{dw} = \frac{1}{w}.$$

$y = \ln P$.

$$\frac{dy}{dp} = \frac{1}{p}$$

$$\frac{dy}{dx} = \frac{dy}{dp} \cdot \frac{dp}{dw} \cdot \frac{dw}{dv} \cdot \frac{dv}{du} \frac{du}{dx}.$$

By putting values

we $\frac{dy}{dx}$.

Q44: $f(x) = \begin{cases} 2x - x^3 - 1 & , x \geq 0 \\ x - \frac{1}{x+1} & , x < 0 \end{cases}$

Sol:- For differentiable;

L.H.L = R.H.L

$$L_1 = \lim_{h \to 0^-} (0-h) - \frac{1}{(0-h)+1}$$

$$= \lim_{h \to 0^-} 0 - h - \frac{1}{-h+1}$$

$$\Rightarrow \frac{-1}{1} = -1$$

Love birds

$$L_2 = \lim_{h \to 0^+} 2(0+h) - (0+h)^3 - 1$$

$$= 2(0) - (0) - 1$$

$$L_2 = -1$$

L.H.L = R.H.L

So The f(x) is differential.

$$f(x) = \begin{cases} 2x - x^3 - 1 & x \geq 0 \\ x - \dfrac{1}{x+1} & , x < 0 \end{cases}$$

$$l_1 = \lim_{h \to 0^-} \frac{(0-h) - \dfrac{1}{(0-h)+1} + 1}{(0-h) - 0}$$

$$= \lim_{h \to 0^-} \frac{-h - \dfrac{1}{-h+1} + 1}{-h}$$

$$\phi = \frac{0 - \dfrac{1}{-1} + 1}{0}$$

$$\frac{\dfrac{h^2 - h - 1 - h + 1}{(-h+1)}}{-h}$$

$$= \frac{h^2 - 2h}{(-h+1)(-h)}$$

$$\frac{\cancel{h}(h-2)}{(-h+1)(-\cancel{h})} = \frac{h-2}{-(-h+1)}$$

$$\frac{0-2}{-(-0+1)} = 2$$

Ex # 3.7

Q18:- $\cos r + \cot \theta = r\theta$

Sol:- $\frac{dr}{d\theta} = ?$

$\Rightarrow \frac{d}{d\theta}(\cos r + \cot\theta) = \frac{d}{d\theta}(r\theta)$

$-\sin r \cdot \frac{dr}{d\theta} - \operatorname{cosec}^2\theta = \theta\frac{dr}{d\theta} + r$

$\theta\frac{dr}{d\theta} + \sin r\frac{dr}{d\theta} = -r - \operatorname{cosec}^2\theta$

$\frac{dr}{d\theta}(\theta + \sin r) = -r - \operatorname{cosec}^2\theta$

$\boxed{\frac{dr}{d\theta} = \frac{-r - \operatorname{cosec}^2\theta}{\theta + \sin r}}$ Ans

Q23:- $2\sqrt{y} = x - y$

Sol:- $\frac{d^2y}{dx^2} = ?$

$\frac{d}{dx}(2\sqrt{y}) = \frac{d}{dx}(x - y)$

$\frac{2}{2\sqrt{y}}\frac{dy}{dx} = 1 - \frac{dy}{dx}$

$\frac{dy}{dx}\left(\frac{2}{2\sqrt{y}} + 1\right) = 1$

$\frac{dy}{dx}\left(\frac{1+\sqrt{y}}{\sqrt{y}}\right) = 1$

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$$\frac{dy}{dx} = \frac{\sqrt{y}}{1+\sqrt{y}}$$

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(\frac{\sqrt{y}}{1+\sqrt{y}}\right)$$

$$\frac{d^2y}{dx^2} = \frac{\frac{1}{2\sqrt{y}} \cdot \frac{dy}{dx}(1+\sqrt{y}) - \sqrt{y} \cdot \frac{1}{2\sqrt{y}}\frac{dy}{dx}}{(1+\sqrt{y})^2}$$

$$= \frac{\frac{1}{2\sqrt{y}}\frac{dy}{dx}(1+\sqrt{y}) - \frac{\sqrt{y}}{2\sqrt{y}}\frac{dy}{dx}}{(1+\sqrt{y})^2}$$

$$= \frac{\frac{(1+\sqrt{y})}{2\sqrt{y}}\frac{dy}{dx} - \frac{1}{2}\frac{dy}{dx}}{(1+\sqrt{y})^2}$$

$$= \frac{\left(\frac{\cancel{1+\sqrt{y}}}{\cancel{2\sqrt{y}}}\right)\left(\frac{\cancel{\sqrt{y}}}{\cancel{1+\sqrt{y}}}\right) - \frac{1}{2}\left(\frac{\sqrt{y}}{1+\sqrt{y}}\right)}{(1+\sqrt{y})^2}$$

$$= \frac{1 - \frac{\sqrt{y}}{2+2\sqrt{y}}}{(1+\sqrt{y})^2} \Rightarrow \frac{\frac{2+2\sqrt{y}-\sqrt{y}}{2(1+\sqrt{y})}}{(1+\sqrt{y})^2}$$

$$\frac{d^2y}{dx^2} = \frac{2+\sqrt{y}}{2(1+\sqrt{y})^3} \quad \text{Ans.}$$

Q26:- $xy + y^2 = 1$ find $\frac{d^2y}{dx^2}$ at $(0,-1)$.

Sol:-

$$y + x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0.$$

$$\frac{dy}{dx}(x+2y) = -y.$$

$$\frac{dy}{dx} = \left(\frac{-y}{x+2y}\right)$$

$$\frac{d^2y}{dx^2} = \frac{-\frac{dy}{dx}(x+2y) + y\left(1+2\frac{dy}{dx}\right)}{(x+2y)^2}$$

$$= \frac{-(x+2y)\frac{dy}{dx} + y + 2y\frac{dy}{dx}}{(x+2y)^2}$$

$$= \frac{-(x+2y)\left(\frac{-y}{x+2y}\right) + y + 2y\left(\frac{-y}{x+2y}\right)}{(x+2y)^2}$$

$$= \frac{y + y - \frac{2y^2}{x+2y}}{(x+2y)^2} \Rightarrow \frac{y(x+2y)+y(x+2y)-2y^2}{(x+2y)(x+2y)^2}$$

$$= \frac{2xy + 4y^2 - 2y^2}{(x+2y)^3}$$

$$= \frac{2y^2 + 2xy}{(x+2y)^3}$$

Now putting values. $(0, -1)$ x y.

$$= \frac{2(-1)^2 + 2(0)(-1)}{(0+2(-1))^3}$$

$$= \frac{+2+0}{-2} \Rightarrow -1 \quad \text{Ans}$$

Q27:- Find slope at given point

$y^2 + x^2 = y^4 - 2x$ at $(-2, 1)$ & $(-2, -1)$

Sol:- $2y\frac{dy}{dx} + 2x + 4y^3\frac{dy}{dx} - 2$.

$$(4y^3 - 2y)\frac{dy}{dx} = 2x + 2.$$

$$\frac{dy}{dx} = \left(\frac{2x+2}{4y^3 - 2y}\right)$$

at $(-2, 1)$

$$= \frac{2(-2)+2}{4(1)^3 - 2(-2)}$$

& at $(-2, -1)$

$$= \frac{2(-2)+2}{4(-1)^3 - 2(-2)} \quad \text{Ans}$$

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