Problem 1.2

1 kg of fluid is compressed reversibily according to law PV = 0.25 where Pressure in bar and V in m^3/kg. The final volume is 1/4 of initial volume. Calculate the work done on the fluid & sketch the process on PV- graph.

Solution:

Given data

$PV = 0.25$

As pressure in bar

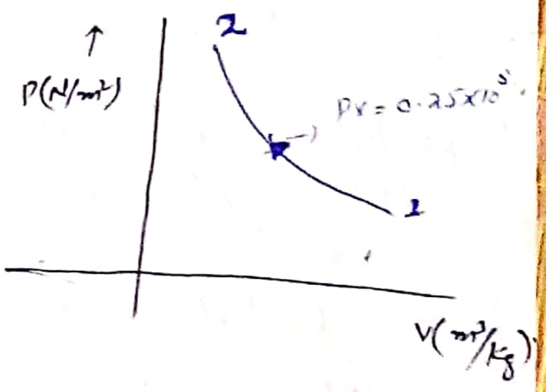
so; $PV = 0.25 \times 10^5$ Pa.

$m = 1$ kg.

$V_2 = \frac{1}{4} V_1$

Calculate $W_{1\to 2} = ?$

(b) Graph



Since we know That;

$$W_{1\to 2} = -m \int_1^2 P\,dv$$

$\because PV = 0.25 \times 10^5$

$$P = \frac{0.25 \times 10^5}{V}$$

$$= -(1) \int_1^2 \frac{0.25 \times 10^5}{V} dv$$

$$= -0.25 \times 10^5 \int_1^2 \frac{1}{V} dv$$

$$= -0.25 \times 10^5 \ln |V|_1^2 \Rightarrow -0.25 \times 10^5 \ln \left|\frac{V_2}{V_1}\right|$$

$$= 0.25 \times 10^5 \left|\frac{\frac{1}{4}V_1}{V_1}\right| \Rightarrow 0.25 \times 10^5 \ln |4| = 0.3465 \times 10^5$$

$\boxed{W_{1-2} = -34657.35 \text{ Nm}}$ Ans.

Scanned with CamScanner

Problem 1.3:-

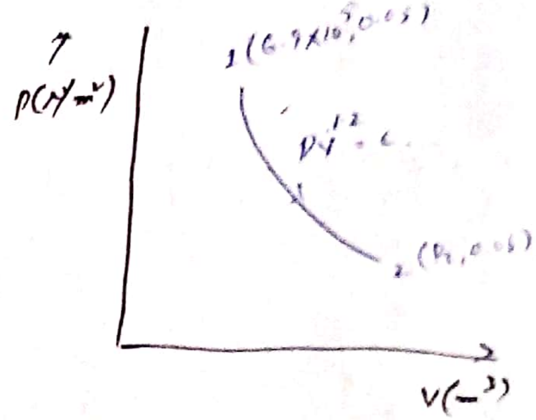
0.05 m^3 of gas at 6.9 bar expands reversibily in a cylinder behind the piston according to law $pV^{1.2}$ = constant until the volume is 0.08 m^3. Calculate The work done by gas & sketch process on PV diagram.

Sol:- Properties at state 1

$P_1 = 6.9 \text{ bar} = 6.9\times10^5 \text{ Pa}$.

Volume, $V_1 = 0.05 m^3$

(b) Graph



Process:- $1 \rightarrow 2$.

$pV^{1.2}$ = Constant

Properties at state 2

Volume, $V_2 = 0.08 m^3$.

Required data,

$W_{1\to2} = ?$

Since we know that;

$$W_{1\to2} = -\int_1^2 P\,dV$$

$\because pV^{1.2} = c \Rightarrow P = \frac{c}{V^{1.2}}$

Finding c.

$P_1V_1^{1.2} = c$

$(6.9\times10^5)(0.05)^{1.2} = c$

$c = 16950.16$

$$= -\int_1^2 \frac{c}{V^{1.2}}\,dV$$

$$= -c\int_1^2 \frac{1}{V^{1.2}}\,dV = -c\int_1^2 V^{-1.2}\,dV$$

$$= \frac{-c}{-0.2}\left(\frac{1}{V^{0.2}}\right)\Big|_1^2$$

$$= \frac{\cancel{-}c}{\cancel{-}0.2}\left(\frac{1}{V_2^{0.2}} - \frac{1}{V_1^{0.2}}\right)$$

By putting values;

$$W_{1\to 2} = \frac{18950.16}{2}\left(\frac{1}{(0.08)^{0.2}} - \frac{1}{(0.05)^{0.2}}\right)$$

$$= 94750.8\left(\frac{1}{0.6034} - \frac{1}{0.5492.8}\right)$$

$$= 94750.8\,(1.6572 - 1.8205)$$

$$\boxed{W_{1\to 2} = -15465.65\ Nm.}\ \text{Ans}$$

Problem 1.4.

1 kg of fluid expands ~~ex~~ reversibility according to linear law from 4.2 bar to 1.4 bar the initial & final volumes are $0.004\,m^3$ & $0.02\,m^3$. The fluid is then cooled reversibily at constant pressure, and finally compressed reversibily according to law $Pv =$ Constant back to initial condition of 4.2 bar and $0.004\,m^3$. Calculate the work in each process & net work of cycle. Sketch the process on PV- diagram.

Solution:- Given data;

Pressure, $P_1 = 4.2\,bar = 4.2\times10^5\,Pa$.

Volume, $V_1 = 0.004\,m^3$

Process :- $P = mV + c$.

~~Properties~~ at state 2

Pressure, $P_2 = 1.4\,bar = 1.4\times10^5\,Pa$.

Scanned with CamScanner

Volume, $V_2 = 0.02 m^3$.

Process $2 \to 3$

$P = c$.

Properties at state 3

Pressure, $P_3 = P_2 = 1.4 \times 10^5 Pa$.

Volume, $V_3 = ?$

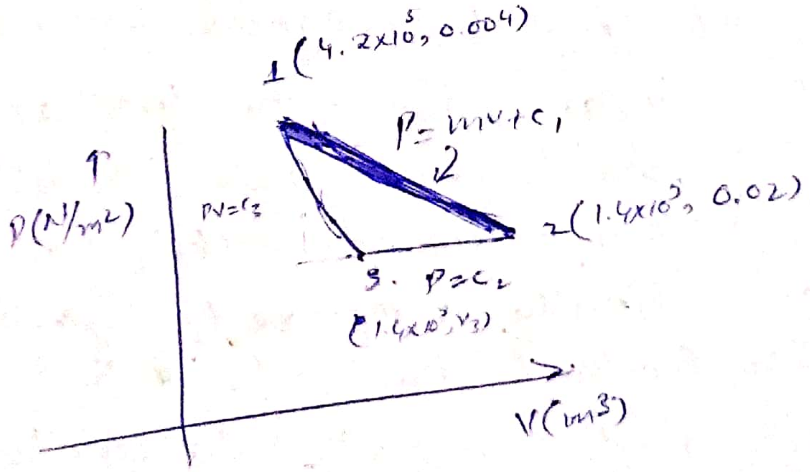
Process $3 \to 1$

$PV = C$.

Required data;

Net work done & work done at each cycle.

Graph



First find $V_3 = ?$

Since;

$P_1 V_1 = P_3 V_3 = c$

$P_1 V_1 = P_3 V_3$.

$\Rightarrow V_3 = \frac{P_1 V_1}{P_3} \Rightarrow \frac{4.2 \times 10^5 \times (0.004)}{1.4 \times 10^5}$

$\boxed{V_3 = 0.012 m^3}$

Scanned with CamScanner

Since;

$W_{net} = W_{1\to2} + W_{2\to3} + W_{3\to1} \to ①.$

As;

$W_{1\to2} = -\int_1^2 PdV \qquad \therefore P = mV + c.$

$= -\int_1^2 (mV+c)dV.$

$= -\left(\int_1^2 mVdV + \int_1^2 cdV\right) \Rightarrow -\left(m\int_1^2 VdV + c\int_1^2 dV\right).$

$= -\left(m\left(\frac{V^2}{2}\right)\Big|_1^2 + c(V)\Big|_1^2\right)$

$= -\left(m\left(\frac{V_2^2}{2} - \frac{V_1^2}{2}\right) + c(V_2 - V_1)\right) \to ①$

Finding values of c & m.

Since; $P = mV + c.$

$P_1 = mV_1 + c$ & $P_2 = mV_2 + c.$

$4.2\times10^5 = m(0.004) + c$, $1.4\times10^5 = m(0.02) + c$

Sub tracting eqns.

$4.2\times10^5 = m(0.004) + \cancel{c}$

$\underline{\overset{+}{-}\ 1.4\times10^5 \overset{+}{=} 0.02m \overset{+}{-} \cancel{c}}$

$(m) - 0.016 = 2.8\times10^5$

1.75×10^5

$\Rightarrow 4.2\times10^5 = 0.004m + c$

$4.2\times10^5 = 0.004(-175\times10^5) + c$

$= -0.7\times10^5 + c$

$c = 4.2\times10^5 + 0.7\times10^5$

$$\boxed{c = 4.9\times10^5}$$

Using these values in eq ①.

$$W_{1\to2} = -\left(-175\times10^5\left(\frac{(0.02)^2}{2} - \frac{(0.004)^2}{2}\right) + 4.9\times10^5(0.02-0.004)\right)$$

$$= 175\times10^5\left(\frac{4\times10^{-4}}{2} - \frac{1.6\times10^{-5}}{2}\right) - 4.9\times10^5\,(0.016)$$

$$= 175\times10^5(2\times10^{-4} - 8\times10^{-6}) - 0.0784\times10^5$$

$$= -0.0448\times10^5$$

$$W_{1\to2} = -4480\ Nm \longrightarrow ②$$

Similarly;

$$W_{2\to3} = -\int_2^3 p\,dv$$

$$= -p\int_2^3 dv \Rightarrow -p(V_3 - V_2)$$

$$= -1.4\times10^5(0.012 - 0.02)$$

$V_3 = 0.012\ m^3$

$V_2 = 0.02\ m^3$

$$= -1.4\times10^5(-8\times10^{-3})$$

$$= 0.0112$$

$$W_{2\to3} = 1120\ Nm \longrightarrow ③$$

Scanned with CamScanner

$$W_{3\to1} = -\int_3^1 p\,dv$$

$$= -\int_3^1 \frac{c}{v}\,dv \qquad \because pv = c.$$

$$= -c\int_3^1 \frac{1}{v}\,dv$$

$$= -c(\ln|v_1| - \ln|v_2|)$$

For $c = ?$

$P_1V_1 = c.$

$4.2\times10^5(0.004) = c.$

$\boxed{c = 1680}$

Using values:

$$W_{3\to1} = 1680\ln\left|\frac{0.004}{0.012}\right|$$

$$= -1680(-1.65)$$

$$\boxed{W_{3\to1} = 1845.67\ Nm}$$

Using all these values in net work done;

$$W_{net} = W_{1\to2} + W_{2\to3} + W_{3\to1}.$$

$$= -4480\ Nm + 1120\ Nm + 1845.67\ Nm.$$

$$\boxed{W_{net} = -1514.39\ Nm}$$ Ans.

Scanned with CamScanner

1.5

A fluid at 0.7 bar occupying $0.09\ m^3$ is compressed reversibly to a pressure of 3.5 bar according to law pv^n = constant. The fluid is than heated reversibly at constant volume until the pressure is 4 bar; the specific volume is than $0.5\ m^3/kg$. A reversible expansion according to a law pv^2 = constant restore the fluid to its initial state. Sketch the process on pv-diagram & calculate.

i. The mass of fluid.

Ans:- Given data;

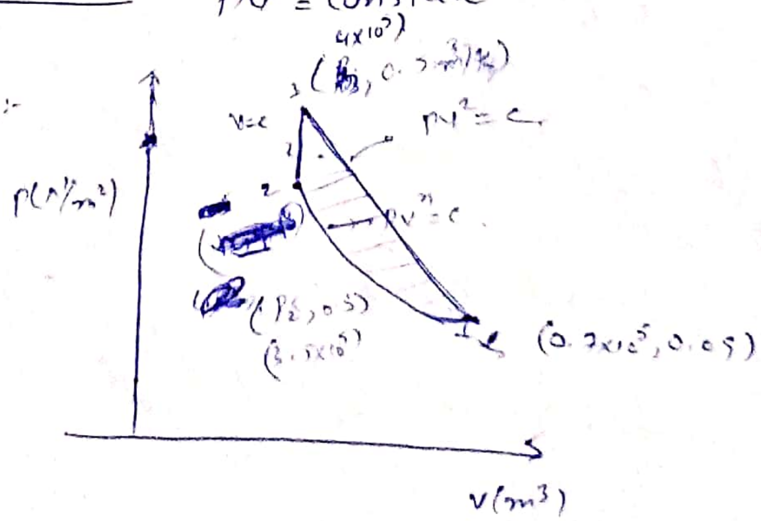
Pressure, $P_1 = 0.7$ bar $= 0.7 \times 10^5$ Pa

Volume, $V_1 = 0.09\ m^3$

Pressure, $P_2 = 3.5 \times 10^5$ Pa

Process 1→2 $\quad pv^n$ = constant

Graph:-



(a) m = ?

Since, $P_1 V_3^2 = c.$

$P_1 V_1^2 = P_3 V_3^2 \Rightarrow \quad V_1 = \sqrt{\dfrac{P_3 V_3^2}{P_1}}$

$V_1 = \sqrt{\dfrac{4\times10^5 (0.5)^2}{0.7}} \Rightarrow \quad \boxed{V_1 = 1.1952\ m^3/kg}$

Since;

$$v_1 = \frac{V_1}{m}$$

$$m = \frac{V_1}{v_1} \Rightarrow \frac{0.09}{1.1952} \Rightarrow \boxed{m = 0.075\,kg}$$

(ii) Value of n:-

As; $PV^n = C$

$$P_1 V_1^n = P_2 V_2^n.$$

$$(6.7\cancel{\times 10^5})(1.1952)^n = (3.\cancel{5\times 10^5})(0.5)^n.$$

$$(1.1952)^n = 5(0.5)^n.$$

Take log (ln) on b/s.

$$\ln(1.1952)^n = \ln(5(0.5)^n).$$

$$n\ln(1.1952) = \ln 5 + n\ln 0.5.$$

$$n(0.178) = 1.6094 + (-0.6931)n.$$

$$n = 1.8469$$

or

$$\boxed{n = 1.847}$$

(iii) Net work done:-

Net work done is;

$$W_{net} = W_{1\to 2} + W_{2\to 3} + W_{3\to 1}. \longrightarrow (1)$$

$$W_{1\to 2} = -\int_1^2 P\,dV$$

$$= -\int \frac{c}{V^{1.847}} dv$$

$\because$ Since;

$PV^n = c$

$P = \frac{c}{V^n}$

$P = \frac{c}{V^{1.847}}$

$$= -c \int V^{-1.847} dv$$

$$= \frac{-c}{-0.847}\left(\frac{1}{V_2^{0.847}} - \frac{1}{V_1^{0.847}}\right).$$

$\because c = ?$

$P_1 V_1^n = c$

$c = (0.7\times10^5)(0.09)^{1.847}$

$\boxed{c = 819.56}$

$$= \frac{-819.56}{-0.847}\left(\frac{1}{(0.0375)^{0.847}} - \frac{1}{(0.09)^{0.847}}\right).$$

$$= 967.60(16.13598 - 7.68699)$$

$\boxed{W_{1\to2} = 8175.2379} \longrightarrow 1$

$V_2 = ?$

Since $v_2 = 0.5$

$v_2 = \frac{V_2}{m}$.

$V_2 = v_2 m$.

$V_2 = (0.5)(0.075)$

$\boxed{V_2 = 0.0375\, m^3}$

$W_{2\to3} = 0$

As fixed volume.

$$W_{3\to1} = -\int_3^1 P dv$$

$\because PV^2 = c$

$P_1 V_1^2 = c$

$(0.7\times10^5)(0.09)^2 = c$

$\boxed{c = 567}$

$$= -\int_3^1 \frac{c}{V^2} dv.$$

$$= \frac{-c}{-1}\left(\frac{1}{V_1} - \frac{1}{V_3}\right)$$

$$= c\left(\frac{1}{V_1} - \frac{1}{V_3}\right)$$

$$= 567\left(\frac{1}{0.09} - \frac{1}{0.0375}\right)$$

Scanned with CamScanner

$= 567(11.11 - 26.66)$

$W_{3\to1} = 567(-15.56)$

$W_{3-1} = -8820.63$

So the net work done is,

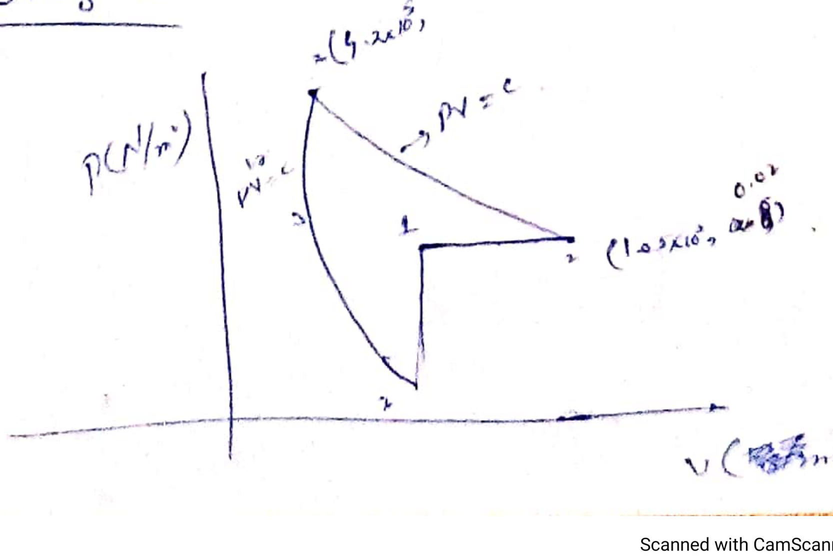
$W_{net} = 8175.23796 - 8820.63$

$\boxed{W_{net} = -645.39 \text{ Nm}}$ Ans

Pb-1.6

A fluid is heated reversibily at a constant pressure of 1.05 bar until it has a specific volume of $0.1 m^3/kg$. It is than compress reversibly according to law $PV=C$ to a pressure of 4.2 bar. Than allowed to expand reversibily according to a law $PV^{1.7}=C$ and finally heated at constant volume back to initial conditions. The work done in constant pressure process is -515 Nm and the mass of fluid present is 0.2 kg. Calculate net work done & sketch PV diagram.

Sol:- PV diagram:-



Since $\cancel{v_2 = 0.2}$ $v_2 = \frac{V_2}{m} \Rightarrow V_2 = m v_2 = 0.1 \times 0.2 = 0.02$

$$\boxed{V_2 = 0.02\, m^3}$$

Properties at state 1

P_1 = Pressure = $1.05 \times 10^5\ N/m^2$

Volume $V_1 = ?$

Process 1→2

$P = c$.

Properties at state 2

Pressure, $P_2 = 1.05 \times 10^5\ N/m^2$.

$V_2 = 0.02 m^3$

Work done = $W_{1\to 2} = -515\ Nm$.

Finding $V_1 = ?$

Since; $W_{1-2} = -\int_1^2 p\, dv$

$$-515 = P \int_1^2 dV \Rightarrow P(V_2 - V_1).$$

$$\frac{515}{1.05 \times 10^5} = (0.02 - V_1) \Rightarrow 0.04904 - 0.02 = -V_1$$

$$\boxed{V_1 = 0.01505\, m^3}$$

Properties at state 3

Pressure, $P_3 = 4.2$ bar.

$V_3 = ?$

Process 2-3

$PV = c$.

Since; $P_2 V_2 = c$.

$(\cancel{4}\,1.05 \times 10^5)(0.02) = c$.

$\boxed{c = 2100}$

$P_3 V_3 = c$

$(4.2 \times 10^5)(V_3) = 2100$

$\boxed{V_3 = 0.005\,m^3}$

$$W_{2\to3} = -\int_2^3 P\,dV$$

$\because PV = c$

$P = \frac{c}{V}$.

$= -c \ln\left|\frac{V_3}{V_2}\right|$.

$= -2100 \ln\left|\frac{0.005}{0.02}\right| \Rightarrow -2100(-1.3862)$

$\boxed{W_{2\to3} = 2911.2181\ N/m}$

Properties at state 4

Pressure, $P_4 = ?$

Volume, $V_4 = V_1 = 0.01505\,m^3$

Process $3\to4$

$PV^{1.7} = c$.

$P_3 V_3^{1.7} = c$.

$(4.2 \times 10^5)(0.005)^{1.7} = c$

$\boxed{c = 51.46}$

$P_4 V_4^{1.7} = C \Rightarrow P_4 = \frac{51.46}{(0.01505)^{1.7}} \Rightarrow \boxed{P_4 = 64225.22\ N/m^2}$

$W_{3\to4} = -\int P\,dV$

Scanned with CamScanner

$$= -c\int_3^4 \frac{1}{V^{1.7}}$$

$PV^{1.7} = C$

$P = \frac{C}{V^{1.7}}$

$$= \frac{C}{0.7}\left(\frac{1}{V_4^{0.7}} - \frac{1}{V_3^{0.7}}\right)$$

$$= \frac{51.46}{0.7}\left(\frac{1}{(0.01509)^{0.7}} - \frac{1}{(0.005)^{0.7}}\right)$$

$$\boxed{W_{3\to4} = -1615.48\ Nm}$$

$W_{4\to1} = 0$ since volume is constant.

$$W_{net} = W_{1\to2} + W_{2\to3} + W_{3\to4} - W_{4\to1}$$

$$= -515 + 2911.2187 - 1615.48 + 0$$

$$\boxed{W_{net} = 780.79\ Nm}$$

[Pb-1.7] In an air compression the compression takes place at a constant internal energy and 50 KJ of heat are rejected to cooling water ~~[illegible]~~ for every kg. Calculate work done of compression per kg.

Sol:- Given data;

$\Delta U = 0$

$\Delta Q = -50\,kJ$

$W = ?$

Scanned with CamScanner

Since,

$\Delta W + \Delta Q = \Delta U$

$\Delta W - 50 = 0$

$\boxed{\Delta W = 50 KJ}$

Pb-1.8

In the compression stroke of gas engine the work done on gas by piston is 70 kJ/kg and heat rejected to cooling water is 42 KJ/kg. Calculate change of specific internal energy stating whether it is gain or loss.

Sol:- Given data:

Work done $= \Delta W = 70 kJ/kg$.

Heat rejected $= \Delta Q = -42 kJ/kg$.

$\Delta U = ?$

Sol:- Since; $\Delta W + \Delta Q = \Delta U$.

$70 - 42 = \Delta U$.

$\boxed{\Delta U = 28 kJ/kg}$

Since the value is positive

So heat is gaining.

/Pb-1.9/

A mass of gas at initial pressure of 28 bar, and with internal energy of 1500 KJ, is contained in well insulated cylinder of volume $0.06 m^3$. The gas is allowed to expand behind a piston until its internal energy is 1400 KJ; the law of expansion is pv^2 = constant. Calculate:

(i) The work done.

(ii) The final volume.

(iii) The final pressure

Sol:- Given data

$U_1 = 1500 KJ$.

$U_2 = 1400 KJ$.

$\Delta Q = 0$ ∵ As well insulated.

(i) $\Delta W = ?$

$$\Delta W + \Delta Q = \Delta U.$$

$$\Delta W + 0 = U_2 - U_1.$$

$$\Delta W = 1400 - 1500.$$

$$\boxed{\Delta W = -100 KJ}.$$

(ii) Final volume $= V_2 = ?$

Pb # 1.5 :-

$W = 70\ kJ/kg$.

$Q = -42\ kJ/kg$.

$U = ?$ $= 70 - 42 \Rightarrow$ 28 kJ/kg

gain in inte...

Pb # 1.7

$\Delta Q = 50\ kJ$.

$\Delta U = 0$.

$W = ?$

As $\Delta Q + \Delta W = \cancel{U}^{0}$

$\Delta W = -\Delta Q$.

$= -50\ kJ/kg$.

work input 50 kJ/kg.

Pb # 1.6

$P = 1.05\ bar$

$V_2 = 0.1\ m^3/kg$.

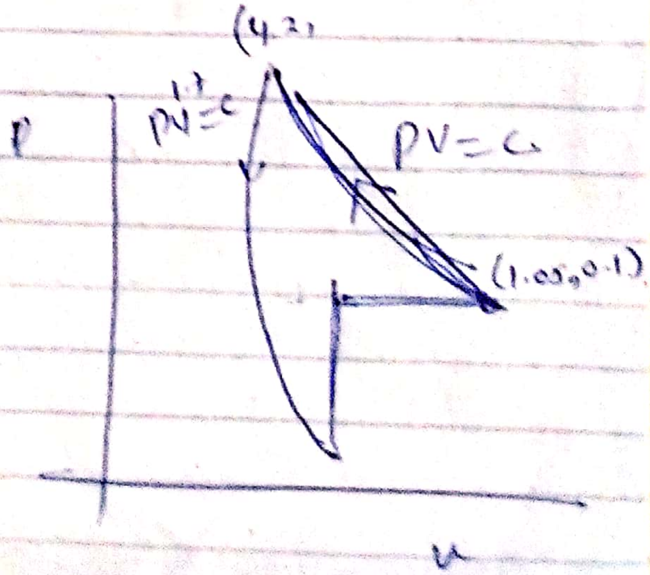
Process 1-2

$P = c$

Process 2-3

$P_v = c$

$W_{1-2} = -515\ Nm$.



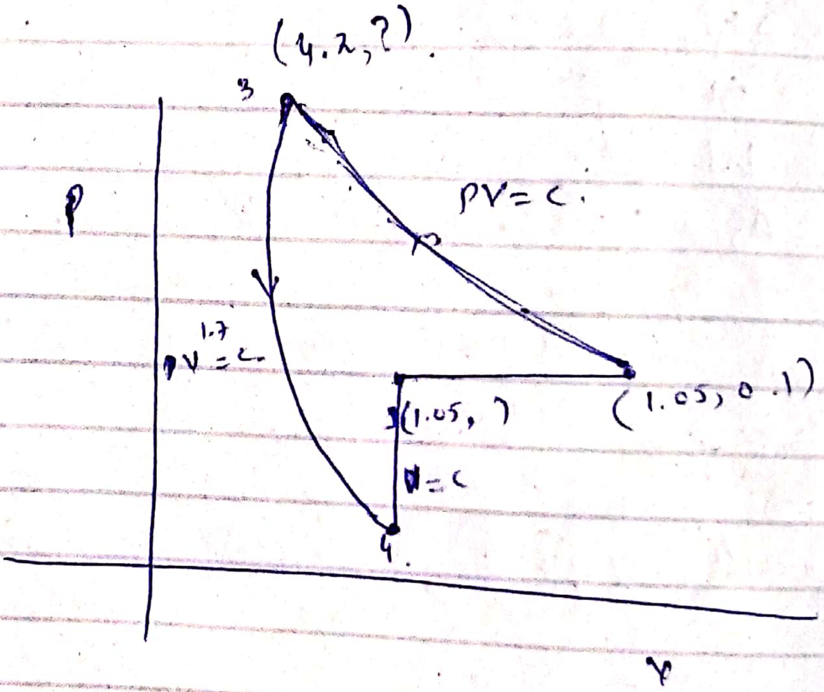
$m = 0.2 \text{ kg}$

$W_{net} = ?$

$W_{1-2} = -m \int P dv$

$515 \quad = -mc(v_2 - v_1)$

Pb 1.96



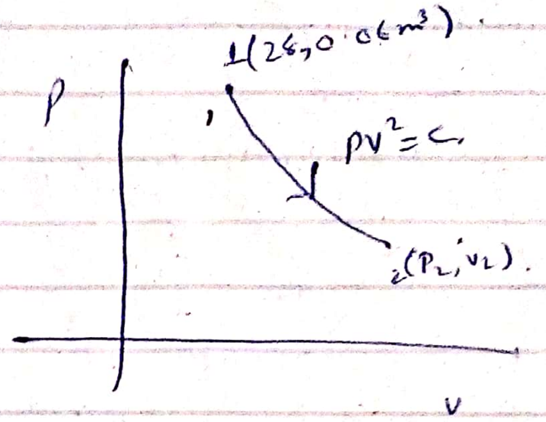
Pb - 1.9

$P_1 = 28$ bar

$U_1 = 1500$ KJ/Kg

$V = 0.06 m^3$.

$U_2 = 1400$ KJ.



Process:-

$PV^2 = \text{Const}$.

(a) Work done.

(b) Final Volume.

(c) Final Pressure

Sol:- As from first law of thermodynamics

$$\Delta W + \Delta Q = \Delta U.$$

As well insulated so $\Delta Q = 0$.

$$\Delta W = \Delta U.$$

$$\Delta W = 1400 - 1500.$$

$$\boxed{\Delta W = -100 \text{ KJ/Kg}}$$

Now to find V_2.

As ;- $-\int P dv = W_{1\to 2}$.

$C = PV^2$.

$P = \frac{C}{V^2}$.

$$-100 = c \int \frac{1}{V^2} dv.$$

$$100 = -c\left(\frac{1}{\;} - \frac{1}{\;}\right)$$

$P_1 V_1^2 = c_1$

$c = 10050.$

$$-100 = c\left(\frac{1}{V_2} - \frac{1}{V_1}\right)$$

$$-100\times10^3 = 10050\left(\frac{1}{V_2} - \frac{1}{0.06}\right)$$

$$-9.92\times10^{?} = \frac{1}{V_1} - \frac{1}{0.06}$$

$$\frac{1}{V_1} = \text{[illegible]}$$

$$V_1 = 0.148\,m^3$$

Now $\cancel{P_2 = ?}$

$\cancel{P_2 V_2^2 = c}$

$\cancel{P_2 = \frac{10080}{0.148}} \Rightarrow$

$P = 4.60\times10^5\,Pa$

$\approx 4.6\,bar$

$$P_1 V_1 = P_2 V_2.$$

$$P_2 = \frac{P_1 V_1^2}{V_2} \Rightarrow \frac{(25\times10^5)(0.08)^2}{(0.148)^2}$$

Ex Problems

Pb # 2.2

$v = 0.03 m^3$

$x = 1$

$P = 17$ bar

$m = ?$

$h = ?$

Sol - Table # 2. $P = 17$ bar.

$v = v_g$.

$v = 0.11664$

~~$v = 0.27268$~~.

$$m = \frac{V}{v} = \frac{0.03}{\cancel{0.27268}\ 0.11664} \Rightarrow$$

$m = 0.25 kg$

$h = h_g$.

$h = 6.396$. Table # 2. $P = 7$ bar

$$H = m(\cancel{6.396}\ 2793.4)$$

$$H = 0.25(\cancel{6.396}\ 2793.4)$$

$$\boxed{H = 717.3 kJ}$$

Pb #2.3

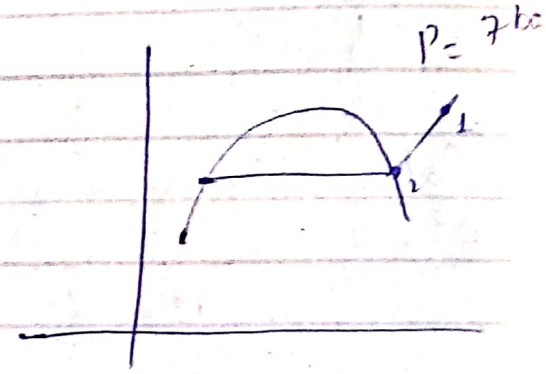
$P_1 = 7$ bar

$T = 250°C$.

$P =$ constant

T at which droplet form?

Heat rejection = ?



Sol:-

Table # 2

T at which droplet form is 165°C.

heat rejection for Isobaric process

$$Q = h_2 - h_1.$$

$h_1 = 2954$ KJ/kg

from table # 4
$P = 7$ bar
$T = 250$ C

$h_2 = h_g$.

Table # 2. P_2 7 bar.

$h_2 = 2762$ KJ/kg.

$$Q = h_2 - h_1$$

$$= 2762 - 2954$$

$$\boxed{Q = -192 \text{ KJ/kg.}}$$

Ans.

Ex # 2.2:-

$P = 7\,bar$

$h = 2600\ KJ/Kg.$

$v = ?$

$x = ?$

$u = ?$

Table # 2.

As ~~$2600 = (1-x)(1.992) + x(2762)$~~

~~2600~~

$$2600 = (1-x)697.1 + x(2762)$$

$$1902.9 =$$

$$x = 0.92.$$

0.91.

$$v = (1-0.92)\,v_f + x\,v_g.$$

$$v = 0.2515$$

$$u = h - Pv.$$

$$= 2600 - \frac{7\times10^5(0.2515)}{1000}.$$

$$u = 2600 - 176.05$$

$$\boxed{u = 2423.95\ KJ/Kg}$$

Scanned with CamScanner

Exp #2 3

$P = 110\ bar$

$v = 0.0196\ m^3/kg$

$T = ?$ $h = ?$ & $U = ?$

From Table #2

$P = 110\ bar$

$v = 0.016007\ m^3/kg < 0.0196\ m^3/kg$

So it means steam is in superheated reg.

$\boxed{T = 350°C}$

$h = ?$

at ~~200~~ 110 bar &

$T = 350°C$

$\boxed{h = 2889.6}$

$U = ?$

$U = h - Pv$

$$U = 2889.6 - \frac{110 \times 10^5 \times 0.0196}{1000}$$

$\boxed{U = 2674\ KJ/kg}$ Ans