Example:- 1 Chap # 1 Partial Derivatives.

If $f(x,y) = \frac{x+y}{2x-y}$, show that;

$$\lim_{x\to 0}\left[\lim_{y\to 0} f(x,y)\right] \neq \lim_{y\to 0}\left[\lim_{x\to 0} f(x,y)\right].$$

Solution:-

As; $f(x,y) = \frac{x+y}{2x-y}$

So,

$$\lim_{x\to 0}\left[\lim_{y\to 0} f(x,y)\right] = \lim_{x\to 0}\left[\lim_{y\to 0}\left(\frac{x+y}{2x-y}\right)\right].$$

$$= \lim_{x\to 0}\left(\frac{x}{2x}\right) \Rightarrow \lim_{x\to 0}\left(\frac{1}{2}\right) = \frac{1}{2} \longrightarrow ①.$$

Also,

$$\lim_{y\to 0}\left[\lim_{x\to 0} f(x,y)\right] = \lim_{y\to 0}\left[\lim_{x\to 0}\left(\frac{x+y}{2x-y}\right)\right].$$

$$= \lim_{y\to 0}\left(\frac{y}{-y}\right)$$

$$= \lim_{y\to 0}(-1) = -1 \longrightarrow ②$$

Hence proved;

$$\lim_{x\to 0}\left[\lim_{y\to 0} f(x,y)\right] \neq \lim_{y\to 0}\left[\lim_{x\to 0} f(x,y)\right] \text{ Ans}$$

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Example # 03

Evaluate $\lim_{\substack{x \to 0 \\ y \to 0}} (x^3 + y^3)$.

Solution:-

$$l_1 = \lim_{y \to 0} \left(\lim_{x \to 0} (x^3 + y^3) \right).$$

$$l_1 = \lim_{y \to 0} (y^3) = 0$$

$$l_2 = \lim_{x \to 0} \left(\lim_{y \to 0} (x^3 + y^3) \right).$$

$$= \lim_{x \to 0} (x^3) = 0.$$

for l_3 put $y = mx$.

$$l_3 = \lim_{x \to 0} \left(x^3 + (mx)^3 \right).$$

$$= \lim_{x \to 0} \left(x^3 + m^3 x^3 \right) \Rightarrow \lim_{x \to 0} \left(x^3 + mx^3 \right).$$

$$l_3 = 0.$$

For l_4 put $\cancel{x}y = mx^2$.

$$l_4 = \lim_{x \to 0} \left(x^3 + (mx^2)^3 \right).$$

$$= \lim_{x \to 0} \left(x^3 + mx^6 \right).$$

$$= 0 + 0.$$

$$l_4 = 0$$

Since $L_1 = l_2 = l_3 = l_4 = 0$ So limit exist with value zero.

Example # 04

Evaluate $\lim_{\substack{x \to 0 \\ y \to 0}} \frac{y^2 - x^2}{y^2 + x^2}$, $x \neq 0$, $y \neq 0$.

Solution;

$$l_1 = \lim_{x \to 0} \left(\lim_{y \to 0} \frac{y^2 - x^2}{y^2 + x^2} \right).$$

$$= \lim_{x \to 0} \left(\frac{-x^2}{x^2} \right) \Rightarrow \lim_{x \to 0} (-1) = -1$$

$$l_2 = \lim_{y \to 0} \left(\lim_{x \to 0} \left(\frac{y^2 - x^2}{y^2 + x^2} \right) \right) \Rightarrow \lim_{y \to 0} \left(\frac{y^2}{y^2} \right)$$

$$= \lim_{y \to 0} (1) = 1$$

As $f_1 \neq f_2$ thus limit does not exist.

Example # 05

Evaluate $\lim_{\substack{x \to 0 \\ y \to 0}} \frac{x^3 - y^3}{x^2 + y^2}$, $x \neq 0$, $y \neq 0$

Solution:-

$$l_1 = \lim_{x \to 0} \left(\lim_{y \to 0} \frac{x^3 - y^3}{x^2 + y^2} \right)$$

$$= \lim_{x \to 0} \left(\frac{x^3}{x^2} \right)$$

$$= \lim_{x \to 0} (x) = 0.$$

for $l_2 = \lim_{y \to 0} \left(\lim_{x \to 0} \frac{x^3 - y^3}{x^2 + y^2} \right)$

$= \lim_{y \to 0} \left(\frac{-y^3}{\ldots} \right)$

$$\lim_{y \to 0} \left(-y^2/y \right) \Rightarrow \lim_{y \to 0} (-y) = 0$$

For l_3 put $y = mx$.

$$l_3 = \lim_{x \to 0} \left(\frac{x^3 - (mx)^2}{x^2 + (mx)^2} \right)$$

$$= \lim_{x \to 0} \left(\frac{x^3 - m^2x^2}{x^2 + m^2x^2} \right)$$

$$\lim_{x \to 0} \left(\frac{x^2(x - m^2)}{x^2(1 + m^2)} \right) = \lim_{x \to 0} \left(\frac{x - m^2}{1 + m^2} \right)$$

Applying limit

$$= 0/1+m^2 = 0$$

For l_4 put $y = mx^2$

$$l_4 = \lim_{x \to 0} \left(\frac{x^3 - (mx)^2}{x^2 + (mx)^2} \right) \Rightarrow \lim_{x \to 0} \left(\frac{x^3 - m^2x^2}{x^2 + m^2x^2} \right)$$

$$= \lim_{x \to 0} \frac{x^2(x - m^2 x)}{x^2(1 + m^2)} \Rightarrow \lim_{x \to 0} \frac{(x - m^2 x)}{1 + m^2}$$

$$= \lim_{x \to 0} \frac{x(1 - m^2)}{1 + m^2} = \frac{0}{1 + m^2}$$

$\cancel{So}$ $l_4 = 0$

As $l_1 = l_2 = l_3 = l_4$ Hence limit exist

with values zero

Example #106

Evaluate $\lim\limits_{\substack{x \to 1 \\ y \to 2}} \dfrac{x^2+2y}{x+y^2}$, $x \neq 0, y \neq 0$.

Solution:-

$$l_1 = \lim_{x \to 1} \left(\lim_{y \to 2} \frac{x^2+2y}{x+y^2} \right).$$

$$= \lim_{x \to 1} \left(\frac{x^2+4}{x+4} \right).$$

$$= \lim_{x \to 1} \left(\frac{x^2+4}{x+4} \right). = \left(\frac{1+4}{1+4} \right) = 1.$$

$$l_2 = \lim_{y \to 2} \left(\lim_{x \to 1} \frac{x^2+2y}{x+y^2} \right).$$

$$= \lim_{y \to 2} \left(\frac{1+2y}{1+y^2} \right).$$

$$= \lim_{y \to 2} \left(\frac{1+2y}{1+y^2} \right) \quad \because \text{Apply limit.}$$

$$l_2 = \frac{1+(2)(2)}{1+(2)^2}.$$

$$= \frac{1+4}{1+4} = 1.$$

As $l_1 = l_2$ Thus limit exist

Exp #7

Evaluate $\lim_{\substack{x\to\infty \\ y\to 3}} \dfrac{2x-3}{x^3+4y^3}$

Sol:- $l_1 = \lim_{x\to\infty}\left(\lim_{y\to 3} \dfrac{2x-3}{x^3+4y^3}\right)$.

$= \lim_{x\to\infty}\left(\dfrac{2x-3}{x^3+4(3)^3}\right)$.

$= \lim_{x\to\infty}\left(\dfrac{2x-3}{x^3+108}\right)$.

$= \lim_{x\to\infty}\left(\dfrac{\cancel{x^3}\left(\frac{2}{x^2}-3/x^3\right)}{\cancel{x^3}\left(1+108/x^3\right)}\right)$.

Apply limit.

$l_1 = \left(\dfrac{\frac{2}{\infty} - 3/\infty^2}{1+108/\infty}\right) = \dfrac{0-0}{1+0} = 0.$

For $l_2 = \lim_{y\to 3}\left(\lim_{x\to\infty}\left(\dfrac{2x-3}{x^3+4y^3}\right)\right)$.

$= \lim_{y\to 3}\left(\lim_{x\to\infty} \dfrac{\cancel{x^3}\left(2/x^2 - 3/x^3\right)}{\cancel{x^3}\left(1+4y^3/x^2\right.}\right)$.

$= \lim_{y\to 3}\left(\dfrac{2/\infty^2 - 3/\infty^3}{1+4y^3/\infty^2}\right)$.

$= \lim_{y\to 3}(0)$.

$= 0.$

As $l_1 = l_2$ so limit exist with value 0.

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Exp#06 Evaluate $\lim\limits_{\substack{x\to\infty \\ y\to 2}} \dfrac{xy+4}{x^2+2y^2}$.

Sol:- For $l_1 = \lim\limits_{x\to\infty}\left(\lim\limits_{y\to 2} \dfrac{(xy+4)}{x^2+2y^2}\right)$.

$= \lim\limits_{x\to\infty}\left(\lim\limits_{y\to 2}\left(\dfrac{xy+4}{x^2+2y^2}\right)\right)$

$= \lim\limits_{x\to\infty}\left(\dfrac{2x+4}{x^2+8}\right)$.

$= \lim\limits_{x\to\infty}\left(\dfrac{\cancel{x^2}(2/x+4/x^2)}{\cancel{x^2}(1+8/x^2)}\right)$.

$= \left(\dfrac{2/\infty+4/\infty^2}{1+8/\infty^2}\right) \Rightarrow 0$.

For $l_2 = \lim\limits_{y\to 2}\left(\lim\limits_{x\to\infty}\left(\dfrac{xy+4}{x^2+2y^2}\right)\right)$

$= \lim\limits_{y\to 2}\left(\lim\limits_{x\to\infty}\left(\dfrac{\cancel{x^2}(y/x+4/x^2)}{x^2(1+2y^2/x^2)}\right)\right)$.

$= \lim\limits_{y\to 2}\left(\dfrac{y/\infty+4/\infty^2}{1+2y^2/\infty^2}\right)$.

$= \lim\limits_{y\to 2}\left(\dfrac{0+0}{1+0}\right) = \lim\limits_{y\to 2}(0)$

$= 0$.

As $l_1 = l_2$ Hence Limit exist with value zero.

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Excercise 1.1

Q4:- $\lim\limits_{\substack{x\to 0\\ y\to 0}} \dfrac{xy}{y-x^2}$; $x \neq 0,\ y \neq 0$.

Solution:-

$$l_1 = \lim_{x\to 0}\left(\lim_{y\to 0}\left(\frac{xy}{y-x^2}\right)\right)$$

$$= \lim_{x\to 0}\left(\frac{0}{-x^2}\right) = 0.$$

$$l_2 = \lim_{y\to 0}\left(\lim_{x\to 0}\left(\frac{xy}{y-x^2}\right)\right).$$

$$= \lim_{y\to 0}\left(\frac{0}{y}\right) = \lim_{y\to 0}(0) = 0.$$

For l_3 put $y = mx$.

$$l_3 = \lim_{x\to 0}\left(\frac{(mx)x}{mx - x^2}\right).$$

$$= \lim_{x\to 0}\left(\frac{mx^2}{mx - x^2}\right) = \lim_{x\to 0}\left(\frac{\cancel{x}\, mx}{\cancel{x}(m-x)}\right)$$

$$l_3 = \lim_{x\to 0}\left(\frac{mx}{m-x}\right). = \frac{0}{m-0} = 0.$$

For l_4 put $y = mx^2$.

$$l_4 = \lim_{x\to 0} \frac{x(mx^2)}{mx^2 - x^2} \Rightarrow \lim_{x\to 0}\frac{(mx^3)}{mx^2 - x^2}.$$

$$= \lim_{x\to 0} \frac{\cancel{x^2}(mx)}{\cancel{x^2}(m-1)} \Rightarrow \lim_{x\to 0}\frac{mx}{(m-1)}.$$

$$= \frac{0}{m-1} = 0.$$

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Q7:- $\lim_{x \to 0} \frac{x^3 + 2y^3}{x^2 + 4y^2}$; $x \neq 0, y \neq 0$.

Sol:-

$$l_1 = \lim_{x \to 0} \left(\lim_{y \to 0} \frac{x^3 + 2y^3}{x^2 + 4y^2} \right)$$

$$= \lim_{x \to 0} \left(\frac{x^3}{x^2} \right) = \lim_{x \to 0} (x) = 0.$$

$$l_2 = \lim_{y \to 0} \left(\lim_{x \to 0} \left(\frac{x^3 + 2y^3}{x^2 + 4y^2} \right) \right) = \lim_{y \to 0} \left(\frac{\cancel{2}y^{\cancel{3}}}{\cancel{4}_2 \cancel{y^2}} \right)$$

$$= \lim_{y \to 0} \left(\frac{1}{2} y \right) = \frac{1}{2}(0) = 0$$

For l_3 put $y = mx$.

$$l_3 = \lim_{x \to 0} \left(\frac{x^3 + 2(m^3 x^3)}{x^2 + 4m^2 x^2} \right) \Rightarrow \lim_{x \to 0} \left(\frac{\cancel{x^2}(x + 2m^3 x)}{\cancel{x^2}(1 + 4m^2)} \right).$$

$$= \lim_{x \to 0} \frac{(x + 2m^3 x)}{1 + 4m^2} \Rightarrow \frac{0}{1 + 4m^2} = 0.$$

For l_4 put $y = mx^2$.

$$l_4 = \lim_{x \to 0} \left(\frac{x^3 + 2(mx^2)^3}{x^2 + 4(mx^2)^2} \right) \Rightarrow \lim_{x \to 0} \left(\frac{x^3 + 2m^3 x^6}{x^2 + 4m^2 x^4} \right)$$

$$= \lim_{x \to 0} \left(\frac{\cancel{x^2}(x + 2m^3 x^4)}{\cancel{x^2}(1 + 4m^2 x^2)} \right).$$

$$= \frac{0 + 0}{1 + 0} = \frac{0}{1} = 0$$

Since;

$l_1 = l_2 = l_3 = l_4$ so limit exist with value 0.

Example #08

$$\lim_{\substack{x\to 0\\ y\to 0}} \frac{x^2y^3}{x^2+y^2} \quad x\neq 0,\ y\neq 0$$

Sol.-

$$l_1 = \lim_{x\to 0}\left(\lim_{y\to 0} \frac{x^2y^3}{x^2+y^2}\right).$$

$$= \lim_{x\to 0}\left(\frac{0}{x^2+0}\right) = \lim_{x\to 0}(0) = 0.$$

$$l_2 = \lim_{y\to 0}\left(\lim_{x\to 0} \frac{x^2y^3}{x^2+y^2}\right) = \lim_{y\to 0}\left(\frac{0}{0+y^2}\right)$$

$$l_2 = \lim_{y\to 0}(0)$$

$$l_2 = 0$$

For l_3 put $y = mx$

$$l_3 = \lim_{x\to 0}\left(\frac{x^2(mx)^3}{x^2+m^2x^2}\right) \Rightarrow \frac{m^3x^5}{x^2+m^2x^2}.$$

$$l_3 = \lim_{x\to 0} \frac{\cancel{x^2}(m^3x^3)}{\cancel{x^2}(1+m^2)} = \lim_{x\to 0}\left(\frac{m^3x^3}{1+m^2}\right).$$

$$l_3 = \frac{0}{1+m^2} = 0.$$

For l_4 put $y = mx^2$.

$$l_4 = \lim_{x\to 0} \frac{x^2(mx^2)^3}{x^2+(mx^2)^2} \Rightarrow \frac{mx^8}{x^2+mx^4} \Rightarrow \frac{\cancel{x^2}(mx^6)}{\cancel{x^2}(1+mx^2)}$$

$$l_4 = \lim_{x\to 0}\left(\frac{mx^6}{1+mx^2}\right) = \frac{0}{1+0} = \frac{0}{1} = 0.$$

As $l_1 = l_2 = l_3 = l_4$ Hence limit exist with value zero.

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Example #10

Discuss Continuity of $f(x,y) = \begin{cases} \frac{x}{\sqrt{x^2+y^2}}, & x\neq 0, y\neq 0 \\ 2, & x=0, y=0. \end{cases}$

at origin.

Solution:- at origin i.e $(0,0)$

$$l_1 = \lim_{x\to 0}\left(\lim_{y\to 0} \frac{x}{\sqrt{x^2+y^2}}\right).$$

$$= \lim_{x\to 0}\left(\frac{x}{\sqrt{x^2}}\right) = \lim_{x\to 0}\left(\frac{x}{x}\right) = \lim_{x\to 0}(1).$$

$$= 1.$$

$$l_2 = \lim_{y\to 0}\left(\lim_{x\to 0} \frac{x}{\sqrt{x^2+y^2}}\right) = \left(\frac{0}{\sqrt{y^2}}\right) = \lim_{y\to 0}(0) = 0.$$

As $l_1 \neq l_2$ Thus limit not exist so

not Continous at origin.

As if we put $y = mx$,

$$l_3 = \lim_{x\to 0} \frac{mx}{\sqrt{x^2+m^2x^2}} \Rightarrow \lim_{x\to 0}\left(\frac{\cancel{x}(m)}{\cancel{x}\sqrt{1+m^2}}\right)$$

$$= \frac{m}{\sqrt{1+m^2}}$$

For different values of m we get different result. So give not unique solution.

Thus not Continous at $x=0, y=0$. (origin).

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Exp# 11:-

Show that function discontinue at $(2,-2)$

$$f(x,y)=\begin{cases}\dfrac{x^2+xy+x+y}{x+y}, & (x,y)\neq(2,-2)\\ 4 & (x,y)=(2,-2)\end{cases}$$

Sol:-

$$l_1 = \lim_{x\to 2}\left(\lim_{y\to -2}\left(\frac{x^2+xy+x+y}{x+y}\right)\right)$$

$$= \lim_{x\to 2}\left(\lim_{y\to -2}\frac{x(x+y)+1(x+y)}{(x+y)}\right)$$

$$= \lim_{x\to 2}\left(\lim_{y\to -2}\frac{(x+y)(x+1)}{(x+y)}\right)$$

$$= \lim_{x\to 2}\left(\lim_{y\to -2}(x+1)\right)$$

$$= \lim_{x\to 2}(x+1) = 2+1 = 3.$$

~~$l_2 = \lim_{x}$~~ or

$$l_1 = \lim_{(x,y)\to(2,-2)}(x+1) = 2+1 = 3.$$

Since; $l_2 = 4.$

$l_1 \neq l_2.$

Thus limit does not exist at

$x = 2,-2.$

$$\lim_{(2,-2)} f(x,y) \neq f(2,-2).$$

Q5.2 $f(x,y) = \begin{cases} \dfrac{x^2+2y}{x+y^2} \\ 1 \end{cases}$ at Point (1, 2)

when $x=1, y=2$

Sol:- ~~lim~~

$$l_1 = \lim_{x \to 1} \left(\lim_{y \to 2} \left(\frac{x^2+2y}{x+y^2} \right) \right).$$

$$l_1 = \lim_{x \to 1} \left(\frac{x^2+4}{x+4} \right).$$

$$l_1 = \frac{1+4}{1+4} = \frac{5}{5} = 1.$$

For $l_2 = \lim_{y \to 2} \left(\lim_{x \to 1} \left(\frac{x^2+2y}{x+y^2} \right) \right).$

$$= \lim_{y \to 2} \left(\frac{1+2y}{1+y^2} \right).$$

$$= \left(\frac{1+2(2)}{1+4} \right).$$

$$= \frac{5}{5} = 1.$$

$f(1,2) = 4.$

Thus function is continue at (1, 2) Ans

Q6:- show that;

$$f(x,y) = \begin{cases} 2x^2+y & (x,y) \neq (1,2) \\ 0, & (x,y) = (1,2) \end{cases}$$ is discontinuous at $(1,2)$

Sol:- we have to find l_1, l_2 & $l_3 = f(1,2) = 0$.

$l_3 = f(1,2) = 0$. (Given)

Now $l_1 = \lim_{x \to 1} \left(\lim_{y \to 2} (2x^2+y) \right)$.

$= \lim_{x \to 1} (2x^2+2)$

$= \lim_{x \to 1} (2x^2+2) = (2+2) = 4$.

$l_2 = \lim_{y \to 2} \left(\lim_{x \to 1} (2x^2+y) \right) = \lim_{y \to 2} (2+y)$

$= \lim_{y \to 2} (2+y) = (2+2) = 4$.

As $l_1 = l_2 \neq l_3$.

Thus not continous at $(1,2)$. Ans

Exp-13 $u = \sin^{-1}\frac{x}{y} + \tan^{-1}\frac{y}{x}$

Find $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = ?$

Sol:-

$$\frac{\partial u}{\partial x} = \frac{1}{\sqrt{1-\left(\frac{x}{y}\right)^2}} \cdot \frac{\partial}{\partial x}\left(\frac{x}{y}\right) + \frac{1}{1+\left(\frac{y}{x}\right)^2}\frac{\partial}{\partial x}\left(\frac{y}{x}\right)$$

$$= \frac{1}{\sqrt{\frac{y^2-x^2}{y^2}}} \cdot \frac{1}{y} + \frac{1}{\frac{x^2+y^2}{x^2}} \cdot \left(-\frac{y}{x^2}\right)$$

$$= \frac{1}{\frac{\sqrt{y^2-x^2}}{y}} \cdot \frac{1}{y} + \frac{x^2}{x^2+y^2}\left(-\frac{y}{x^2}\right)$$

$$= \frac{1}{\sqrt{y^2-x^2}} + \frac{(-y)}{x^2+y^2}$$

$$x\frac{\partial u}{\partial x} = \frac{x}{\sqrt{y^2-x^2}} - \frac{yx}{x^2+y^2} \longrightarrow ①$$

$$\frac{\partial u}{\partial y} = \frac{1}{\sqrt{1-\left(\frac{x}{y}\right)^2}} \cdot \left(-\frac{x}{y^2}\right) + \frac{1}{1+\left(\frac{y}{x}\right)^2} \cdot \left(\frac{1}{x}\right).$$

$$= \frac{1}{\frac{\sqrt{y^2-x^2}}{y}} \cdot \left(-\frac{x}{y^2}\right) + \frac{1}{\frac{x^2+y^2}{x^2}} \cdot \frac{1}{x}$$

$$= \frac{-x}{y\sqrt{y^2-x^2}} + \frac{x}{x^2+y^2}$$

$$y\frac{\partial u}{\partial y} = \frac{-xy}{y\sqrt{y^2-x^2}} + \frac{xy}{x^2+y^2} \Rightarrow \frac{-x}{\sqrt{y^2-x^2}} + \frac{xy}{x^2+y^2} \longrightarrow ②$$

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \cancel{\frac{x}{\sqrt{y^2-x^2}}} - \cancel{\frac{yx}{x^2+y^2}} - \cancel{\frac{x}{\sqrt{y^2-x^2}}} + \cancel{\frac{xy}{x^2+y^2}} = 0 \text{ Ans}$$

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Sup #10

For $y = f(x+at) + g(x-at)$

Prove that,

$$\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2} \longrightarrow ①$$

Sol– L.H.S

$$\frac{\partial^2}{\partial t^2} y = ?$$

$$\frac{\partial}{\partial t} y = a f'(x+at) - a g'(x-at)$$

$$\frac{\partial^2 y}{\partial t^2} = a^2 f''(x+at) + a^2 g''(x-at) \longrightarrow ①$$

R.H.S $= a^2 \frac{\partial y}{\partial x^2} = ?$

$$\frac{\partial y}{\partial x} = f'(x+at) + g'(x-at)$$

$$\frac{\partial^2 y}{\partial x^2} = f''(x+at) + g''(x-at)$$

$$a^2 \frac{\partial^2 y}{\partial x^2} = a^2 f''(x+at) + a^2 g''(x-at) \longrightarrow ②$$

$$= a^2 \left[f''(x+at) + g''(x-at) \right]$$

$$= a^2 \frac{\partial^2 y}{\partial x^2} \longrightarrow ②$$

From eq ① & ②

L.H.S = R.H.S

Proved

Q4# $f(x,y) = \begin{cases} x^3+y^3 & x \neq 0, y \neq 0 \\ 0 & x=0, y=0 \end{cases}$ at origin.

Sol:-

$$l_1 = \lim_{x \to 0} \left(\lim_{y \to 0} x^3+y^3 \right).$$

$$= \lim_{x \to 0} (x^3) = 0.$$

$$l_2 = \lim_{y \to 0} \left(\lim_{x \to 0} (x^3+y^3) \right) = \lim_{y \to 0} (y^3) = 0.$$

For l_3 put $y = mx$.

$$l_3 = \lim_{x \to 0} \left(x^3 + m^3 x^3 \right).$$

$$= 0.$$

For l_4 put $y = mx^2$.

$$l_4 = \lim_{x \to 0} \left(x^3 + (mx^2)^3 \right).$$

$$= \lim_{x \to 0} (x^3 + m^3 x^6).$$

$$= \lim_{x \to 0} x^3 + \lim_{x \to 0} m^3 x^6.$$

$$= 0 + 0 = 0.$$

As $l_1 = l_2 = l_3 = l_4 = f(0,0)$

Thus function is continue at origin.

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Excercise 1.2

Q3:- $f(x,y) = \begin{cases} \dfrac{x^3y^3}{x^3+y^3} & \text{when } x \neq 0, y \neq 0 \text{ at origin} \end{cases}$

Sol:- $l_1 = \lim\limits_{x\to 0}\left(\lim\limits_{y\to 0} \dfrac{x^3y^3}{x^3+y^3}\right) = \lim\limits_{x\to 0}\left(\dfrac{0}{x^3}\right) = 0.$

$l_2 = \lim\limits_{y\to 0}\left(\lim\limits_{x\to 0} \dfrac{x^3y^3}{x^3+y^3}\right) = \lim\limits_{y\to 0}\left(\dfrac{0}{y^3}\right) = \lim\limits_{y\to 0}(0) = 0.$

For l_3: Put $y = mx$.

$l_3 = \lim\limits_{x\to 0} \dfrac{x^3(mx)^3}{x^3+(mx)^3} \Rightarrow \dfrac{mx^6}{x^3+mx^3} = \lim\limits_{x\to 0} \dfrac{\cancel{x^3}(mx^3)}{\cancel{x^3}(1+m)}$

$= \lim\limits_{x\to 0} \dfrac{mx^3}{1+m} = \boxed{0}/_{1+m} = 0.$

For l_4: Put $y = mx^2$.

$l_4 = \lim\limits_{x\to 0} \dfrac{x^3(mx^2)^3}{x^3+(mx^2)^3} \Rightarrow \lim\limits_{x\to 0} \dfrac{mx^8}{x^3+m^2x^6}.$

$= \lim\limits_{x\to 0} \dfrac{\cancel{x^3}(mx^5)}{\cancel{x^3}(1+m^2x^3)}).$

$= 0$

Thus continous at origin.

Example #18

If $z = \tan(y+ax) + (y-ax)^{3/2}$ then show that

$$\frac{\partial^2 z}{\partial x^2} = \frac{a^2 \partial^2 z}{\partial y^2}$$

$\frac{3}{2} - 1$

$\frac{3-2}{2} = \frac{1}{2}$

Sol:- As; $z = \tan(y+ax) + (y-ax)^{3/2}$

$$\frac{\partial z}{\partial x} = \sec^2(y+ax).(a) + \frac{3}{2}(y-ax)^{1/2}(-a)$$

$$= a\sec^2(y+ax) - a\frac{3}{2}(y-ax)^{1/2}$$

$$\frac{\partial^2 z}{\partial x^2} = a(2)\sec(y+ax).\sec(y+ax).\tan(y+ax)(a)$$

$$-\frac{1}{2}a\left(\frac{3}{2}\right)(y-ax)^{-1/2}(-a)$$

$$= 2a^2\sec^2(y+ax).\tan(y+ax) + \frac{3}{4}a^2(y-ax)^{-1/2} \rightarrow ①$$

Now;

$$\frac{\partial z}{\partial y} = \sec^2(y+ax) + \frac{3}{2}(+y-ax)^{1/2}$$

$$\frac{\partial^2 z}{\partial y^2} = 2\sec(y+ax).\sec(y+ax).\tan(y+ax) + \frac{3}{2}\left(\frac{1}{2}\right)(y-ax)^{-1/2}$$

$$= 2\sec^2(y+ax)\tan(y+ax) + \frac{3}{4}(y-ax)^{-1/2}$$

$$a^2\frac{\partial^2 z}{\partial y^2} = 2a^2\sec^2(y+ax)\tan(y+ax) + \frac{3}{4}a^2(y-ax)^{-1/2} \rightarrow ②$$

Since eq ① = eq ② L.H.S = R.H.S

Example #19

If $u = e^{xyz}$ find value of $\frac{\partial^3 u}{\partial x \partial y \partial z}$

Sol:- As given

$$u = e^{xyz}.$$

$$\frac{\partial u}{\partial z} = e^{xyz}\left(\partial/\partial z\,(xyz)\right) \Rightarrow e^{xyz}(xy)$$

$$\frac{\partial}{\partial y} \cdot \frac{\partial u}{\partial z} = e^{xyz}(x) + xy\, e^{xyz} \cdot xz.$$

$$\frac{\partial^2 u}{\partial y \partial z} = x e^{xyz} + x^2 y z e^{xyz}$$

$$= e^{xyz}(x + x^2 y t).$$

$$\frac{\partial}{\partial x} \frac{\partial^2 u}{\partial y z} = e^{xyz}(1 + 2xyz) + (x + x^2 yz) e^{xyz}(yz).$$

$$= e^{xyz}(1 + 2xyz + xyz + x^2 y^2 z^2).$$

$$\frac{\partial^3 u}{\partial x \partial y \partial z} = e^{xyz}(1 + 3xyz + x^2 y^2 z^2).$$

Ans

Exp# 3 Page # 28

if $z = f(x,y)$ & $x = e^u + e^{-v}$, $y = e^{-u} \pm e^v$.

Prove That;

$$\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y}.$$

Sol:- $L.H.S = \frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} \rightarrow$ ①.

$$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial u} \rightarrow ②.$$

$$\frac{\partial x}{\partial u} = \frac{\partial}{\partial u}\left(e^u + e^{-v}\right) = e^u.$$

$$\frac{\partial y}{\partial u} = \frac{\partial}{\partial u}\left(e^u + e^{-v}\right) = -e^{-u}.$$

Put values in eq ②

$$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x}\cdot e^u + \frac{\partial z}{\partial y}\left(-e^{-u}\right) \Rightarrow e^u\frac{\partial z}{\partial x} - e^{-u}\frac{\partial z}{\partial y}.$$

Also; $\frac{\partial z}{\partial v} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial v} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial v}$

$$\frac{\partial x}{\partial v} = -e^{-v}$$

$$\frac{\partial y}{\partial v} = e^{-v}.$$

By putting values. $\frac{\partial z}{\partial v} = \frac{\partial z}{\partial x}\left(-e^{-v}\right) + \frac{\partial z}{\partial y}\left(e^{-v}\right)$

Put values in eq ①.

$$L.H.S = e^u\frac{\partial z}{\partial x} - e^{-u}\frac{\partial z}{\partial y} + \frac{\partial z}{\partial x}e^{-v} + \frac{\partial z}{\partial y}\left(e^{-v}\right)$$

$$= \frac{\partial z}{\partial x}\left(e^u + e^{-v}\right) - \frac{\partial z}{\partial y}\left(e^{-u} - e^{-v}\right)$$

$$= \frac{\partial z}{\partial x}(x) - \frac{\partial z}{\partial y}(y) \Rightarrow x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y}$$

$\boxed{\text{Proved}}$

$\boxed{L.H.S = R.H.S}$ Ans

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[illegible] If $u = (1-2xy+y^2)^{-1/2}$ prove that

$$x\frac{\partial u}{\partial x} - y\frac{\partial u}{\partial y} = y^2u^3.$$

Sol:- Given that; $u = (1-2xy+y^2)^{-1/2}$.

$$\frac{\partial u}{\partial x} = -\frac{1}{2}(1-2xy+y^2)^{-3/2}(-2y).$$

$$= y(1-2xy+y^2)^{-3/2}$$

$$x\frac{\partial u}{\partial x} = xy/(1-2xy+y^2)^{3/2}.$$

$$\frac{\partial u}{\partial y} = -1/2\,(1-2xy+y^2)^{-3/2}(-2\cancel{y}^{x}+2y)$$

$$= -\frac{1}{2}(1-2xy+y^2)^{-3/2}(-\cancel{2})(x-y)$$

$$= \frac{x-y}{(1-2xy+y^2)^{3/2}}$$

$$y\frac{\partial u}{\partial y} = \frac{y(x-y)}{(1-2xy+y^2)^{3/2}}$$

$$x\frac{\partial u}{\partial x} - y\frac{\partial u}{\partial y} = \frac{xy}{(1-2xy+y^2)^{3/2}} - \frac{y(x-y)}{(1-2xy+y^2)^{3/2}}$$

$$= \frac{\cancel{xy}-\cancel{xy}+y^2}{(1-2xy+y^2)^{3/2}} = y^2\left((1-2xy+y^2)^{-1/2}\right)^3$$

$$= y^2(u)^3 \Rightarrow \therefore \underline{y^2u^3 \text{ proved}}$$

Ans.

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Exp#5 If $w = f(x,y)$, $x = r\cos\theta$, $y = r\sin\theta$

show that,

$$\left(\frac{\partial w}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial w}{\partial \theta}\right)^2 = \left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2$$

Sol:- As,

$$\frac{\partial w}{\partial r} = \frac{\partial w}{\partial x}\cdot\frac{\partial x}{\partial r} + \frac{\partial w}{\partial y}\cdot\frac{\partial y}{\partial r} \rightarrow (1)$$

$$\frac{\partial x}{\partial r} = \cos\theta. \qquad \frac{\partial y}{\partial r} = \sin\theta$$

$$\frac{\partial w}{\partial r} = \frac{\partial w}{\partial x}\cdot\cos\theta + \frac{\partial w}{\partial y}\sin\theta \rightarrow (2)$$

$$\frac{\partial w}{\partial \theta} = \frac{\partial w}{\partial x}\cdot\frac{\partial x}{\partial \theta} + \frac{\partial w}{\partial y}\cdot\frac{\partial y}{\partial \theta} \rightarrow (3)$$

$$= \frac{\partial x}{\partial \theta} = -r\sin\theta \quad , \quad \frac{\partial y}{\partial \theta} = r\cos\theta$$

$$\frac{\partial w}{\partial \theta} = -\frac{\partial w}{\partial x} r\sin\theta + \frac{\partial w}{\partial y} r\cos\theta$$

$$\frac{1}{r}\frac{\partial w}{\partial \theta} = -\frac{\partial w}{\partial x}\sin\theta + \frac{\partial w}{\partial y}\cos\theta \rightarrow (4)$$

Squaring & add of (2) & (4)

$$\left(\frac{\partial w}{\partial r}\right)^2 + \left(\frac{1}{r}\frac{\partial w}{\partial \theta}\right)^2 = \left(\frac{\partial w}{\partial x}\cos\theta + \frac{\partial w}{\partial y}\sin\theta\right)^2 + \left(\left(-\frac{\partial w}{\partial x}\sin\theta\right) + \frac{\partial w}{\partial y}\cos\theta\right)$$

$$= \left(\frac{\partial w}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2 (\sin$$

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Exp # 06

if $u = u(y-z, z-x, x-y)$ prove that,

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0 \longrightarrow (A)$$

Sol:- let $r = y - z$, $s = z - x$ & $t = x - y$

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial r}\cdot\frac{\partial r}{\partial x} + \frac{\partial u}{\partial s}\cdot\frac{\partial s}{\partial x} + \frac{\partial u}{\partial t}\cdot\frac{\partial t}{\partial x} \longrightarrow (1)$$

$$= \frac{\partial u}{\partial r}(0) + \frac{\partial u}{\partial s}(-1) + \frac{\partial u}{\partial t}(1)$$

$$= -\frac{\partial u}{\partial s} + \frac{\partial u}{\partial t} \longrightarrow (a)$$

$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial r}\cdot\frac{\partial r}{\partial y} + \frac{\partial u}{\partial s}\frac{\partial s}{\partial y} + \frac{\partial u}{\partial t}\frac{\partial t}{\partial y}$$

$$= \frac{\partial u}{\partial r}(1) + \frac{\partial u}{\partial s}(0) + \frac{\partial u}{\partial t}(-1)$$

$$= \frac{\partial u}{\partial r} - \frac{\partial u}{\partial t} \longrightarrow (b)$$

$$\frac{\partial u}{\partial z} = \frac{\partial u}{\partial r}\frac{\partial r}{\partial z} + \frac{\partial u}{\partial s}\frac{\partial s}{\partial z} + \frac{\partial u}{\partial t}\frac{\partial t}{\partial z}$$

$$= \frac{\partial u}{\partial r}(-1) + \frac{\partial u}{\partial s}(1) + \frac{\partial u}{\partial t}(0)$$

$$= -\frac{\partial u}{\partial r} + \frac{\partial u}{\partial s} \longrightarrow (c)$$

Put all values in eq (A)

$$\Rightarrow \cancel{-\frac{\partial u}{\partial s}} + \cancel{\frac{\partial u}{\partial t}} + \cancel{\frac{\partial u}{\partial r}} - \cancel{\frac{\partial u}{\partial t}} - \cancel{\frac{\partial u}{\partial r}} + \cancel{\frac{\partial u}{\partial s}} = 0$$

Proved

Exp # 16

If $z = e^{ax+by} \cdot f(ax-by)$ prove that;

$$b\frac{\partial z}{\partial x} + a\frac{\partial z}{\partial y} = 2abz.$$

Sol:- As $z = e^{ax+by} \cdot f(ax-by)$.

$$\frac{\partial z}{\partial x} = e^{ax+by} \cdot f'(ax-by)(a) + f(ax-by)e^{ax+by} \cdot a$$

$$b\frac{\partial z}{\partial x} = abe^{ax+by} \cdot f'(ax-by) + ab\, f(ax-by)\, e^{ax+by}.$$

Now;

$$\frac{\partial z}{\partial y} = e^{ax+by} \cdot f'(ax-by)(-b) + f(ax-by)e^{ax+by} \cdot b.$$

$$a\frac{\partial z}{\partial y} = -ab(e^{ax+by}) f'(ax-by) + ab\, f(ax-by)e^{ax+by}.$$

$$b\frac{\partial z}{\partial x} + a\frac{\partial z}{\partial y} = abe^{ax+by} \cancel{f'(ax-by)} + abf(ax-by)e^{ax+by} - ab(e^{ax+by})\cancel{f'(ax-by)} + abf(ax-by)e^{ax+by}.$$

$$= ab\, f(ax-by)e^{ax+by} + abf(ax-by)e^{ax+by}.$$

$$\because f(ax-by)e^{ax+by} = z.$$

$$= ab(f(ax-by)e^{ax+by}).$$

$$= abz. \quad \text{Ans. } \boxed{\text{Proved}}!$$

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