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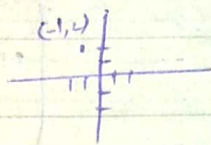
Shift Ex:- 4.1

Name.....subject.....Examiner's Award.....

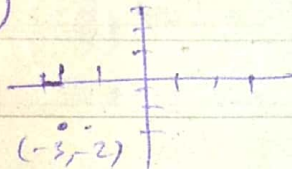
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Q1:- Plot the following point.

(b) $(-1, 2)$

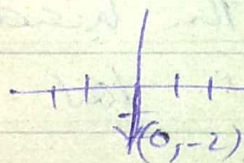


(c) $(-3, -2)$



Q2:- Sketch

(a) $\begin{bmatrix} 0 \\ -2 \end{bmatrix}$

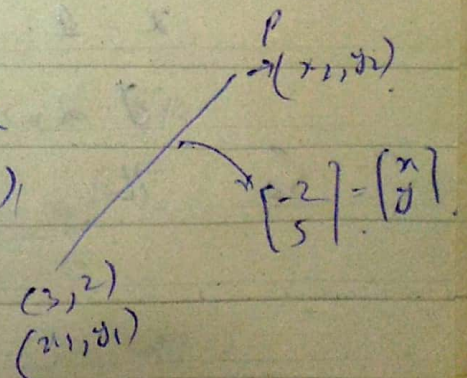


Q3:- Determine head of a vector $\begin{bmatrix} -2 \\ 5 \end{bmatrix}$ whose tail is at $(3, 2)$. make a sketch.

Sol:- Let $P(x_2, y_2)$ be head of a vector $u = \begin{bmatrix} -2 \\ 5 \end{bmatrix}$ also $P_1(x_1, y_1) = P_1(3, 2)$

is tail as given;

$$P_1P_2 = u = \begin{bmatrix} x_2 - 3 \\ y_2 - 2 \end{bmatrix} = \begin{bmatrix} -2 \\ 5 \end{bmatrix}$$

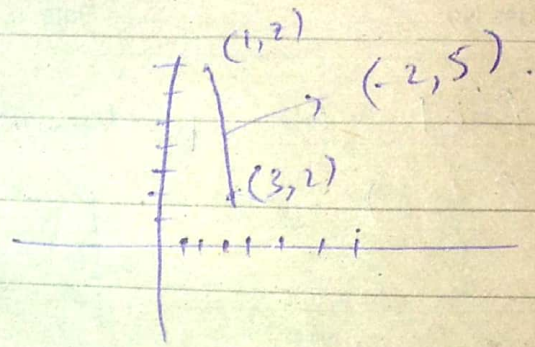


By comparing;

$$x_2 - 3 = -2 \Rightarrow x_2 = 1$$

$$y_2 - 2 = 5$$

$$y_2 = 7$$



So the desired head of vector is $(1, 7)$ Ans

Q4:-

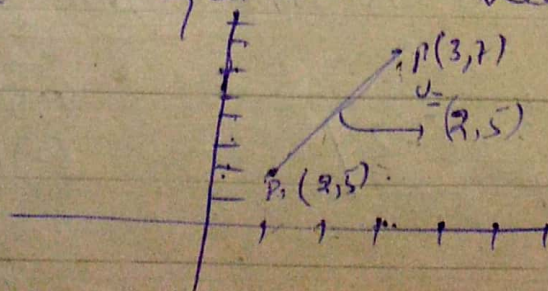
Sol:- Let $P(x, y)$ be the head of ~~tail~~ vector $u = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$ also $P_1(1, 2)$ is tail so;

$$P_1 P_2 = u \Rightarrow \begin{bmatrix} x-1 \\ y-2 \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$$

$$x - 1 = 2 \Rightarrow x = 3$$

$$y - 2 = 5 \Rightarrow y = 7$$

So the required head is $P = (3, 7)$



Q7:- Let $u = (1, 2)$, $v = (-3, 4)$, $w = (w_1, 4)$ & $x = (-2, x_2)$

Find w_1 & x_2

(a) $|w| = 2u$

$$(w_1, 4) = 2(1, 2)$$

$$(w_1, 4) = (2, 4) \Rightarrow \boxed{w_1 = 2}$$

(b) $w + x = u$

$$(w, 4) + (-2, x_2) = (1, 2)$$

$$w - 2, 4 + x_2 = (1, 2)$$

$$w - 2 = 1$$

$$4 + x_2 = 2$$

$$\boxed{w = 3}$$

$$\boxed{x_2 = -2}$$

Ans.

Q8:-

Sol:- Suppose $(-5, 6)$ is the linear comb. of $(1, 2)$ & $(3, 4)$ then there exist c_1 & c_2 such that;
 $c_1(1, 2) + c_2(3, 4) = (-5, 6)$

$$c_1 + 3c_2 = -5 \Rightarrow 2c_1 + 6c_2 = -10$$

$$2c_1 + 4c_2 = 6 \Rightarrow \underline{5.2c_1 + 4c_2 = 6}$$

$$2c_2 = -16$$

$$\boxed{c_2 = -8}$$

$$\text{Q} \Rightarrow c_1 + 3(-8) = -5 \Rightarrow c_1 = 24 - 5$$

$$\boxed{c_1 = 19}$$

As system is consistent so $(-5, 6)$ is L.C
of $(1, 2)$ & $(3, 4)$.

Q14:-

Sol:- Suppose C_1 & C_2 are L.C then;

$$C_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} + C_2 \begin{bmatrix} 3 \\ 4 \end{bmatrix} = 0$$

$$C_1 + 3C_2 = 0 \quad \text{--- (1)} \Rightarrow \quad 2C_1 + 6C_2 = 0$$

$$2C_1 + 4C_2 = 0$$

$$\oplus \quad 2C_1 + 4C_2 = 0$$

$$\Rightarrow 2C_1 = 0 \Rightarrow C_1 = 0$$

Q10 $\Rightarrow$

$$C_1 = 0$$

It is not possible both $C_1 = C_2 = 0$.

$$A = \frac{1}{2} \left| \det \begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{bmatrix} \right|$$

$$A = \frac{1}{2} \begin{vmatrix} 0 & 3 & 1 \\ 0 & 0 & 1 \\ 4 & 0 & 1 \end{vmatrix}$$

$$\Rightarrow -3(-4) + 1(0)$$

By Pythagoras Theorem.

$$= (12) \frac{1}{2} \Rightarrow 6 \text{ Square unit.}$$

$$(BC)^2 = (AB)^2 + (AC)^2$$

$$(\sqrt{4^2 + 3^2})^2 = (\sqrt{3^2})^2 + (\sqrt{4^2})^2 \Rightarrow \sqrt{16+9} = \sqrt{9+16}$$

$$\text{Area} = \frac{1}{2} (3 \times 4) = 6 \text{ Square unit} \Rightarrow 25 = 25$$

Ex:- 4.1

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Q17:- Area of parallelogram.

$$A = \left| \det \begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{bmatrix} \right| \rightarrow \text{①}$$

(2, 3), (5, 3)
(4, 5), (7, 5)

Area of parallelogram

$$A = A_{T_1} + A_{T_2} \rightarrow \text{①}$$

$$A_{T_1} = \frac{1}{2} \left| \det \begin{bmatrix} 2 & 3 & 1 \\ 5 & 3 & 1 \\ 2 & 5 & 1 \end{bmatrix} \right|$$

$$= \frac{1}{2} \left| 2(3-5) - 3(5-7) + 1(25-21) \right| \Rightarrow \frac{1}{2} \left| 2(-2) - 3(-2) + 1(4) \right|$$

$$= \frac{1}{2} \left| -4 + 6 + 4 \right| \Rightarrow \boxed{A_{T_1} = 3 \text{ Sq unit}}$$

$$\begin{bmatrix} 2 & 3 & 1 \\ 4 & 5 & 1 \\ 2 & 5 & 1 \end{bmatrix}$$

$$A_{T_2} = \frac{1}{2} \left| 2(5-5) - 3(4-7) + 1(20-35) \right|$$

$$= \frac{1}{2} \left| -3(-3) + 1(-15) \right| \Rightarrow \frac{1}{2} \left| +9 - 15 \right| \Rightarrow \frac{+6-3}{2}$$

$$\boxed{A_{\square} = 3 + 3 \Rightarrow 6 \text{ Square unit}}$$

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Q20:- $\vec{x} = (5, 0)$

Sol:- The unit vector is given as;

$$\hat{x} = \frac{\vec{x}}{\|\vec{x}\|} \rightarrow \text{①}$$

$$\vec{x} = (5, 0)$$

$$\|\vec{x}\| = \sqrt{25} = 5$$

$$\hat{x} = \frac{1}{5}(5, 0) \Rightarrow (1, 0) = \text{unit vector}$$

Ans $\sqrt{1^2 + 0^2} = 1$ Ans

(b) $(-2, -3)$

$$\hat{x} = \frac{\vec{x}}{\|\vec{x}\|} = \frac{(-2, -3)}{\sqrt{4+9}}$$

$$\hat{x} = \left(\frac{-2}{\sqrt{13}}, \frac{-3}{\sqrt{13}} \right) \text{ Ans}$$

$\hat{x} = 1$

$$\sqrt{\left(\frac{-2}{\sqrt{13}}\right)^2 + \left(\frac{-3}{\sqrt{13}}\right)^2}$$

$$= \sqrt{\frac{4+9}{13}} = \sqrt{\frac{13}{13}} = 1$$

Q21:- Find cosine of angle

① $u = (-3, -4), v = (4, -3)$

Sol:- As cosine of angle is given

$$\cos \theta = \frac{u \cdot v}{\|u\| \|v\|}$$

$$u \cdot v = (-3, -4) \cdot (4, -3) = -12 + 12 = 0$$

$$\cos \theta = 0$$

$$u \cdot v = (-3, -4) \cdot (4, -3) = -12 + 12 = 0$$

$$\cos \theta = 0 \Rightarrow \cos \theta = 0$$

② $u = (2, 1), v = (-2, -1)$

$$\cos \theta = \frac{u \cdot v}{\|u\| \|v\|} = \frac{(2, 1) \cdot (-2, -1)}{\sqrt{2^2 + 1^2} \cdot \sqrt{(-2)^2 + (-1)^2}}$$

$$= \frac{-4 - 1}{\sqrt{5} \cdot \sqrt{5}} = \frac{-5}{5} = -1$$

$$\cos \theta = -1 \text{ Ans}$$

(b) $u = (2, 2)$, $v = (4, -5)$

Sol:- $\cos \theta = \frac{u \cdot v}{\|u\| \|v\|} \Rightarrow \frac{8 - 10}{\sqrt{2^2 + 2^2} \cdot \sqrt{4^2 + (-5)^2}}$

$\frac{-2}{2\sqrt{2} \cdot \sqrt{41}} = \frac{-1}{\sqrt{2} \cdot \sqrt{41}} \Rightarrow \frac{-1}{\sqrt{82}} \Rightarrow \frac{-1}{\sqrt{82}}$

Q2:- Show that;

$i \cdot i = j \cdot j = 1$

As $i = |i| \cos \theta$

Since $\cos \theta = 0 = 0$ so,

$\|i\| = 1$

$i \cdot i = \|i\| \|i\| \cos \theta$
 $= 1(1)$

$\therefore 1$ Ans. $\frac{1}{2}$ same as for $j \cdot j = 1$

Q3:- $u_1 = (1, 2)$, $u_2 = (0, 1)$, $u_3 = (-2, -4)$, $u_4 = (-2, 1)$, $u_5 = (2, 4)$
 $u_6 = (-6, 3)$

Q Sol:- For orthogonal: $\theta = 90^\circ$

$u \cdot v = \|u\| \|v\| \cos \theta \Rightarrow \boxed{u \cdot v = 0} \rightarrow \text{Condition}$

$u_1 \cdot u_4 = (-2 + 2) = 0$. u_1 & u_4 are orthogonal.

$u_1 \cdot u_6 = -6 + 6 = 0$

$\sqrt{5} \cdot \sqrt{20} = \sqrt{100} = 10$
 $\sqrt{5} \cdot \sqrt{1} = \sqrt{5}$

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⑥ In same direction: $\theta = 0^\circ$

$$U \cdot V = \|U\| \|V\| \cos \theta$$

$$U \cdot V = \|U\| \|V\| \rightarrow \text{conditions}$$

$$U_1 \text{ \& } U_5 \Rightarrow U_1 \cdot U_5 = 2 + 8 = 10$$

$$\|U_1\| \|U_5\| = \sqrt{5} \cdot \sqrt{20} = 10$$

> same direction

⑦ For opposite direction = ?

$$U \cdot V = -\|U\| \|V\|$$

for $\theta = 180^\circ$

Qas:- $U = (a, 4)$ { $V = (2, 5)$ } // $a = ?$

Sol:- As Parallel i.e. $\theta = 0^\circ$ or 180°

$$\cos U \cdot V = \pm \|U\| \|V\|$$

$$2a + 20 = \sqrt{a^2 + 16} \cdot \sqrt{4 + 25}$$

$$2a + 20 = \sqrt{a^2 + 16} \cdot \sqrt{29}$$

$$2a + 20 = \sqrt{(a^2 + 16)29}$$

$$(2a + 20)^2 = (a^2 + 16)29$$

$$29a^2 - 4a + 4$$

$$(2a+20) = (a^2+16)29$$

$$\frac{4}{8.0}$$

⑧

$$\begin{array}{r} 5 \\ 16 \\ 25 \\ \hline 144 \\ 32x \\ \hline 5284 \\ 524 \\ \hline 464 \end{array}$$

$$4a^2 + 400 + 80a = 29a^2 + 464$$

$$29a^2 - 4a^2 + 464 - 400 - 80a = 0$$

$$25a^2 - 80a + 64 = 0$$

$$a = -125$$

$$a = \frac{4}{5}$$

$$a = \pm \frac{4}{5} \Rightarrow \text{Ans}$$

$$4a^2 + 400 + 80a = 29a^2 + 464$$

$$29a^2 - 4a^2 + 464 - 400 - 80a = 0$$

$$25a^2 - 80a + 64 = 0$$

$$(a, 2) \quad (a, -2)$$

sol: for orthogonal;

$$U \cdot V = \|U\| \|V\| \cos 90^\circ$$

$$U \cdot V = 0$$

$$a^2 - 4 = 0$$

$$\Rightarrow a = \pm 2 \Rightarrow \text{Ans}$$

$$[3 \quad -2]$$

Q30

Q25:-

$$(1, 4) - (2, 5)$$

Sol:- For parallel $\theta = 0^\circ$

$$u \cdot v = ||u|| ||v|| \rightarrow \textcircled{1}$$

$$2a + 20 = \sqrt{a^2 + 16} \cdot \sqrt{4 + 25}$$

$$2a + 20 = \sqrt{a^2 + 16} \cdot \sqrt{29}$$

$$2\sqrt{(2a + 20)} = (\sqrt{29(a^2 + 16)})^2$$

$$(2a + 20)^2 = 29a^2 + 384$$

$$4a^2 + 40a + 80a - 24a^2 - 384 = 0$$

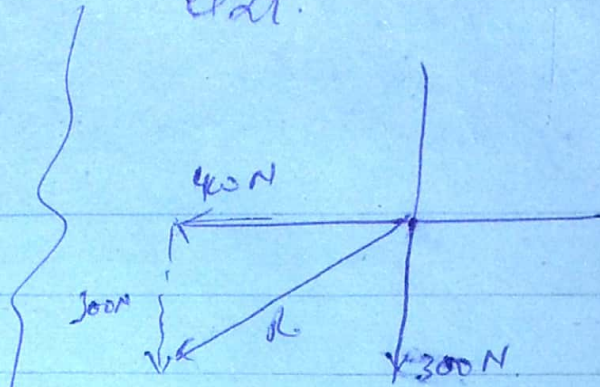
$$24a^2 - 4a^2 + 384 - 40a - 80a = 0$$

$$20a^2 - 16 - 80a = 0$$

$$5a^2 - 4 - 20a = 0$$

$$5a^2 - 20a - 4 = 0$$

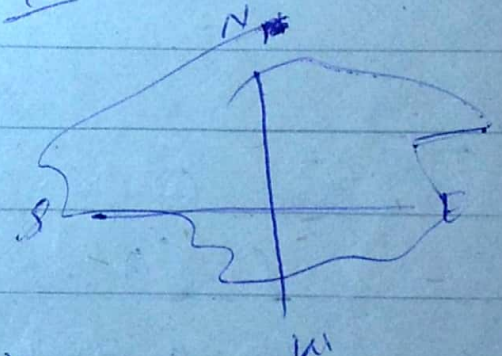
Q29



$$R^2 = (400)^2 + (300)^2$$

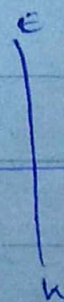
$$= 160000 + 90000$$

$$\begin{array}{r} 29 \\ 16 \\ \hline \end{array}$$



$$\begin{array}{r} 24 \\ 12 \\ \hline 144 \\ 24 \times \\ \hline 384 \end{array}$$

$$384$$



Cauchy Schwarz Inequality

$$|u \cdot v| \leq \|u\| \|v\|$$

Left side absolute & real no.
Right side denotes length & square

Triangle Inequality

$$\|u+v\| \leq \|u\| + \|v\|$$

It means length of any side of triangle not exceed than sum of other sides (length).

Ex# 4.2

Q3:- (a) $u+x=v$. Find $a, b \in \mathbb{C}$.

Soln

$$\begin{bmatrix} a \\ -1 \\ b \end{bmatrix} + \begin{bmatrix} 3 \\ c \\ 2 \end{bmatrix} = \begin{bmatrix} -3 \\ -1 \\ 3 \end{bmatrix}$$

$$\Rightarrow a+3 = -3 \Rightarrow a = -6$$

$$c-1 = -1$$

$$b+2 = 3$$

$$c = 0$$

$$b = 1$$

Q4:- (b) $u+x=v$. Find $a, b, c \in \mathbb{C}$.

$$\begin{cases} (a, -3, -6, b) + (2, c, d, 4) = (4, -1, -2, 3) \end{cases}$$

$$a+2 = 4 \Rightarrow a = 2$$

$$-3+c = -1$$

$$c = 2$$

$$d-6 = -2 \Rightarrow d = 4$$

$$b+4 = 3$$

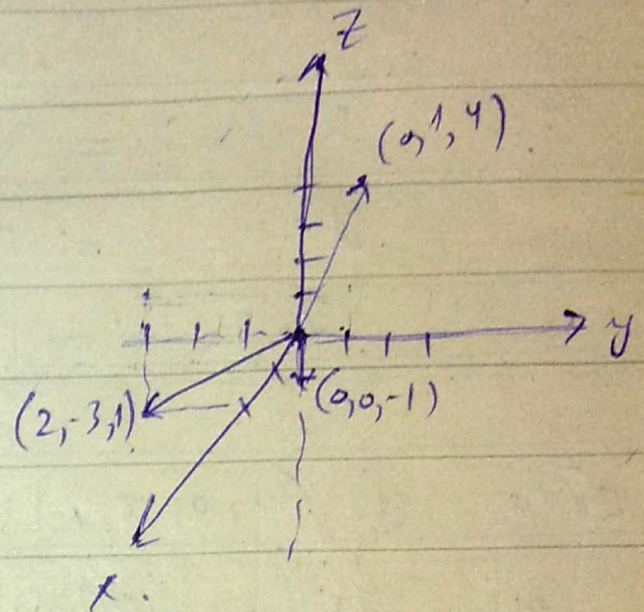
$$\Rightarrow b = -1$$

So, $(a, b, c, d) = (2, -1, 2, 4)$ Ans

Q:- Q7:- ⑥ $(2, -3, 1)$

⑤ $(0, 1, 4)$

⑦ $(0, 0, -1)$



Q:- ⑧ $(1, 1, 3) \{ (0, 0, 1) \}$

Sol:- Let $U = (1, 1, 3)$ is tail & $V = (0, 0, 1)$ is head
of a vector $W(x, y, z)$. So;

$$W = UV = (-1, -1, -2)$$

$$\text{or } W = UV = (1, 1, 2) \quad \underline{\text{Ans}}$$

Q:- Determined head & $(3, 4, -1)$ Tail $(1, -2, 3)$.

Sol:- Let $P_1(x, y, z)$ be tail $P_2 = \text{head}$ $P_1 P_2 = U$

of vector ~~$(3, 4, -1)$~~ $(2, 1, 2)$ whose

$$(x-1, y+2, z-3) = (1, -2, 3)$$

Tail ~~$(1, 0)$~~ vector $(3, 4, -1)$

$$(x=2), (y=-4), (z=6)$$

whose tail is $(1, -2, 3)$ so head is $(2, -4, 6)$ Ans

Q₂: Vector, $P_1 = (3, 4, -1)$

$$\text{tail} = (1, -2, 3)$$

$$\text{head} = ?$$

Sol:- Let $P_2(x, y, z)$ be head of a vector $(3, 4, -1)$ whose tail is $(1, -2, 3)$ Then;

$$P_1 = P_2 - T \Rightarrow (3, 4, -1) = (x-1, y+2, z-3)$$

$$x-1=3 \Rightarrow x=4$$

$$y+2=4 \Rightarrow y=2$$

$$z-3=-1 \Rightarrow z=2$$

So the head is

$$(4, 2, 2) \text{ Ans}$$

Q₁₀: (a) $(0, 0, 3, 4)$. Let $v = (0, 0, 3, 4)$

$$\|v\| = \sqrt{(0)^2 + (0)^2 + (3)^2 + (4)^2}$$

$$= 5 \text{ Am.}$$

Q₁₁: Find the distance?

$$(0, 0, 2), (-3, 0, 0)$$

$$\|P_1 - P_2\| = \sqrt{(-3)^2 + (2)^2} = \sqrt{9+4} = \sqrt{13} \text{ Am.}$$

$$(4, 2, -1, 6), (4, 3, 1, 5)$$

$$\text{Sol:- Let } P = (4, 2, -1, 6) \text{ \& } P_1 = (4, 3, 1, 5)$$

$$\|P - P_1\| = \sqrt{}$$

$$\|P - P_1\| = \sqrt{(1)^2 + (2)^2 + (-1)^2} = \sqrt{6} \text{ Am.}$$

Q14:-

Sol:-

Suppose $(2, -2, 3)$ is L.C of vectors $(1, 2, -3)$, $(-1, 1, 1)$ & $(-1, 4, -1)$ then exist c_1, c_2, c_3 such that

$$c_1(-1, 1, 1) + c_2(2, -2, 3) + c_3(1, 2, -3) = (2, -2, 3)$$

$$c_1 - c_2 - c_3 = 2$$

$$+ 2c_1 + c_2 + 4c_3 = -2$$

$$-3c_1 + c_2 - c_3 = 3$$

$$R \rightarrow \left[\begin{array}{ccc|c} 1 & -1 & -1 & 2 \\ 2 & 1 & 4 & -2 \\ -3 & 1 & -1 & 3 \end{array} \right]$$

$$R \rightarrow \left[\begin{array}{ccc|c} 1 & -1 & -1 & 2 \\ 0 & 3 & 6 & -6 \\ 0 & -2 & -4 & 9 \end{array} \right] \begin{array}{l} R_2 - 2R_1 \\ R_3 + 3R_1 \end{array} \Rightarrow \left[\begin{array}{ccc|c} 1 & -1 & -1 & 2 \\ 0 & 1 & 2 & -2 \\ 0 & -2 & -4 & 9 \end{array} \right] \begin{array}{l} \\ \\ \frac{1}{3} \times R_3 \end{array}$$

$$R \rightarrow \left[\begin{array}{ccc|c} 1 & -1 & -1 & 2 \\ 0 & 1 & 2 & -2 \\ 0 & 0 & 0 & 5 \end{array} \right] \begin{array}{l} \\ \\ R_3 + 2R_2 \\ R_3 + 2R_2 \end{array}$$

As $0 \neq 5$.

Since the system is not consistent so,

$(2, -2, 3)$ is not L.C of $(-1, 1, 1), (-1, 4, -1), (1, 2, -3)$.

$$Q_{15}:- c_1 \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} + c_3 \begin{bmatrix} 3 \\ 7 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{aligned} c_1 + c_2 + 3c_3 &= 0 \\ 2c_1 + 3c_2 + 7c_3 &= 0 \\ -c_1 - 2c_2 + c_3 &= 0 \end{aligned} \Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 3 & 0 \\ 2 & 3 & 7 & 0 \\ -1 & -2 & 1 & 0 \end{array} \right]$$

$$Q \quad \left[\begin{array}{ccc|c} 1 & 1 & 3 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & -1 & 4 & 0 \end{array} \right] \begin{array}{l} R_2 - 2R_1 \\ R_3 + R_1 \end{array} \Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 3 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 5 & 0 \end{array} \right] \begin{array}{l} \\ \\ R_3 + R_2 \end{array}$$

$$5x = 0 \Rightarrow x = 0$$

$y = 0$, $z = 0$ All zero so not possible to find any one have real root.

$$Q_{16}:- \sqrt{(1)^2 + (a)^2 + (3)^2 + (2)^2} = 5 \Rightarrow 1 + a^2 + 9 + 4 = 25$$

$$a^2 = 15 \Rightarrow a = \pm \sqrt{15}$$

$$Q_{17}:- a^2 - 2 - 2 - 3a = 0$$

$$a^2 - 3a - 4 = 0 \Rightarrow a^2 - 4a + a - 4 = 0$$

$$a(a-4) + (a-4) \Rightarrow (a-4)(a+1) = 0, a = -1, 4 \text{ Am}$$

Q₂₁:- Find Cosine of Angle:-

$$(c) \quad u = (2, 0, 1); v = (2, 2, -1)$$

$$\cos \theta = \frac{u \cdot v}{\|u\| \|v\|} = \frac{3}{3\sqrt{5}} \Rightarrow \cos \theta = \frac{1}{\sqrt{5}}$$

$$u \cdot v = 4 - 1 = 3$$

$$\|u\| = \sqrt{5}$$

$$\|v\| = \sqrt{9} \Rightarrow 3$$

(b) $U = (1, 2, -1, 3)$, $V = (0, 0, -1, -2)$

Sol:- $\cos \theta = \frac{U \cdot V}{\|U\| \|V\|}$

$U \cdot V = 1 - 6 \Rightarrow -5$

$\|U\| = \sqrt{15}$, $\|V\| = \sqrt{5}$

$= \frac{-5}{\sqrt{15} \cdot \sqrt{5}} \Rightarrow \frac{-5}{\sqrt{75}} \Rightarrow \frac{-5}{5\sqrt{3}} = \frac{-1}{\sqrt{3}}$

Q23:-

Sol:- for orthogonal:- $\theta = 90^\circ$

$\cos \theta = \frac{U \cdot V}{\|U\| \|V\|} \Rightarrow U \cdot V = \frac{0}{\|U\| \|V\|} \Rightarrow U \cdot V = 0$

$U_1 \& U_2 \Rightarrow U \cdot V = -8 + 6 - 6 + 8 = 0$, $U_1 \& U_2$ orthogonal.

$U_1 \& U_6 \Rightarrow U \cdot V = -6 + 6 = 0$

$U_2 \& U_3 \Rightarrow U \cdot V = 4 - 3 + 3 - 4 = 0$

$U_3 \& U_6 \Rightarrow U \cdot V = 3 - 3 = 0$

For parallel:- $\theta = 0^\circ$ or 180°

$$U \cdot V = \pm \|U\| \|V\| \rightarrow (1)$$

$$\begin{array}{r} 120 \\ 30 \\ \hline 000 \\ 360 \times \\ \hline 3600 \end{array}$$

$$U_1 \cdot U_3 = (4, 2, 6, -8) \cdot (-2, -1, -3, 4)$$

$$= -8 - 2 - 18 - 32$$

$$= -60$$

$$\|U\| \|V\| = \sqrt{16+4+36+64} \cdot \sqrt{4+1+9+16}$$

$$= \sqrt{120} \sqrt{30} \Rightarrow \sqrt{3600}$$

$$\|U\| \|V\| = 3600 \Rightarrow U \cdot V$$

For same direction:-

$$\theta = 0^\circ$$

Not found.

$$\textcircled{24} \quad V = (2, 6, 3), \quad V = (1, -2, 1) -$$

$$2 - 2c + 3 = 0$$

$$c = 5/2$$

Q25:- 3- possible find $V = (a, b, c)$.

~~30~~

$$\begin{aligned} a + 2b + c &= 0 \rightarrow \textcircled{1} \\ a - b + c &= 0 \rightarrow \textcircled{2} \end{aligned}$$

subtract each eqn from other

$$\begin{array}{r} a + 2b + c = 0 \\ + \quad a - b + c = 0 \\ \hline 3b = 0 \Rightarrow b = 0 \end{array}$$

eq ① $a + c = 0$

let $c = r$ where $r \in \mathbb{R}$.

$$\boxed{a = -c} \Rightarrow \boxed{a = -r}$$

So;

$$V = (-r, 0, r) \quad \text{for } r=1$$

$$V = (-1, 0, 1) \quad \underline{\text{Ans}}$$

Q26:- $u = (1, 2, 3, -1)$ & $v = (1, 0, -2, 3)$

Sol:- A triangle inequality is;

$$\|u + v\| \leq \|u\| + \|v\| \rightarrow \textcircled{1}$$

$$\|2+2+1+2\| \leq \sqrt{15} + \sqrt{14}$$

$$\sqrt{4+4+1+4}$$

$$\Rightarrow \sqrt{17} \leq$$

$$\Rightarrow 4.12 < 3.87 + 3.74$$

$$4.12 < 7.61 \quad \text{proved}$$

Q27:- Find unit vector.
⑥ $(1, 2, 3, 4)$.

Sol:- As $\hat{x} = \frac{x}{\|x\|} \rightarrow \text{①}$

$$\|x\| = \sqrt{1+4+9+16} \Rightarrow \sqrt{30} \Rightarrow$$

$$\hat{x} = \left(\frac{1}{\sqrt{30}}, \frac{2}{\sqrt{30}}, \frac{3}{\sqrt{30}}, \frac{4}{\sqrt{30}} \right) \underline{\text{Ans.}}$$

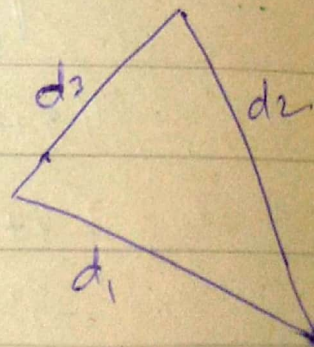
Q31:- Show $P_1(2, 3, -4)$, $P_2(3, 1, 2)$ & $P_3(-3, 0, 4)$ are vertices of isosceles Δ .

Sol:- In isosceles Δ , two sides are of same length & one of other.

$$d_1 = \|P_1 - P_2\| = \sqrt{(1)^2 + (-2)^2 + (6)^2} \Rightarrow \sqrt{1+4+36} \Rightarrow \sqrt{41}$$

$$d_2 = \|P_2 - P_3\| = \sqrt{36 + 1 + 4} \Rightarrow \sqrt{41}$$

$$d_3 = \|P_1 - P_3\| = \sqrt{25 + 9 + 64} \Rightarrow \sqrt{98}$$



As $d_2 = d_1$ while d_3 is different. So it is proved they are vertices of isosceles Δ .

Q32:-

Sol:- To show $P_1(2, 3, -4)$, $P_2(3, 1, 2)$ & $P_3(7, 0, 1)$
is right Δ .

In ~~the~~ right angle Δ

$$(\text{Hyp})^2 = (\text{Base})^2 + (\text{Perp})^2.$$

$$\text{Perp} = \|P_1 - P_2\| = \sqrt{1 + 4 + 36} = \sqrt{41}.$$

$$\text{Base} = \|P_2 - P_3\| = \sqrt{16 + 1 + 1} = \sqrt{18}.$$

$$\text{Hyp} = \|P_1 - P_3\| = \sqrt{25 + 9 + 25} = \sqrt{59}.$$

$$(\text{Hyp})^2 = (\text{Base})^2 + (\text{Perp})^2$$

$$(\sqrt{59})^2 = (\sqrt{18})^2 + (\sqrt{41})^2$$

$$59 = 18 + 41,$$

$$\boxed{59 = 59} \quad \text{proved}$$

Ex 4.3

Linear Transformation:-

A linear transformation is a function $L: \mathbb{R}^n \rightarrow \mathbb{R}^m$ assigning a unique vector $L(u)$ in $\mathbb{R}^m$ to each vector u in $\mathbb{R}^n$ such that

$$(a) \quad L(u+v) = L(u) + L(v)$$

$$(b) \quad L(ku) = kL(u)$$

~~For~~ We can also write above two equations as
 $L(k_1 u_1 + k_2 u_2) = k_1 L(u_1) + k_2 L(u_2)$

$$L(c_1 u_1 + c_2 u_2 + \dots + c_k u_k) = c_1 L(u_1) + c_2 L(u_2) + \dots + c_k L(u_k)$$

Ex # 4.3

$$Q1. (a) \quad L(x, y) = (x+1, y, x+y)$$

Sol:- See copy. (Sorry)

$L: \mathbb{R}^2 \rightarrow \mathbb{R}^3$ defined by

$$L(x, y) = (x+1, y, x+y)$$

To show L is linear transformation

The following two conditions satisfied

(i) $L(u+v) =$

for any $u = (x_1, y_1)$ & $v = (x_2, y_2)$ in $\mathbb{R}^2$,
we have

$$L(u+v) = L(u) + L(v) \rightarrow (1)$$

$$u+v = (x_1+x_2, y_1+y_2)$$

$$L(u+v) = (x_1+x_2+1, y_1+y_2, x_1+x_2+y_1+y_2)$$

Now R.H.S.

$$L(u) = (x_1+1, y_1, x_1+y_1)$$

$$L(v) = (x_2+1, y_2, x_2+y_2)$$

$$L(u) + L(v) = (x_1+x_2+2, y_1+y_2, x_1+y_1+x_2+y_2)$$

Since L.H.S $\neq$ R.H.S So L is not
Linear transformation.

Q:- ⑥ $L\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} 2x - 3y \\ 3y - 2z \\ 2z \end{bmatrix}$

$L: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ defined by;

$$L\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} 2x - 3y \\ 3y - 2z \\ 2z \end{bmatrix}$$

L is lin Trans if

① For any $u = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}$ & $v = \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}$ in $\mathbb{R}^3$ we have

$$L(u+v) = L(u) + L(v) \rightarrow \text{①}$$

$$u+v = \begin{bmatrix} x_1+x_2 \\ y_1+y_2 \\ z_1+z_2 \end{bmatrix}$$

$$L(u+v) = \begin{bmatrix} 2(x_1+x_2) - 3(y_1+y_2) \\ 3(y_1+y_2) - 2(z_1+z_2) \\ 2(z_1+z_2) \end{bmatrix}$$

$$L(u) + L(v) = \begin{bmatrix} 2x_1 - 3y_1 \\ 3y_1 - 2z_1 \\ 2z_1 \end{bmatrix} + \begin{bmatrix} 2x_2 - 3y_2 \\ 3y_2 - 2z_2 \\ 2z_2 \end{bmatrix}$$

$$= \begin{bmatrix} 2(x_1+x_2) - 3(y_1+y_2) \\ 3(y_1+y_2) - 2(z_1+z_2) \\ 2(z_1+z_2) \end{bmatrix}$$

Since $L(u+v) = L(u) + L(v)$ So not L.T.

① For $u = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3$ then

$$L(k(u)) = L(k(u)) = k \cdot L(u)$$

$$L\left(\begin{pmatrix} kx \\ ky \\ kz \end{pmatrix}\right) = k L\left(\begin{pmatrix} x \\ y \\ z \end{pmatrix}\right)$$

Proved L.H.S = R.H.S So L.T.

Ex:- ① $L(x, y) = (x-y, 0, 2x+3)$.

Sol:- $L: \mathbb{R}^2 \rightarrow \mathbb{R}^3$ defined by;

$$L(x, y) = (x-y, 0, 2x+3)$$

L is L.T if

② For any $u = (x_1, y_1) \in \mathbb{R}^2$ and $v = (x_2, y_2) \in \mathbb{R}^2$ Then,

$$L(u+v) = L(u) + L(v) \rightarrow (1)$$

$$L(u+v) = L(x_1+x_2, y_1+y_2)$$

$$= ((x_1+x_2) - (y_1+y_2), 0, 2(x_1+x_2)+3)$$

$$\mathbb{R} \cdot H \cdot S = \mathcal{L}(u) + \mathcal{L}(v)$$

$$\mathcal{L}(u) + \mathcal{L}(v) = (x_1 - y_1, 0, 2x_1 + 3) + (yx_2 - y_2, 0, 2x_2 + 3)$$

$$= (x_1 + x_2 - (y_1 + y_2), 0, 2(x_1 + x_2) + 6)$$

$\mathcal{L} \cdot H \cdot S \neq \mathbb{R} \cdot H \cdot S$ So not Lin Transformation

$$Q4: \textcircled{b} \quad \mathcal{L}\left(\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix}\right) = \begin{bmatrix} u_1 \\ u_1^2 + u_2 \\ u_1 - u_3 \end{bmatrix}$$

sol:- $\mathcal{L}: \mathbb{R}^4 \rightarrow \mathbb{R}^3$ defined by

$\mathcal{L}$ is L.T if

$$\textcircled{a} \text{ For any } u = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} \quad \{ \quad v = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} \text{ in } \mathbb{R}^4 \text{ then}$$

$$\mathcal{L}(u+v) = \mathcal{L}(u) + \mathcal{L}(v)$$

$$u+v = \begin{bmatrix} u_1+v_1 \\ u_2+v_2 \\ u_3+v_3 \\ u_4+v_4 \end{bmatrix}$$

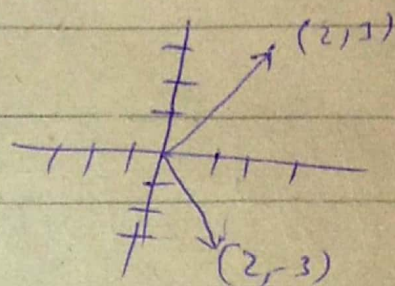
$$\mathcal{L}(u+v) = \begin{bmatrix} u_1+v_1 \\ (u_1+v_1)^2 + u_2+v_2 \\ u_1+v_1 - (u_3+v_3) \end{bmatrix}$$

$$\text{Now } \mathcal{L}(u) + \mathcal{L}(v) = \begin{bmatrix} u_1 \\ u_1^2 + v_1 \\ u_1 - v_3 \end{bmatrix} + \begin{bmatrix} v_1 \\ v_1^2 + v_2 \\ v_1 - v_3 \end{bmatrix} = \begin{bmatrix} u_1 + v_1 \\ u_1^2 + v_1^2 + v_1 + v_2 \\ u_1 + v_1 - (u_3 + v_3) \end{bmatrix}$$

L.H.S $\neq$ R.H.S so not L.T

Q5:- $L: \mathbb{R} \rightarrow \mathbb{R}^2$ defined by;
 $L(x, y) = (x, -y) \quad : \quad P = (2, 3)$

Sol:- Given That;
 $L(x, y) = (x, -y)$
 $L(2, 3) = (2, -3)$



Q4:- (c) $L\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

Sol:- $L: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ defined by $L\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

L is L.T if

(a) $L(u+v) = L(u) + L(v)$

$u = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}, \quad v = \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}$ in $\mathbb{R}^3$ then

$u+v = \begin{bmatrix} x_1+x_2 \\ y_1+y_2 \\ z_1+z_2 \end{bmatrix} \Rightarrow L(u+v) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

According to property 4.1 $T(\mathbb{R}^n) = \mathbb{R}^n$

$L(u) + L(v) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ (proved)

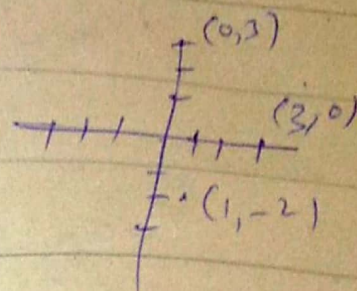
L.H.S = R.H.S

Q6:- $L: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is defined by;

$$L\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x-y \\ 2x+y \end{bmatrix} \quad \text{where } U = (1, -2)$$

Sol:-

$$L\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x-y \\ 2x+y \end{bmatrix}$$



$$L\left(\begin{bmatrix} 1 \\ -2 \end{bmatrix}\right) = \begin{bmatrix} 1 - (-2) \\ 2(1) + (-2) \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$$

Q8:- Counter clock wise rotation through $\frac{2}{3}\pi$.
 $U = (-2, -3)$

Sol:- As Counter Clock wise

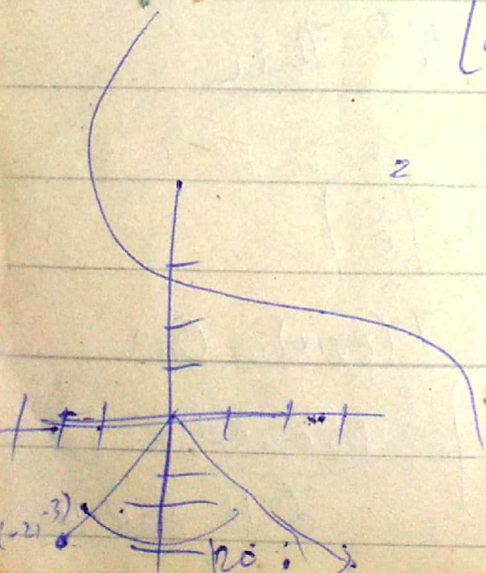
So; $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \quad \because \theta = \frac{2}{3}\pi$

$\frac{2}{3}(180^\circ)$
 $= 120^\circ$

$$A = \begin{bmatrix} -0.5 & -0.866 \\ 0.866 & 0.5 \end{bmatrix} \begin{bmatrix} -2 \\ -3 \end{bmatrix}$$

$$= \begin{bmatrix} -0.5(-2) - (0.866)(-3) \\ 0.866(-2) + 0.5(-3) \end{bmatrix} = \begin{bmatrix} 1 + 2.598 \\ -1.732 - 1.5 \end{bmatrix}$$

$$= \begin{bmatrix} 3.5 \\ -3.232 \end{bmatrix}$$



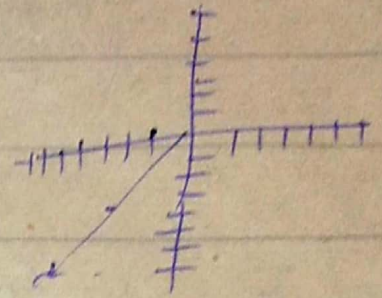
Q10:- $L: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ $Lu = 2u$; $u = (-3, 3)$

Sol:- Given $L: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ defined as;

$$L: \mathbb{R}^2 \rightarrow \mathbb{R}^2 \quad L(u) = 2u$$

$$u = (-3, 3)$$

$$L(u) = 2(-3, 3) = (-6, 6)$$



Q11:- $L: \mathbb{R}^3 \rightarrow \mathbb{R}^2$ defined by;

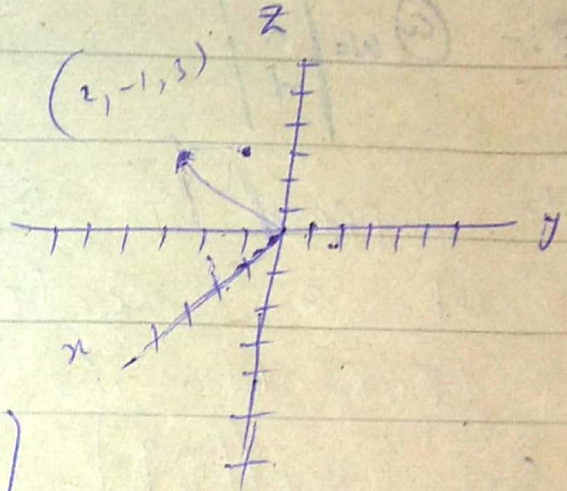
$$L\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x \\ x-y \end{bmatrix} ; u = (2, -1, 3)$$

Sol:- Given that, $L: \mathbb{R}^3 \rightarrow \mathbb{R}^2$ defined as;

$$L\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x \\ x-y \end{bmatrix}$$

For $u = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$

$$L(u) = L\left(\begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

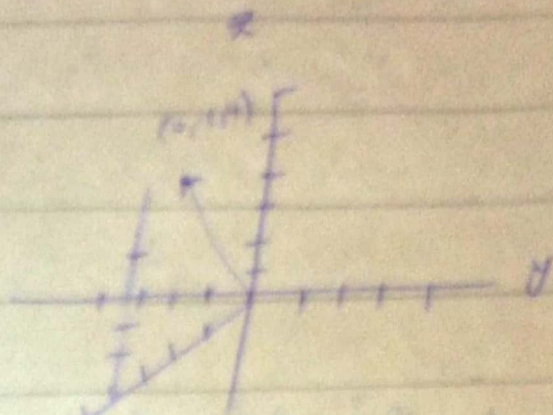


Q12:- $L\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ $\mathbb{R}^3 \rightarrow \mathbb{R}^3$

$U = (0, 2, 4)$

Sol:- $L: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ defined as

$$L\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x+z \\ -x+y \\ z \end{bmatrix}$$



for $U = (0, 2, 4)$

$$L\left(\begin{bmatrix} 0 \\ 2 \\ 4 \end{bmatrix}\right) = \begin{bmatrix} 4 \\ -2 \\ 4 \end{bmatrix}$$

Q14
Q1:- (a) $W = \left\{ \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix} \right\}$

$$\Rightarrow L\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} -x+2y \\ x+y+z \\ 2x-y+z \end{bmatrix}$$

①, ②

Sol:- Suppose W is in range of L if then exist a vector say $U = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ such that,

$$L(U) = W \quad \text{--- (1)}$$

from ① & ②

So, $L\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$

$$= \begin{bmatrix} -x+2y \\ x+y+z \\ 2x-y+z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

$$-x + 2y = 1$$

$$x + y + z = 2$$

$$2x - y + z = -1$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ -1 & 2 & 0 & 2 \\ 2 & -1 & 1 & -1 \end{array} \right]$$

$$R_2 \left[\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 0 & 3 & 1 & 3 \\ 0 & -3 & -1 & -5 \end{array} \right] \begin{array}{l} R_2 + R_1 \\ R_3 - 2R_1 \end{array}$$

$$R_2 \left[\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 0 & 3 & 1 & 3 \\ 0 & 0 & 0 & -2 \end{array} \right] R_3 + R_2$$

$$\text{Since } 0 \neq -2$$

Thus as system is not consistent
So W is not in range of L .

Q11:- $L\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x+z \\ y+z \\ x+2y+2z \end{bmatrix}$ ~~$L\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x+z \\ y+z \\ x+2y+2z \end{bmatrix}$~~

Is W in range L .

(b) $W = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$

Sol:- Suppose W in range of L ~~defined as~~;
if there exist a vector say $U = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ such
that; $L(U) = W \rightarrow \text{①}$.

$$L(U) = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} \rightarrow \text{①} \Rightarrow L\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} \rightarrow \text{①}$$

But $L\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x+z \\ y+z \\ x+2y+2z \end{bmatrix} \rightarrow \text{②}$

From eq ① & ②.

$$\begin{bmatrix} x+z \\ y+z \\ x+2y+2z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$$

$$x+z=2$$

$$y+z=-1$$

$$x+2y+2z=3$$

$$x+2(y+z)=3$$

$$x+2(-1)=3$$

$$x=5$$

$$x+z=2$$

$$z=-3$$

$$y+z=-1$$

$$y=2$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 1 & 1 & -1 \\ 1 & 2 & 2 & 3 \end{array} \right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 1 & 1 & -1 \\ 0 & 2 & 1 & 1 \end{array} \right] R_3 - R_1$$

$$L \left[\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & -1 & 3 \end{array} \right] \text{ e.g. } 2L_1$$

As consistent
so W is
in range of
 L .

$$-z = 3 \Rightarrow z = -3$$

$$y + z = -1$$

$$y = -2$$

$$x + z = 2$$

$$x = 0$$

$$U = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ -2 \\ -3 \end{pmatrix}$$

Justification:-

$$5 + 4 - 6 = 3$$

$$L(U) = W$$

$$\begin{bmatrix} -3 \\ -1 \end{bmatrix} \begin{pmatrix} x+z \\ y+z \end{pmatrix} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$$

Prove

Q14:-
 $\Rightarrow$ ⑥ $W = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$.

Sol:- Given That,

$$L\left(\begin{pmatrix} x \\ y \\ z \end{pmatrix}\right) = \begin{bmatrix} -1 & 2 & 0 \\ 1 & 1 & 1 \\ 2 & -1 & 1 \end{bmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$L\left(\begin{pmatrix} x \\ y \\ z \end{pmatrix}\right) = \begin{bmatrix} -x+2y \\ x+y+z \\ 2x-y+z \end{bmatrix} \rightarrow \textcircled{1}$$

Suppose W is in range of L Then there exist a vector say $U = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ such that,

$$L(U) = W \rightarrow \textcircled{2}$$

$$L\left(\begin{pmatrix} x \\ y \\ z \end{pmatrix}\right) = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} \rightarrow \textcircled{2}$$

$$\begin{bmatrix} -x+2y \\ x+y+z \\ 2x-y+z \end{bmatrix} = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ -1 & 2 & 0 & 1 \\ 2 & -1 & 1 & 2 \end{array} \right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 3 & 1 & 4 \\ 0 & -3 & -1 & -4 \end{array} \right] \begin{array}{l} \\ R_2+R_1 \\ R_3-2R_1 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 3 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{array} \right] R_3+R_2$$

As Infinite case.

So let $z = r$ where $r \in \mathbb{R}$.

$$3y + r = 4 \Rightarrow y = \frac{4-r}{3}$$

$$x + \frac{(4-r)}{3} + r = 3$$

$$x + \frac{4-r+3r}{3} = 3 \Rightarrow x + \frac{2r+4}{3} = 3$$

$$x = \frac{9-2r+4}{3} \Rightarrow x = \frac{13-2r}{3}$$

So, $U = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \frac{13-2r}{3} \\ 4-r \\ 3 \end{pmatrix}$ For $r = 1$.

$$U = \begin{pmatrix} 11/3 \\ 3 \\ 3 \end{pmatrix} \text{ is sol.}$$

As consistent so W is in range of L .

Q14:- (b) $W = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$.

Sol:- As $L(U) = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{bmatrix} -x+2y \\ x+y+z \\ 2x-y+z \end{bmatrix} \rightarrow \textcircled{1}$

Suppose W is in range of L - then there exist a vector say $U = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ such that.

$$L(U) = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = W \rightarrow \textcircled{2}$$

By comparing

$$-x + 2y = 1 \rightarrow (1)$$

$$x + y + z = 3 \rightarrow (2)$$

$$2x - y + z = 2 \rightarrow (3)$$

The augment matrix is;

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ -1 & 2 & 0 & 1 \\ 2 & -1 & 1 & 2 \end{array} \right]$$

Apply elementary row operations.

$$R_2 \rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 3 & 1 & 4 \\ 0 & -3 & 1 & -4 \end{array} \right] \begin{array}{l} R_2 + R_1 \\ R_3 - 2R_1 \end{array}$$

$$R_3 \rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 3 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{array} \right] R_3 + R_2$$

Since $0=0$ so it is case of infinite many solution.

$$\text{eq (1)} \Rightarrow x + 2y = 1 \quad \text{let } \boxed{y = r} \text{ where } r \in \mathbb{R}.$$

$$x + 2r = 1$$

$$\boxed{x = 1 - 2r}$$

$$\text{eq (2)} \quad x + y + z = 3$$

$$1 - 2r + r + z = 3$$

$$\boxed{z = r + 2}$$

$$\text{So, } \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 - 2r \\ r \\ r + 2 \end{bmatrix} \quad \text{For } r = 1 \Rightarrow \begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix}$$

As system is consistent so it is in range.

Justification:-

$$u = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$L(u) = \begin{bmatrix} -x+2y \\ x+y+z \\ 2x-y+z \end{bmatrix} \Rightarrow \begin{bmatrix} x+2 \\ -1+1+3 \\ -2+1+3 \end{bmatrix} \Rightarrow \begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix}$$

~~~~~ //

Q16:-

Sol:- Suppose  $w$  is in range of  $L$  then there exist a vector say  $u = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$  such that,  
 $L(u) = w \Rightarrow L\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} a \\ b \\ c \end{bmatrix} \rightarrow (1)$

As given that,

$$L\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x+2y+3z \\ -3x-2y-z \\ -2x+2z \end{bmatrix} \rightarrow (2)$$

By

Comparing

$$\begin{bmatrix} x+2y+3z \\ -3x-2y-z \\ -2x+2z \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ -3 & -2 & -1 \\ -2 & 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

The augmented matrix is,



$$R_1 \left[ \begin{array}{ccc|c} 1 & 2 & 3 & a \\ -3 & -2 & -1 & b \\ -2 & 0 & 2 & c \end{array} \right]$$

$$R_2 \left[ \begin{array}{ccc|c} 1 & 2 & 3 & a \\ 0 & 4 & 8 & b+3a \\ 0 & 4 & 8 & c+2a \end{array} \right] \begin{array}{l} R_2 + 3R_1 \\ R_3 + 2R_1 \end{array}$$

$$R_3 \left[ \begin{array}{ccc|c} 1 & 2 & 3 & a \\ 0 & 4 & 8 & b+3a \\ 0 & 0 & 0 & c+2a-b-3a \end{array} \right] R_3 - R_2$$

Since  $0 = c + 2a - b - 3a$

So,  $-a - b + c = 0$ .

is desired eqn relating  $a, b$  &  $c$ .

Q17:- Let  $L: \mathbb{R}^2 \rightarrow \mathbb{R}^2$  be linear transformation such that,  
 $L(i) = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$  &  $L(j) = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$ .  
 Find  $L\left(\begin{bmatrix} 4 \\ -3 \end{bmatrix}\right)$ .

Sol:- As given that,

$$L\left(\begin{bmatrix} 4 \\ -3 \end{bmatrix}\right) = L(i) = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \quad \& \quad L(j) = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

Since,  $\begin{bmatrix} 4 \\ -3 \end{bmatrix} = 4 \begin{bmatrix} 1 \\ 0 \end{bmatrix} - 3 \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$$= 4\hat{i} - 3\hat{j}$$

$$\therefore L = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ -3 \end{bmatrix}$$

Now;  $L\begin{bmatrix} 4 \\ -3 \end{bmatrix} = L(4\hat{i} - 3\hat{j}) = 4L(\hat{i}) - 3L(\hat{j})$   
 $= 4\begin{bmatrix} 2 \\ 3 \end{bmatrix} - 3\begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 8+3 \\ 12-6 \end{bmatrix} = \begin{bmatrix} 11 \\ 6 \end{bmatrix}$  Any



$$L\left(\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}\right) = L\left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix}\right)$$

$$L\left(\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}\right) = L\left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right) + L\left(\begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix}\right)$$

$$L\left(\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}\right) = L\left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right) + L\left(\begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix}\right)$$

$$L\left(\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix}$$

$$L\left(\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}\right) = \begin{bmatrix} 1+0 \\ 2+2 \\ 3+3 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 6 \end{bmatrix}$$

$$L\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x & 0 & 1 \\ -x & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Ex:  $L: \mathbb{R}^3 \rightarrow \mathbb{R}^3$  defined as;

$$L\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x+z \\ -x+y \\ x \end{bmatrix}$$

According to condition  $L(x) = 0$

$$\begin{bmatrix} x+z \\ -x+y \\ x \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \boxed{x=0}, \boxed{z=0}, \boxed{y=0}$$



Q<sub>21</sub>:- ②  $L(x, y) = (-x, y)$

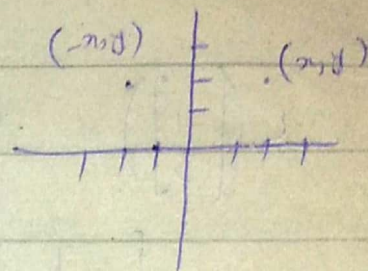
Sol:-  $L: \mathbb{R}^2 \rightarrow \mathbb{R}^2$  defined as;

$$L(x, y) = (-x, y)$$

Now for any  $v = (x, y)$  in  $\mathbb{R}^2$  we have;

$$L(x, y) = (-x, y)$$

So L.T. with respect to y-axis.



③  $L(x, y) = (-y, -x)$

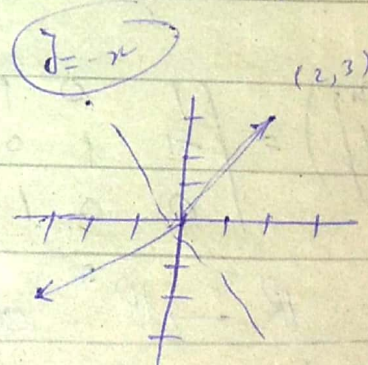
Sol:-  $L: \mathbb{R}^2 \rightarrow \mathbb{R}^2$  defined as;

$$L(x, y) = (-y, -x)$$

Now for  $v = (x, y) = (2, 3)$  - then; in  $\mathbb{R}^2$  we have;

$$L(2, 3) = (-3, -2)$$

So in fig.



So reflection w.r.t

$$y = -x$$



Q24:-  $L: \mathbb{R}^2 \rightarrow \mathbb{R}^1$  defined by  $L(x, y) = \sin x + \sin y$ .

Sol:-  $L: \mathbb{R}^2 \rightarrow \mathbb{R}^2$  defined as;

$$L(x, y) = \sin x + \sin y.$$

$L$  is L.T. if the following conditions satisfied

① For any  $u = (x_1, y_1) \in v = (x_2, y_2)$  in  $\mathbb{R}^2$   
then;

$$L(u+v) = L(u) + L(v).$$

$$u+v = (x_1+x_2, y_1+y_2)$$

$$L(u+v) = \sin(x_1+x_2) + \sin(y_1+y_2).$$

Now R.H.S:-  $L(u) + L(v) = \sin x_1 + \sin y_1 + \sin x_2 + \sin y_2$ .

Since L.H.S  $\neq$  R.H.S so not L.T.

Q:- Find the standard matrix representing  $L$ .

Q25:-  $L: \mathbb{R}^2 \rightarrow \mathbb{R}^2$  is reflection w.r.t  $y$ -axis.

Sol:- For any  $u = \begin{bmatrix} x \\ y \end{bmatrix}$  we have  $L(u) = L\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} -x \\ y \end{bmatrix}$

is reflection w.r.t  $y$ -axis.

The standard matrix of  $L$  is given as;

$$L = [L(e_1) \ L(e_2)] \rightarrow \text{①} \quad e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

$$L(e_1) = \begin{bmatrix} -1 \\ 0 \end{bmatrix}; L(e_2) = L\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

$\therefore \text{①} \Rightarrow L = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$  is the matrix (standard)  
for ref w.r.t  $y$ -axis



Q. 26:-  $L: \mathbb{R}^2 \rightarrow \mathbb{R}^2$  is defined by;

$$L\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x-y \\ x+y \end{bmatrix}$$

Sol:- For any  $v = \begin{bmatrix} x \\ y \end{bmatrix}$  we have  $L(v) = L\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x-y \\ x+y \end{bmatrix}$ .  
The standard matrix is given as;

$$A = [L(e_1) \quad L(e_2)] \rightarrow \text{①}$$

$$e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$L(e_1) = L\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$L(e_2) = L\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \text{ is the desired stand. Matrix.}$$

Q. 27:-  $L: \mathbb{R}^2 \rightarrow \mathbb{R}^2$  is count clock through  $\pi/4$ .

Sol:- For any  $v = \begin{bmatrix} x \\ y \end{bmatrix}$  we have

$$L\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \quad \because \theta = \pi/4$$

Stand matrix is given as;

$$A = [L(e_1) \quad L(e_2)] \rightarrow \text{①}$$

$$e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$L(e_1) = \begin{bmatrix} \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{2}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} 0 \\ \sqrt{2} \end{bmatrix}$$

$$L(e_2) = \begin{bmatrix} 0 - \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$$

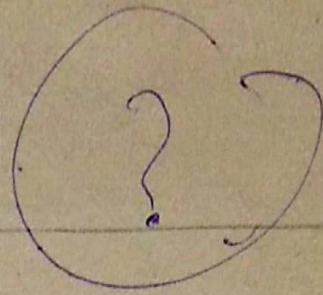
$$\therefore A = \begin{bmatrix} 0 & -\frac{1}{\sqrt{2}} \\ \sqrt{2} & \frac{1}{\sqrt{2}} \end{bmatrix}$$



Q27:- The standard matrix is;

Sol:-

$$L\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} \sin \theta & -\cos \theta \\ \cos \theta & \sin \theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$



$$A = \begin{bmatrix} \sin \pi/4 & -\cos \pi/4 \\ \cos \pi/4 & \sin \pi/4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \Rightarrow \begin{bmatrix} 1/\sqrt{2}x - 1/\sqrt{2}y \\ 1/\sqrt{2}x + 1/\sqrt{2}y \end{bmatrix}$$

---

Q27:-

Sol:-

The standard matrix is given as;

$$A = \begin{bmatrix} L(e_1) & L(e_2) & \cancel{L(e_3)} \end{bmatrix}$$

$$e_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \Rightarrow L(e_1) = \begin{bmatrix} 1/\sqrt{2} \\ 0 \\ 0 \end{bmatrix}$$

$$e_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \Rightarrow L(e_2) = \begin{bmatrix} -1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \text{ Ans.}$$

---



Q.10:-  $L: \mathbb{R}^3 \rightarrow \mathbb{R}^3$   $L(0) = 20$

Sol:- The standard matrix is given.

$$A = \begin{bmatrix} L(e_1) & L(e_2) & L(e_3) \end{bmatrix} \rightarrow \text{①}$$

$$e_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \Rightarrow L(e_1) = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} \text{ from q ②}$$

$$e_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \Rightarrow L(e_2) = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} \text{ from q ②}$$

$$e_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \Rightarrow L(e_3) = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} \text{ from q ②}$$

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \quad \underline{\text{Ans}}$$

①31:-  $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$

② Code the message Send him money.

Sol:- To code the message.

Send him money.

$$\begin{array}{ccccccc} 19 & 5 & 14 & 4 & 8 & 9 & 13 & 13 & 15 & 14 & 5 & 25 \\ \hline x_1 & & x_2 & & x_3 & & x_4 & & & & & \end{array}$$

$$AX_1 = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 19 \\ 5 \\ 14 \end{bmatrix} \Rightarrow \begin{bmatrix} 19+10+42 \\ 19+5+28 \\ 0+5+28 \end{bmatrix} = \begin{bmatrix} 71 \\ 52 \\ 33 \end{bmatrix}$$

$$AX_2 = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 8 \\ 9 \\ 13 \end{bmatrix} = \begin{bmatrix} 8+18+39 \\ 8+9+26 \\ 0+9+26 \end{bmatrix} = \begin{bmatrix} 65 \\ 43 \\ 35 \end{bmatrix}$$



$$AX_1 = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 13 \\ 13 \\ 15 \end{bmatrix}$$

$$= \begin{bmatrix} 13+26+45 \\ 13+13+30 \\ 0+13+30 \end{bmatrix} = \begin{bmatrix} 84 \\ 36 \\ 43 \end{bmatrix}$$

$$AX_4 = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 14 \\ 5 \\ 25 \end{bmatrix} \Rightarrow \begin{bmatrix} 14+10+75 \\ 14+5+50 \\ 0+5+50 \end{bmatrix} = \begin{bmatrix} 99 \\ 69 \\ 61 \end{bmatrix}$$

The code message is;

71 52 33 65 43 35 84 56 43 99 69 61

⑥ Decode the message;

67 44 91 49 39 19 113 76 62 109 65 55  
 $x_1$   $x_2$   $x_3$   $x_4$

Since  $A^{-1} = \frac{1}{\Delta} \begin{bmatrix} \dots \end{bmatrix}$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} \quad A^{-1} = \begin{bmatrix} 1 & 2 & 3 & | & 1 & 0 & 0 \\ 1 & 1 & 2 & | & 0 & 1 & 0 \\ 0 & 1 & 2 & | & 0 & 0 & 1 \end{bmatrix}$$

$$R_1 \begin{bmatrix} 1 & 2 & 3 & | & 1 & 0 & 0 \\ 0 & -1 & -1 & | & -1 & 1 & 0 \\ 0 & 1 & 2 & | & 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_3} \begin{bmatrix} 1 & 2 & 3 & | & 1 & 0 & 0 \\ 0 & 1 & 1 & | & 1 & -1 & 0 \\ 0 & -1 & -1 & | & -1 & 1 & 0 \end{bmatrix} \xrightarrow{-1 \times R_2}$$

$$R_1 \begin{bmatrix} 1 & 2 & 3 & | & 1 & 0 & 0 \\ 0 & 1 & 1 & | & 1 & -1 & 0 \\ 0 & 0 & 1 & | & 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_3 \leftrightarrow R_2} \begin{bmatrix} 1 & 2 & 3 & | & 1 & 0 & 0 \\ 0 & 1 & 0 & | & 2 & -2 & -1 \\ 0 & 0 & 1 & | & -1 & 1 & 1 \end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_3}$$



$$R_1 \left[ \begin{array}{ccc|ccc} 1 & 0 & 3 & -3 & 4 & 2 \\ 0 & 1 & 0 & 2 & -2 & 1 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array} \right] \quad R_1 - 2R_2$$

End of

$$R_2 \left[ \begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & 2 & -2 & 1 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array} \right] \quad R_2 - 3R_3$$

chapter

$$\text{So } A^{-1} = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix}$$

$$-3 \cdot (-1)$$

$$-3$$

$$4-3$$

$$2-1$$

$$2-1$$

$$128$$

$$129$$

$$85$$

$$18$$

So;

$$A^{-1}X_1 = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 67 \\ 44 \\ 41 \end{bmatrix} \Rightarrow \begin{bmatrix} 0+44-41 \\ 134-88-41 \\ -67+44+41 \end{bmatrix} \Rightarrow \begin{bmatrix} 3 \\ 5 \\ 18 \end{bmatrix}$$

$$A^{-1}X_2 = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 49 \\ 39 \\ 19 \end{bmatrix} \Rightarrow \begin{bmatrix} 39-19 \\ 98-78-19 \\ -49+39+19 \end{bmatrix} = \begin{bmatrix} 20 \\ 1 \\ 9 \end{bmatrix}$$

$$\frac{138}{113} = 2.5$$

$$A^{-1}X_3 = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 113 \\ 76 \\ 62 \end{bmatrix} \Rightarrow \begin{bmatrix} 76-62 \\ 226-152-62 \\ -113+76+62 \end{bmatrix} = \begin{bmatrix} 14 \\ 12 \\ 25 \end{bmatrix}$$

$$\frac{138}{113} = 2.5$$

$$A^{-1}X_4 = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 164 \\ 69 \\ 55 \end{bmatrix} = \begin{bmatrix} 69-55 \\ 208-138-55 \\ -164+69+55 \end{bmatrix} \Rightarrow \begin{bmatrix} 14 \\ 15 \\ 20 \end{bmatrix}$$

$$\frac{138}{113} = 2.5$$

$$\frac{138}{113} = 2.5$$

Decode Message is

3 5 18 20 9 14 12 25 14 15 20

CERTAINLY NOT

Ans



## Chapter #05

### Applications of Vector in $\mathbb{R}^n$ i.e. $\mathbb{R}^2 \& \mathbb{R}^3$ .

∴ Cross product of vectors give a vector.

While dot product give a scalar i.e. a number.

$$\therefore u \times (v \times w) = (u \cdot w)v - (u \cdot v)w$$

$$\therefore (u \times v) \times w = (w \cdot u)v - (w \cdot v)u$$

→ Commutative & associative law does not hold for vector multiplication i.e. (for cross product)

$$\rightarrow (u \times v) \cdot w = u \cdot (v \times w)$$

$$(u \times v) \cdot w = \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$$

$$\|u \times v\| = \|u\| \|v\| \sin \theta$$

This area is in space:-

$$\text{Area of triangle } A_T = \frac{1}{2} \|u \times v\|$$

For e.g. Find area of  $\Delta$  with  $P(2,2,4)$ ,  $P_2(-1,0,5)$

$$\{P_3(3,4,3)\}. \quad \text{Sol} = \text{Pr. } A_3 \quad A_T = \frac{1}{2} \|u \times v\|$$

$$u = \vec{P_1 P_2} = (-3, -2, 1) \Rightarrow -3\hat{i} - 2\hat{j} + \hat{k}$$

$$v = \vec{P_1 P_3} = (1, 2, -1) \Rightarrow \hat{i} + 2\hat{j} - \hat{k}$$



$$U \times V = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & -2 & +1 \\ 1 & 2 & -1 \end{vmatrix} \Rightarrow \hat{i}(2-2) - \hat{j}(3-1) + \hat{k}(-6+2)$$

$$= 0\hat{i} - 2\hat{j} - 4\hat{k}$$

$$\|U \times V\| = \sqrt{4+16} \Rightarrow 2\sqrt{5}$$

$$A_T = \frac{1}{2} \|U \times V\| \Rightarrow \frac{1}{2} (2\sqrt{5})$$

$$A_T = \sqrt{5} \text{ square unit}$$

(\*) Area of Parallelogram:-

$$A = \|U \times V\|$$

Volume of Parallelepiped:-

$$V = |U \cdot (V \times W)|$$

where

$$U \cdot (V \times W) = \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$$



$$\boxed{Ex: -5.1}$$

Q1. (a)  $u = 2\hat{i} + 3\hat{j} + 4\hat{k}, v = -\hat{i} + 3\hat{j} - \hat{k}.$

Soln-  $u \times v = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ -1 & 3 & -1 \end{vmatrix}$

$$= \hat{i}(-3-12) - \hat{j}(-2+4) + \hat{k}(6+3)$$

$$= -15\hat{i} - 2\hat{j} + 9\hat{k}. \quad \underline{\text{Ans}}$$

Q2:- (a)  $u = 2\hat{j} + \hat{k} \quad \& \quad v = 3u.$

Soln-  $u = 2\hat{j} + \hat{k}, \quad v = 3(2\hat{j} + \hat{k}) \Rightarrow v = 6\hat{j} + 3\hat{k}$

$$u \times v = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 2 & 1 \\ 0 & 6 & 3 \end{vmatrix} \Rightarrow \hat{i}(6-6) - \hat{j}(0) + \hat{k}(0)$$

$$= 0\hat{i} + 0\hat{j} + 0\hat{k} \quad \underline{\text{Ans}}$$

For Question #3 Properties 9 - 11 rough (d) are;

(a)  $u \times v = -(v \times u)$

(b)  $u \times (v + w) = u \times v + u \times w$

(c)  $(u + v) \times w = u \times w + v \times w$

(d)  $c(u \times v) = (cu) \times v = u \times (cv)$

(e)  $u \cdot (v \times w) = (u \times v) \cdot w$



Q. Verify each of cross product  
orthogonal to both  $u$  &  $v$

Q. 1.  $u = (1, 0, 1)$ ,  $v = (2, 3, -1)$   
 $u = \hat{i} + \hat{k}$ ,  $v = 2\hat{i} + 3\hat{j} - \hat{k}$

$$u \times v = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 1 \\ 2 & 3 & -1 \end{vmatrix}$$

$$= \hat{i}(-3) - \hat{j}(-1-2) + \hat{k}(3)$$

$$= -3\hat{i} + 3\hat{j} + 3\hat{k}$$

So, the vector  $u$  is orthogonal to

$u \times v$  is  $u \cdot (u \times v) = 0$   
 $(\hat{i} + \hat{k}) \cdot (-3\hat{i} + 3\hat{j} + 3\hat{k}) = 0$   
 $2\hat{i} \cdot \hat{k} \quad 2 - 1 = 0$   
 $1 \neq 0$

So not orthogonal  
 for  $v \cdot (-3\hat{i} + 3\hat{j} + 3\hat{k}) = 0$   
 $-6 + 9 - 3 = 0$

$0 = 0$   
 So orthogonal.  $v$  is orthogonal

Q. 2. Verify 4.2 for pair of vector in Ex. 1  
 $u = \hat{i} - \hat{j} + 2\hat{k}$ ,  $v = 3\hat{i} - \hat{j} + \hat{k}$

Sol. Using 4.2  $\|u \times v\| = \|u\| \|v\| \sin \theta \rightarrow \text{①}$



$$U \times V = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 2 \\ 3 & -4 & 1 \end{vmatrix} \Rightarrow \hat{i}(-1-8) - \hat{j}(1-6) + \hat{k}(-4+3)$$

$$= -7\hat{i} + 5\hat{j} - \hat{k}$$

$$\|U \times V\| = \sqrt{(-7)^2 + (5)^2 + (-1)^2} \Rightarrow \sqrt{49 + 25 + 1} \Rightarrow \sqrt{75} \\ \Rightarrow 5\sqrt{3}$$

$$\|U\| = \sqrt{6}, \|V\| = \sqrt{26}$$

$$\therefore \textcircled{1} \Rightarrow 5\sqrt{3} = \sqrt{6} \sqrt{26} \sin \theta$$

$$\frac{5\sqrt{3}}{\sqrt{156}} = \sin \theta$$

$$\sin \theta = \frac{8.66}{12.49} \Rightarrow \theta = \sin^{-1} \frac{8.66}{12.49}$$

$$\theta = 43.89^\circ$$

$$\therefore \textcircled{2} \Rightarrow$$

$$5\sqrt{3} = \sqrt{6} \sqrt{26} \sin 43.89^\circ$$

$$8.66 = 8.66$$

Proved



Q11:- Sol: Let  $\vec{P}_1, \vec{P}_2 = \vec{U} = 2\hat{i} + 3\hat{j} - \hat{k}$ ,  $\vec{P}_2, \vec{P}_3 = \vec{V} = \hat{i} + 2\hat{j} + 2\hat{k}$

$$A = \frac{1}{2} \|\vec{U} \times \vec{V}\|$$

$$\vec{U} \times \vec{V} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & -1 \\ 1 & 2 & 2 \end{vmatrix}$$

$$A = \frac{\sqrt{90}}{2} \text{ Ans}$$

$$= \hat{i}(6+2) - \hat{j}(4+1) + \hat{k}(4-3)$$

$$= 8\hat{i} - 5\hat{j} + \hat{k}$$

$$\|\vec{U} \times \vec{V}\| = \sqrt{64+25+1}$$

$$= \sqrt{90}$$

Q11:-  $A_{\text{area}} = \|\vec{U} \times \vec{V}\|$

$$A_{\text{area}} = \frac{\sqrt{126}}{2} \text{ Ans square unit}$$

$$\vec{U} \times \vec{V} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & -2 \\ 3 & -1 & -1 \end{vmatrix}$$

Q12:- Find volume of parallelepiped.

Sol:- Consider a parallelepiped with vertex at origin & edges is,

$$= \hat{i}(-3+2) - \hat{j}(-4+6) + \hat{k}(4-9)$$

$$= -\hat{i} - 5\hat{j} - 5\hat{k}$$

Volume of parallelepiped

is given as,

$$\|\vec{U} \times \vec{V}\| = \sqrt{1+25+25}$$

$$= \sqrt{51}$$

Ans

$$V = \vec{U} \cdot (\vec{V} \times \vec{W})$$

$$\begin{vmatrix} 2 & -1 & 0 \\ 1 & -2 & -2 \\ 3 & -1 & 1 \end{vmatrix} \Rightarrow 2(-2-2) + 1(1+6)$$

$$= 2(-4) + 7$$

$$= -8 + 7$$

$$\vec{U} = 2\hat{i} - \hat{j}$$

$$\vec{V} = \hat{i} - 2\hat{j} - 2\hat{k}$$

$$\vec{W} = 3\hat{i} - \hat{j} + \hat{k}$$

$$\vec{U} = (2, -1, 0)$$

$$\vec{V} = (1, -2, -2)$$

$$\vec{W} = (3, -1, 1)$$

or  $\vec{U} \times \vec{V} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 0 \\ 1 & -2 & -2 \end{vmatrix}$

$$= \hat{i}(2) - \hat{j}(-4) + \hat{k}(-4)$$

$$= 2\hat{i} + 4\hat{j} - 4\hat{k}$$

$$(\vec{U} \times \vec{V}) \cdot \vec{W} = (2\hat{i} + 4\hat{j} - 4\hat{k}) \cdot (3\hat{i} - \hat{j} + \hat{k})$$

$$= 6 - 4 - 4 = -2$$

Ans



## Line Equation (In $\mathbb{R}^2$ ).

$$= \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0.$$

or  $ax + by + c = 0 \rightarrow (1).$

Where  $a = y_1 - y_2$ ,  $b = x_2 - x_1$ , &  $c = x_1 y_2 - y_1 x_2$ .

## Line In $\mathbb{R}^3$ :-

For line in  $\mathbb{R}^3$  we need to find one point & its direction.

§ Parameter eqn of line in  $\mathbb{R}^3$ .

$$x = x_0 + at$$

$$y = y_0 + bt$$

$$z = z_0 + ct$$

Symmetric eqn of line.

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$$



### Example #03

Find parametric eqn of line through point  $P_0(2, 3, -4)$  &  $P_1(3, -2, 5)$ .

Sol:-

First we find parallel vector to that line i.e.

$$U = P_0P_1 = (1, -5, 9)$$

As  $U = (a, b, c)$  by comparing;

$$a = 1, \quad b = -5, \quad \& \quad c = 9$$

So,  $x = x_0 + at$

Where  $x_0 = 2$

$$x = 2 + t$$

$$y = 3$$

$$y = 3 - 5t$$

$$z_0 = -4$$

$$z = -4 + 9t$$

→ Parametric eqn of line

### Planes in $\mathbb{R}^3$ :-

To determine a plane in  $\mathbb{R}^3$  we need to find a point & normal vector to plane.

The eqn of plane in  $\mathbb{R}^3$  is;

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$



Q Exp #6 Find eqn of plane passing through points  
~~(3, 4, -3)~~  $P_1(2, -2, 1)$ ,  $P_2(-1, 0, 3)$ ,  $P_3(5, -3, 4)$

Sol:- To find eqn of plane we need one point i.e.  $P_1(2, -2, 1)$  & one normal vector  $\vec{n}$ .  
Since  $\vec{n} = |\vec{U} \times \vec{V}| \rightarrow \text{①}$

$$\vec{U} = \vec{P_1 P_2} = (-3, 2, 2), \quad \vec{V} = \vec{P_1 P_3} = (3, -1, 3)$$

$$\vec{n} = |\vec{U} \times \vec{V}| \Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 2 & 2 \\ 3 & -1 & 3 \end{vmatrix} \Rightarrow \hat{i}(6+2) - \hat{j}(-9-6) + \hat{k}(3-6) \\ = 8\hat{i} + 15\hat{j} - 3\hat{k}$$

$(x, y, z)$

Let  $P$  be arbitrary point in plane then,  
 $\vec{n} \cdot \vec{PP_1} = 0$

$$8\hat{i} + 15\hat{j} - 3\hat{k} \cdot (x-2, y+2, z-1) = 0$$

$$(8, 15, -3) \cdot (x-2, y+2, z-1) = 0$$

$$8x - 16 + 15y + 30 - 3z + 3 = 0$$

$$8x + 15y - 3z + 17 = 0$$

$\hookrightarrow$  is required plane eqn.



Ex 10) Find two planes whose intersection is line

$$\begin{aligned}x &= -2 + 3t \\y &= 3 - 2t \\z &= 5 + 4t\end{aligned}$$

Sol We can write above eqn as.

$$\frac{x+2}{3} = t, \quad \frac{y-3}{-2} = t, \quad \frac{z-5}{4} = t.$$

By comparing;

$$\frac{x+2}{3} = \frac{y-3}{-2} = \frac{z-5}{4}.$$

Now to find two planes compare any two we get.

$$\frac{x+2}{3} = \frac{y-3}{-2}, \quad \frac{y-3}{-2} = \frac{z-5}{4}$$

$$-2(x+2) = 3(y-3)$$

$$4(y-3) = -2(z-5)$$

$$-2x-4 = 3y-9$$

$$2x+3y-9+4=0$$

$$2x+3y-5=0$$

one plane eqn.

$$4y-12 = -2z+10$$

$$4y+2z-12-10=0$$

$$4y+2z-22=0$$

Ans.



Exp #10.

Find the parametric eqn of line of intersection of planes.

$$\pi_1 = 2x + 3y - 2z + 4 = 0.$$

$$\pi_2 = x - y + 2z + 3 = 0.$$

Sol:- The given eqns of planes are;

$$\pi_1 = 2x + 3y - 2z + 4 = 0 \rightarrow (1)$$

$$\pi_2 = x - y + 2z + 3 = 0 \rightarrow (2)$$

Add eq (1) & (2).

$$2x + 3y - 2z + 4 = 0$$

$$x - y + 2z + 3 = 0$$

---


$$3x + 2y + 7 = 0$$

$$3x = -7 - 2y$$

$$x = \frac{-7 - 2y}{3}$$

Let  $y = r$ .

where  $r \in \mathbb{R}$

$$\left( \frac{-7 - 2r}{3} \right) + 2y + 7 = 0$$

$$2y + \frac{2(-7 - 2r)}{3} = 0 \Rightarrow 2y = \frac{14 + 2r}{3}$$

$$y = \frac{-14 - 2r}{6}$$



$$4 \Rightarrow x - y + 2z + 3 = 0.$$

$$-\frac{7-2z}{3} - \left(\frac{-14+2z}{6}\right) + 2z + 3 = 0.$$

$$2z = \frac{-14 - 4z + 14 - 2z + 18}{6} = 0.$$

$$2z = \frac{-6z + 18}{6} = 0 \Rightarrow 2z - \frac{4(z+3)}{2} = 0$$

$$z = \frac{z-3}{2}$$

So the symmetric eqn of line is;

$$x = \frac{-7-2z}{3}, \quad y = \frac{-14+2z}{6}, \quad z = \frac{z-3}{2} \quad \text{Ans}$$

Ex:- 5.2

Q1. @  $P_1(-2, -3), P_2(3, 4)$ .

Sol:- Since in  $\mathbb{R}^2$  So eqn of line in  $\mathbb{R}^2$  is

$$\Rightarrow \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0 \Rightarrow \text{---} \text{---} \text{---}$$

$$P_1(x_1, y_1) = P(-2, -3)$$

$$x_1 = -2$$

$$y_1 = -3$$

$$\Rightarrow \begin{vmatrix} x & y & 1 \\ -2 & -3 & 1 \\ 3 & 4 & 1 \end{vmatrix} = 0$$

$$P_2(x_2, y_2) = P(3, 4)$$

By expanding R<sub>2</sub> we get line eqn.

$$x_2 = 3, \quad y_2 = 4$$



Q3:- state which of the following points are on line.

$$x = 3 + 2t$$

$$y = -2 + 3t$$

$$z = 4 - 3t$$

⑥ (1, 0, -2)

Sol:-

As given that,

$$x = 3 + 2t$$

$$y = -2 + 3t$$

$$z = 4 - 3t$$

$$\frac{x-3}{2} = t$$

$$\frac{y+2}{3} = t$$

$$\frac{z-4}{-3} = t$$

By comparing all.

$$\frac{x-3}{2} = \frac{y+2}{3} = \frac{z-4}{-3} \rightarrow \text{①}$$

As (1, 0, -2)

$$\frac{1-3}{2} = \frac{0+2}{3} = \frac{-2-4}{-3}$$

for  $x=1$   
 $y=0$

$z=-2$

$$\frac{-2}{2} = \frac{2}{3} = \frac{-6}{-3}$$

Invalid case

$$-1 \neq \frac{2}{3} \neq 2$$

Since not equal so points are not on

Q5:- ⑥  $P_0(3, 4, -2)$ ,  $u(4, -5, 2)$ . Find parametric

Sol:- As parametric eqn is;

$$u = (a, b, c)$$

$$x = x_0 + at, \quad y = y_0 + bt, \quad z = z_0 + ct \quad a=4, b=-5, c=2$$

$$x = 3 + 4t$$

$$P_0(x_0, y_0, z_0)$$

$$y = 4 - 5t \quad \{ \quad z = -2 + 2t \quad \text{Ans}$$

$$P_0(3, 4, -2)$$

$$x_0=3, y_0=4, z_0=-2$$



Q6:- Find the line passing through given points.

(b)  $(-3, -2, -2), (5, 5, 4)$ .

Sol:- As for line in  $\mathbb{R}^3$  we need one point & one parallel vector.

So; let  $P_0 = (-3, -2, -2)$  &  $P_1 = (5, 5, 4)$   
 $x_0, y_0, z_0$                        $x_1, y_1, z_1$

So,  $\vec{v} = \vec{P_0 P_1} = (5+3, 5+2, 4+2)$

$\vec{v} = (8, 7, 6)$   
 $a, b, c,$

So parametric eqn is;

$$x = x_0 + at, \quad y = y_0 + bt, \quad z = z_0 + ct$$

$$x = -3 + 8t, \quad y = -2 + 7t, \quad z = -2 + 6t$$

is required parametric eqn.

Q7:- For ex. 6 find eqn in symmetric form.

Sol:- As in symmetric form, we have;

(b)  $\frac{x-x_0}{a} = \frac{y-y_0}{b} = \frac{z-z_0}{c} \rightarrow \text{①}$

By putting values:

$$\frac{x+3}{8} = \frac{y+2}{7} = \frac{z+2}{6} \quad \text{Ans}$$

is required symmetric eqn.



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Q8:- State which of the following points on plane.  
 $3(x-2) + 2(y+3) - 4(z-4) = 0$ .  
 (b)  $(1, -2, 3)$ .

Sol:- Given plane is,  
 $3(x-2) + 2(y+3) - 4(z-4) = 0 \rightarrow \text{①}$ .

Q The point lies then it will satisfy eq ①.  
 Since  $P(x, y, z) = (1, -2, 3)$  put in eq ①.  
 $3(1-2) + 2(-2+3) - 4(3-4) = 0$   
 $3(-1) + 2(1) - 4(-1) = 0$   
 $-3 + 2 + 4 = 0$

$3 \neq 0$  invalid case.

So the above points not lies on plane.

Q9:- Find eqn of plane? @  $(-2, 3, 4)$ ,  $n = (0, 1, -3)$ .

Sol:- Eqn of plane is  $a(x-x_0) + b(y-y_0) + c(z-z_0) = 0$   
 here  $a=0, b=1, c=-3$  {  $x_0=-2, y_0=3, z_0=4$  }  $\rightarrow \text{①}$

eq ①  $\Rightarrow 0(x+2) - 1(y-3) - 3(z-4)$

$-y+3-3z+12=0$

$-y-3z+15=0$  Required plane



Q10:- Find eqn of planes passing through give 3 points.  
 @ (0, 1, 2), (3, -2, 5), (2, 3, 4)

Sol:- For the eqn of plane we need one point & one normal vector.

Here point is  $P_0(0, 1, 2)$

$$x_0 = 0, y_0 = 1, z_0 = 2$$

Now to find normal vector.

$$U = \vec{P_1 P_0} = (3, -3, 3)$$

$$V = \vec{P_2 P_0} = (2, 2, 2)$$

$$\vec{n} = U \times V = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -3 & 3 \\ 2 & 2 & 2 \end{vmatrix} \quad \text{Expand by } R_1$$

$$= \hat{i}(-6-6) - \hat{j}(6-6) + \hat{k}(6+6)$$

$$= -12\hat{i} + 12\hat{k}$$

Let  $P(x, y, z)$  be arbitrary point in plane

&  $P_0(0, 1, 2)$  is fixed point. Then;

$$\vec{PP_0} = (x, y-1, z-2)$$

Since both vector are  $\perp$  so its dot product will be equal to zero.

$$\vec{U} \cdot \vec{PP_0} = 0$$

$$(-12, 12) \cdot (-12\hat{i} + 0\hat{j} + 12\hat{k}) \cdot (x\hat{i} + (y-1)\hat{j} + (z-2)\hat{k}) = 0$$



$$-12x + 0j + 12(x-2) = 0.$$

$$-12x + 12x - 24 = 0 \quad \text{Ans}$$

Q11:- Find Parametric eqn of line intersecting given plane  
(b)  $3x - 2y - 5z + 4 = 0$  &  $2x + 3y + 4z + 8 = 0.$

Sol:- Given planes are;

$$3x - 2y - 5z + 4 = 0. \rightarrow (1)$$

$$2x + 3y + 4z + 8 = 0. \rightarrow (2)$$

Multiply 2 to eq (1) & (3) to eq (2)

$$6x - 4y - 10z + 8 = 0$$

$$+ 6x + 9y + 12z + 24 = 0.$$

$$\begin{array}{r} \ominus \quad \ominus \quad \ominus \quad \ominus \\ \hline \end{array}$$

$$-13y - 22z - 16 = 0 \rightarrow$$

$$13y = -22z - 16.$$

$$y = \frac{-22z - 16}{13}$$

$$\text{Let } z = r$$

where  
 $r \in \mathbb{R}$

$$y = \frac{-22r - 16}{13}$$

$$3x - 2\left(\frac{-22r - 16}{13}\right) - 5r + 4 = 0$$

$$33x + 44r + 32 - 65r + 52 = 0$$

$$3x$$

$$\frac{+44r + 32 - 65r + 52}{13} = 0$$



$$3x - \frac{21y + 84}{13} = 0 \Rightarrow 3x = \frac{21y + 84}{13}$$

$$x = \frac{3(7y - 28)}{13}$$

$$x = \frac{7y - 28}{13}$$

is required parametric eqn of line.

Q12:- Find a pair of planes whose intersection is given line.

(a)  $x = 2 - 3t, y = 3 + t, z = 2 - 4t$ .

$$\frac{x-2}{-3} = \frac{y-3}{1} = \frac{z-2}{-4}$$

By comparing we get

$$\frac{x-2}{-3} = \frac{y-3}{1} \Rightarrow -3(y-3) = x-2$$

$$-3y + 9 = x - 2 \Rightarrow x - 2 + 3y - 9 = 0$$

$$x - 2 + 3y - 9 = 0 \Rightarrow x + 3y - 11 = 0$$

$$\frac{y-3}{1} = \frac{z-2}{-4} \Rightarrow -4(y-3) = z-2$$

$$4y - 12 + 2 - z = 0$$

$$4y - z - 10 = 0 \quad \text{Ans.}$$



Q13:- Are  $(2, 3, -2)$ ,  $(4, -2, -3)$  &  $(0, 8, -1)$  on the same line?

Soln:- The points are on same line if

$$\begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix} = 0.$$

$$\begin{vmatrix} 2 & 3 & -2 \\ 4 & -2 & -3 \\ 0 & 8 & -1 \end{vmatrix} = 0.$$

$$2(2 + 24) - 3(-4) - 2(32).$$

$$52 + 12 - 64.$$

$$64 - 64 = 0.$$

∴ Thus on same line



Q15- Find the point of intersection.

$$\begin{aligned} x &= 2-3s \\ y &= 3+2s \\ z &= 4+2s \end{aligned}$$

{

$$\begin{aligned} x &= 5+2t \\ y &= 1-3t \\ z &= 2+t \end{aligned}$$

Sol:- Let  $P(x, y, z)$  be the point of intersection

By comparing

$$2-3s = 5+2t \Rightarrow 3s+2t+3=0 \rightarrow (1)$$

$$3+2s = 1-3t \Rightarrow 3t+2s+2=0 \rightarrow (2)$$

$$4+2s = 2+t \Rightarrow 2s-t+2=0 \rightarrow (3)$$

eq (1) & (3)

$$3t+2s+2=0$$

$$4t+2s+t=0$$

$$4t=0 \Rightarrow t=0$$

eq (1)  $\Rightarrow 3s+2(0)+3=0$

$$3s=-3 \Rightarrow s=-1$$

Put values in eq

$$x = 2-3(-1)$$

$$x = 5$$

$$y = 3+2(-1)$$

$$= 1$$

$$z = 4+2(-1)$$

$$z = 2$$

So the point of intersection is

$$P(x, y, z) = P(5, 1, 2) \text{ Ans}$$



Q16:- Which of the following pair of lines are  $\perp$ .

$$\textcircled{a} \quad \left. \begin{aligned} x &= 2+2t \\ y &= -3-3t \\ z &= 4+4t \end{aligned} \right\} \rightarrow \textcircled{1}$$

$$\left. \begin{aligned} x &= 2+t \\ y &= 4-t \\ z &= 5-t \end{aligned} \right\} \rightarrow \textcircled{2}$$

Sol:- As the lines are given in parametric form so we find parallel vectors to both lines.

$$\begin{array}{llll} \text{Parallel vector of } \textcircled{1} & \Rightarrow & u = [2 \ -3 \ 4] \\ \text{" " " " } & & v = [1 \ -1 \ -1] \end{array}$$

Since if the lines are  $\perp$  then their parallel vector will also  $\perp$ . i.e. dot product  $u \cdot v = 0$ .

$$2 + 3 - 4 = 0$$

$$5 - 4 = 0$$

$$1 \neq 0$$

As not satisfied so the given pairs of lines are not perpendicular.



Q17. Show the following parametric eqn define same line.

$$\left. \begin{aligned} x &= 2+3t \\ y &= 3-2t \\ z &= -1+4t \end{aligned} \right\} \text{--- (1)}$$

$$\left. \begin{aligned} x &= 1-9t \\ y &= 5+6t \\ z &= -5-12t \end{aligned} \right\} \text{--- (2)}$$

Sol:- As parametric eqn are given so we will find parallel vector to both lines.

U vector parallel to line 1.

$$u = (3 \quad -2 \quad 4)$$

$$v = (-9 \quad 6 \quad -12)$$

If Suppose both lines are parallel then their corresponding vector  $u$  &  $v$  also parallel. & Their cross product will be zero.

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -2 & 4 \\ -9 & 6 & -12 \end{vmatrix} = 0$$

$$\hat{i}(24-24) - \hat{j}(-36+36) + \hat{k}(18-18) = 0$$

$$0\hat{i} + 0\hat{j} + 0\hat{k} = 0$$

As L.H.S = R.H.S Thus the given parametric eqn define same line.



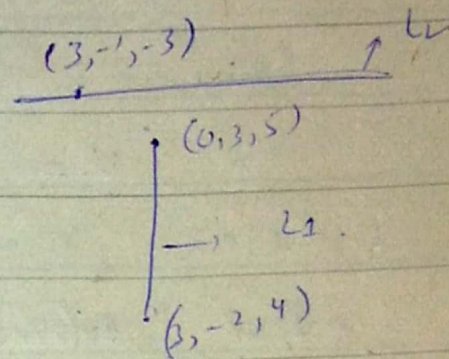
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Q18:- Find parametric eqn of line passing through the point  $(3, -1, -3)$  &  $\perp$  to line passing through the point  $(3, -2, 4)$  &  $(0, 3, 5)$ .

Sol:- Parallel vector to  $L_1$  is,

$$V = P_1 P_2 = [3 \ -5 \ -1]$$



Now find parallel vector to  $L_2$  i.e.  
 $U = (a, b, c)$

As they are  $\perp$  so dot product will be zero.  
 $(a \ b \ c) \cdot (3 \ -5 \ -1) = 0$

$$3a - 5b - c = 0 \rightarrow (2)$$

$$U = \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix} \text{ to}$$

$$a = 1$$

$$b = 1$$

$$c = -2$$

Satisfy above eqn.

Thus parametric eqn is,

$$x = 3 - t, \quad y = -1 + t, \quad z = -3 - 2t$$

Ans.



Q19:- Find eqn of plane passing through  $(-2, 3, 4)$  &  $\perp$  to line passing through points  $(4, -2, 5)$  &  $(0, 2, 4)$

Sol:- Since eqn of plane is,  
 $a(x-x_0) + b(y-y_0) + c(z-z_0) = 0$ .  
 $\hookrightarrow$  ①

$$P(-2, 3, 4) = P(x_0, y_0, z_0).$$

Now find  $\vec{a}, b, c$

So,

$$\vec{u} = \vec{P_1 P_2} = \begin{pmatrix} 4 & -4 & 1 \end{pmatrix}.$$

a      b      c

$$\hookrightarrow \text{①} \Rightarrow a(x+2) - 4(y-3) + (z-4) = 0.$$

$$4x + 8 - 4y + 12 + z - 4 = 0.$$

$$4x - 4y + z + 16 = 0 \quad \underline{\text{required plane eqn.}}$$

Q20:- Find point of intersection of line

$$x = 2 - 3t$$

$$y = 4 + 2t$$

$$z = 3 - 5t$$

& plane  $2x + 3y + 4z + 8 = 0$ .  
 $\hookrightarrow$  ①

Sol:-  $x = 2 - 3t \rightarrow$  ①,  $y = 4 + 2t \rightarrow$  ②,  $z = 3 - 5t \rightarrow$  ③

Put eq ①, ② & ③ in eq ①

$$2(2-3t) + 3(4+2t) + 4(3-5t) + 8 = 0.$$

Point of intersection

We will find  $t$  then put in eq ①, ②, & ③ to get



Q2:- Find a plane containing the line.

$$\left. \begin{aligned} x &= 3+2t \\ y &= 4-3t \\ z &= 5+4t \end{aligned} \right\} \rightarrow L_1 \quad \left\{ \begin{aligned} x &= 1-2t \\ y &= 7+4t \\ z &= 1-3t \end{aligned} \right\} \rightarrow L_2$$

Sol:- Parallel vector to  $L_1$  is;  
 $u = [2 \ -3 \ 4]$

Parallel vector to  $L_2$  is;  
 $v = [-2 \ 4 \ -3]$

Since we need normal vector. So,

$$u \times v = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 4 \\ -2 & 4 & -3 \end{vmatrix} \Rightarrow -7\hat{i} - 2\hat{j} + 2\hat{k}$$

Let  $P$  be arbitrary point  $P(x, y, z)$

$\{ P_1(3, 4, 5) \}$  is fixed then;

$$\vec{P_1P_2} = (x-3, y-4, z-5)$$

Since  $n \perp \vec{P_1P_2}$  so dot product = 0.

$$\vec{n} \cdot \vec{P_1P_2} = (-7 \ -2 \ 2) \cdot (x-3, y-4, z-5) \\ -7(x-3) - 2(y-4) + 2(z-5) = 0$$

$$-7x + 21 - 2y + 8 + 2z - 10 = 0$$

$$-7x - 2y + 2z + 19 = 0$$

is the required Ans.  
plane.



Q22:- Find a plane passes through  $(2, 4, -3)$  is parallel to plane  $-2x + 4y - 5z + 6 = 0$ .

(Sol): The point on desired plane is  $P_0(2, 4, -3)$ .

$(2, 4, -3)$

The plane is parallel to  $-2x + 4y - 5z + 6 = 0$  plane.

normal to this given plane is;

$$\vec{n} = [-2 \quad 4 \quad -5]$$

Since  $\vec{n}$  is  $\perp$  to given plane so it will also  $\perp$  required plane.

Let  $P = (x, y, z)$  be arbitrary point on desired plane then;

$$v = \vec{P_0P} = (x-2, y-4, z+3)$$

Then

$$\vec{P_0P} \cdot \vec{n} = 0$$

$$(-2 \quad 4 \quad -5) \cdot (x-2, y-4, z+3) = 0$$

$$-2(x-2) + 4(y-4) + (-5)(z+3) = 0$$

$$-2x + 4 + 4y - 16 - 5z - 15 = 0$$

$$-2x + 4y - 5z - 27 = 0$$

or

$$2x - 4y + 5z + 27 = 0 \text{ is required}$$

plane i.e. parallel to  $-2x + 4y - 5z + 6$  plane