Ex # 4.3

GOVERNMENT OF KHYBER PAKHTUNKHWA
ESTABLISHMENT DEPARTMENT.

(b) $L\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x+y \\ y \\ x-z \end{bmatrix}$

Sol:- For L to be in L.T The two Conditions of L.T. should be satisfied

Condition # 01

$$L(U+V) = L(U)+L(V) \longrightarrow (i)$$

Since the domain is in $\mathbb{R}^3$ so U & V will be

$$U = \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix}, \quad V = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

$$U+V = \begin{bmatrix} U_1+V_1 \\ U_2+V_2 \\ U_3+V_3 \end{bmatrix}$$

$$L(U+V) = \begin{bmatrix} U_1+V_1+U_2+V_2 \\ U_2+V_2 \\ U_1+V_1-(U_3+V_3) \end{bmatrix} \longrightarrow (ii)$$

Now R.H.S of eq (i) will be,

$$L(U)+L(V) = \begin{bmatrix} U_1+U_2 \\ U_2 \\ U_1-U_3 \end{bmatrix} + \begin{bmatrix} V_1+V_2 \\ V_2 \\ V_1-V_3 \end{bmatrix}$$

$$= \begin{bmatrix} U_1+U_2+V_1+V_2 \\ U_2+V_2 \\ U_1-U_3+V_1-V_3 \end{bmatrix}$$

$$= \begin{bmatrix} U_1+U_2+V_1+V_2 \\ U_2+V_2 \\ U_1+V_1-(U_3+V_3) \end{bmatrix}$$

Since L.H.S = R.H.S so Condition Satisfied

Now apply Condition # 02

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$\rightarrow$ $L(2u) = 2L(u)$

$$2u = \begin{bmatrix} 2u_1 \\ 2u_2 \\ 2u_3 \end{bmatrix}$$

$$L(2u) = \begin{bmatrix} 2u_1 + 2u_2 \\ 2u_2 \\ 2u_1 - 2u_3 \end{bmatrix} \rightarrow$$

$$2L(u) = \begin{bmatrix} u_1 + u_2 \\ u_2 \\ u_1 - u_3 \end{bmatrix}$$

$$2L(u) = \begin{bmatrix} 2u_1 + 2u_2 \\ 2u_2 \\ 2u_1 - 2u_3 \end{bmatrix}.$$

L.H.S = R.H.S.

Since both condition satisfied so L is linear

Q3

(c) $L(x, y) = (x - y, 0, 2x+3)$.

Sol:- ~~L~~ $L(u+v) = L(u) + L(v)$.

$u = (x_1, y_1)$, $v = (x_2, y_2)$.

$u+v = (x_1 + x_2, \ y_1 + y_2)$.

$$L(u+v) = (x_1 + x_2 - (y_1 + y_2), 0, 2(x_1 + y_1) + 3)$$

$$= (x_1 + x_2 - (y_1 + y_2), 0, 2x_1 + 2y_1 + 3) \longrightarrow ①.$$

$$L(u) + L(v) = (x_1 - y_1, 0, 2x_1 + 3) + (x_2 - y_2, 0, 2x_2 + 3)$$

$$= (x_1 - y_1 + (x_2 - y_2), 0, 2x_1 + 3 + 2x_2 + 3)$$

$$= (x_1 - y_1 + (x_2 - y_2), 0, 2x_1 + 2x_2 + 6)$$

Condition # 1 not satisfy so non linear.

Q3 (a) $L(x, y, z) = (x + y, 0, 2x - z)$.

$U = (x_1, y_1, z_1)$, $V = (x_2, y_2, z_2)$

$U + V = (x_1 + x_2, y_1 + y_2, z_1 + z_2)$.

$L \cdot (U + V) = \left(x_1 + x_2 + y_1 + y_2, 0, 2x_1 + 2x_2 - (z_1 + z_2)\right)$ — ①

$L(U) + L(V) = (x_1 + y_1, 0, 2x_1 - z_1) + (x_2 + y_2, 0, 2x_2 - z_2)$

$= (x_1 + y_1 + (x_2 + y_2), 0, 2x_1 - z_1 + 2x_2 - z_2)$

$= (x_1 + x_2 + y_1 + y_2, 0, 2x_1 + 2x_2 - (z_1 + z_2))$

L.H.S = R.H.S Condition satisfy.

Condition # 2.

$L(cU) = c\,L(U)$.

$cU = (cx_1, cy_1, cz_1)$.

$L(cU) = [cx_1 + cy_1, 0, 2cx_1 - cz_1)$.

$L(U) = [x_1 + y_1, 0, 2x_1 - z_1)$.

$c\,L(U) = [cx_1 + cy_1, 0, 2cx_1 - cz_1)$.

L.H.S = R.H.S L is linear.

Q#4 © $L\left(\begin{bmatrix}x\\y\\z\end{bmatrix}\right) = \begin{bmatrix}0\\0\\0\end{bmatrix}$

$U=\begin{bmatrix}x_1\\y_1\\z_1\end{bmatrix}, \quad v=\begin{bmatrix}x_2\\y_2\\z_2\end{bmatrix}$

$L(U+V) = L(U)+L(V)$

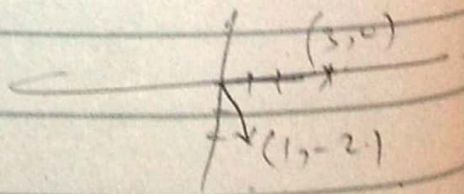
$L\begin{bmatrix}x_1+x_2\\y_1+y_2\\z_1+z_2\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}+\begin{bmatrix}0\\0\\0\end{bmatrix}$

$=\begin{bmatrix}0\\0\\0\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$

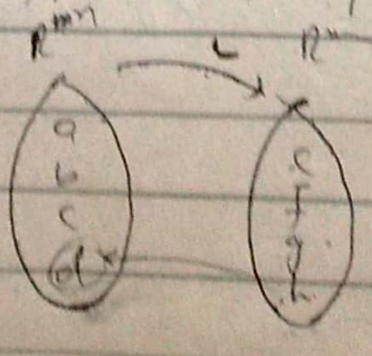
Q6 $L: R^2 \to R^2$ $L\left(\begin{bmatrix}x\\y\end{bmatrix}\right)=\begin{bmatrix}1&-1\\2&1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}$

$P=(1,-2)$

$L(U)=\begin{bmatrix}x-y\\2x+y\end{bmatrix}$



$L(P)=\begin{bmatrix}1-(-2)\\2(1)+(-2)\end{bmatrix} \Rightarrow \begin{bmatrix}3\\0\end{bmatrix}$



Q13 (b) $L\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x+z \\ y+z \\ x+2y+2z \end{bmatrix} \rightarrow (1)$

$L\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = w = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} \rightarrow (2)$

Sol:- Let w is in range of L, then must exist a vector say $u = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$.

$L(u) = w$

$L\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$

$\begin{bmatrix} x+z \\ y+z \\ x+2y+2z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$

$z = -3$

$x+z = 2$

$y+z = -1$

$x+2y+2z = 3$

$z = 2-x \rightarrow *$

$y+2-x = -1$

$y-x = -3 \rightarrow (*)$

$y = -3+x$

$x+2(x-3)+2(2-x) = 3$

$x+2x-6+4-2x = 3$

$\boxed{x = 5}$

$5+2(2)+2z = 3$

$2z = 3-9$

$\boxed{z = -3}$

Justification:-

$L\left[\begin{pmatrix} x \\ y \\ z \end{pmatrix}\right] =$

$x+2(y+z) = 3$

$x+2(-1) = 3$

$x = 5$

So $z = 2-5$

$z = -3$

$y = 2$

Consistent

w in range of L

Q14.- $L\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} -1 & 2 & 0 \\ 1 & 1 & 1 \\ 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$.

$w = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$.

Solution:- $\begin{bmatrix} -x+2y \\ x+y+z \\ 2x-y+z \end{bmatrix} = L\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right)$

$L\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = w = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$.

Thus w is not in range of L.

$$-x+2y = 1$$
$$x+y+z = 2$$
$$2x-y+z = -1.$$

$$x+y+z = 2$$
$$\underset{(-)}{+}2x \underset{(+)}{-} y \underset{(-)}{+} z = \underset{(+)}{-}1$$
$$\overline{\qquad\qquad\qquad\qquad}$$
$$-\cancel{x}+2\cancel{y} = 3.$$
$$\underset{(+)}{-}\cancel{x}\underset{(-)}{+}2\cancel{y} = \underset{(-)}{+}1$$
$$\overline{\qquad\qquad\qquad\qquad}$$
$$0 \neq 2$$

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Q16:- $L\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x+2y+3z \\ -3x-2y-z \\ -2x+2z \end{bmatrix}$ $\quad w = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$.

$$L\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} 1 & 2 & 3 \\ -3 & -2 & -1 \\ -2 & 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Suppose w is in range of L Then $L(u) = w$ for some $u = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$.

$$\underset{A}{\begin{bmatrix} 1 & 2 & 3 \\ -3 & -2 & -1 \\ -2 & 0 & 2 \end{bmatrix}} \underset{u}{\begin{bmatrix} x \\ y \\ z \end{bmatrix}} = \underset{w}{\begin{bmatrix} a \\ b \\ c \end{bmatrix}}$$

The augmented matrix is;

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & a \\ -3 & -2 & -1 & b \\ -2 & 0 & 2 & c \end{array}\right].$$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & 3 & a \\ 0 & 4 & 8 & b+3a \\ 0 & 4 & 8 & c+2a \end{array}\right] \begin{matrix} R_2+3R_1 \\ R_3+2R_1 \end{matrix} \Rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 3 & a \\ 0 & 4 & 8 & b+3a \\ 0 & 0 & 0 & b \end{array}\right] R_3-R_2$$

$c+2a-b-3a$

$-a-b+c$

or

$a+b-c=0$.

Since the system is consistant i.e. w lies in range of L iff $a+b-c=0$ which is desired eqn relating a, b & c.

Q17:- $L: \mathbb{R}^2 \to \mathbb{R}^2$.

$L(i) = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$, & $L(j) = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

Find $L\left(\begin{bmatrix} 4 \\ -3 \end{bmatrix}\right)$.

Solution:- $\begin{bmatrix} 4 \\ -3 \end{bmatrix} = 4\begin{bmatrix} 1 \\ 0 \end{bmatrix} - 3\begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

$= 4\hat{i} - 3\hat{j}$.

$L\left(\begin{bmatrix} 4 \\ -3 \end{bmatrix}\right) = L(4\hat{i} - 3\hat{j}) \Rightarrow L(4\hat{i}) - L(3\hat{j})$.

$= 4\,L(\hat{i}) - 3\,L(\hat{j})$.

$= 4\begin{bmatrix} 2 \\ 3 \end{bmatrix} - 3\begin{bmatrix} -1 \\ 2 \end{bmatrix}$.

$= \begin{bmatrix} 8 \\ 12 \end{bmatrix} + \begin{bmatrix} 3 \\ -6 \end{bmatrix} \Rightarrow \begin{bmatrix} 11 \\ 6 \end{bmatrix}$ Ans.

Q20:-

$L\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $u = (0, -2, 4)$.

$L\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x + z \\ -x + y \\ z \end{bmatrix} \longrightarrow ①$.

As X in $\mathbb{R}^3$ so $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

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$$\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} -y \\ -x \end{bmatrix}$$

$$L(X) = L\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = 0 \longrightarrow (2)$$

$$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} -x \\ -y \end{bmatrix}$$

from eq (1) & (2)

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$L\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = 0$$

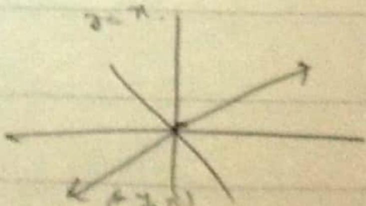
$$x + z = 0 \Rightarrow x = 0 \longrightarrow (1)$$
$$-x + y = 0 \qquad y = 0$$
$$z = 0 \Rightarrow z = 0 \longrightarrow (1)$$

$$X = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

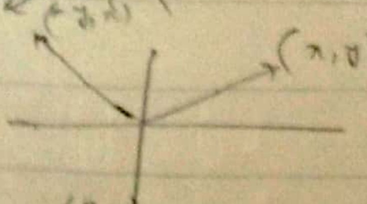
which are all X satisfying $L(X) = 0$.

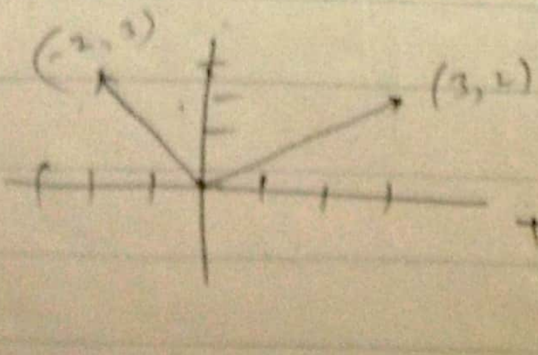
Q2 (b) $L(+x, +y) = (-x, -y)$

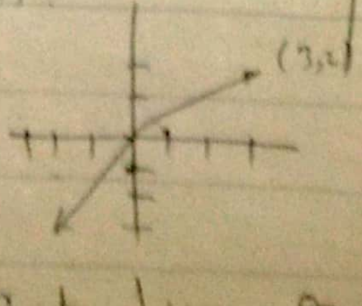
w.r.t line $y = x$.

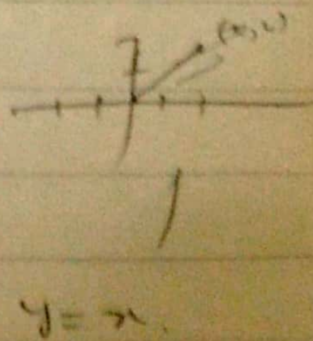


(c) $L(x, y) = (-y, x)$









$L(x, y) = (y, x)$ w.r.t line $y = x$.

$L(x, y) = (-y, -x)$

Q23 :- $L: R^2 \to R^2$ defined by $L(x,y) = (x+y+1, x-y)$

Sol:- to find whether L is Lin T. The following condition should be satisfied.

(a) $L(u+v) = L(u) + L(v)$.

u+v $\quad u = (x_1, y_1)$

$\quad v = (x_2, y_2)$.

$u+v = (x_1+x_2, y_1+y_2)$.

$L(u+v) = (x_1+x_2+y_1+y_2+1, x_1+y_1-(x_2+y_2))$

$L(u) = (x_1+y_1+1, x_1-y_1) + (x_2+y_2+1, x_2-y_2)$

$= (x_1+y_1+x_2+y_2+2, x_1-y_1+x_2-y_2)$

$= (x_1+y_1+x_2+y_2+2, x_1+x_2-(y_1+y_2))$

L.H.S $\neq$ R.H.S

It is not linear

Q 24:- $L: \mathbb{R}^2 \to R$ $\quad L(x,y) = \sin x + \sin y$.

Sol:-

$U = (x_1, y_1)$, $V = (x_2, y_2)$.

$\cancel{L(x_1)} +$ $L(U+V) = L(U) + L(V)$.

$U+V = (x_1+x_2, y_1+y_2)$

$L(U+V) = \sin(x_1+x_2) + \sin(y_1+y_2)$

$L(U) = \sin(x_1) + \sin y_1$.

$L(V) = \sin x_2 + \sin y_2$

$L(U) + L(V) = \sin(x_1) + \sin y_1 + \sin(x_2) + \sin(y_2)$.

$= \sin(x_1) + \sin x_2 + \sin y_1 + \sin y_2$.

$= \cancel{\sin(x_1+x_2)} + \sin(y_1+y_2)$.

L.H.S $\neq$ R.H.S so not L.T.

Condition # 02

$L(KU) = KL(U)$

$KU = \sin KU \rightarrow \sin K(x_1, y_1)$

$L(\sin KU) = \sin Kx_1 + \sin Ky_1$.

$= K(\sin x_1 + \sin y_1)$.

$K\,L \sin U = L(\sin x_1, y_1)$

$= K(\sin x_1, \sin y_1)$

$= K(\sin x_1 + \sin y_1)$. L.H.S = R.H.S

Thus L is linear

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Q_{26} - $L\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x-y \\ x+y \end{bmatrix}$.

Sol:- ~~The standard~~ for any $u = \begin{bmatrix} x \\ y \end{bmatrix}$ we have

$$L(u) = L\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x-y \\ x+y \end{bmatrix}.$$

The standard matrix of L here is given as,

$$~~ls~~ \; A = \begin{bmatrix} L(e_1) & L(e_2) \end{bmatrix} \longrightarrow ①.$$

$$e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \Rightarrow L(e_1) = \begin{bmatrix} 1-0 \\ 1+0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

$$L(e_2) = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 0-1 \\ 0+1 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & -1 \\ 1 & +1 \end{bmatrix}$$ Ans. is desired standard matrix eqn.

~~Q_{28} - $L: \mathbb{R}^2 \to \mathbb{R}^2$~~ $\pi/3$.

$$L : \mathbb{R}^3 \to \mathbb{R}^3.$$

Q29:- $L\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x-y \\ x+z \\ y-z \end{bmatrix}$

Sol:- For any $U = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ as in $\mathbb{R}^3$ we have

$$L(U) = L\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x-y \\ x+z \\ y-z \end{bmatrix}.$$

The standard matrix given here is;

$$A = \begin{bmatrix} L(e_1) & L(e_2) & L(e_3) \end{bmatrix}. \longrightarrow (1)$$

$$L(e_1) = L\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1-0 \\ 1+0 \\ 0+0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$L(e_2) = L\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 0-1 \\ 0+0 \\ 1-0 \end{bmatrix} \Rightarrow \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}.$$

$$L(e_3) = L\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 0-0 \\ 0+1 \\ 0-1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}.$$

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$ Ans. desired standard matrix.

Q30:- $L : \mathbb{R}^3 \to \mathbb{R}^3 \quad L(U) = -2u.$

Sol:- Since $\mathbb{R}^3$ so let $U = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$$L(U) = -2\begin{bmatrix} x \\ y \\ z \end{bmatrix} \Rightarrow \begin{bmatrix} -2x \\ -2y \\ -2z \end{bmatrix}$$ is dilation in opposite direction.

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The standard desired matrix is;

$$A = \begin{bmatrix} L(e_1) & L(e_2) & L(e_3) \end{bmatrix} \rightarrow ①$$

$$L(e_1) = L\begin{bmatrix}1\\0\\0\end{bmatrix} \Rightarrow \begin{bmatrix}-2\\0\\0\end{bmatrix}$$

$$L(e_2) = L\begin{bmatrix}0\\1\\0\end{bmatrix} \Rightarrow \begin{bmatrix}0\\-2\\0\end{bmatrix}$$

$$L(e_3) = L\begin{bmatrix}0\\0\\1\end{bmatrix} \Rightarrow \begin{bmatrix}0\\0\\-2\end{bmatrix}$$

eq ① ⇒ $A = \begin{bmatrix}-2 & 0 & 0\\0 & -2 & 0\\0 & 0 & -2\end{bmatrix}$ Ans is required standard matrix.

Q31:-

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26

Send him money.

19 5 14 4 8 9 13 13 15 14 5 25.

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$$\underbrace{19 \quad 5 \quad 14}_{x_1} \quad \underbrace{4 \quad 8 \quad 9}_{x_2} \quad \underbrace{13 \quad 13 \quad 15}_{x_3} \quad \underbrace{14 \quad 5 \quad 25}_{x_4}$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} \quad \& \quad A^{-1} = \begin{pmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{pmatrix}$$

$$AX_1 = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 19 \\ 5 \\ 14 \end{bmatrix} = \begin{bmatrix} 19+10+42 \\ 19+5+28 \\ 0+5+28 \end{bmatrix} = \begin{bmatrix} 71 \\ 52 \\ 33 \end{bmatrix}.$$

$$AX_2 = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 4 \\ 8 \\ 9 \end{bmatrix} \Rightarrow \begin{bmatrix} 4+16+27 \\ 4+8+18 \\ 0+8+18 \end{bmatrix} = \begin{bmatrix} 47 \\ 30 \\ 26 \end{bmatrix}.$$

$$AX_3 = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 13 \\ 13 \\ 15 \end{bmatrix} \Rightarrow \begin{bmatrix} 13+26+45 \\ 13+13+30 \\ 0+13+30 \end{bmatrix} = \begin{bmatrix} 84 \\ 56 \\ 43 \end{bmatrix}.$$

$$AX_4 = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 14 \\ 5 \\ 25 \end{bmatrix} \Rightarrow \begin{bmatrix} 14+10+75 \\ 14+5+50 \\ 0+5+50 \end{bmatrix} = \begin{bmatrix} 99 \\ 69 \\ 55 \end{bmatrix}.$$

Code of message send him money is;

71 52 33 47 30 26 84 56 43 99 69 55.

69   76   152   138   67   88
55   62   62   55   78   41
124   138   214   193   208   129
104   113   193
22   25   15

Decode the message 67 44 41 ~~89~~ 49 39 19 113 76 62 104 69 55 :

$$A^{-1}X_1 = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix}\begin{pmatrix} 67 \\ 44 \\ 41 \end{pmatrix} = \begin{bmatrix} 0+44-41 \\ 134-88-41 \\ -67+44+41 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \\ 18 \end{bmatrix}.$$

$$A^{-1}X_2 = \begin{pmatrix} 0 & +1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{pmatrix}\begin{pmatrix} 49 \\ 39 \\ 19 \end{pmatrix} = \begin{pmatrix} 0+39+(-19) \\ 98-78-19 \\ -49+39+19 \end{pmatrix} = \begin{pmatrix} 20 \\ 1 \\ 9 \end{pmatrix}.$$

$$A^{-1}X_3 = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix}\begin{pmatrix} 113 \\ 76 \\ 62 \end{pmatrix} = \begin{pmatrix} 76-62 \\ 226-152-62 \\ -113+76+62 \end{pmatrix} = \begin{pmatrix} 14 \\ 12 \\ 25 \end{pmatrix}.$$

$$A^{-1}X_4 = \begin{pmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{pmatrix}\begin{pmatrix} 104 \\ 69 \\ 55 \end{pmatrix} = \begin{bmatrix} 0+69-55 \\ 208-138-55 \\ -104+69+55 \end{bmatrix} = \begin{bmatrix} 14 \\ 15 \\ 22 \end{bmatrix}$$

The decode message will be;

3 5 18 20 1 9 14 12 25 14 15 22

C E R T A I N L Y N O W.

So decode message is

Certainly now.

Q32. $A = \begin{bmatrix} 5 & 3 \\ 2 & 1 \end{bmatrix}$ $A^{-1} = \begin{bmatrix} -1 & 3 \\ 2 & -5 \end{bmatrix}$.

encode: work hard.

23 15 18 11 8 1 18 4.

$AX_1 = \begin{bmatrix} 5 & 3 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 23 \\ 15 \end{bmatrix} \Rightarrow \begin{bmatrix} 115 + 45 \\ 46 + 15 \end{bmatrix} = \begin{bmatrix} 160 \\ 61 \end{bmatrix}$.

$AX_2 = \begin{bmatrix} 5 & 3 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 18 \\ 11 \end{bmatrix} \Rightarrow \begin{bmatrix} 90 + 33 \\ 36 + 11 \end{bmatrix} \Rightarrow \begin{bmatrix} 123 \\ 47 \end{bmatrix}$.

$AX_3 = \begin{bmatrix} 5 & 3 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 8 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 40 + 3 \\ 16 + 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 43 \\ 17 \end{bmatrix}$.

$AX_4 = \begin{bmatrix} 5 & 3 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 18 \\ 4 \end{bmatrix} \Rightarrow \begin{bmatrix} 90 + 12 \\ 36 + 4 \end{bmatrix} \Rightarrow \begin{bmatrix} 102 \\ 40 \end{bmatrix}$.

So code is; 160 61 123 47 43 17 102 40.

Decoding:- $\begin{bmatrix} -1 & 3 \\ 2 & -5 \end{bmatrix}\begin{bmatrix} 93 \\ 36 \end{bmatrix} \Rightarrow \begin{bmatrix} -93 + 108 \\ 186 - 180 \end{bmatrix} = \begin{bmatrix} 15 \\ 6 \end{bmatrix}$.

$A^{-1}X_2 = \begin{bmatrix} -1 & 3 \\ 2 & -5 \end{bmatrix}\begin{bmatrix} 60 \\ 21 \end{bmatrix} \Rightarrow \begin{bmatrix} -60 + 63 \\ 120 - 105 \end{bmatrix} \Rightarrow \begin{bmatrix} 3 \\ 15 \end{bmatrix}$.

$A^{-1}X_3 = \begin{bmatrix} -1 & 3 \\ 2 & -5 \end{bmatrix}\begin{bmatrix} 159 \\ 60 \end{bmatrix} \Rightarrow \begin{bmatrix} -159 + 180 \\ 318 - 300 \end{bmatrix} = \begin{bmatrix} 21 \\ 18 \end{bmatrix}$.

$A^{-1}X_4 = \begin{bmatrix} -1 & 3 \\ 2 & -5 \end{bmatrix}\begin{bmatrix} 110 \\ 43 \end{bmatrix} \Rightarrow \begin{bmatrix} -110 + 129 \\ 220 - 215 \end{bmatrix} \Rightarrow \begin{bmatrix} 19 \\ 5 \end{bmatrix}$.

Decode message is; 15 6 3 15 21 18 19 5.

O F C O U R S E.