Combined Stresses

Pb # 918 A compressive load $P = 12$ Kips is applied as shown in fig, at a point 1" to the right and 2" above the centroid of a rectangular section for which $h = 10$ in and $b = 6"$. Compute the stress at each corner & location of N.A.

Given data;

$P = 12$ kips

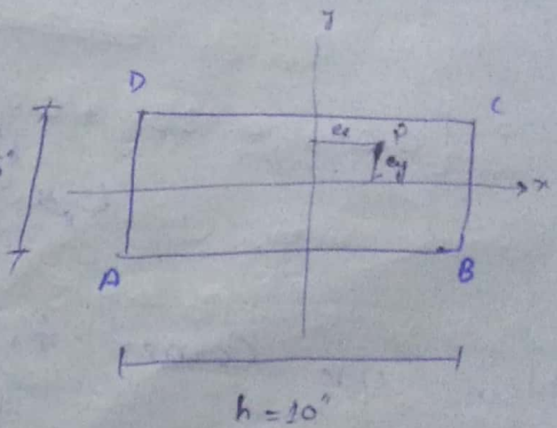
$e_x = 1"$

$e_y = 2"$

$h = 10"$

$b = 6"$

Figure:-



Required data:-

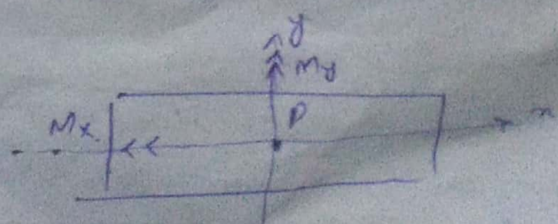
(a) σ_T at each Corner i.e A, B, C & D.

(b) Location of N. A.

Solution:-

Step # 01

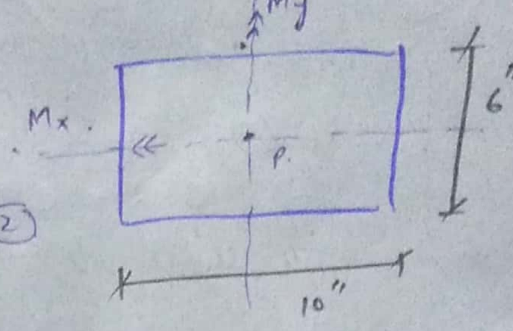
Bring the applied force to centroid of rectangular section.



Step #02

Write eqns of stresses for positive quadr

$$\delta_T = \delta_a + \delta_{fx} + \delta_{fy} \rightarrow ①$$



$$\delta_T = \frac{P}{A} + \frac{M_x \cdot y}{I_x} + \frac{M_y (x)}{I_y} \rightarrow ②$$

By putting values in eq ②

∴ Take compression as +ve.

$$\delta_T = \frac{12\times10^3}{60} + \frac{24\times10^3}{180} y + \frac{12\times10^3 (x)}{500}$$

Since $P = 12$ Kips.

$A = 10\times6$

$= 60 in^2$

$$\delta_T = [0.2 + 0.133y + 0.024x] \times 10^3 \rightarrow ③$$

$M_x = P.e_y$

$= 12\times10^3 \times 2$

$= 24\times10^3$ Kipsin

Now use Corner A, B, C & D to find stresses on those corners.

$M_y = Pe_x$

$= 12\times10^3 \times (1)$

$= 12$ Kip. inch

	x	y	δ_T (Ksi)
A	-5	-3	-0.319
B	5	-3	-0.079
C	5	3	0.719
D	-5	3	0.479

$I_x = \frac{10(6)^3}{12}$

$= 180 in^4$

$I_y = \frac{6(10)^3}{12}$

$= 500 in^4$.

Note :- -ve signs that those corners are in tension while +ve signs show those corners in Compression.

Location of Neutral axis:-

Since at N.A $\delta_T = 0$ so use eq ③.

eq ③ $\Rightarrow$ $\delta_T = [0.2 + 0.133y + 0.024x] \times 10^3$

$\therefore \delta_T = 0$

$$0 = 0.2 + 0.133y + 0.024x \longrightarrow ④.$$

To find x & y - intercepts.

For x intercept:-

Put $y = 0$

$x = x_i$

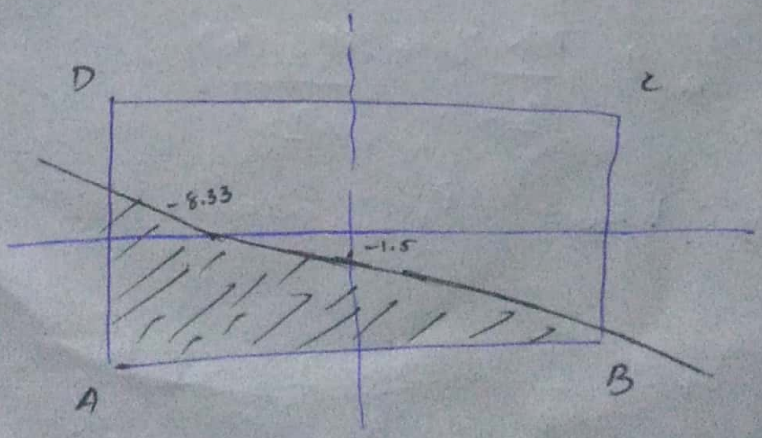
$$0.133(0) + 0.2 + 0.024(x_i) = 0$$

$$x_i = \frac{-0.2}{0.024} \Rightarrow \boxed{x_i = -8.33''}$$

For y intercept put $x = 0$ & $y = y_i$

$$y_i = \frac{-0.2}{0.133} \Rightarrow \boxed{y_i = -1.5''}$$

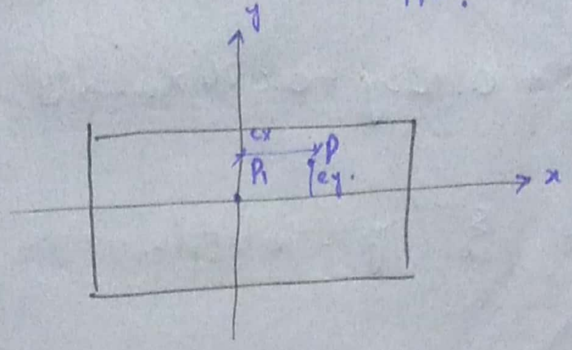
Diagram:-



Pb # 919 From the data in Pb # 918 what additional load applied at centroid is necessary so that no tensile stress will occur/exist anywhere on the cross section.

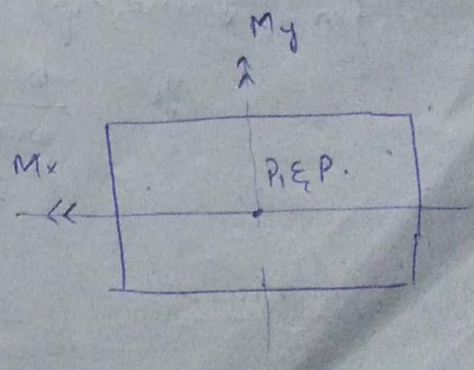
$P_1 = ?$

Figure:-



Solution:-

To find ~~centroid~~ load at centroid such that whole cross section is under compression. As we see from previous solution of Pb # 918 that corner A & B are in tension so we will consider those two corner.



As from eq (3).

$$\sigma_T = \frac{P}{A} + \frac{M_x y}{I_x} + \frac{M_y (x)}{I_y}$$

But here P_1 load also create ~~to~~ stress

So,

$$\sigma_T = \frac{P}{A} + \frac{M_x y}{I_x} + \frac{M_y (x)}{I_y} + \frac{P_1}{A} \longrightarrow (A).$$

By putting values in eq (A).

$$\sigma_T = \left(\frac{12}{60} + \frac{24}{180}y + \frac{12x}{500}\right) \times 10^{-2} + \frac{P_1 \times 10^{-3}}{60}.$$

To find P_1 put $\sigma_T = 0$

$$0 = 0.2 + 0.133y + 0.024x + 0.0167P_1$$

$$0 = 0.2 + 0.133y + 0.024x + 0.0167P_1 \quad \rightarrow (1)$$

For corner A (−5, −3)

$$0 = 0.2 + 0.133(-3) + 0.024(-5) + 0.0167P_1$$

$$0.0167P_1 - 0.319 = 0$$

$$P_1 = \frac{0.319}{0.0167}$$

$$\boxed{P_1 = 19.1 \text{ Kips}}$$

For corner B (5, −3).

$$0 = 0.2 + 0.133(-3) + 0.024(5) + 0.0167P_1$$

$$0.0167P_1 - 0.079 = 0.$$

$$\boxed{P_1 = 4.73 \text{ kips}}$$

We will consider $\boxed{P_1 = 19.1 \text{ kips}}$ since due to this force the whole cross section will be under compression.

$P = 12$ Kips

$e_x = 1''$

$e_y = 2''$

$A = 60 \text{ in}^2$.

$M_x = P \cdot e_y$

$= 24$ kips inch

$M_y = P \cdot e_x$

$= 12$ Kips inch

$I_x = \frac{10(6)^3}{12}$

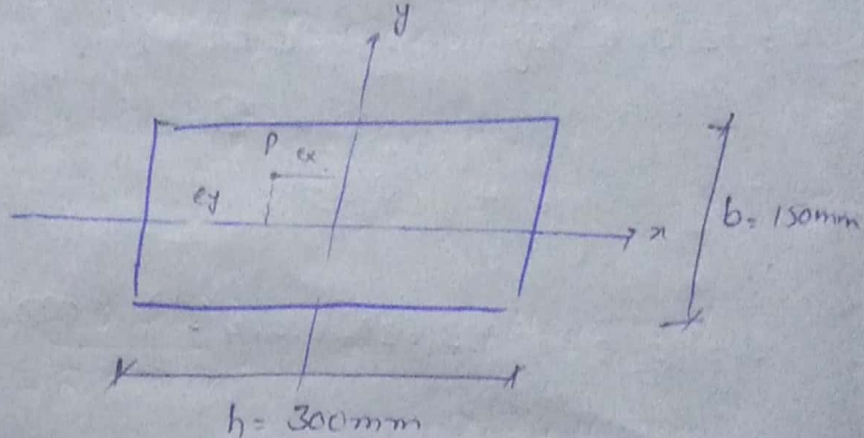
$= 180 \text{ in}^4$.

$I_y = \frac{6(10)^3}{12}$

$= 500 \text{ in}^4$.

Pb# 920:- A compressive load $P = 100\,kN$ is applied as shown in fig at a point 70mm to the left and 30mm to above centroid of rectangular section for which $h = 300mm$ & $b = 150mm$. What additional load, acting normal to cross section at its centroid, will eliminate tensile stress anywhere over cross section?

Figure:-



Given data;

$P = 100\,KN$

$e_x = 70mm$

$e_y = 30mm$

$h = 300mm$ & $b = 150mm$.

Required data:-

To find P_1 such that whole cross section is under compression.

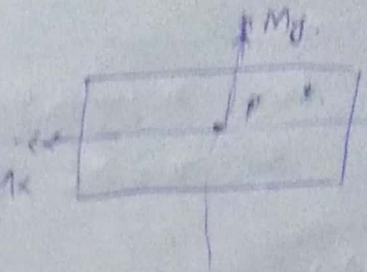
Solution:-

step #1 is to bring load $P = 100\,kN$ to centroid and solve for that so that we find at which corner there is tension & at which there is compression.

Step: Write eqns of stresses for positive quadrant

$\delta_T = \delta_a + \delta_{fx} - \delta_{fy} \longrightarrow (1)$

By solving (1)



$$\delta_T = \frac{P}{A} + \frac{Mx(y)}{Ix} - \frac{My(x)}{Iy} \longrightarrow (2)$$

$P = 100kN$

$A = 300 \times 150 = 45000 \, mm^2$

$Mx = Pey \Rightarrow 100kN \times 30 = 3000 \, kNmm$

$My = Pex \Rightarrow 100kN \times 70 = 7000 \, kNmm$

$Ix = \frac{bh^3}{12} = \frac{300(150)^3}{12} \Rightarrow 84375000 \, mm^4$

$Iy = \frac{hb^3}{12} = \frac{150(300)^3}{12} \Rightarrow \quad Iy = 337500000 \, mm^4$

By putting values in eq (2)

$$\delta_T = \left(\frac{100}{45000} + \frac{3000\,y}{84375000} - \frac{7000\,x}{337500000} \right) \times 10^3$$

$$\delta_T = (2.22 \times 10^{-3} + 3.55 \times 10^{-5} y - 2.074 \times 10^{-5} x) \times 10^3 \longrightarrow (A)$$

	x	Y	δ_T (MPa)
A	-150	-75	2.6685
B	150	75	-3.55
C	150	75	1.77
D	-150	75	7.99

Negative sign show that we have tension at one corner i.e at B.

Location of N.A:- (Additional) or optional

For x intercept put $y=0$ & $x = x_i$ & put $\delta_T = 0$.

of (A) ⇒

$$\delta_T = (2.22\times10^{-3} + 3.55\times10^{-5}y - 2.074\times10^{-5}x)\,10^{3}.$$

$$0 = 2.22 + 3.55\times10^{-2}y - 2.074\times10^{-2}x.$$

Put $y=0$.

$$x_i = \frac{-2.22}{-2.074\times10^{-2}} \Rightarrow \boxed{x_i = 107\,mm}$$

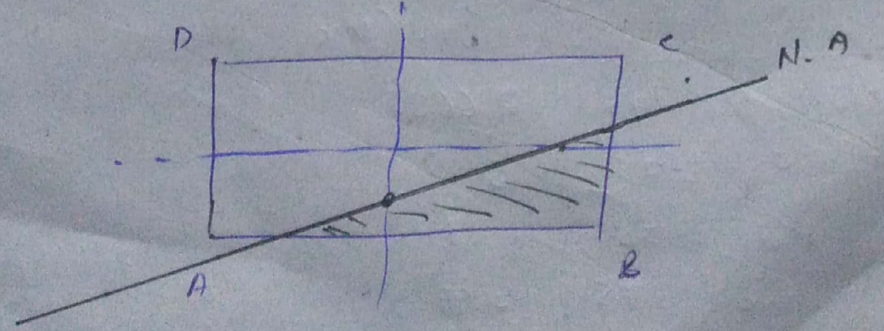
$$\boxed{x_i = +107\,mm}$$

For y intercept put $x=0$.

$$0 = 2.22 + 3.55\times10^{-2}y.$$

$$y_i = \frac{-2.22}{3.55\times10^{-2}} \Rightarrow \boxed{y_i = -62.53\,mm}$$

Sketch:- (optional).



Now consider load P_1 that acts on centroid so the required eqn will be;

$$\delta_T = (\delta_a)_P + \delta_{fx} - \delta_{fy} + (\delta_a)_{P_1}.$$

$$\delta_T = \frac{P}{A} + \frac{M_x y}{I_x} - \frac{M_y (x)}{I_y} + \frac{P_1}{A}.$$

By putting values.

$$\delta_T = \frac{100\times10^3}{45000} + \frac{3000\times10^3 y}{84375000} - \frac{7000\times10^3 x}{337500000} + \frac{1}{45000}P_1$$

$$\delta_T = 2.22 + 3.55\times10^{-2} y - 2.074\times10^{-2} x + \frac{1}{45000}P_1$$

to find P_1 such that $\delta_T = 0$.
Also put $x = 150$ & $y = -75$ for point B.

$$0 = -3.55 + \frac{1}{45000}P_1$$

$$\frac{1}{mm^2}\frac{1}{45000}P_1 = 3.55\ N/mm^2.$$

$$P_1 = 3.55\frac{N}{mm^2}\times 45000\ mm^2.$$

$$\boxed{P_1 = 159750N}$$

or

$$\boxed{P_1 = 160KN}$$ Ans

Pb #921:- Calculate & Sketch Kern of W360x122

Solution:- From Book we know the followings about W360x122.

Depth = Height = 363 mm

Width = 257 mm.

Area = 15500 mm^2.

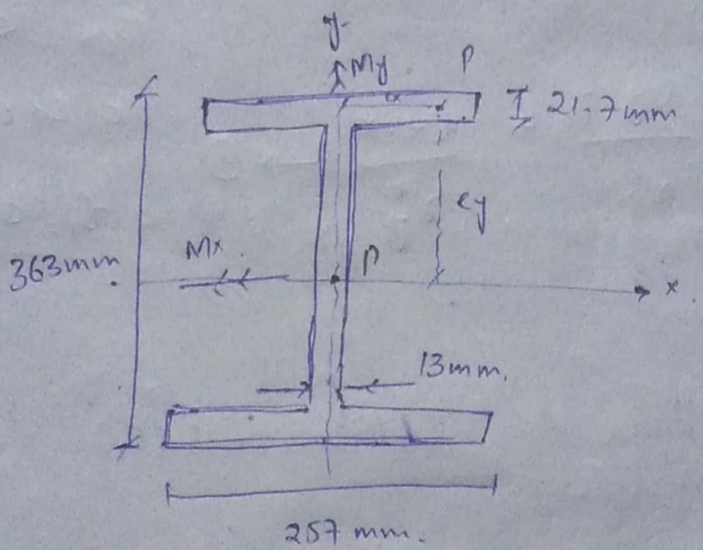
$I_x = 365 \times 10^6\ mm^4$.

$I_y = 61.5 \times 10^6\ mm^4$

Web thickness = 13 mm.

Flange thickness = 21.7 mm

Figure:-



Step#1.
Transfer load to centroid.

Step#2
Write eqns of stresses for +ve quadrant.

as;

$$\delta_T = \delta_p + \delta_{f_x} + \delta_{f_y}$$

$$\delta_T = \frac{P}{A} + \frac{M_x\, y}{I_x} + \frac{M_y \times x}{I_y} \longrightarrow (1)$$

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$$\sigma_T = \frac{P}{A} + \frac{P.e_y.y}{I_x} + \frac{P.e_x(x)}{I_y}.$$

$$\sigma_T = \frac{P}{A} + \frac{Pe_y.y}{\cancel{I_x}\, r_x^2 A} + \frac{Pe_x.x}{r_y^2 A}.$$

But $\sqrt{I_x/A} = r.$

$I_x = r_x^2 A.$

$$0 = \frac{P}{A}\left[1 + \frac{e_y.y}{r_x^2} + \frac{e_x x}{r_y^2}\right].$$

For Kern of a section $\sigma_T = 0.$

$$0 = 1 + \frac{e_y.y}{r_x^2} + \frac{e_x.x}{r_y^2} \rightarrow (1)$$

<u>Now For Corner A:-</u> $(\overset{x}{-128.5}\,;\overset{y}{-181.5}).$

Put Values in eq (1)

$$0 = 1 + \frac{e_y(-181.5)}{23548.4} + \frac{e_x(-128.5)}{3967.74}$$

$r_x^2 = \frac{I_x}{A} = 23548.4\ mm^2$

$0 = 1 - 7.71\times 10^{-3} e_y - 0.0324\, e_x$

$r_y^2 = \frac{I_y}{A} = 3967.7\ mm^2$

For $e_x = 0$, $e_y = ?$

~~e_y (-1/0.0324)~~

$0 = 1 - 7.71\times10^{-3} e_y \Rightarrow e_y = -1/-7.71\times10^{-3}$

$\boxed{e_y = 129.74\,mm}$

For $e_x = ?$ $e_y = 0$.

$0 = 1 - 0.0324\,e_x \Rightarrow e_x = \frac{-1}{-0.0324}$

$\boxed{e_x = 30.86\,mm}$

For Corner B:- $(128.5, -181.5)$.

eq ① ⇒

$$0 = 1 + \frac{e_y(-181.5)}{23548.4} + \frac{e_x(128.5)}{3967.74}$$

$$0 = 1 - 7.71\times10^{-3}\,e_y + 0.0324\,e_x$$

For e_x:-

$e_x = \frac{-1}{0.0324}$

$\boxed{e_x = -30.86\,mm}$

For $e_y = ?$

$e_y = \frac{-1}{-7.71\times10^{-3}}$

$\boxed{e_y = 129.74\,mm}$

For Corner C:- $(128.5, 181.5)$

Using eq ① By Putting values we get.

$$0 = 1 + 7.71\times10^{-3}\,e_y + 0.0324\,e_x$$

For e_x:-

$e_x = \frac{-1}{0.0324}$

For $e_y = ?$

$e_y = \frac{-1}{+7.71\times10^{-3}}$

$\boxed{e_x = -30.86\,mm}$ $\boxed{e_y = -129.74\,mm}$

For Corner D:- $(-128.5, +181.5)$

For e_x:-

$$e_x = \frac{-1}{-0.0324}$$

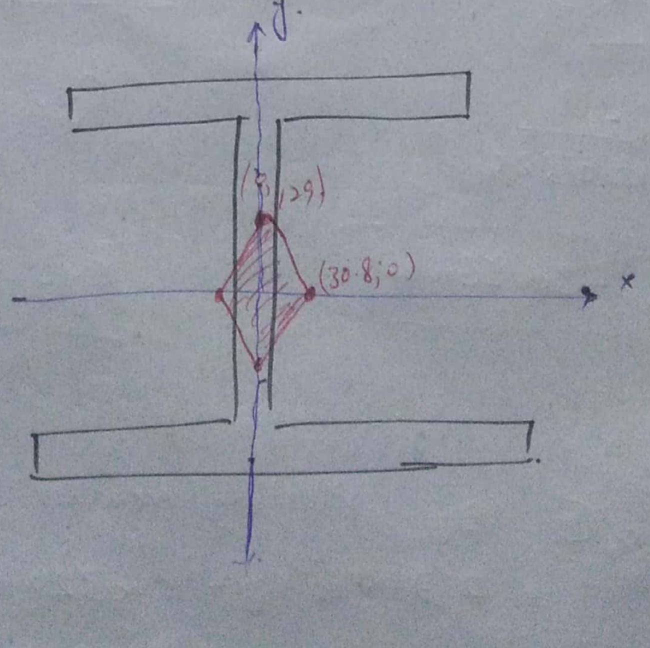
For e_y:-

$$e_y = \frac{-1}{7.71\times10^{-3}}$$

$\boxed{e_x = 30.86\,mm}$, $\boxed{e_y = -129.74\,mm}$

Let Consider Figure:-

Fig:- 7



Pb# 922:-

Calculate & sketch the Kern of W10x112:

Solution:-

From book we get the following d

For W10x112.

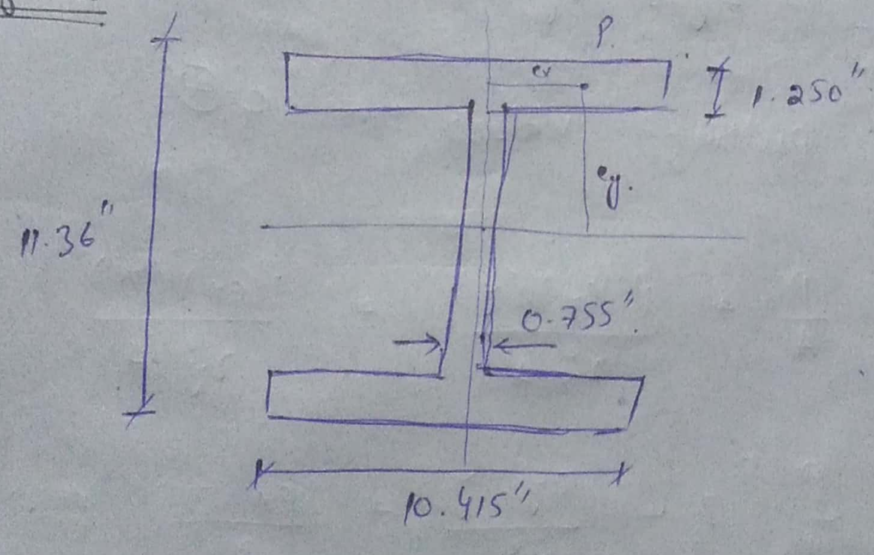
Depth or height = 11.36" Web thickness = 0.755"

Area = 32.9 in^2. Flange width = 10.415".

$I_x = 716 \ in^4$ Flange thickness = 1.250".

$I_y = 236 \ in^4$.

Figure:-



Consider the first quadrant for load P.

$$S_T = S_a + S_{fx} + S_{fy} \longrightarrow ①.$$

By putting values.

$$S_T = \frac{P}{A} + \frac{M_x(y)}{I_x} + \frac{M_y(x)}{I_y}$$

To find kern, $\delta_T = 0$.

$$0 = \frac{P}{A} + \frac{M_x(y)}{I_x} + \frac{M_y(x)}{I_y} \rightarrow (A)$$

$M_x = P(e_y)$

$M_y = P(e_x)$

Since; $r = \sqrt{\frac{I}{A}} \Rightarrow \boxed{r^2 = I/A}$

$I_x = r_x^2 A$.

$I_y = r_y^2 A$.

By putting values in eq (A).

$$0 = \frac{P}{A} + \frac{P(e_y)(y)}{r_x^2 A} + \frac{P(e_x)(x)}{r_y^2 A}$$

$$0 = \frac{P}{A}\left[1 + \frac{e_y \cdot y}{r_x^2} + \frac{e_x(x)}{r_y^2}\right]$$

$$0 = 1 + \frac{y e_y}{r_x^2} + \frac{x e_x}{r_y^2} \longrightarrow (B)$$

For Corner A $(-5.2075, -5.36)$.

$$0 = 1 + \frac{e_y(-5.36)}{21.763} + \frac{e_x(-5.2075)}{3.03}$$

$r_x^2 = \frac{I_x}{A}$

$= \frac{716}{32.9}$

$0 = 1 - 0.2463\, e_y - 1.72\, e_x \rightarrow ©$

For e_x:-

Put $e_y = 0$

$e_x = \frac{-1}{-1.72}$

$\boxed{e_x = 0.58''}$

For e_y:-

Put $e_x = 0$.

$e_y = \frac{-1}{-0.2463}$.

$\boxed{e_y = 4.06 \text{ in}}$

For Corner B:- $(5.2075, -5.36)$

$0 = 1 - 0.2463 e_y + 1.72 e_x$

For e_x:-

Put $e_x = 0$.

$e_x = \frac{-1}{1.72} \Rightarrow$

$\boxed{e_x = -0.58''}$

For e_y:-

Put $e_y = 0$.

$e_y = \frac{-1}{-0.2463}$

$\boxed{e_y = 4.06''}$

For Corner C:- $(5.2075, 5.36)$:-

For e_y:-

Put $e_x = 0$.

$e_y = \frac{-1}{0.2463}$

$\boxed{e_y = -4.06''}$

For e_x:-

Put $e_y = 0$

$e_x = \frac{-1}{1.72}$

$\boxed{e_x = -0.58''}$

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For Corner D $(-5.2075, 5.36)$

eq (B) $\Rightarrow$ $0 = 1 + \frac{y e_y}{r_x^2} + \frac{x e_x}{r_y^2}$.

By putting values.

$0 = 1 + 0.2463 e_y - 1.72 e_x \rightarrow$ (D)

<u>For e_x:-</u>

Put $e_y = 0$.

$e_x = \frac{-1}{\cancel{0.2463}\ 1.72}$.

$\boxed{e_x = -0.58''}$.

<u>For e_y:</u>

Put $e_x = 0$.

$e_y = \frac{-1}{-0.2463}$

$\boxed{e_y = 4.06''}$

Ans

<u>Sketch:-</u>