

Chapter # 11

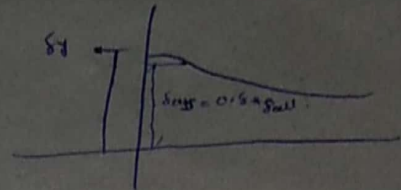
Columns

Pb # 1101 Select the lightest I shape that can be used as a column 7m long to support an axial load of 450 kN with a factor of safety 3. Assume (a) Both ends hinged & (b) One end fixed & other end hinged. Use $\sigma_{pi} = 200 \text{ MPa}$ & $E = 200 \text{ GPa}$.

Solution:- When both ends hinged:-

$$\sigma_{assume} = 0.8 \times \sigma_{all} \quad \text{--- (1)}$$

First select short column
whose $l/r = 0$.



$$F.S = 5/3$$

$$\sigma_{all} = \left(1 - \frac{(l/r)^2}{2C_c^2}\right) \frac{\sigma_{yp}}{F.S} \quad \text{--- (2)}$$

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_{yp}}} \Rightarrow C_c = \sqrt{\frac{2 \times \pi^2 \times 200 \times 10^3}{200}} = 140.42$$

$$\text{Q.1} \Rightarrow \sigma_{all} = \left(1 - \frac{0}{2C_c^2}\right) \frac{200}{5/3} \Rightarrow \sigma_{all} = 120 \text{ MPa}$$

$$\text{Q.2} \Rightarrow \sigma_{ass} = 0.8 \times 120 \Rightarrow \sigma_{ass} = 96 \text{ MPa}$$

$$P = \sigma_{all} \times A$$

$$450 \times 10^3 = \frac{450 \times 10^3}{96 \times 10^6} \times A \Rightarrow A = 468.75$$

$$A = 4687.5 \text{ mm}^2$$

$A = 4687.5 \text{ mm}^2$
 So select a W section whose area is
 compatible with this area.

Let select W250x39

$$A = 4920 \text{ mm}^2$$

$$r_{\min} = 34.7 \text{ mm}$$

$$(L/r) = \frac{1 \times 7 \times 1000}{34.7}$$

$$\boxed{L/r = 201.7}$$

Now $C_c = \sqrt{\frac{2 \pi^2 E F}{S_y}}$

$$= \sqrt{\frac{2 \times 3.14^2 \times 200 \times 10^3}{250}}$$

$$\boxed{C_c = 79.246}$$

As $L/r > C_c$ so long column.

Use

$$S_{all} = \frac{12}{23} \frac{\pi^2 E}{(L/r)^2}$$

$$= \frac{12}{23} \times \frac{(3.14)^2 \times 200 \times 10^3}{(201.7)^2} \Rightarrow S_{all} = 25.28 \text{ MPa}$$

$$P = S_{all} \times A$$

$$P_w = 25.28 \times 4920 \Rightarrow$$

$$\boxed{P_w = 124.4 \text{ kN}}$$

$$As P_{cal} < P_{given}$$

so this section is inadequate.

Let select

~~W10x~~

W250x73.

9280

$$A = 9280 \text{ mm}^2$$

$$r_{\min} = 64.7 \text{ mm}$$

$$l/r = \frac{1 \times 7000}{64.7}$$

$$\boxed{l/r = 108.2}$$

$$C_c = \sqrt{\frac{2\pi^2 E}{S_y}} = \sqrt{\frac{2 \times (3.14)^2 \times (200 \times 10^3)}{200}}$$

$$C_c = 79.246$$

As $l/r > C_c$ So Long Column.

$$A3; \quad \sigma_{all} = \left(\frac{12}{23} \frac{\pi^2 E}{(l/r)^2} \right)$$

$$= \frac{12}{23} \times \frac{(3.14)^2 \times (200 \times 10^3)}{(108.2)^2}$$

$$\sigma_{all} = 87.88 \text{ MPa}$$

$$A3; \quad \sigma_{all} = P/A$$

$$P = \sigma_{all} \times A$$

$$= 87.88 \times 9280$$

$$\boxed{P_{cal} = 815.5 \text{ kN}}$$

As $P_{cal} \geq P_{given}$ So
W250x73 is our required section.

$$A_0 = A_{cal} = 4687.5 \text{ mm}^2$$

So let section $W 460 \times 46$

$$A = 8140 \text{ mm}^2$$

$$r_{min} = 48.1$$

First find

$$l_e/r = \frac{0.7 \times 7 \times 1000}{48.1}$$

As hinge is
fixed
so $K = 0.7$

$$l_e/r = 101.87 \rightarrow \textcircled{A}$$

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_{pi}}}$$

$$= \sqrt{\frac{2 \times (3614)^2 \times 200 \times 10^3}{200}}$$

$$C_c = 140.4$$

As $C_c > l_e/r$ so Short or Intermediate column.

Use;

$$\sigma_{al} = \left(1 - \frac{l_e/r}{2C_c}\right) \frac{\sigma_{yp}}{F.S} \rightarrow \textcircled{B}$$

$$F.S = 5/3 + 3/8 \left(\frac{(l_e/r)^2}{C_c} \right) - \frac{(l_e/r)^3}{8C_c^3}$$

$$= 5/3 + \frac{3}{8} \frac{101.87}{140.4} - \frac{(101.87)^3}{8(140.4)^3}$$

$$= 5/3 + 0.2721 - 0.0477$$

$$\boxed{F.S = 1.89}$$

$$\sigma_{al} = \left(1 - \frac{101.87}{2(140.4)^2}\right) \left(\frac{200}{1.89}\right)$$

$$\boxed{\sigma_{al} = 105.54 \text{ MPa}}$$

Pb # 1102

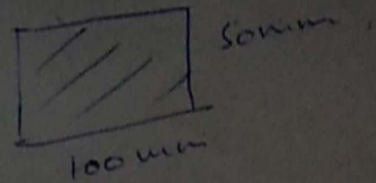
Given data:
 $A = 50 \text{ mm by } 100 \text{ mm}$

1) $L = ?$

$E = 10 \text{ GPa}$

$\delta_{PL} = 30 \text{ MPa}$

(ii) F.S. = 2
 length = ?



Sol:- First to find
 length = ?

As we know

$$L_e/r = \sqrt{\frac{\lambda^2 E}{\delta_{PL}}} = \sqrt{\frac{(3.14)^2 (10 \times 10^3)}{30}}$$

$$L_e/r = 57.33$$

$$L_e = r \times 57.33$$

$$L_e = 14.43 \times 57.33$$

$$L_e = 827.48 \text{ mm}$$

Fixed ends,
 so, $L_e = \frac{1}{2}L$

$$\frac{1}{2}L = 827.48 \text{ mm}$$

$$L = 1654.97 \text{ mm}$$

$$L = 1.65 \text{ m}$$

To find

$$r = \sqrt{\frac{I}{A}}$$

$$r = \sqrt{\frac{100 \times 50^3}{12 \times 100 \times 50}}$$

$$r = \sqrt{\frac{50^2}{12}}$$

$$r = 14.43 \text{ mm}$$

(b) Find the critical load P_{cr}

$$A_3 \quad P_{cr} = \frac{\pi^2 EI}{(L_e)^2} \quad \rightarrow (A)$$

$$\cancel{P_{cr}} = \frac{\pi^2 (10 \times 10^3) (50^3 \times 100)}{(82.746)^2}$$

$$A_3 \quad L = 2.5 \text{ m}$$

$$L_e = \frac{1}{2} L = \frac{2.5}{2} = 1.25 \text{ m}$$

$$\therefore (A) \Rightarrow P_{cr} = \frac{\pi^2 (10 \times 10^3) (50^3 \times 100)}{12 (1.25)^2 \times 10^3}$$

$$P_{cr} = 65730.67 \text{ N}$$

$$A_4 \quad P_{allowable} = \frac{P_{cr}}{2}$$

$$= \frac{65730.67 \text{ N}}{2}$$

$$P_{all} = 32865.33 \text{ N}$$

or
?

$$P_{all} = 32.86 \text{ kN}$$

Ans

Prob 1103 Given data:

$$L = 6'$$

$$A = \frac{3}{4}'' \times 2''$$

A₁ act as hinged column about axis \perp to $2''$
A₂ act as fixed column about axis \perp to $\frac{3}{4}''$

$$P_{safe} = ?$$

$$F.S = 2$$

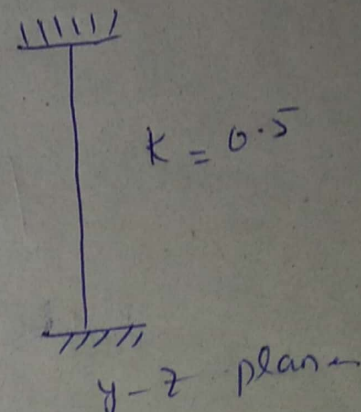
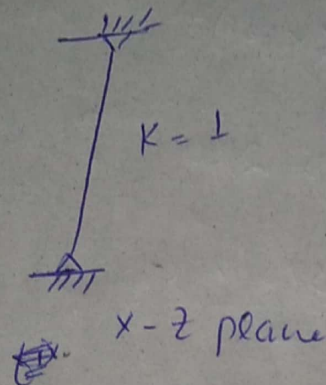
$$E = 10.3 \times 10^6 \text{ psi}$$

Sol:- A₁ act as hinged
column about axis \perp to

$2''$

So the buckling will
not produce in y-plane
it will act as hinged
column in x-z plane.

Similarly buckling will also reduce for fixed
support in y-axis. So buckling will produce
in just y-z plane.



The P_{cr} required to produce buckling in x-z plane is,

$$P_{cr} = \frac{\pi^2 EI}{(L_e)^2}$$

$$= \frac{\pi^2 \times 10.3 \times 10^6 \times \frac{(3/4)(2)^3}{12}}{(1 \times 6 \times 12)^2}$$

$$P_{cr} = 9804 \text{ lb.}$$

$$P_{all} = \frac{P_{cr}}{F.S} = \frac{9804 \text{ lb.}}{2}$$

$$\boxed{P_{all} = 4902 \text{ lb}}$$

For buckling produce in y-z plane.

$$P_{cr} = \frac{\pi^2 EI}{(L_e)^2}$$

$$= \frac{\pi^2 \times 10.3 \times 10^6 \times \frac{(2)(3/4)^3}{12}}{(0.5 \times 6 \times 12)^2}$$

$$\boxed{P_{cr} = 5515.2 \text{ lb}}$$

$$P_{all} = \frac{P_{cr}}{F.S} = \frac{5515.2 \text{ lb.}}{2}$$

$$\boxed{P_{all} = 2757.6 \text{ lb}}$$

for safest load select $P = 2757.6 \text{ lb}$ $\underline{\text{Ans}}$

Pb # 1104

For

Given data:

Rounded end

$$P = 20 \text{ Kips}$$

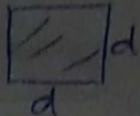
$$K = 1$$

$$L = 10 \text{ ft.}$$

$$E = 29 \times 10^6 \text{ psi}$$

Solution -

To find length of each side

As square = 

So,

Since,
$$P_{cr} = \frac{\pi^2 EI}{L^2}$$

$$I = \frac{PL^2}{\pi^2 E}$$

$$d^4 = \frac{12 P_{cr} L^2}{\pi^2 E}$$

For square.

$$I = \frac{d(d)^3}{12}$$

$$= \frac{d^4}{12}$$

$$d^4 = \frac{12 \times 20 \times 10^3 \times (10 \times 12)^2}{(3.14)^2 \times (29 \times 10^6)}$$

$$d^4 = 12.08$$

$$d = (12.08)^{1/4}$$

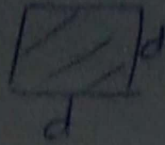
$$\boxed{d = 1.86''}$$

So each side

of square is 1.86". Ans

Repeat Prob 1104 for column made of wood
for which $E = 1.6 \times 10^6 \text{ psi}$.

Sol.



to find $d = ?$

$$\text{As, } P_{cr} = \frac{\pi^2 EI}{L_e^2}$$

$$I = \frac{P_{cr} L_e^2}{\pi^2 E}$$

$$\text{Since for square } I = \frac{d(d)^3}{12} \\ = \frac{d^4}{12}$$

$$\frac{d^4}{12} = \frac{P_{cr} L_e^2}{\pi^2 E}$$

$$d^4 = \frac{12 P_{cr} L_e^2}{\pi^2 E}$$

$$d^4 = \frac{12 \times 20 \times 10^3 (120)^2}{(3.14)^2 \times (1.6 \times 10^6)}$$

$$d^4 = 219.1$$

$$d = (219.1)^{1/4}$$

$$\boxed{d = 3.84''} \text{ Ans}$$

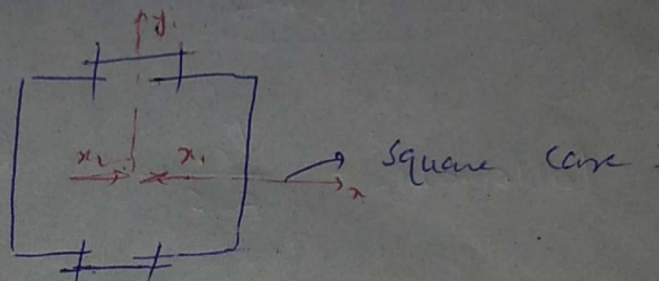
Pb #1106

Two $C310 \times 45$ channels are latticed together, so they have equal moment of inertia about principal axis. Determine the minimum length of column having this section assuming pinned ends. $E = 200 \text{ GPa}$ and proportional limit of 240 MPa . What safe load will be column carry for length of 12 m with factor of safety of 2.5 .

Solution:- Already solved!

Pb #1107:- Repeat Pb #1106 assuming that one end is fixed & other is hinged.

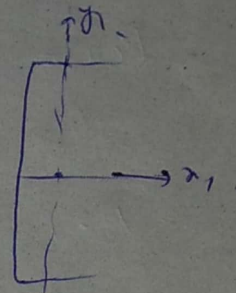
Solution:- As we have this case that $I_x = I_y$.



As $I_{x_1} = 67.3 \times 10^6 \text{ mm}^4$

$A = 5690 \text{ mm}^2$

$I_{x_2} = 2.12 \times 10^6 \text{ mm}^4$



As Symmetric about x -axis.

So, $I_x = 2I_{x_1}$

$= 2(67.3 \times 10^6 \text{ mm}^4)$

$I_x = I_y = 134.6 \times 10^6 \text{ mm}^4$

To find slenderness ratio

$$l/r = \sqrt{\frac{\pi^2 E}{\rho_L}}$$

$$= \sqrt{\frac{(3.14)^2 \times 200 \times 10^3}{240}}$$

$$l/r = 90.6$$

$$l = 90.6 \times r$$

$$= 90.6 \times \sqrt{\frac{I}{A}}$$

$$= 90.6 \times \sqrt{\frac{134.6 \times 10^6}{5696 \times 2}}$$

$$l = 9.8 \text{ m}$$

As; $P_{allowable} = \frac{P_{cr}}{F.S.} \rightarrow \textcircled{A}$

$$P_{cr} = \frac{\pi^2 EI}{(l)^2}$$

As; $l = KL$

$$= 0.7 \times 12 \text{ m}$$

$$= 8.4 \text{ m}$$

As Fixed-hinged

$$K = 0.7$$

$$P_{cr} = \frac{(3.14)^2 \times 200 \times 10^3 \times 134.6 \times 10^6}{(8.4 \times 1000)^2}$$

$$P_{cr} = 3761627.438 \text{ N}$$

$$P_{cr} = 3761.627 \text{ kN}$$

$$P_{cr} = 3761.6 \text{ kN}$$

$$\begin{aligned} \text{Q. (A)} \Rightarrow P_{\text{allowable}} &= \frac{P_{cr}}{2} \\ &= \frac{3761.6}{2.5} \end{aligned}$$

$$P_{\text{allowable}} = 1504.64 \text{ kN}$$

Pb #1108:-

Select the lightest W shape that will act as column 8m long with hinged end and support an axial load of 270 kN. Assume that the proportional limit is 200 MPa & $E = 200 \text{ GPa}$.
 $F.O.S = 2.5$

Solution:- Given data
 $l = 8 \text{ m}$
 Hinged ends.
 $P = 270 \text{ kN}$
 $\sigma_{PL} = 200 \text{ MPa}$
 $E = 200 \text{ GPa}$
 $F.O.S = 2.5$

$$\text{Also, } P_{cr} = F.O.S \times P_{\text{allowable}}$$

$$P_{cr} = 2.5 \times 270$$

$$P_{cr} = 675 \text{ kN}$$

As we know that;

$$P_{cr} = \frac{\pi^2 E I}{l^2}$$

$$I = \frac{P_{cr} l^2}{\pi^2 E}$$

$$I = \frac{675 \times 10^3 \times (1.8 \times 1000)^2}{(3.14)^2 \times 200 \times 10^9}$$

Ans: $l = 1$

~~$I = 21.9 \times 10^6$~~

~~$I = 21.9 \times 10^6 \text{ mm}^4$~~

Ans; $I = \frac{P_{cr} l^2}{\pi^2 E}$

$$I = \frac{675 \times 10^3 \times (8)^2}{(3.14)^2 \times (200 \times 10^9)}$$

$$I = 2.19 \times 10^{-5} \text{ m}^4 \Rightarrow 21.9 \times 10^{-6} \text{ m}^4$$

or $I = 21.9 \times 10^6 \text{ mm}^4$

But we also know that,

$$l/r = \sqrt{\frac{\pi^2 E}{A_{pl}}}$$

$$l/r = \sqrt{\frac{(3.14)^2 \times (200 \times 10^3)}{200}}$$

$$l/r = 99.29$$

To find $r = ?$

Ans; $l/r = 99.29$

$$r = \frac{4}{99.29}$$

$$r = \frac{(8 \times 1000 \times 1)}{99.29}$$

$$r = 80.6 \text{ mm}$$

So the selection (selected) section has

$$I \approx r > I_{cal.}$$

$$\{ r_{cal} < r_{given}$$

So To looking $W_{250 \times 67}$ section the above two criteria are satisfied whose

$$I = 22.2 \quad \{ \quad r = 51 \text{ mm}$$

So the lightest weight I section is $W_{250 \times 67}$.

Pb # 1109:- Select the lightest W-shape that will act as column 40' long with fixed ends and support an axial load of 150 Kips with factor of safety of 2. Assume that proportional limit is 30 Ksi. Use $E = 29 \times 10^6 \text{ psi}$. What set of criteria determines the section.

Sol:- As given $P_{all} = 150 \text{ Kips}$

$$As, P_{cr} = F.O.S \times P_{all}$$

$$= 2 \times 150 \text{ Kips}$$

$$P_{cr} = 300 \text{ Kips}$$

Now find $I = ?$

$$\text{Ans } P_{cr} = \frac{\pi^2 EI}{L^3}$$

$$I = \frac{P_{cr} \times L^3}{E \pi^2}$$

$$I = \frac{(300 \times 1000) \times (40^3 \times 10^{-9})}{29 \times 10^6 \times (3.14)^2}$$

$$I = 0.6419 \text{ ft}^4$$

$$\boxed{I = 870.2 \text{ in}^4} \Rightarrow \boxed{I = 8702.6 \text{ in}^4}$$

Now check for $\gamma = ?$

$$\text{Ans, } L/\gamma = \sqrt{\frac{\pi^2 E}{\rho PL}}$$

$$L/\gamma = \sqrt{\frac{(3.14)^2 (29 \times 10^6)}{(3 \times 1000)}}$$

$$\boxed{L/\gamma = 308.7}$$

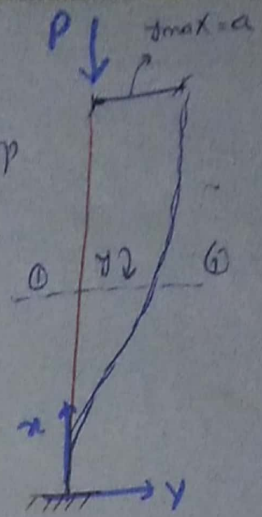
To find $\gamma = ?$ $\gamma = \frac{L}{308.7} = (0.5 \times 40 \times 12)$

$$\gamma = \frac{(20 \times 12)}{308.7} \Rightarrow \boxed{\gamma = 0.78 \text{ in}}$$

Centrically loaded Column:-

Consider a column of length L on which load P is applied at top whose one end is fixed and other end is free as shown.

When the load on column become equal to critical load the column will deflect as shown (buckle)



The maximum buckling (deflection) is y_{\max} as shown.

$$\text{i.e. } y_{\max} = a.$$

Consider section ①-① ^{down} ~~top~~ area (portion).

$$M = P(a - y) \quad \text{where } y' = (a - y).$$

From eqn of elastic curve.

$$EI \frac{d^2y}{dx^2} = M = P(a - y).$$

General solution of deflected shape of column.

$$y = A \sin k'x + B \cos k'x + a \rightarrow (1)$$

$$\text{where } k' = \sqrt{\frac{P}{EI}}.$$

To find value of A & B
apply boundary condition
 $x=0$; $y=0$

$$\text{Eq ①} \Rightarrow 0 = A(0) + B(1) + a$$

$$\boxed{B = -a}$$

Similarly

$$\text{When } x=0 \quad \frac{d}{dx} y = 0$$

$$\frac{d}{dx} y = \frac{d}{dx} \text{Eq ①} = A \cos k'x + B(-\sin k'x)$$

Apply condition.

$$0 = A \Rightarrow \boxed{A = 0}$$

Eq ① \Rightarrow Put values in ①

$$y = -a \cos k'x + a \rightarrow \text{②}$$

We also know that

$$\text{When } x=L \quad y = y_{\max} = a$$

$$\text{Eq ②} \Rightarrow y_{\max} = a = -a \cos k'L + a$$

$$a = -a \cos k'L + a$$

$$-a \cos k'L = 0 \rightarrow 0$$

When $a=0$, it means there is no buckling we not interest in that case

so, we consider

$$\cos k'L = 0.$$

To satisfy eq (3)
we will consider
that,
 $\cos k'L = 0,$

$$\cos(n\pi/L) = 0.$$

$$\therefore k'L = n\pi/L$$

$$\text{Since } k' = \sqrt{\frac{P}{EI}}.$$

$$\sqrt{\frac{P}{EI}} L = n\pi/L$$

Taking square,

$$\frac{PL^2}{EI} = \frac{n^2 \pi^2}{4}.$$

$$P = \frac{n^2 \pi^2 EI}{(2L)^2}.$$

$$\text{Since } L_e = 2L.$$

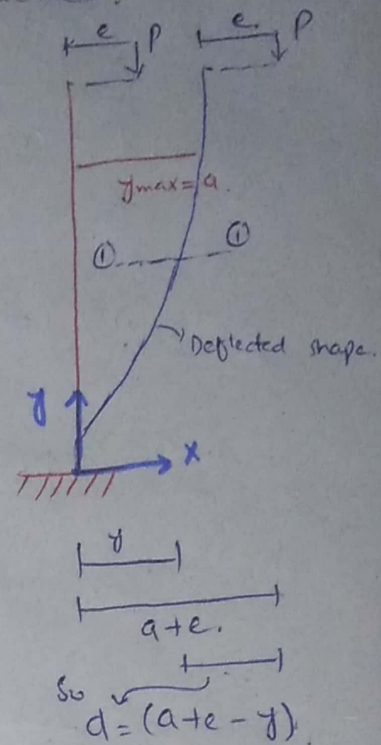
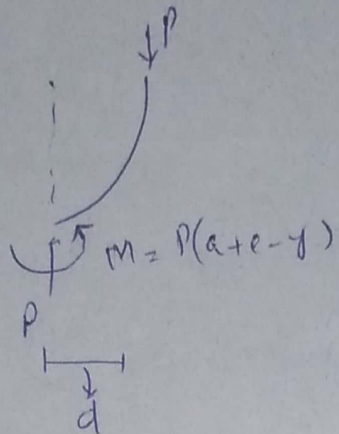
$$P = \frac{n^2 \pi^2 EI}{(L_e)^2}$$

Eccentrically loaded Column:-

Consider a column of length L whose one end is fixed and other is free.

Let the load is applied at eccentricity " e " when $P = P_{cr}$ the column will buckle, as shown.

Consider section ①-①



From eqn of elastic curve

$$EI \frac{d^2 y}{dx^2} = M = P(a + e - y).$$

General solution of deflected shape of column is

$$y = A \sin k'x + B \cos k'x + a + e \rightarrow (1)$$

$$\text{where } k' = \sqrt{\frac{P}{EI}}$$

Apply Initial (boundary) conditions.

When $x = 0$, $y = 0$.

$$(1) \Rightarrow 0 = A(0) + B(1) + a + e$$

$$\boxed{B = -a - e}$$

$$B = -(a + e)$$

$$\text{When } x = 0, \frac{dy}{dx} = 0.$$

$$\frac{dy}{dx} = ?$$

$$\text{①} \Rightarrow \frac{dy}{dx} = A \cos k'x - B \sin k'x \rightarrow \text{②}$$

$$0 = A (1)$$

$$\boxed{A = 0}$$

$$\text{③} \Rightarrow$$

$$y = -(a + e) \cos k'x + a + e \rightarrow \text{③}$$

③ gives buckled shape of deformed column.

$$\text{When } x = L, y = y_{\max} = a.$$

$$y = y_{\max} = y = a = -(a + e) \cos k'L + a + e.$$

$$a = -(a + e) \cos k'L + a + e$$

$$0 = -(a + e) \cos k'L + e \rightarrow \text{④}$$

$$(a + e) \cos k'L = e.$$

Divide by $\cos k'L$.

$$a + e = \frac{e}{\cos k'L} \Rightarrow a + e = \sec k'L (e) \rightarrow \text{⑤}$$

$$-a \cos k'L - e \cos k'L + e = 0$$

$$a \cos k'L = -e \cos k'L + e = e - e \cos k'L$$

$$a \cos k'L = e (\cos k'L + 1)$$

$$a \cos k'L = e (1 - \cos k'L)$$

Divide by $\cos k'L$.

$$a = e \left(\frac{1}{\cos k'L} - \frac{\cos k'L}{\cos k'L} \right)$$

$$a = e (\sec k'L - 1)$$

Since $y_{\max} = a$.

$$y_{\max} = a = e [\sec k'L - 1] \rightarrow \textcircled{6}$$

Eq ⑥ show equation of deflected column. which shows that deflection is max when either e is maximum or $[\sec k'L - 1]$ is max. As e is constant so we have possibility is;

$$\sec k'L - 1 = \infty$$

∞ for max.

$$\sec k'L = 1 + \infty$$

$$\therefore 1 + \infty = \infty$$

$$\sec k'L = \infty$$

$$\sec\left(n\pi/2\right) = \infty$$

$$\text{So, } kL = n\pi/2$$

$$\text{As } k' = \sqrt{\frac{P}{EI}}$$

$$\sqrt{\frac{P}{EI}} L = n\pi/2$$

$$\frac{P}{EI} L^2 = \frac{n^2 \pi^2}{4}$$

$$P = \frac{n^2 \pi^2 EI}{(2L)^2}$$

$$\therefore L_e = 2L$$

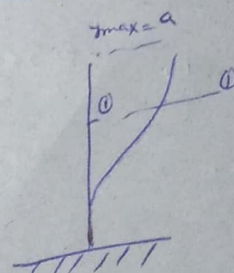
$$P_{cr} = \frac{n^2 \pi^2 EI}{L_e^2}$$

Result:- When the column is concentrically loaded or eccentrically it will buckle at same critical load.
 → The deformation in previous case will be more than normal one.

Stresses for eccentrically loaded column:-

Consider eccentrically loaded column as shown

$$TP \cdot M = P(a + e - \delta)$$



Let assume compression +ve.

$$\delta_{\max} = \frac{P}{A} + \frac{Mc}{I}$$

$$= \frac{P}{A} + \frac{M}{I/c}$$

$$= \frac{P}{A} + \frac{M}{s}$$

But; $M = P e \sec k' L$

$$\boxed{M = \frac{P}{A} + \frac{P e \sec k' L}{s}} \rightarrow (A)$$

is called secant formula used for all kind of column whether short or long. also valid for eccentrically & concentrically loaded column.

For short column.

$$L \rightarrow 0$$

$$\sec 0 = 1$$

$$\{ (A) \Rightarrow \boxed{M = \frac{P}{A} + \frac{Pe}{s}} \rightarrow (2)$$

$\{ (2)$ is used for analysis of short column.

Design of Column:-

As we have studied how to find the critical load corresponding for short, long & intermediate column ~~are~~ the corresponding stresses to critical load for these columns using physics or some empirical relation.

We also know how to find critical load for column whose one end is fixed at which load is applied ~~for~~ eccentrically or concentrically.

As in practically there are a lot of uncertainty in placing load, strength of material etc.

For e.g. If in empirical method i.e. to design a column load is applied at 2" from its centre but may be practically the load may be applied at 2.2" or 2.3" which give uncertainty. \therefore due to poor workmanship.

\rightarrow So different design code take these uncertainty & are replaced \therefore taken account by Factor of safety.

i. Structural stability research Council:- (SSRC).

For steel structures there is research council that develop their own empirical formula.

ii) American Institute of Steel Construction:-
(AISC)

iii) Allowable stress design:-
(ASD)

These two are used to design column (steel)
→ Those columns that are used in building design.

To find allowable stress in column:-

First we have to find slenderness ratio
i.e. $(L/r) =$

Now compare it with critical i.e. " C_c "

If $(L/r) > C_c$

∴ C_c is critical slenderness ratio.

Then used this formula.

∴ When $L/r > C_c$ (long column)

$$\sigma_{all} = \frac{12}{23} \frac{\pi^2 E}{(L/r)^2}$$

∴ $12/23 = F.S.$

∴ When $L/r < C_c$ (short or intermediate column)

$$\sigma_{allowable} = \left[1 - \frac{(L/r)^2}{2C_c} \right] \frac{\sigma_{cr}}{F.O.S}$$

Factor of safety:-

Factor of safety can vary & can be

Calculated as;

$$F.O.S = \frac{5}{3} + \frac{3}{8} \frac{L_e}{r/c_c} - \frac{(L_e/r)^3}{8c_c^3}$$

Critical Slenderness Ratio:-

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_{yp}}}$$

Note #

These all formulae for columns which are concentrically loaded & those are building structures (column) of steel.

(±) Design of Eccentrically load Column:-

We have two approaches.

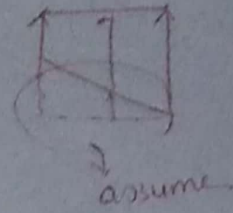
i) Maximum load approach:-

According to this approach effect of buckling deflection on moment arm is eliminated by limiting the maximum stresses to the allowable stresses computed from column formula.

$$\sigma_{max} = \frac{P}{A} + \frac{M}{S} \rightarrow \text{①}$$

Then σ_{max} will be compare with σ_{axial} of

material. We ignore the bending stresses for material.



(2) Interaction formula:-

In this case the axial stresses will be compared with axial strength of material & bending stresses ^(calculated) with bending stresses of material.

As max stresses is,

$$\sigma_{\max} = P/A$$

The area required to resist these stresses are

$$A_a = \frac{P}{(\sigma_a)_{\max}}$$

If δ_B is max bendinging moment so the area required to resist these bending moment

is;

$$A = \frac{M}{(\delta_B)_{\max} \times r^2}$$

Total area required will be;

$$A \geq A_a + A_B$$

$$= \frac{P}{(\sigma_a)_{\max}} + \frac{M}{(\delta_B)_{\max} \times r^2}$$

Divide A (P by A)

$$= \frac{P/A}{(\sigma_a)_{\max}} + \frac{M/Ay^2}{\sigma_b \max} \leq 1.$$

$$= \frac{\sigma_a}{(\sigma_a)_{\max}} + \frac{\sigma_b}{(\sigma_b)_{\max}} \leq 1.$$

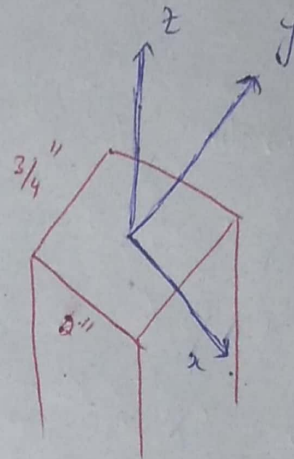
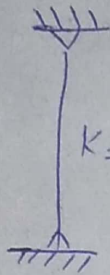
This AISC formula in which both axial & flexural stresses contribute.

According to AISC.
When the axial stresses are equal or less than 15% of axial strength of material
- Then we can use this interaction formula
But if large so not used this formula.

Pb# 1103 An aluminum strut 6' long has a rectangular section $\frac{3}{4}"$ by 2". A bolt through each end secures the strut so that it acts a hinged column about an axis perpendicular to 2" dimension and fixed end column about an axis perpendicular to $\frac{3}{4}"$. Determine the safe central load using a factor of safety of 2 & $E = 10.3 \times 10^6$ psi.

Solution:-

Consider the rectangular section of given dimension as shown.



For hinge column as buckling is restricted about y -axis.

So buckling can take place in x - z plane or about y -axis.

So to find safe load.

as;

$$P_{cr} = \frac{\pi^2 EI}{(Le)^2}$$

$$P_{cr} = \frac{(3.14)^2 \times 10.3 \times 10^6 \times \frac{(3/4)(2)^3}{12}}{(1 \times 6 \times 12)^2}$$

$$P_{cr} = 9804 \text{ lb}$$

$$P_{allow} = \frac{P_{cr}}{F.O.S}$$

$$= \frac{9804}{2} \Rightarrow \boxed{P_{allow} = 4902 \text{ lb}}$$

Similarly consider fixed ended column about an axis \perp to $\frac{3}{4}"$ which will prevent buckling about x-axis or buckling will take place in

* y-z plane

So, at this case.

$$P_{cr} = \frac{\pi^2 EI}{(L_e)^2}$$

$$L_e = K \times L$$

$$= \frac{\pi^2 \times 10.3 \times 10^6 \times \left(\frac{3}{4}\right)^3 \times 2}{(0.5 \times 6 \times 12)^2}$$

$$\boxed{P_{cr} = 5515.2 \text{ lb}}$$

$$P_{allowable} = \frac{P_{cr}}{2}$$

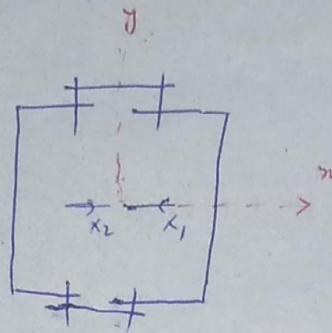
$$= \frac{5515.2}{2}$$

$$\boxed{P_{allowable} = 2757.6 \text{ lb}}$$

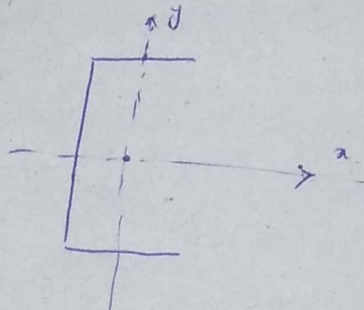
In order to prevent buckling in both ended consider $\boxed{P = 2757.6 \text{ lb}}$
i.e. Safest value

Pb #1106 Two $C_{310 \times 45}$ channels are latticed together so that they have equal moment of inertia about P.A. Determine the minimum length of column having this section assuming pinned ends. $E = 200 \text{ GPa}$ and proportional limit of 240 MPa . What safe load column carry for a length of 12 m with a factor of safety of 2.5 ?

Sol:- As two $C_{310 \times 45}$ channels are latticed together so that they have equal moment of inertia so to join them either in form of circle or square so this condition satisfy.



Consider one C section.



The properties of $C_{310 \times 45}$ section are

$$A = 5690 \text{ mm}^2$$

$$I_{x1} = 67.3 \times 10^6 \text{ mm}^4$$

$$I_{y1} = 2.12 \times 10^6 \text{ mm}^4$$

As given channel section is symmetric about X-axis so I_{xx} for both section when combine can be written as;

$$I_x = 2I_{x_1}$$

$$I_x = 2 \times 67.3 \times 10^6$$

$$I_x = 134.6 \times 10^6 \text{ mm}^4$$

As square section so,

$$I_x = I_y = 134.6 \times 10^6 \text{ mm}^4 \quad (\text{given condition})$$

As we know that,

$$S_{cr} = \frac{\pi^2 E}{(l_e/r)^2}$$

$$l_e/r = \sqrt{\frac{\pi^2 E}{S_{cr}}}$$

$$\text{Put } S_{cr} = S_{pl}$$

$$l_e/r = \sqrt{\frac{\pi^2 E}{S_{pl}}}$$

By putting values

$$l_e/r = \sqrt{\frac{\pi^2 \times 200 \times 10^3}{240}}$$

$$\Rightarrow l_e/r = 90.6$$

$$l_e = r \times 90.6$$

$$r = \sqrt{\frac{I}{A}}$$

$$l_e = 90.6 \times \sqrt{\frac{134.6 \times 10^6}{5690 \times 2}}$$

$$r = \sqrt{\frac{134.6 \times 10^6}{5690 \times 2}}$$

As two channel section.

$$l_e = 9.8 \text{ m}$$

So if $l_e > 9.8 \text{ m}$ it will consider long column.

if $l_e < 9.8 \text{ m}$ - that will short or inter medial column.

To find $P_{allowable} = ?$

$$P_{allowable} = \frac{P_{cr}}{F.O.S.} \rightarrow (1)$$

As;

$$P_{cr} = \frac{\pi^2 EI}{(l_e)^2}$$

$$A_3 \quad l_e = KL \\ = 1 \times 12 \text{ m}$$

So;

$$P_{cr} = \frac{\pi^2 \times 200 \times 10^3 \times 134.6 \times 10^6}{(1 \times 12 \times 1000)^2}$$

$$P_{cr} = 1843197.444$$

$$\text{Eq (1)} \Rightarrow P_{allowable} = \frac{P_{cr}}{F.O.S.}$$

$$= \frac{1843197.444}{2.5}$$

$$P_{allowable} = 732278.9 \text{ N}$$

or

$$P = 732.278 \text{ kN}$$

Ans.

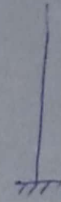
Pb # 1115:- A $W_{14 \times 82}$ section is used as a column with an effective length of 30'. Use AISC specification compute the maximum load that can be easily applied use $\sigma_{yp} = 50 \text{ ksi}$ & $E = 29 \times 10^6 \text{ psi}$.

Solution:-

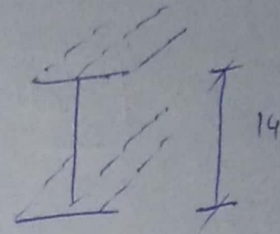
$P = ?$

cts;

$$P = \frac{\pi^2 EI}{L_e^2} \rightarrow \textcircled{1}$$



W-section



As other items are constant just P depend on I & I depend on " r ". So consider that r which has smallest value.

So, $r = 2.48 \text{ in.}$

First find $\frac{L_e}{r}$.

$$\frac{L_e}{r} = \frac{30 \times 12}{2.48} \Rightarrow \boxed{\frac{L_e}{r} = 145.16}$$

Now find critical slenderness ratio.

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_{yp}}} = \sqrt{\frac{2\pi^2 \times 29 \times 10^6}{50 \times 10^3}}$$

$$\boxed{C_c = 107}$$

As $\frac{L_e}{r} > C_c$ So this is long column

cto

$$\delta_{al} = P/A$$

$$P = \delta_{al} \times A \rightarrow \textcircled{ii}$$

$$\delta_{al} = \frac{12}{23} \times \frac{\lambda^2 E}{(L/r)^2}$$

$$\therefore L/r = 145.16$$

$$= \frac{12}{23} \times \frac{(3.14)^2 \times 29 \times 10^6}{(145.16)^2}$$

$$\boxed{\delta_{al} = 7.09 \text{ Ksi}}$$

Put in $\textcircled{ii} \Rightarrow$

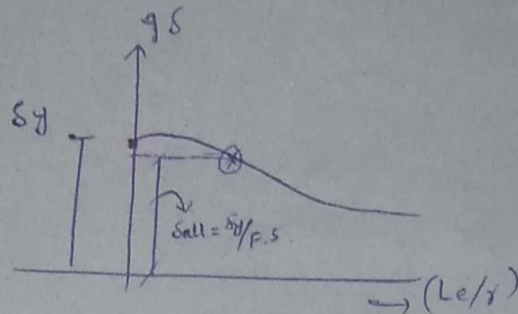
$$P = \delta_{al} \times A$$

$$= 7.09 \times 24.1$$

$$\boxed{P = 170.8 \text{ Kips.}} \quad \underline{\underline{\text{Ans}}}$$

Pb #1122:- Select the lightest I shape, according to AISC specification, that can be used as column to support an axial load of 420 kN on the effective length of 4m. Use AISC specification with $\sigma_{yp} = 250 \text{ MPa}$ & $E = 200 \text{ GPa}$.

Solution:-



For short column. Consider it to find allowable stress.

As $\sigma_{\text{assume}} = 0.8 \times \sigma_{\text{all}} \rightarrow \text{①}$.

As for short column from graph.

$\frac{L}{r} = 0$.

As for short column.

$$\sigma_{\text{all}} = \left[1 - \frac{(L/r)^2}{2C_c^2} \right] \frac{\sigma_{yp}}{F.S} \rightarrow \text{②}$$

Find $\sigma_{yp} = ?$

$\sigma_{yp} = 250 \text{ MPa}$. (Given)

$F.S = ?$ As $F.S = \frac{5}{3} + \frac{3}{8} \frac{L/r}{C_c} - \frac{(L/r)^3}{8C_c^3}$

By putting values.

$$F \cdot S = \frac{S}{3} + \frac{3}{8} \frac{0}{C_c} - \frac{0}{8C_c}$$

$$\boxed{F \cdot S = \frac{S}{3}}$$

A₃ for short column.

$$\sigma_{all} = \left(1 - \frac{(l_e/r)^2}{2C_c^2}\right) \frac{\sigma_{yp}}{F \cdot S}$$

$$= \left(1 - \frac{0}{2C_c^2}\right) \frac{250}{5/3}$$

$$\therefore \sigma_{yp} = 225 \text{ MPa} \quad (\text{given})$$

$$\sigma_{all} = 150 \text{ MPa}$$

$$\sigma_{ass} = 0.8 \times 150$$

$$\boxed{\sigma_{ass} = 120 \text{ MPa}}$$

As we have to find dimension;
i.e. Area = ?

$$\text{Since } P_{all} = \frac{P}{A}$$

$$A = \frac{P}{P_{all}}$$

$$= \frac{420 \times 10^3}{120 \times 10^6}$$

$$A = 3.5 \times 10^{-3} \text{ m}^2$$

$$\boxed{A = 3500 \text{ mm}^2}$$

As; $A = 3500 \text{ mm}^2$

Now at end of book by looking properties of section consider a section whose area is compatible with 3500 mm^2 .

Let consider

W150x30

Where $A = 3790 \text{ mm}^2$

$\{ r_{\min} = 38.3 \text{ mm}$

Now find l/r & C_c .

$$l/r = \frac{4 \times 10^3}{38.3} = 104.4$$

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_{yp}}} = \sqrt{\frac{2\pi^2 \times 200 \times 10^3}{250 \times 10^6}} = 125.66$$

As; $l/r < C_c$ so use short column formula

i.e. $\sigma_{all} = \left(1 - \frac{(l/r)^2}{2C_c^2}\right) \frac{\sigma_{yp}}{F.S} \rightarrow (1)$

$$F.S = \frac{5}{3} + \frac{3}{8} \frac{(l/r)^2}{C_c^2} - \frac{(l/r)^3}{8C_c^3}$$

$$= \frac{5}{3} + \frac{3}{8} \frac{(104.4)^2}{125.66^2} - \frac{(104.4)^3}{8(125.66)^3}$$

$$\boxed{F.S = 1.91}$$

∴ (1) $\sigma_{all} = \left(1 - \frac{(104.4)^2}{2(125.66)^2}\right) \times 250 \Rightarrow \boxed{\sigma_{all} = 85.67 \text{ MPa}}$

Since; $P = \sigma_{all} \times A = 85.67 \times 3790 \Rightarrow \boxed{P = 324.689 \text{ kN}}$

As $P_{\text{cal}} < P_{\text{axial}}$ So not this section.
 Consider $W_{150 \times 30}$ is inadequate.

Consider $W_{200 \times 36}$ whose $A = 4580 \text{ mm}^2$ & $r_{\text{min}} = 40.8 \text{ mm}$

for this also find l/r & C_c .

$$l/r = \frac{40 \times 10^3}{40.8} = 98.04$$

$$C_c = \sqrt{\frac{2\pi^2 E}{S_{yp}}} = \sqrt{\frac{2\pi^2 (200 \times 10^3)}{250}}$$

$$C_c = 125.6$$

As $C_c > l/r$.

So use short column formula;

$$S_{\text{all}} = \left(1 - \frac{(l/r)^2}{2C_c^2}\right) \frac{S_{yp}}{F.S} \rightarrow \textcircled{\text{iii}}$$

$$F.S = 5/3 + \frac{3}{8} \frac{(l/r)^2}{C_c} - \frac{(l/r)^3}{8C_c^3}$$

$$= 5/3 + \frac{3}{8} \frac{(98.04)^2}{125.6} - \frac{(98.04)^3}{8(125.6)^3}$$

$$F.S = 1.90$$

$$\textcircled{\text{iii}} \Rightarrow S_{\text{all}} = \left(1 - \frac{(98.04)^2}{2(125.6)^2}\right) \frac{280}{1.90}$$

$$S_{\text{all}} = 91.53 \text{ MPa}$$

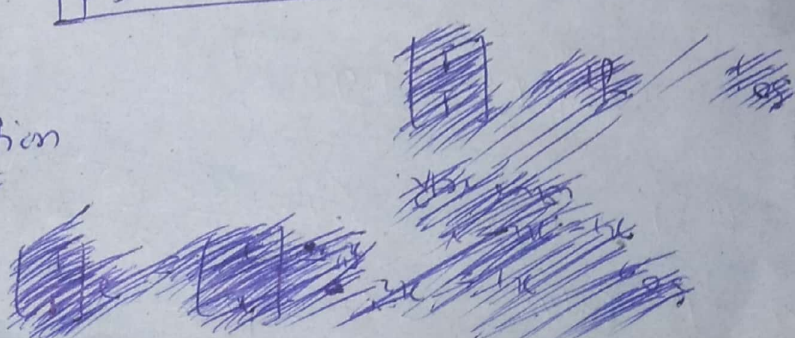
$$P = S_{\text{all}} \times A \Rightarrow 91.53 \times 4580$$

$$P = 419.2 \text{ kN}$$

$$\approx 420 \text{ kN} = P$$

So $W_{200 \times 36}$ is

our required section
 whose $A = 4580 \text{ mm}^2$



Pb # 1110 :-

Use AISC Specification.

Page = ?

Section = W360x122.

- (a) Hinged ends. length 9 m.
(b) Built in and unsupported length 9 m.
(c) Built in ends & length 9 m braced at midpoint.

Use $\sigma_{yp} = 380 \text{ MPa}$ & $G = 200 \text{ GPa}$.

Solution:- From property of W360x122.

$$A = 15500 \text{ mm}^2$$

$$r = 63 \text{ mm}$$

(a) For Hinged end of length 9 m.
So; $\frac{L}{r} = \frac{9 \times 1000 \times 1}{63}$

$$\boxed{\frac{L}{r} = 142.85}$$

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_{yp}}} = \sqrt{\frac{2(3.14)^2 (200 \times 10^3)}{380}}$$

$$C_c = 702$$

As $\frac{L}{r} > C_c$ So long column & use

$$\sigma_{all} = \frac{12}{23} \frac{\pi^2 E}{(\frac{L}{r})^2}$$

$$\sigma_{all} = \frac{12}{23} \times \frac{(3.14)^2 \times 200 \times 10^3}{(142.85)^2}$$

$$\boxed{\sigma_{all} = 50.4 \text{ MPa}}$$

$$\text{Ans; } P = S_{all} \times A$$

$$P = 50.4 \times 15500$$

$$\boxed{P = 781.2 \text{ kN}}$$

(b) Built ends:-

Ans; for built end (fixed) $k = \frac{1}{2}$

So;

$$l/r = \left(\frac{k \times L}{r} \right) = \frac{0.5 \times 10 \times 1000}{63}$$

$$\boxed{l/r = 79.36}$$

$$\text{And } C_c = \sqrt{\frac{2\pi^2 E}{P}} = 102$$

Ans $C_c \geq l/r$ so short or
intermediate column.

So used

$$S_{all} = \left[1 - \frac{(l/r)^2}{2C_c^2} \right] \frac{S_y}{F.S.}$$

$$= \left[1 - \frac{(79.36)^2}{2(102)^2} \right] \frac{380}{F.S.} \rightarrow \textcircled{1}$$

$$F.S. = \frac{S_y}{3} + \frac{3(l/r)^2}{8C_c^2} - \frac{(l/r)^3}{8C_c^3}$$

$$= \frac{380}{3} + \frac{3(79.36)^2}{8(102)^2} - \frac{(79.36)^3}{8(102)^3}$$

$$\boxed{F.S. = 1.90}$$

① ⇒

$$\sigma_{all} = \left[1 - \frac{(79.36)^2}{2(102)^2} \right] \frac{380}{1.9}$$

$$\sigma_{all} = 139 \text{ MPa.}$$

$$\text{Since, } P = \sigma_{all} \times A$$

$$= 139 \text{ N/mm}^2 \times 15500 \text{ mm}^2$$

$$\boxed{P = 2161.5 \text{ kN}}$$

② Built in of length 10m & braced at midpoint.

As braced at mid point so the column is behaving like hinged-fixed support of length 5m.

$$\text{So, } l_{e/r} = \frac{k \times L}{63} = \frac{0.7 \times 5 \times 1000}{63}$$

$$l_{e/r} = 55.6$$

$$\& C_c = 102$$

As $C_c > l_{e/r}$ so Short Column.

$$\text{Use } \sigma_{all} = \left(1 - \frac{(l_{e/r})^2}{2C_c^2} \right) \frac{\sigma_{yp}}{F.S} \rightarrow \text{①}$$

$$F.S = 1.5 + \frac{3(l_{e/r})}{8C_c} - \frac{(l_{e/r})^3}{8C_c^3}$$

$$\boxed{F.S = 1.85}$$

$$\sigma_{all} = \left(1 - \frac{(L/r)^2}{2C_c^2}\right) \frac{\sigma_{yp}}{F.S.}$$

$$= \left(1 - \frac{(55.6)^2}{2(102)^2}\right) \frac{380}{1.85}$$

$$\sigma_{all} = 175 \text{ MPa}$$

$$P = \sigma_{all} \times \text{Area}$$

$$= 175 \text{ N/mm}^2 \times 15500 \text{ mm}^2$$

$$P = 27125 \text{ kN} \quad \text{Ans}$$

Pb # 1111 :-

Select w lightest shape.

$$P = 90 \text{ kips.}$$

$$L = 15'$$

Use AISC specification

$$\sigma_{yp} = 36 \text{ ksi}$$

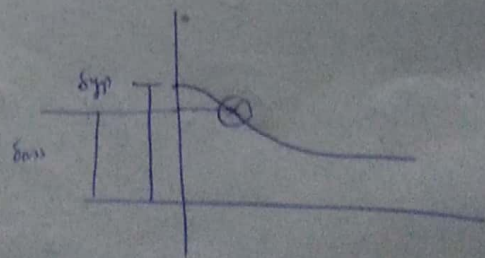
$$E = 29 \times 10^6 \text{ psi}$$

$$\sigma_{all} = \sigma_{all} = 0.8 \times \sigma_{all} \rightarrow (1)$$

To find $\sigma_{all} = ?$

consider $L/r = 0$

$$\text{So, } F.S. = 5/3$$



$$\delta_{all} = \left(1 - \frac{(le/r)^2}{2C_c^2}\right) \frac{\delta_{yp}}{F.S}$$

$$= \left(1 - \frac{0}{2C_c^2}\right) \frac{36 \times 10^3}{5/3}$$

$$\delta_{all} = 21.6 \text{ Ksi}$$

$$\delta_{ass} = 0.8 \times 21.6 \text{ ksi}$$

$$\delta_{ass} = 17.28 \text{ Ksi}$$

$$A = \frac{P}{\delta_{ass}} = \frac{90 \times 10^3}{17.28 \times 10^3}$$

$$A = 5.21 \text{ in}^2$$

So By trial method.

First try:-

let select W6x21.

$$A = 6.16 \text{ in}^2$$

$$r_{min} = 1.26 \text{ in}$$

To find le/r

$$= \frac{15 \times 12}{1.26} = 143$$

$$C_c = \sqrt{\frac{2\pi^2 E}{F_{yp}}} = \sqrt{\frac{2(3.14)^2 (29 \times 10^6)}{36 \times 10^3}}$$

$$C_c = 126 \quad A_3$$

As $l/r > C_c$ so long column

use

$$\sigma_{all} = \frac{12}{23} \frac{\pi^2 E}{(l/r)^2}$$

$$= \frac{12}{23} \times \frac{(3.14)^2 \times (29 \times 10^6)}{(143)^2}$$

$$\boxed{\sigma_{all} = 7.30 \text{ ksi}}$$

So, $P = \sigma_{all} \times A$

$$= 7.30 \times 616$$

$$\boxed{P = 45 \text{ kips}}$$

Since $P_{cal} < P_{given}$ so discard

this column

Consider $W8 \times 25$ section

$$A = 82.5 \text{ in}^2 \quad \& \quad r_{min} = 1.62 \text{ in}$$

So, first $l/r = \frac{15 \times 12}{1.62} = 111.1$

$$\& \quad C_c = 126$$

Since $C_c > l/r$ so short or intermediate column

Use

$$\sigma_{all} = \left(1 - \frac{(l/r)^2}{2C_c^2}\right) \frac{\sigma_{yp}}{F.S.} \rightarrow (A)$$

$$F.S. = \frac{5}{3} + \frac{3}{8} \frac{l/r}{C_c} - \frac{(l/r)^3}{8C_c^3}$$
$$= \frac{5}{3} + \frac{3}{8} \frac{(111.1)}{126} - \frac{(111.1)^3}{8(126)^3}$$

$$F.S. = 1.91$$

$$\sigma_{all} = \left(1 - \frac{(111.1)^2}{2(126)^2}\right) \frac{36 \times 10^3}{1.91}$$

$$\boxed{\sigma_{all} = 11.53 \text{ Ksi}}$$

AS.

$$P = \sigma_{all} \times A$$

$$= 11.53 \times (8.25)$$

$$\boxed{P = 95.12 \text{ Kips}}$$

cb given $P = 90 \text{ Kips}$

So, $P_{cal} > P_{given}$

So Consider 141/8 x 28 section.

Pb # 1112 :-

$$L = 5m$$

Built in ends.

a) of cross section is circular with radius 40mm.

(b) 50mm square

Required $l/r = ?$

Solution:-

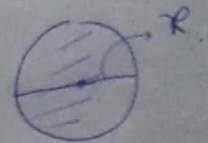
As built in ends (fixed support)

$$\text{So } K = 0.5$$

(a) For Circular cross section of $R = 40mm$.

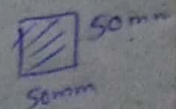
$$\text{As, } \frac{l}{r} = \frac{K \times L}{r} = \frac{0.5 \times 5 \times 1000}{\sqrt{\frac{I}{A}}}$$

$$\frac{l}{r} = \frac{2500}{\sqrt{\frac{\pi R^4}{4\pi R^2}}}$$



$$= \frac{2500}{R/2} \Rightarrow \frac{5000}{R} = \frac{5000}{40}$$

$$\text{So, } \boxed{\frac{l}{r} = 125}$$



(b) $\frac{l}{r} = ?$ for square of 50mm.

$$\frac{l}{r} = \frac{0.5 \times 5 \times 1000}{\sqrt{\frac{50(50)^3}{12(50)(50)}}} = \frac{2500}{\sqrt{50^3/12}}$$

$$\boxed{\frac{l}{r} = 173.61} \text{ Ans.}$$

Pb # 1113 :-

Given data:

$$L = 12'$$

one fixed & other hinged

$$K = 0.7$$

Find u/r

(a) Circular with radius 2" $\{ (b) 2.5$ in square

Sol: (a) For Circular section.

$$u/r = \frac{K \times L}{r} = \frac{0.7 \times 12}{\sqrt{\frac{I}{A}}}$$

$$= \frac{0.7 \times 12}{\sqrt{\frac{\pi r^4}{4\pi r^2}}}$$

$$= \frac{0.7 \times 12 \times 12}{\sqrt{\frac{r^2}{4}}} = \frac{100.8}{r/2} = \frac{100.8}{2/2}$$

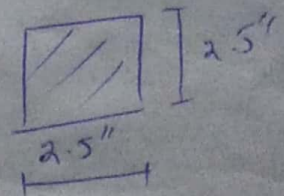
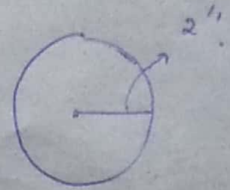
$$\boxed{u/r = 100.8}$$

(b) For 2.5 in² Square

$$u/r = \frac{K L}{r} = \frac{0.7 \times 12 \times 12}{\sqrt{\frac{I}{A}}}$$

$$= \frac{100.8}{\sqrt{\frac{b^4}{12b^2}}} = \frac{100.8}{b} = \frac{100.8}{\frac{(2.5)^2}{12}}$$

$$\boxed{u/r = 139.67} \text{ Ans}$$



Pb #1114:- Determine the maximum length l of a $W250 \times 167$ section used as a hinged end column to support a load of 1600 kN . Use AISC specification with $\sigma_{yp} = 380 \text{ MPa}$ & $E = 200 \text{ GPa}$.

Sol:- Given data

$$P = 1600 \text{ kN}$$

As section is;

$W250 \times 167$

From properties table

$W250 \times 167$

$$A = 21,300 \text{ mm}^2$$

Now first step is to find σ .

$$\text{As } \sigma = \frac{P}{A} = \frac{1600 \times 10^3 \text{ N}}{21300 \text{ mm}^2} \Rightarrow \boxed{\sigma = 75.11 \text{ MPa}}$$

Now find $C_c = ?$

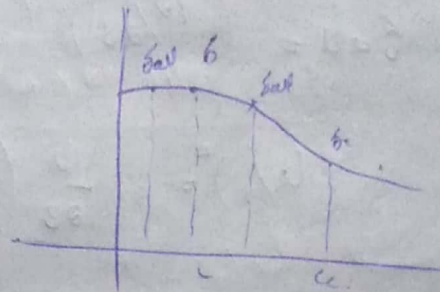
$$\text{As } C_c = \sqrt{\frac{2\pi^2 E}{\sigma_{yp}}} = \sqrt{\frac{2(3.14)^2 (200 \times 10^9)}{380 \times 10^6}}$$

$$\boxed{C_c = 101.87}$$

To σ_c on basis of C_c

$$\text{As } \sigma = \frac{12}{23} \frac{\pi^2 E}{C_c^2}$$

$$\sigma = \frac{12}{23} \frac{(3.14)^2 (200 \times 10^9)}{(101.87)^2}$$



$$\delta_c = 10.05 \text{ MPa}$$

$$\delta > \delta_c$$

So, use

$$\delta_{all} = \left(\frac{12}{23} \right) \left(\frac{\pi^2 E}{(l_e/r)^2} \right)$$

$$\delta_{all} = \delta$$

$$(l_e/r)^2 = \frac{12}{23} \left(\frac{\pi^2 E}{\delta_{all}} \right)$$

$$= \frac{12}{23} \frac{(3.14)^2 (200 \times 10^9)}{75.11 \times 10^6}$$

$$l_e/r = \sqrt{13697.61}$$

$$l_e/r = 117$$

$$l_e = 117 \times r$$

$$r = 68.1 \text{ mm}$$

$$l_e = 117 \times 68.1$$

$$l_e = 7967.7 \text{ mm}$$

$$l_e = 7.9 \text{ m}$$

As hinged
end column
so $k=1$

or

$$L \times k = 7.9 \text{ m}$$

$$L \times 1 = 7.9 \text{ m}$$

$$L = 7.9 \text{ m} \quad \text{Ans}$$

Pb # 1115 :-

Given data

W14x82 section

$$L = 30 \text{ ft.}$$

$$S_{yp} = 50 \text{ Ksi}$$

$$E = 29 \times 10^6 \text{ psi}$$

Use AISC Specification calculate $P = ?$

Sol:-
2 As use AISC specification;

So;

$$L/r = \frac{30 \times 12}{2.48}$$

$$r_{min} = 2.48''$$

$$\boxed{L/r = 145.2}$$

$$C_c = \sqrt{\frac{2\pi^2 E}{S_{yp}}}$$

$$= \sqrt{\frac{2(3.14)^2(29 \times 10^6)}{50 \times 10^3}}$$

$$\boxed{C_c = 106.94}$$

As $L/r > C_c$ so large column

Since;

$$S_{all} = \frac{12}{23} \frac{\pi^2 E}{(L/r)^2}$$

$$\sigma_{all} = \frac{12}{23} \frac{\lambda^2 E}{(L/r)^2}$$

$$= \frac{12}{23} \times \frac{(3.14)^2 (29 \times 10^6)}{(145.2)^2}$$

$$\sigma_{all} = 7075.83 \text{ psi}$$

$$\text{or } \sigma_{all} = 7.075 \text{ ksi}$$

To find $P_{safe} = ?$

$$\text{As } \sigma_{all} = P/A$$

$$A = 24.1 \text{ in}^2$$

$$P_{safe} = \sigma_{all} \times A$$

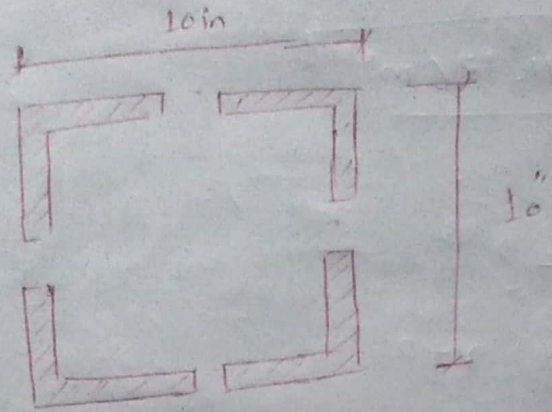
$$= 7.075 \times 24.1$$

$$P_{safe} = 170.5 \text{ kip}$$

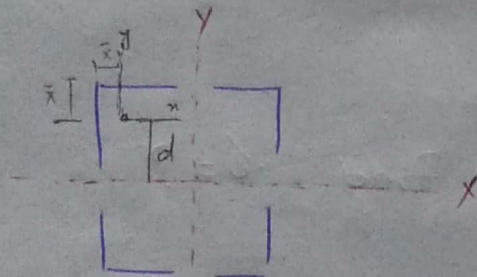
Ans

Pb # 1117:- Four $4 \times 4 \times \frac{1}{2}$ in angles are latticed together to form the column section shown in fig. Use AISC specification determine the maximum length at which a 200 kips load can be safely supported. Use $\phi_{yp} = 60 \text{ ksi}$ & $E = 29 \times 10^6 \text{ psi}$.

Figure:-



Solution:- First find geometrical moment of Inertia. Which can be calculated as;



$$A = 3.75 \text{ in}^2$$

As from property table we found Centroid is at $\bar{x} = 1.18''$, $I_x = 5.56 \text{ in}^4$.
Since $4 \times 4 \times \frac{1}{2}$ both same so from both direction i.e. $\bar{x} = \bar{y} = 1.18''$.

So, $I_x = (I_x + Ad^2) \times 4$ As four L sections.

$$I_x = (5.56 + 3.75(5 - 1.18)^2) \times 4$$

$$I_x = (586 + 54.7215) \times 4$$

$$\boxed{I_x = 241.126 \text{ in}^4}$$

Similarly

$$I_y = (I_y + Ad^2) \times 4$$

$$\boxed{I_y = 241.126 \text{ in}^4}$$

So, $L/r = ?$

First find $r = ?$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{241.126}{3.75 \times 4}}$$

$$\boxed{r = 8.018 \text{ in}}$$

$L/r = ?$

First step is to find $\delta = ?$

So $\delta = P/A$

$$= \frac{200 \times 10^3}{4 \times 3.75}$$

$$\boxed{\delta = 13.33 \text{ Ksi}}$$

Now find

$C_c = ?$

So $C_c = \sqrt{\frac{2\pi^2 E}{\delta_{yp}}}$

$$C_c = \sqrt{\frac{2(3.14)^2 (29 \times 10^6)}{(60 \times 10^3)}}$$

$$C_c = 97.62$$

Now find " δ " on basis of C_c

As,

$$\delta_{cc} = \frac{12}{23} \frac{\pi^2 E}{C_c^2}$$

$$= \frac{12}{23} \frac{(3.14)^2 (29 \times 10^6)}{(97.62)^2}$$

$$\delta_{cc} = 15.65 \text{ kS.}$$

As $\delta_{cc} > \delta$. So now error & trial method.

Prob # 1118 A steel column with effective length of 10m is fabricated from two C250x45 channels latched together so that the section has equal moment of inertia about the principal axis. Determine the safe load P using AISC specification. Use $\sigma_{yp} = 380 \text{ MPa}$ & $E = 200 \text{ GPa}$.

Solution:-

Given - L_{eff}

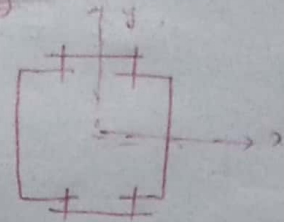
$$L_e = 10 \text{ m}$$

Section = C250x45

$$\sigma_{yp} = 380 \text{ MPa}$$

$$E = 200 \text{ GPa}$$

Figure:-



From the table, Properties of C250x45 are

$$A = 5670 \text{ mm}^2$$

$$I_x = 42.8 \times 10^6 \text{ mm}^4$$

$$I_y = 1.6 \times 10^6 \text{ mm}^4$$

$$r_x = 86.9 \text{ mm}$$

$$r_y = 16.3 \text{ mm}$$

Note # For built up section radius of gyration should be found manually.

$$\text{As, } r_x = \sqrt{\frac{2 I_x}{2A}} \Rightarrow \text{For Combined Section.}$$

$$= \sqrt{\frac{2(42.8 \times 10^6)}{2(5670)}}$$

$$\boxed{r_x = 86.9 \text{ mm}} = r_y$$

Now;

$$u_{rmin} = \frac{10 \times 100}{(86.9)^2}$$

$$|u/r = 115.1| = 57.54$$

To find $c_c = ?$

$$c_c = \sqrt{\frac{2\pi^2 E}{\delta_{yp}}} = \sqrt{\frac{2(3.14)^2 (200 \times 10^9)}{(380 \times 10^6)}}$$

$$|c_c = 101.87|$$

$$95 \\ u/r > c_c$$

$$\delta_{all} = \left(1 - \frac{(u/r)^2}{2c_c^2}\right) \frac{\delta_{yp}}{F.S} \rightarrow \textcircled{1}$$

$$\delta_{all} = \frac{12 \pi^2 E}{23 (u/r)^2}$$

$$= \frac{12 (3.14)^2 (200 \times 10^9)}{23 (115.1)^2}$$

$$F.S = \frac{5}{3} + \frac{3}{8} \frac{u/r}{c_c} - \frac{1}{8} \frac{(u/r)^3}{c_c^3}$$

$$\delta_{all} = 77.65 \text{ MPa}$$

$$= \frac{5}{3} + 0.2118 - 0.022525$$

$$P_{ult} = 77.65 \times 5670$$

$$|F.S = 1.86|$$

$$|P_{ult} = 440 \text{ kN}|$$

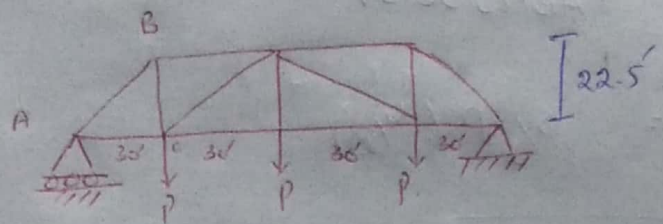
$$\delta_{all} = \left(1 - \frac{(57.54)^2}{2(101.87)^2}\right) \frac{380 \times 10^6}{1.86}$$

$$\delta_{all} = 171.71$$

$$P =$$

Pb # 1119 In the bridge truss as shown, the member AC is composed of two C9x20 channels latched together so that equal moment of inertia about axis of symmetry. If safe load P on the truss is govern by strength of member AC. Compute P using AISC specification with $\sigma_{yp} = 36 \text{ ksi}$ & $E = 29 \times 10^6 \text{ psi}$.

Figure:-

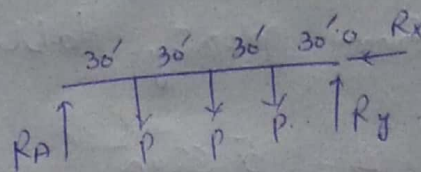


Solution:-
First step is to find reaction in member AC.

+ \curvearrowleft - \curvearrowright .

$$\sum M_A = 0$$

$$R_A \times 120 = P(90) + P(60) + P(30)$$



$$120 R_A = 180P$$

$$R_A = \frac{180}{120} P$$

$$R_A = \frac{3}{2} P$$

To find force in member AC using joint method.

Draw FBD of joint A.

$$\sum F_y = 0 \quad \uparrow + \quad \downarrow -$$

$$F_{AB} \sin 36.87 = R_A$$

$$F_{AB} = \frac{3/2 P}{3/5}$$

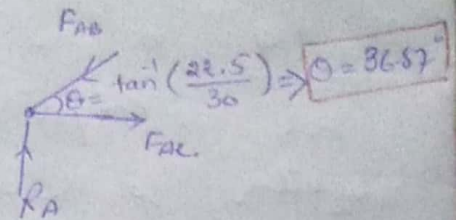
$$\boxed{F_{AB} = \frac{5}{2} P}$$

$$\sum F_x = 0 \quad (\rightarrow + \quad \leftarrow -)$$

$$F_{AC} = F_{AB} \cos 36.87$$

$$= \frac{5}{2} P \left(\frac{4}{5} \right)$$

$$\boxed{F_{AC} = 2P}$$



Now; Consider two C9x20 section.

From property table of C9x20.

$$A = 5.88 \text{ in}^2, \quad I_x = 60.9 \text{ in}^4, \quad I_y = 2.42 \text{ in}^4$$

$$I_x = 2 \times I_{x1}$$

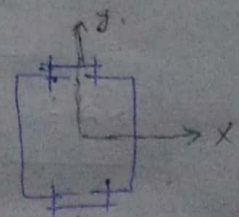
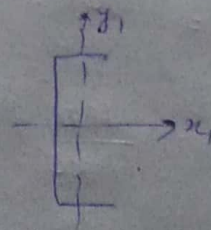
$$= 2 \times 60.9$$

$$\boxed{I_x = 121.8 \text{ in}^4}$$

As square section

$$\boxed{I_x = I_y}$$

Now to find I_y .



$$u/r = \frac{K \times L}{r} = \frac{0.7 \times 30 \times 12}{\sqrt{\frac{I}{A}}} = \frac{252}{5.88}$$

$$u/r = 55.3688 \quad 48.3$$

$$C_c = \sqrt{\frac{2\pi^2 E}{S_{yp}}} = \sqrt{\frac{2(3.14)^2 (29 \times 10^6)}{36 \times 10^3}}$$

$$C_c = 126$$

As $C_c > u/r$

$$\delta_{all} = \left(1 - \frac{u/r}{2C_c}\right) \frac{S_{yp}}{F.S.} \rightarrow \text{①}$$

$$F.S. = \frac{5}{3} + \frac{3}{8} \frac{u/r}{C_c} - \frac{1}{8} \frac{(u/r)^3}{C_c^3}$$

$$= \frac{5}{3} + 0.333 \times 0.1648 - 0.010667$$

$$F.S. = 1.82 \quad 1.91$$

$$\delta_{all} = \left(1 - \frac{55.3688}{2(126)^2}\right) \frac{36 \times 10^3}{1.82} \Rightarrow \delta_{all} = 1909.51 \text{ psi}$$

$$P = \delta_{all} \times A = 1909 \times 5.88 = \underline{112.3 \text{ Kips}}$$

Pb# 1116:-

Section $W_{310 \times 52}$ (Hinged ends)

use AISC specification to find P_c ?

(a) $L = 10m$ (b) $L = 14m$

Use $\sigma_{yp} = 250 \text{ MPa}$ & $E = 200 \text{ GPa}$.

Sol:- To find P_c .

$$L/r = \frac{L \times k}{r} = \frac{10 \times 1000}{39.3} = 254.45$$

Use $r = r_{min}$.

from properties of $W_{310 \times 52}$.

$$A = 6670 \text{ mm}^2$$

$$I_x = 118 \times 10^6 \text{ mm}^4, \quad I_y = 10.3 \times 10^6 \text{ mm}^4$$

$$r_x = 133 \text{ mm}, \quad r_y = 39.3 \text{ mm}$$

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_{yp}}} = \sqrt{\frac{2(3.14)^2 (200 \times 10^9)}{250 \times 10^6}}$$

$$\boxed{C_c = 125.6}$$

As $L/r > C_c$.

so long column.

$$\sigma_{all} = \left(\frac{12 \pi^2 E}{23 (L/r)^2} \right) = \frac{12}{23} \frac{(3.14)^2 (200 \times 10^9)}{(254.45)^2}$$

$$\boxed{\sigma_{all} = 15.89 \text{ MPa}}$$

$$\text{At } P = \sigma_{all} \times A$$

$$= 15.89 \times 6670 \Rightarrow \boxed{P = 105.98 \text{ kN}}$$

$$\text{For } L = 14 \text{ m}$$

$$k/r = \frac{14 \times 1 \times 1000}{39.3} = 356.23$$

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_{yp}}}$$

$$C_c = \sqrt{\frac{2(3.14)^2 (200 \times 10^3)}{250 \times 10^6}}$$

$$\boxed{C_c = 125.6}$$

At $k/r \geq C_c$ So Long Column.

$$\text{Using } \sigma_{all} = \frac{12}{23} \frac{\pi^2 E}{(k/r)^2}$$

$$= \frac{12}{23} \times \frac{(3.14)^2 (200 \times 10^9)}{(356.23)^2}$$

$$\sigma_{all} = 8.11 \text{ MPa}$$

$$P = \sigma_{all} \times A = 8.11 \times 6670$$

$$\boxed{P = 54.093 \text{ kN}} \text{ Ans}$$

Pb # 1122:- Select the lightest W that can be used as a column to support an axial load of $P = 420 \text{ kN}$ on an effective length of 4 m . Use AISC specification with $\sigma_{yp} = 250 \text{ MPa}$ & $E = 200 \text{ GPa}$.

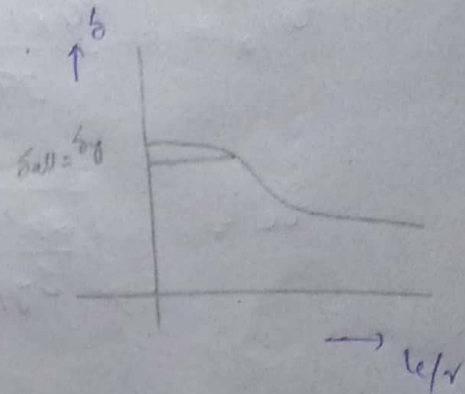
Sol:- Given data:

$$P = 420 \text{ kN}$$

$$l_e = 4 \text{ m}$$

$$\sigma_{yp} = 250 \text{ MPa}$$

$$E = 200 \text{ GPa}$$



Required data

Lightest W shape = ?

$$\sigma_{\text{assume}} = 0.8 \times \sigma_{\text{all}} \rightarrow \textcircled{A}$$

Consider ~~long~~ ^{short} column.

Since for ~~long~~ ^{short} column,

$$l_e/r = 0$$

$$\text{So, } F_s = \frac{5}{3} + \frac{3}{8} \frac{l_e/r}{c} - \frac{1}{8} \frac{(l_e/r)^3}{c^3}$$

$$\text{So, } \boxed{F_s = \frac{5}{3}}$$

$$\sigma_{\text{all}} = \sigma_{yp}$$

$$\sigma_{\text{all}} = \left(1 - \frac{(l_e/r)^2}{2c^2} \right) \frac{\sigma_{yp}}{F_s}$$

$$\sigma_{\text{all}} = \frac{250}{5/3}$$

$$\boxed{\sigma_{\text{all}} = 150 \text{ MPa}}$$

$$e.g. \textcircled{A} \Rightarrow \sigma_{\text{ass}} = \sigma_{\text{all}} \times 0.8$$

$$\sigma_{\text{ass}} = 0.8 \times 150$$

$$\boxed{\sigma_{\text{ass}} = 120 \text{ MPa}}$$

To find area:

$$\text{As } \sigma_{\text{all}} = P/A$$

$$A = \frac{P}{\sigma_{\text{all}}} = \frac{420 \times 10^3}{120 \times 10^6}$$

$$\boxed{A = 3500 \text{ mm}^2}$$

By checking to area of different section & compare with this calculated area;

Let select W150x30 section whose area is

$$A = 3790 \text{ mm}^2$$

$$r_{\text{min}} = 38.3 \text{ mm}$$

$$\text{So, } l/r = \frac{4 \times 10^3}{38.3} = 104.4$$

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_{yp}}} = \sqrt{\frac{2(3.14)^2 (200 \times 10^9)}{(250 \times 10^6)}}$$

$$\boxed{C_c = 125.66}$$

As $c_c > l_e/r$.

So short or intermediate column.

Using;

$$\sigma_{all} = \left(1 - \frac{(l_e/r)^2}{2c_c^2}\right) \frac{\sigma_{yp}}{F.S.}$$

$$= \left(1 - \frac{(104.4)^2}{2(125.66)^2}\right) \frac{250 \times 10^6}{1.91}$$

$$\boxed{\sigma_{all} = 85.67 \text{ MPa}}$$

To find F.S.

$$F.S. = 5/3 + 3/8 \frac{l_e/r}{c_c} + 1/8 \frac{(l_e/r)^3}{c_c^3}$$

$$F.S. = 5/3 + 3/8 \left(\frac{104.4}{125.66}\right) + 1/8 \frac{(104.4)^3}{(125.66)^3}$$

$$\boxed{F.S. = 1.91}$$

Now to find P.

$$\text{As; } \sigma_{all} = P/A$$

$$P = \sigma_{all} \times A$$
$$= 85.67 \times 3790$$

$$P_{cal} = 324.67 \text{ kN}$$

As $P_{cal} < P_{given}$ So this not
Compatible & 150×30 is inadequate.

Let select
W200 x 36 section;

Where $A = 4580 \text{ mm}^2$ & $r_{\min} = 40.8 \text{ mm}$.

$$\text{So, } l/r = \frac{4 \times 1000}{40.8} = \boxed{l/r = 98.04}$$

$$\text{Since } \boxed{C_c = 125.66}$$

As $l/r < C_c$. So short column.

$$\text{Using } \sigma_{all} = \left(1 - \frac{(l/r)^2}{2C_c^2}\right) \frac{\sigma_{yp}}{F.S} \rightarrow \textcircled{B}$$

$$F.S = 5/3 + 3/8 \frac{l/r}{C_c} - \frac{1}{8} \frac{(l/r)^3}{C_c^3}$$

By putting values.

$$\boxed{F.S = 1.90}$$

$$\sigma_{all} = \left(1 - \frac{(98.04)^2}{2(125.66)^2}\right) \frac{250 \times 10^6}{1.9}$$

$$\boxed{\sigma_{all} = 91.53 \text{ MPa}}$$

$$\text{Using } P = \sigma_{all} \times A$$

$$= 91.53 \times 4580$$

$$= 419.2 \text{ kN} \approx 420 \text{ kN}$$

So select

W200 x 36 section.

Pb # 1123:-

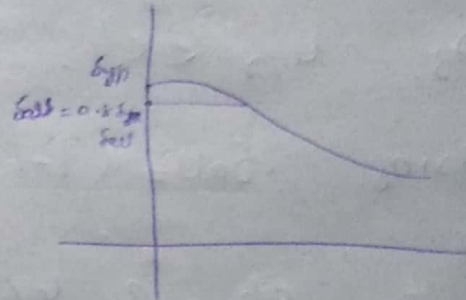
Select the lightest W shape that can be used as a column, according to AISC specification, to support an axial load of 700 kN on an effective length of 5.5 m.

Assume $\sigma_{yp} = 250 \text{ MPa}$ $E = 200 \text{ GPa}$.

Solution:-

Consider

$$\sigma_{\text{assume}} = 0.8 \sigma_{\text{all}} \rightarrow \text{①}$$



Consider short column

$$\text{with } l/r = 0$$

As for short or intermediate column.

$$\sigma_{\text{all}} = \left(1 - \frac{(l/r)^2}{2c_c^2} \right) \frac{\sigma_{yp}}{F.S} \rightarrow \text{②}$$

As

$$F.S = \frac{5}{3} + \frac{3}{8} \frac{(l/r)^2}{c_c^2} - \frac{1}{8} \frac{(l/r)^4}{c_c^4}$$

$$\boxed{F.S = \frac{5}{3}}$$

$$\sigma_{\text{all}} = \frac{250}{5/3}$$

$$\boxed{\sigma_{\text{all}} = 150 \text{ MPa}}$$

$$\begin{aligned} \text{eqn} \Rightarrow \sigma_{\text{ass}} &= 0.8 \times \sigma_{\text{all}} \\ &= 0.8 \times 150. \end{aligned}$$

$$\boxed{\sigma_{\text{ass}} = 120 \text{ MPa}}$$

To find area:-

$$\text{As } \sigma_{\text{all}} = P/A$$

$$A = \frac{P}{\sigma_{\text{all}}} = \frac{700}{120}$$

$$A = \frac{700 \times 10^3}{120 \times 10^6}$$

$$\boxed{A = 5833.33 \text{ mm}^2}$$

By comparing areas of different sections to get suitable section for $P = 700 \text{ kN}$.

Let select $\text{W}250 \times 80$.

$$\text{With area} = 10,200 \text{ mm}^2 \quad \left\{ \begin{array}{l} r_{\text{min}} = 65 \text{ mm} \end{array} \right.$$

$$\frac{L}{r} = \frac{5.5 \times 1000}{65} \Rightarrow \boxed{\frac{L}{r} = 84.61}$$

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_{yp}}} \Rightarrow C_c = \sqrt{\frac{2(3.14)^2 (200 \times 10^9)}{250 \times 10^6}}$$

$$\boxed{C_c = 125.6}$$

As $l_e/r < C_c$ So short or Intermediate Column.

$$\text{Use } \sigma_{all} = \left(1 - \frac{(l_e/r)^2}{2C_c^2}\right) \frac{\sigma_{yp}}{F.S} \rightarrow (B)$$

$$F.S = \frac{5}{3} + \frac{3}{8} \frac{l_e/r}{C_c} - \frac{1}{8} \frac{(l_e/r)^3}{C_c^3}$$

$$F.S = \frac{5}{3} + \frac{3}{8} \frac{(84.61)}{(125.6)} - \frac{1}{8} \frac{(84.61)^3}{(125.6)^3}$$

$$= \frac{5}{3} + 0.2526 -$$

$$\boxed{F.S = 1.88}$$

$$\sigma_{all} = \left(1 - \frac{(84.61)^2}{2(125.6)^2}\right) \frac{250 \times 10^6}{1.88}$$

$$\sigma_{all} = 102.8 \text{ MPa}$$

To find $P_w = ?$

$$P_w = \sigma_{all} \times A$$

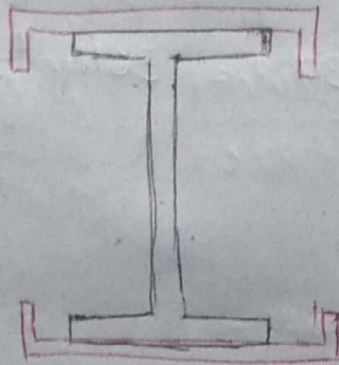
$$= 102.8 \times 10200$$

$$\boxed{P_{cal} = 1048.56 \text{ kN}}$$

As $P_{cal} > P_{given}$,

So this section can be used to support an axial load of 700 kN
So section is W150x80 Ans

Pb # 1125 A hinged end steel column 30 ft long is fabricated from W8x31 section & two C12x30 channel section are arranged as shown. Determine the safe load using AISC specification with $\sigma_{yp} = 36 \text{ ksi}$ & $E = 29 \times 10^6 \text{ psi}$.



Properties

of W8x31 section

$$A = 9.13 \text{ in}^2$$

$$\text{Depth} = 8 \text{ in}$$

$$T_{\text{web}} = 0.285 \text{ in}$$

$$F_{\text{width}} = 7.995 \text{ in}$$

$$F_{\text{fl}} = 0.435 \text{ in}$$

$$I_x = 110 \text{ in}^4$$

$$I_y = 37.1 \text{ in}^4$$

Properties of C12x30

$$A = 8.82 \text{ in}^2$$

$$\text{Depth} = 12 \text{ in}$$

$$\text{Web thickness} = 0.510 \text{ in}$$

$$F_{\text{width}} = 3.170 \text{ in}$$

$$F_{\text{fl}} = 0.501 \text{ in}$$

$$I_x = 162 \text{ in}^4$$

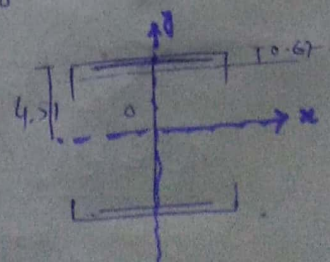
$$I_y = 5.14 \text{ in}^4$$

First find I_x & I_y about global axis.

$$I_x = 2(I_y + Ad^2) + 1(I_x)$$

$$I_x = 2(5.14 + 8.82(4.51 - 0.67)^2) + 110$$

$$+ 110$$



$$\boxed{I_x = 380.39 \text{ in}^4}$$

To find I_y

$$I_y = 2(I_x)_{\text{each}} + 1[I_y]_{\text{web}}$$

$$= 2[162] + 37.1 = 361.1 \text{ in}^4$$

$$\boxed{I_y = 361.1 \text{ in}^4}$$

To find $l/r = ?$

First find $r = ?$

I should be I_{\min}

$$I_{\min} = I_y$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{I_{\min}}{A}}$$

$$r = \sqrt{\frac{361.1}{28.77}}$$

$A = \text{Total area}$

$$A = 2 \times 8.82 + 9.13$$

$$A = 28.77 \text{ in}^2$$

$$\boxed{r = 3.67 \text{ in}}$$

$$l/r = \frac{30 \times 1 \times 12}{3.67} \Rightarrow \boxed{l/r = 98.1}$$

$$C_c = ? \quad \text{As } C_c = \sqrt{\frac{2\pi^2 E}{\sigma_{yp}}}$$

$$C_c = \sqrt{\frac{2(314)^2(29 \times 10^6)}{36 \times 10^3}}$$

$$C_c = 126.11$$

As $C_c > l/r$ so short or intermediate column.

So, Use

$$\sigma_{all} = \left(1 - \frac{(l/r)^2}{2C_c^2}\right) \frac{\sigma_{yp}}{F.S.} \rightarrow \textcircled{A}$$

$$F.S. = \frac{5}{3} + \frac{3}{8} \frac{l/r}{C_c} - \frac{1}{8} \frac{(l/r)^3}{C_c^3}$$

$$F.S. = \frac{5}{3} + \frac{3}{8} \frac{(98.1)}{(126.11)} - \frac{1}{8} \frac{(98.1)^3}{(126.11)^3}$$

$$\boxed{F.S. = 1.835}$$

$$\textcircled{A} \Rightarrow \sigma_{all} = \left(1 - \frac{(98.1)^2}{2(126.11)^2}\right) \frac{36 \times 10^3}{1.835}$$

$$\boxed{\sigma_{all} = 13.683 \text{ ksi}}$$

$$\sigma_{all} = P/A$$

$$P = \sigma_{all} \times A$$

$$P = 13.683 \times 10^3 \times 26.77$$

$$\boxed{P = 366 \text{ kips}} \quad \underline{\text{Ans}}$$

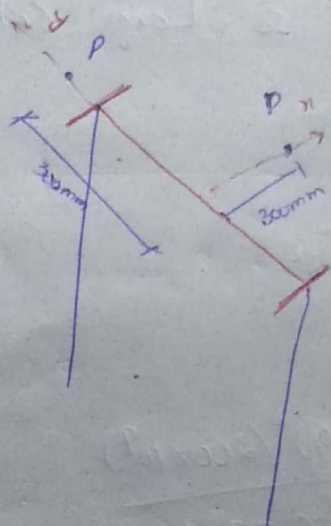
Pb#11134:- A W1360X122 section is used as a column with an effective length of 10m. Determine the max load that can be carried at eccentricity of 300mm.

Should the load be placed on X or Y axis?

Assume $\sigma_{yp} = 290 \text{ MPa}$ & $E = 200 \text{ GPa}$.

Solution:-

Figure.



As from properties table, of W1360X122 section.

$$A = 15500 \text{ mm}^2$$

$$d = 363 \text{ mm}$$

$$b_f = 257 \text{ mm}$$

$$I_x = 365 \times 10^6 \text{ mm}^4$$

$$I_y = 61.5 \times 10^6 \text{ mm}^4$$

$$r_x = 153 \text{ mm}$$

$$r_y = 63 \text{ mm}$$

First find L_e/r_{min}

$$L_e/r = \frac{10 \times 1000}{63} = 158.73$$

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_{yp}}}$$

$$= \sqrt{\frac{2(3.14)^2 (200 \times 10^9)}{290 \times 10^6}}$$

$$C_c = 116.67$$

As $L/r > C_c$ so Long Column.

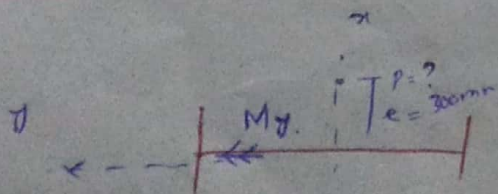
$$\sigma_{all} = \frac{12}{23} \frac{\pi^2 E}{(L/r)^2}$$

$$= \frac{12}{23} \times \frac{(3.14)^2 (200 \times 10^9)}{(158.73)^2}$$

$$\boxed{\sigma_{all} = 40.8 \text{ MPa}}$$

Now we have two possibilities.

Case a:-



Using maximum stress approach.

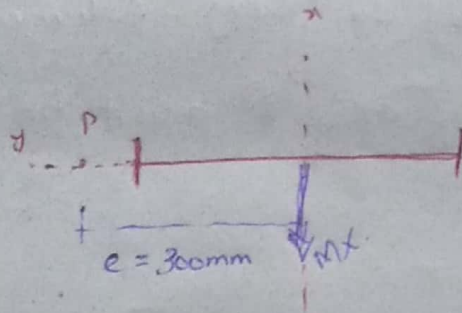
$$\sigma_a + \sigma_f \leq \sigma_{all}$$

$$\frac{P}{A} + \frac{M_e c}{I_x} \leq 40.87$$

$$\frac{P}{15500} + \frac{P(300)(257/2)}{61.5 \times 10^6} = 40.87$$

$$P = 59.117 \text{ kN}$$

Case # b



$$\delta_a + \delta_f \leq \delta_{all}$$

$$\frac{P}{A} + \frac{M_x c}{I_x} \leq 40.8$$

$$\frac{P}{15500} + \frac{P(300)(363/2)}{365 \times 10^6} = 40.8$$

$$P = 191.25 \text{ kN}$$

So select the max safe load i.e. $P = 191.25 \text{ kN}$

Which will apply on y-axis.

Pb # 1139:- Given data.

Section = $C_{310 \times 45}$ (Hinged end column).

$$L = 2.2 \text{ m}$$

$$P = 50 \text{ kN.}$$

$$\sigma_{yp} = 380 \text{ MPa.}$$

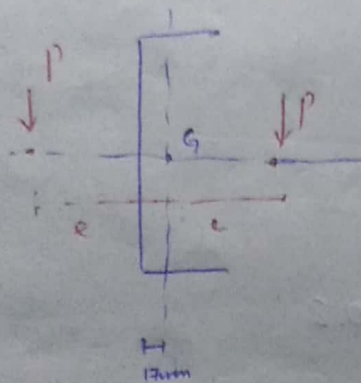
$$\sigma_T = 140 \text{ MPa}$$

$$E = 200 \text{ GPa.}$$

On which side of axis P must be placed?

$$e_{\max} = ?$$

Solution:-



From property of $C_{310 \times 45}$ section.

$$A = 5690 \text{ mm}^2$$

$$I_x = 67.3 \times 10^6 \text{ mm}^4$$

$$I_y = 2.12 \times 10^6 \text{ mm}^4$$

$$r_x = 109 \text{ mm} \quad r_y = 19.3 \text{ mm.}$$

$$x = 17 \text{ mm.} \quad d = 305 \text{ mm} \quad \xi \quad b_f = 80 \text{ mm.}$$

$$\therefore \quad \frac{L}{r}_{\min} = \frac{2.2 \times 1000 \times 1}{19.3} = 114$$

$$C_c = \sqrt{\frac{2\pi^2 E}{S_{yp}}}$$

$$= \sqrt{\frac{2(3.14)^2(200 \times 10^9)}{380 \times 10^6}}$$

$$C_c = 101.87$$

As $l_e/r > C_c$ so large column.

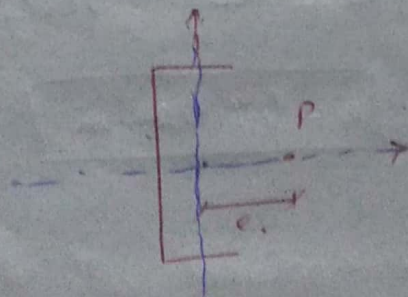
$$\sigma_{all} = \frac{12}{23} \frac{\pi^2 E}{(l_e/r)^2}$$

$$= \frac{12}{23} \frac{(3.14)^2 (200 \times 10^9)}{(114)^2}$$

$$\boxed{\sigma_{all} = 79.16 \text{ MPa}}$$

Case #1

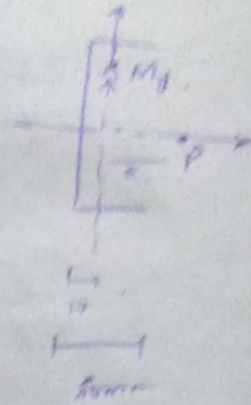
Consider the load applied on right side at distance e as shown.



Based on compression; use max. stress approach;

$$\sigma_a + \sigma_f \leq \sigma_{all}$$

$$\frac{P}{A} + \frac{M_y c}{I_y} \leq \sigma_{all}$$



$$\frac{50 \times 10^3}{5690} + \frac{50 \times 10^3 e (80 - 17)}{2.12 \times 10^6} = 79.24$$

$$\boxed{e = 47.42 \text{ mm}}$$

Based on Tension:

$$\sigma_a - \sigma_b \leq \sigma_{all}$$

$$\frac{P}{A} - \frac{M_y c}{I_y} \leq \sigma_t$$

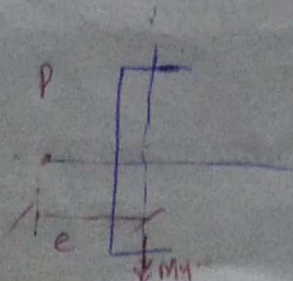
$$\frac{50 \times 10^3}{5690} - \frac{50 \times 10^3 e (17)}{2.12 \times 10^6} = -140$$

$$\boxed{e = 370.6 \text{ mm}}$$

So the max. safe value to prevent buckling

& tensile failure is $\boxed{e = 47.42 \text{ mm}}$

Case #b let load is applied at ~~right~~ left



Based on compression

$$\sigma_a + \sigma_f \leq \sigma_{all}$$

$$\frac{P}{A} + \frac{M_y c}{I_y} \leq \sigma_{all}$$

$$\frac{50 \times 10^3}{5690} + \frac{50 \times 10^3 (e) (17)}{3.12 \times 10^6} = 79.24$$

$$\boxed{e = 175.72 \text{ mm}}$$

Based on tensile stresses.

$$\sigma_a - \sigma_f \leq \sigma_T$$

$$\frac{P}{A} - \frac{M_y c}{I_y} \leq \sigma_T$$

$$\frac{50 \times 10^3}{5690} - \frac{50 \times 10^3 (e) (80 - 17)}{2.12 \times 10^6} = -140$$

$$\boxed{e = 100 \text{ mm}}$$

Select the smallest $\boxed{e = 100 \text{ mm}}$

But in case (a) & case (b) we will consider e_{max} which have largest e value

So select $\boxed{e_{max} = 100 \text{ mm}}$ Ans

Pb #1136 :- Given data

Dimension = 2" by 3"

$$L_e = 5'$$

$e = 5"$ (from the weaker axis).

$$\sigma_{yp} = 36 \text{ ksi}$$

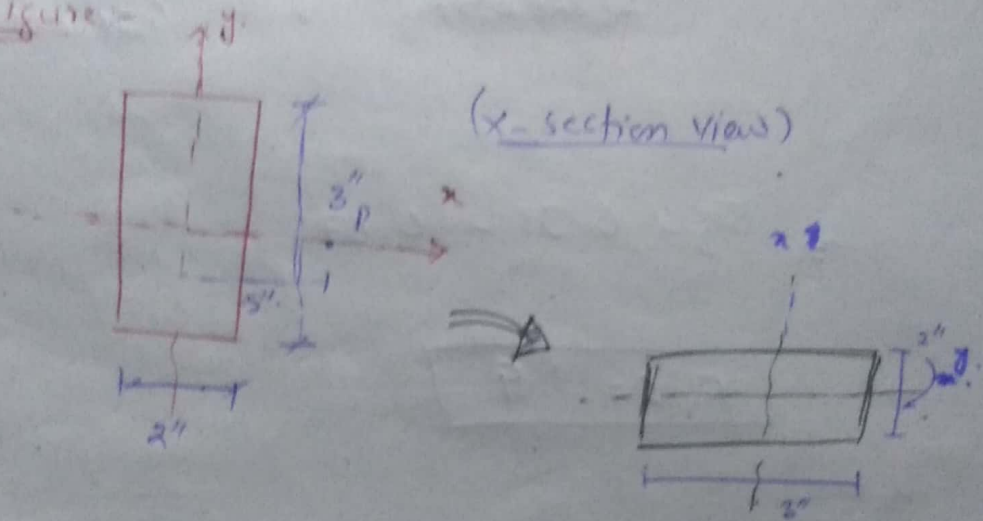
$$P_a = 11 \text{ kips}$$

$$E = 29 \times 10^6 \text{ psi}$$

$$P_e = ?$$

Sol:-

Figure -



As $I_x > I_y$ so y-axis weaker axis

$$\text{So from } L_e/r = \frac{5 \times 12}{\sqrt{I/A}} = \frac{60}{\sqrt{\frac{3(2)^2}{12(3)(2)}}}$$

$$= \frac{60}{\sqrt{4/12}} = \frac{60}{0.577}$$

$$\boxed{L_e/r = 103.9}$$

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_{yp}}} = \sqrt{\frac{2(3.14)^2 (29 \times 10^6)}{36 \times 10^3}}$$

$$\boxed{C_c = 126.035}$$

As $c_c > l_e/r$ so ~~large~~ small column or intermediate

So,

$$\sigma_{all} = \left(1 - \frac{(l_e/r)^2}{2c_c^2}\right) \frac{\sigma_{yp}}{F.S} \rightarrow (1)$$

$$F.S = \frac{5}{3} + \frac{3(l_e/r)}{8c_c} - \frac{1(l_e/r)^3}{8c_c^3}$$

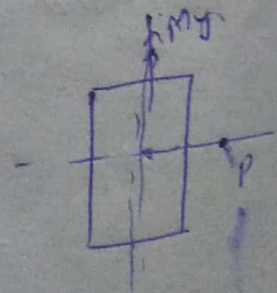
$$= \frac{5}{3} + \frac{3(103.9)}{8(126.035)} - \frac{1}{8} \left(\frac{(103.9)^3}{(126.035)^3} \right)$$

$$= \frac{5}{3} + 0.31 - 0.07$$

$$\boxed{F.S = 1.9}$$

$$\sigma_{all} = \left(1 - \frac{(103.9)^2}{2(126.035)^2}\right) \frac{3 \times 10^3}{1.9}$$

$$\boxed{\sigma_{all} = 12.51 \text{ Ksi}}$$



Using max. stress approach.

$$\sigma_a + \sigma_f \leq \sigma_{all}$$

$$\frac{EP}{A} + \frac{M_y c}{I_y} = \sigma_{all}$$

$$\frac{P_c + 11}{2 \times 3} + \frac{P_c \times 5 \times (1) \times 12^4}{3(2)^3} = 12.51 \times 10^3$$

$$\frac{P_e + 11}{2 \times 3} + \frac{P_e \times 5 \times 2 \times 4}{2 \times 2 \times 2} = 12.51 \times 10^3$$

$$\frac{P_e + 11 + 3(P_e \times 5)}{6} = 12.51 \times 10^3$$

$$P_e + 11 + 15P_e = 75.06 \times 10^3$$

$$16P_e = 74.06$$

$$P_e = 4000 \text{ lb.}$$

Pb #1134:- Given data.

$L = 8'$ (Fixed - Free)

Pb #1137:- Given data

$L = 8'$ (Fixed - Free end)

Sign whose c.g. is $2''$ from axis of pipe.

$$d_{out} = 4.5''$$

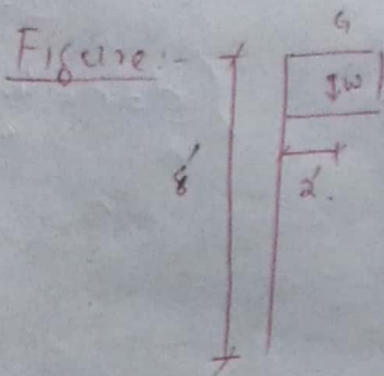
$$Area = 3.174 \text{ in}^2$$

$$I = 7.233 \text{ in}^4$$

$W = ?$

$$S_{yp} = 50 \text{ Ksi}$$

$$E = 29 \times 10^6 \text{ psi}$$



Solution:- Since;

$$L/r = \frac{8 \times 2 \times 12}{\sqrt{\frac{7.233}{3.174}}}$$

$$\Rightarrow \boxed{L/r = 127.2}$$

$$C_c = \sqrt{\frac{2\pi^2 E}{S_{yp}}} = \sqrt{\frac{2(3.14)^2 (29 \times 10^6)}{50 \times 10^3}}$$

$$\boxed{C_c = 106.94}$$

As $L/r > C_c$ so long Column.

Since;

$$S_{all} = \frac{12}{23} \frac{\pi^2 E}{(L/r)^2}$$

$$= \frac{12}{23} \frac{(3.14)^4 (29 \times 10^6)}{(127.2)^4}$$

$$\boxed{\sigma_{all} = 9220.12 \text{ Psi}}$$

Using max stress approach

$$\sigma_a + \sigma_f \leq \sigma_{all}$$

$$\therefore \sigma_f = \frac{Mc}{I}$$

$$\left(W = \frac{P}{A} + \frac{W(2 \times 12) \times \left(\frac{4.5}{2} \right)}{7.233} \leq 9.22 \text{ ksi} \right) \left\{ \begin{array}{l} \sigma_f = \frac{P \times d(c)}{I} \\ \frac{d}{2} = c = \frac{4.5}{2} \end{array} \right.$$

$$\frac{W}{3.174} + 7.465W \leq 9.22$$

$$0.31501W + 7.465W = 9.22 \times 10^3$$

$$7.78W = 9.22 \times 10^3$$

$$\boxed{W = 1185.1 \text{ lb}}$$

$$\text{or } \boxed{W = 1.185 \text{ Kips}} \quad \text{Ans}$$

Pb #1138:- Given data

$$\text{Section} = W_{360 \times 134} = W_{360 \times 134}$$

$$l_e = 6 \text{ m}$$

$$P_a = 260 \text{ kN}$$

$$P_e = 220 \text{ kN} \text{ (applied on minor axis)}$$

$$e_{\text{max}} = ? , \sigma_{yp} = 250 \text{ MPa} \quad \& \quad E = 200 \text{ GPa}$$

Solution:- From the property of $W_{360 \times 134}$

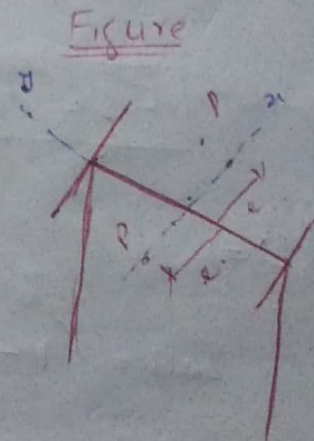
$$A = 17100 \text{ mm}^2$$

$$d = 356 \text{ mm}$$

$$b_f = 369 \text{ mm}$$

$$r_x = 156$$

$$r_y = 94 \text{ mm}$$



First find $l_e/r_{\text{min}} = ?$

$$l_e/r_{\text{min}} = \frac{6 \times 1000}{94}$$

$$\boxed{l_e/r = 63.83}$$

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_{yp}}} = \sqrt{\frac{2(3.14)^2 (200 \times 10^9)}{250 \times 10^6}}$$

$$\boxed{C_c = 125.6}$$

As $C_c > l_e/r$ so Short or Intermediate column.

$$\sigma_{all} = \left(1 - \frac{(l/r)^4}{2c_c^4}\right) \frac{\sigma_{yp}}{F.S.} \rightarrow (1)$$

$$F.S. = \frac{5}{3} + \frac{3(l/r)}{8c_c} - \frac{(l/r)^3}{8c_c^3}$$

$$= \frac{5}{3} + \frac{3}{8} \frac{(63.83)}{125.6} - \frac{(63.83)^3}{8(125.6)^3}$$

$$= \frac{5}{3} + 0.19 - 0.0164$$

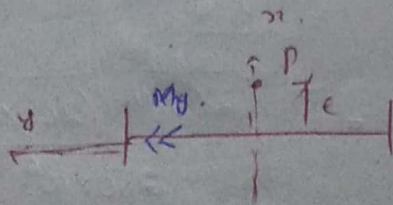
$$\boxed{F.S. = 1.84}$$

$\{ \Rightarrow$

$$\sigma_{all} = \left(1 - \frac{(63.83)^2}{2(125.6)^2}\right) \frac{250}{1.84}$$

$$\sigma_{all} = 118.32 \text{ MPa}$$

Case a



Using max stress approach

$$\sigma_a + \sigma_f \leq \sigma_{all}$$

$$\frac{P}{A} + \frac{M_y c}{I_y} < \sigma_{all}$$

$$\frac{(260 + 220) \times 10^3}{17100} + \frac{0.020 \times 10^3 \times e (369/2)}{415 \times 10^6} = 118.32$$

$$26.07 MR + 0.097e = 118.32$$

$$0.097e = 90.25$$

$$e = \frac{90.25}{0.097}$$

$$\boxed{e = 930.4 \text{ mm}} \quad \text{Ans}$$

Pb # 1139:- Given data

Section ~~is~~ C₃₁₀ × 45 (Hinged end column).

Already solved!

Pb

Pb # 1141 :- Given data

Section W14x90.

$$L = 30'$$

$$P_a = 65 \text{ Kips}$$

$$P_e = 90 \text{ Kips (acting on Y-axis)}$$

$$e_{\max} = ?$$

Using max. stress approach & AISC specification

$$s_{yp} = 50 \text{ Ksi} \quad \& \quad E = 29 \times 10^6 \text{ psi.}$$

Sol:- From the property table, the W14x90 has;

$$A = 26.5 \text{ in}^2$$

$$d = 14.02 \text{ in}$$

$$d_f = 14.520''$$

$$I_x = 999 \text{ in}^4$$

$$r_x = 6.14 \text{ in}$$

$$I_y = 362 \text{ in}^4$$

$$r_y = 3.70 \text{ in.}$$

Using AISC specification.

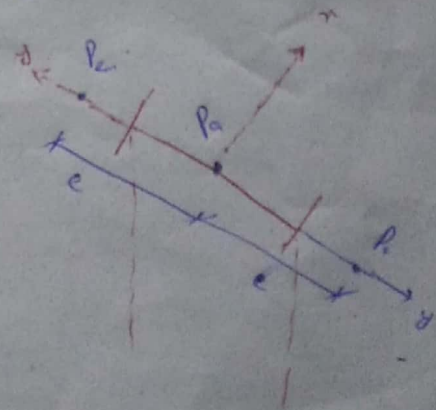
$$L/r_{\min} = \frac{30 \times 12 \times 1}{3.70}$$

$$L/r_{\min} = 97.3$$

$$C_c = \sqrt{\frac{2\pi^2 E}{s_{yp}}}$$

$$= \sqrt{\frac{2(3.14)^2 (29 \times 10^6)}{50 \times 10^3}}$$

$$C_c = 106.9$$



As $C_c > l/r$ so short or intermediate column.

$$\sigma_{all} = \left(1 - \frac{(l/r)^2}{2C_c^2}\right) \frac{\sigma_{yp}}{F.S.} \quad \text{--- (1)}$$

$$F.S. = \frac{5}{3} + \frac{3(l/r)}{8C_c} - \frac{(l/r)^3}{8C_c^3}$$

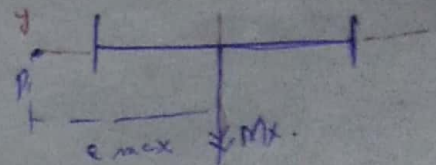
$$= \frac{5}{3} + \frac{3}{8} \frac{(97.3)}{106.9} - \frac{(97.3)^3}{8(106.9)^3}$$

$$= \frac{5}{3} + 0.341 - 0.09425$$

$$\boxed{F.S. = 1.9}$$

$$\text{Eq (1)} \Rightarrow \sigma_{all} = \left(1 - \frac{(97.3)^2}{2(106.9)^2}\right) \frac{50 \times 10^3}{1.9}$$

$$\boxed{\sigma_{all} = 15.415 \text{ Ksi}}$$



Using max stress approach.

$$\sigma_a + \sigma_f \leq \sigma_{all}$$

$$\frac{P}{A} + \frac{M_x c}{I_x} = \sigma_{all}$$

$$\frac{(65 + 90) \times 10^3}{26.5} + \frac{90 \times e \times (14.02)}{999} = 15.415 \times 10^3$$

$$5.85 \times 10^3 + 631.53e = 15.415 \times 10^3$$

$$631.53e = 9.565 \times 10^3$$

$$\boxed{e = 15.14 \text{ in}} \quad \underline{\text{Ans}}$$

Pb #1139 A C310x45 channel is used as a hinged end column 2.2m long. How far from centre can a load of 50kN be placed on X-axis. Assume $\sigma_{yp} = 380 \text{ MPa}$, ϵ tensile stress to be limited to 140 MPa. On which side of Y axis must the load be applied? Use $E = 200 \text{ GPa}$.

Given data.

$$L = 2.2 \text{ m}$$

Hinged end, $K = 1$

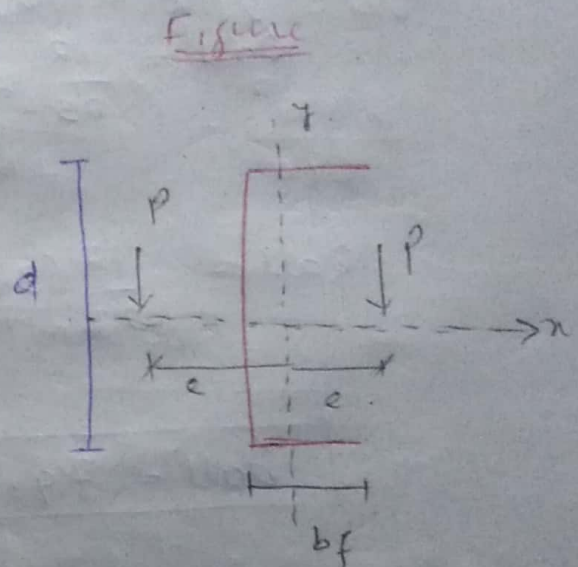
$$P = 50 \text{ kN}$$

$$\sigma_{yp} = 380 \text{ MPa}$$

$$\sigma_t = 140 \text{ MPa}$$

Required data

$$e_{\max} = ?$$



From property, table of C310x45 section.

$$A = 5690 \text{ mm}^2, I_x = 67.3 \times 10^6 \text{ mm}^4, I_y = 2.12 \times 10^6 \text{ mm}^4$$

$$r_x = 109 \text{ mm}, r_y = 19.3 \text{ mm}, \bar{x} = 17 \text{ mm}$$

$$d = 305 \text{ mm}, b_f = 80 \text{ mm}$$

Solution:-

First find $L/r = ?$

$$L/r = \frac{2.2 \times 1000 \times 1}{19.3}$$

$$L/r = 114$$

$$C_c = \sqrt{\frac{2\lambda^2 E}{\sigma_{yp}}} = \sqrt{\frac{2(3.14)^2 (200 \times 10^9)}{(38 \times 10^6)}}$$

$$C_c = 101.82$$

∴ $l/r > C_c$ long column.

Use

$$\sigma_{all} = \frac{12}{23} \frac{\lambda^2 E}{(l/r)^2}$$

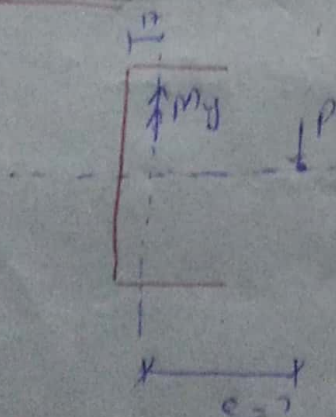
$$\sigma_{all} = \frac{12}{23} \frac{(3.14)^2 (200 \times 10^9)}{(114)^2}$$

$$\sigma_{all} = 79.18 \text{ MPa}$$

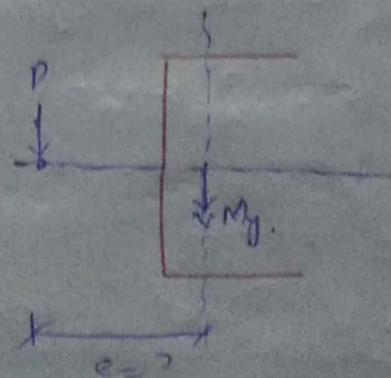
Using Max. stress approach.

Two Cases

Case #1



Case #2



Consider Case #1

Based on compression:-

Consider Compression side

$$\delta_a + \delta_f \leq \delta_{all}$$

$$P/A + \frac{M_y \times c}{I_y} \leq 79.24$$

$$\frac{56 \times 10^3}{5690} + \frac{50 \times 10^3 \times (e)(80-17)}{2.12 \times 10^6} = 79.24$$

$$\boxed{e = 47.42 \text{ mm}}$$

Based on tension side:-

$$\delta_a - \delta_f \leq \delta_{all}$$

$$P/A - \frac{M_y \times c}{I_y} < -140$$

$$\frac{56 \times 10^3}{5690} - \frac{50 \times 10^3 (e)(17)}{2.12 \times 10^6} = -140$$

$$\boxed{e = 370.6 \text{ mm}}$$

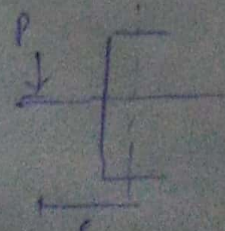
Consider the smallest value of e
to prevent buckling & tension failure so
 $\boxed{e = 47.42 \text{ mm}}$

Case #02 Load applied on left side

Based on compression:-

$$\delta_a + \delta_f \leq \delta_{all}$$

$$P/A + M_y(c)/I_y = \delta_{all}$$



$$\frac{50 \times 10^3}{5690} + \frac{50 \times 10^3 (e)(17)}{2.12 \times 10^6} = 79.24$$

$$\boxed{e = 175.72 \text{ mm}}$$

On-tension side:-

$$\sigma_a - \sigma_f < -140$$

$$\frac{50 \times 10^3}{5690} - \frac{(50 \times 10^3)e(63)}{2.12 \times 10^6} < -140$$

$$\boxed{e = 100 \text{ mm}}$$

Here in these case Consider the smallest value i.e. $\boxed{e = 100 \text{ mm}}$

But for

max. safe e :-

Consider $\boxed{e = 100 \text{ mm}}$

Max. eccentricity is $e = 100 \text{ mm}$ { - the load should applied on left of x-axis.