Statistics

Lect # 10

Conditional Probability :-

The probability of an experiment in which we restrict the solution space of experiment by providing more information.

For example :-

A dice is thrown what is probability that 4 will appear given that even number appear.

→ As we give extra Information that the 4 probability find with in event having even number

Since the sample space is from 1 to 6 but we restrict our domain to even number i.e 2, 4, 6.

② If two coins are tossed if one head came what is probability that on second tossed tail occured.

So here in both example conditional probability is used as extra Information is given.

Mathematically, Probability of event A that B occu

$P(A/B) = \frac{P(A \cap B)}{P(B)}$

∴ Event B should not be Impossible event.
For e.g If die is thrown what is probability of zero come?
→ Ans Impossible event.

Proof :-

As $A \subseteq S$ so; $A = A \cap S$

S = Sample Space

$P(A) = \frac{n(A)}{n(S)} = \frac{n(A \cap S)}{n(S)}$.

As we restrict the domain i.e Sample space. So In this case Sample space will be B.

∴ Where B is the information i.e given which restrict the sample space

So; $P(A/B) = \frac{n(A \cap B)}{n(B)}$.

∴ These Probability eqn should satisfy those three axioms.

Now divide by $n(S)$ both numerator & denumerator.

$P(A/B) = \frac{n(A \cap B)/n(S)}{n(B)/n(S)}$

$$\boxed{P(A/B) = \frac{P(A \cap B)}{P(B)}}$$

Similarly;

$$\boxed{P(B/A) = \frac{P(A \cap B)}{P(A)}}$$

∴ $P(A/B)$ can be read as probability of A given B.

~~Exam~~ Multiplication law:-

If A & B are events from sample space

so;

$$\boxed{P(A\cap B) = P(A/B) * P(B)}$$

or

$$\boxed{P(A\cap B) = P(B/A) * P(A)}$$

For three events:-

$$\boxed{P(A\cap B\cap C) = P(A)\, P(B/A)\, P(C/A\cap B)}$$

Example # 6.20:-

A box contain 15 items, 4 of which are defective & 11 are good. Two items are selected. what is probability that first is good & second is defective?

Solution:- As here word "&" used so used multiplication law;

Since $P(A\cap B) = P(A)\, P(B/A) \longrightarrow ①$.

let A represent event representing good items & B represent event representing bad (defective) items.

Probability of good item is;

$$P(A) = \frac{n(A)}{n(s)} = \frac{11}{15}.$$

$P(B/A)$ is probability of selecting defective after good is;

$n(S) = 15$

$$P(B/A) = \frac{n(B/A)}{n(S)}$$

As
$n(S) = 15$
But when one is selected
so $n(S) = 14$.

$\because$ $n(B/A)$ = no. of defective items remaining

$$= \frac{4}{14}.$$

eq ① $\Rightarrow$

$$P(A \cap B) = \frac{11}{15} * \frac{4}{14}.$$

$$= \frac{44}{210}.$$

$$\boxed{P(A \cap B) = 0.16}$$ Ans

Example # 6.17

Two coins are tossed. What is the conditional probability that two heads result, given that there is atleast one head.

Solution:- As the sample space is;

$$S = \{HH, HT, TH, TT\}.$$

$$n(S) = 4.$$

$$A = \{HH\}, \quad B = \{HT, TH, HH\}.$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{4}$$

$$P(B) = \frac{n(B)}{n(S)} = 3/4.$$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = 1/4.$$

So use

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{1/4}{3/4}.$$

$$\boxed{P(A/B) = 1/3}$$

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Example # 6.18

A man tosses two fair dices. What is Conditional probability that the sum of two dice will be 7, given that (i) the sum is odd (ii) The sum is greater than 6 (iii) The two dice has same outcome?

Solution:- As conditional Probability is;

$$P(X/Y) = \frac{P(X \cap Y)}{P(Y)} \longrightarrow ①.$$

let A represent event in which sum is 7.

$A = \{(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)\}$.

$n(A) = 6$.

As two dices are thrown so;

$n(S) = 6^2 = 36$

$n(S) = 36$

$$P(A) = \frac{n(A)}{n(S)} = \frac{6}{36} = 1/6.$$

Let B represent an event in which we have number whose sum is odd.

$B = \{(1,2), (1,4), (1,6), (2,1), (2,3), (2,5), (3,2), (3,4), (3,6), (4,1), (4,3), (4,5), (5,2), (5,4), (5,6), (6,1), (6,3), (6,5)\}$

$n(B) = 18$.

$$P(B) = \frac{n(B)}{n(S)} = \frac{18}{36} = 1/2$$

As we know from def of conditional probability.

$$P(A/B) = \frac{P(A \cap B)}{P(B)} \rightarrow (1).$$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{6}{36} = \frac{1}{6}.$$

$$\text{eq (1)} \Rightarrow P(A/B) = \frac{1/6}{1/2}$$

$$\boxed{P(A/B) = 1/3}$$

Now finding $P(A/C) = ?$

As $P(A/C) = \frac{P(A \cap C)}{P(C)}.$

Let C represent an event whose sum is greater than 6.

$C = \{ (1,6), (2,5), (2,6), (3,4), (3,5), (3,6), (4,3), (4,5)$
$(4,4), (4,6), (5,2), (5,3), (5,4), (5,5), (5,6), (6,1), (6,2)$
$-- (6,6)$

$$P(C) = \frac{n(C)}{n(S)} = \frac{21}{36} = \frac{7}{12}.$$

$$\frac{n(A \cap C)}{n(S)} = P(A \cap C)$$

$$P(A \cap C) = \frac{6}{36} = 1/6$$

$\{①\} \Rightarrow$ $$P(A/C) = \frac{P(A \cap C)}{P(C)}$$

$$= \frac{1/6}{7/12}$$

$$\boxed{P(A/C) = 2/7}$$

(iii) Let D represent an event which represent same out comes.

$D = \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)\}$

$n(D) = 6$

$$P(D) = \frac{n(D)}{n(S)} = \frac{6}{36} = 1/6$$

$$P(A \cap D) = \frac{n(A \cap D)}{n(S)} = 0$$

$\therefore A \cap D = \phi$

$P(A \cap D) = 0$.

So mutually exclusive so $P(A/D)$ is zero.

So, $$P(A/D) = \frac{0}{1/6}$$

$$\boxed{P(A/D) = 0}$$ Ans

Example #6.19

What is the probability that randomly selected poker hand, contain exactly 3 aces, given that it contains at least 2 aces.

Solution:- As 5 cards (Pokar hand contain 5 cards) are selected randomly where order is not important so we used combination.

So; $n(S) = {}^{52}C_5 = \frac{52!}{5!\,(52-5)!}$

$$\boxed{n(S) = 2598960}$$

let A be event which represent 3 aces.

So; $n(A) = {}^{4}C_3 \times {}^{48}C_2$.

∴ As we have 4 aces in which we have to select 3. But total cards i.e to be selected is 5 so remaining cards are 48 in which we have to select 2 to fullfill the demand → Not requirement of order So used combination.

$n(A) =$

let B represent an event which contain at least 2 aces.

$n(B) =$ 2As & 3Os or 3As & 2Os or 4As & 1Os.

$= {}^4C_2 * {}^{48}C_3 + {}^4C_3 * {}^{48}C_2 + {}^4C_4 * {}^{48}C_1$

∵ Since we have 4 As so move till 4 As.

$n(B) =$

Since we know that;

$$P(A/B) = \frac{P(A \cap B)}{P(B)} \longrightarrow (A).$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{{}^4C_2 * {}^{48}C_3 + {}^4C_3 * {}^{48}C_2 + {}^4C_4 * {}^{48}C_1}{2598960}.$$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} \longrightarrow (B)$$

Since $A \cap B = A$

bcz at least two aces come in definition & 3 aces which event A.

So; $n(A \cap B) = n(A)$.

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{n(A)}{n(S)} = \frac{{}^4C_3 * {}^{48}C_2}{2598960}.$$

Put values in eq (A).

$$P(A/B) = \left(\frac{{}^4C_3 * {}^{48}C_2}{\cancel{2598960}}\right) \div \frac{{}^4C_2 * {}^{48}C_3 + {}^4C_3 * {}^{48}C_2 + {}^4C_4 * {}^{48}C_1}{\cancel{2598960}}$$

$$= \frac{{}^4C_3 * {}^{48}C_2}{{}^4C_2 * {}^{48}C_3 + {}^4C_3 * {}^{48}C_2 + {}^4C_4 * {}^{48}C_1} = \frac{4512}{108,336} = 0.0416$$

$P(A/B) = 0.0416$ Ans

Example # 6.21:-

Two cards are delt from a pack of ordinary playing cards. Find the probability that second card delt is heart?

Solution:- As two cards are delt from a pack of cards.

So possibilities are that first card is heart & second card is also heart.
or first card is not heart & second is heart.

Let H_1 represent event which represent first card is heard.

while H_2 also represent that second card is or can be heart

$H_1 \rightarrow$ First card.

$H_2 \rightarrow$ Second card.

As for second card

$$P(H_2) = P(H_1 \cap H_2) + P(\bar{H}_1 \cap H_2) \rightarrow (A)$$

As we have two possibilities that

(1) first is heart & second is also heart.

(2) first is not heart & second is heart.

$$= P(H_2)\,P(H_1/H_2) + P(\bar{H}_1).\,P(H_2/\bar{H}_1)$$

$$= \frac{13}{52} \cdot \frac{12}{51} + \frac{39}{52} \cdot \frac{13}{51}$$

$$= \frac{1}{17} + \frac{13}{68} = \frac{17}{68}$$

$$\boxed{P(H_2) = \frac{1}{4} \quad \text{Ans}}$$

Example # 6.22:-

Box A contains 5 green & 7 red balls. Box B contains 3 green, 3red & 6 yellow balls. A box is selected at random and a ball is drawn at random from it. What is probability that first ball is green?

Solution:-

$$A \rightarrow 5\ \text{Green}, \quad A \rightarrow 7\ \text{Red}$$

$$B \rightarrow 3\ \text{Green}, \quad B \rightarrow 3\ \text{red}, \quad B \rightarrow 6\ \text{yellow}$$

As we have two boxes so first we have to select one box randomly & than find probability of green ball.

So we have two possibilities i.e.

$P(G) = P(A \cap G) + P(B \cap G) \longrightarrow ①.$

Possibilities:-

i. Probability of Green ball in box A.

ii. Probability of " " " " B.

Using multiplication law.

$P(G) = P(A)\ P(G/A) + P(B)\ P(G/B)$

∴ P(A) represent probability of box A.
P(G/A) represent probability of green ball in box A
& vice versa for P(B) & P(G/B).

$=$

$= \frac{1}{2} \times \frac{5}{12} + \frac{1}{2} \times \frac{3}{12}$

$P(G) = \frac{5}{24} + \frac{3}{24} = \frac{8}{24} = \frac{1}{3}.$

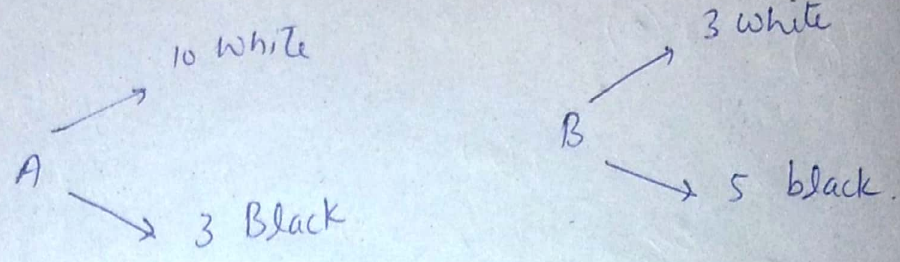
$\boxed{P(G) = 1/3}$

or $\boxed{P(G) = 0.33}$ Ans

Example # 6.23

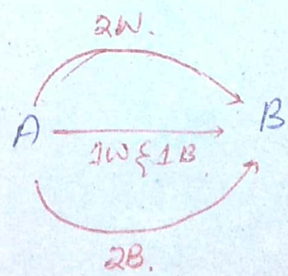
An urn contains 10 white & 3 black balls. Another urn contains 3 white & 5 black balls. Two balls are transfered from first urn & placed in second urn & than one ball is taken from the latter. What is probability that is white ball?

Solution:- As we have two urns.



Now as given that two balls are transfereed from urn A to B.

So there are three possibilities as shown.



Let A represent event that 2 balls are transferred to B from A.

So; A_1 = 2 white balls.

$n(A_1) = {}^{10}C_2$

∴ As any two white balls so; used combination.

$n(S) = {}^{13}C_2$.

So; $P(A_1) = \dfrac{{}^{10}C_2}{{}^{13}C_2} = \dfrac{10!/2!(10-2)!}{13!/2!(10-3)!}$

$$\boxed{P(A_1) = \frac{45}{78}}$$

A_2: 1 w & 1 B.

$n(A_2) = {}^{10}C_1 * {}^{3}C_1$.

$P(A_2) = \dfrac{{}^{10}C_1 * {}^{3}C_1}{{}^{13}C_2} = \dfrac{\frac{10!}{1!(10-1)!} * \frac{3!}{1!(3-2)!}}{13!/2!(13-2)!}$

$$\boxed{P(A_2) = \frac{30}{78}}$$

A_3: 2B.

$n(A_3) = {}^{3}C_2$.

$P(A_3) = \dfrac{n(A_3)}{n(S)} = \dfrac{{}^{3}C_2}{{}^{13}C_2}$.

$= \dfrac{3!/2!(3-2)!}{13!/2!(13-2)!} \Rightarrow \dfrac{3}{78}$.

Now Probability of white ball is;

$P(w) = P(A_1 \cap w) + P(A_2 \cap w) + P(A_3 \cap w)$

Use multiplication law;

$= P(A_1)\, P(W/A_1) + P(A_2)\, P(W/A_2) + P_3(A_3)\, P(W/A_3).$

$= \frac{45}{78} \cdot \frac{5}{10} + \frac{30}{78} \cdot \frac{4}{10} + \frac{3}{78} \cdot \frac{3}{10}.$

$\therefore P(W/A_1)$ = Probability of white balls in B after transfer of two balls in B.

$P(W) = \frac{15}{52} + \frac{2}{13} + \frac{3}{260}$

$\therefore P(W/A_2)$ = Probability of white ball in B.

$= \frac{59}{130}.$

$\boxed{P(W) = 0.4538}$

Example #6.24

A Card is drawn at random from deck of ordinary playing card. What is the probability that it is a diamond, a face card or king?

Solution:- As we have 3 events & there used or.

So;

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$ ⟶ ①.

Let A represent event which contain diamond.

$P(A) = n(A)/n(S) = 13/52$. $\therefore n(S) = 52$.

B represent event which contain face card

$$P(B) = \frac{n(B)}{n(S)} = \frac{12}{52}$$

C represent event which contain King.

$$P(C) = \frac{n(C)}{n(S)} = \frac{4}{52}$$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = 3/52$$

$$P(A \cap C) = \frac{n(A \cap C)}{n(S)} = 1/52$$

$$P(B \cap C) = \frac{n(B \cap C)}{n(S)} = 4/52$$

$$P(A \cap B \cap C) = 1/52 = \frac{n(A \cap B \cap C)}{n(S)}$$

By putting values;

$$P(A \cup B \cup C) = 13/52 + \frac{12}{52} + \frac{4}{52} - 3/52 - \frac{1}{52} - 4/52 + 1/52$$

$$= \frac{13+12+4-3-1-4+1}{52}$$

$$= \frac{22}{52} = 0.423 \quad \text{Ans}$$

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multiplication law.

Example # 6.25

Three urns of the same appearance are given as follow.

Urn A contains 5 red & 7 white balls

Urn B contains 4 red & 3 white balls.

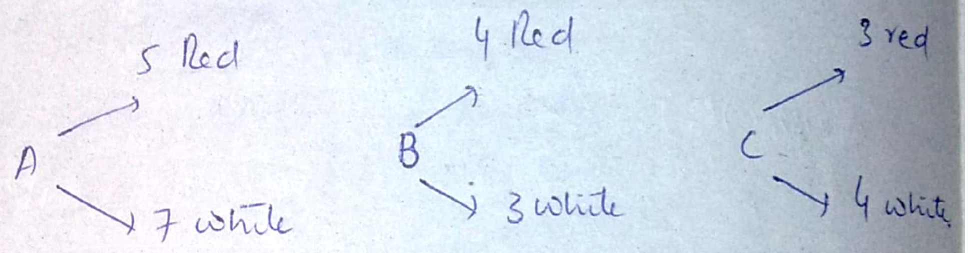
Urn C contain 3 red & 4 white balls.

An urn is selected at random & the ball is drawn from urn.

i- What is probability That ball drawn is red?

ii. If the ball is drawn is red, what is probability That it came from A (urn).

Solution:-



(i)

$$P(R) = P(A \cap R) + P(B \cap R) + P(C \cap R)$$

use multiplication law.

$$P(R) = P(A) \cdot P(R/A) + P(B)\, P(R/B) + P(C) \cdot P(R/C).$$

$\therefore$ $P(A)$ represent Probability of box selection.

$P(R/A)$. represent probability of selecting red ball

(iii) Find $P(A \cap B)$ & $P(B)$.
in Urn A.

so;

$$P(R) = \frac{1}{3} \cdot \frac{5}{12} + \frac{1}{3} \cdot \frac{4}{7} + \frac{1}{3} \cdot \frac{3}{7}$$

$$= \frac{119}{252} = 0.4722.$$

(ii) We need probability of urn A is selected, such that red ball is drawn.

so; $P(A/R) = ?$

Since we know;

$$P(A/R) = \cancel{P(A)} \cdot \frac{P(A \cap R)}{P(R)} \cdot \longrightarrow ①$$

$P(A \cap R) =$ Probability that urn A is selected & red ball is drawn.

$$= \frac{1}{3} * \frac{5}{12} = \frac{5}{36}.$$

$$P(R) = 0.4722.$$

eq ① => $P(A/R) = \dfrac{5/36}{0.4722}$

$$\boxed{P(A/R) = 0.294}$$ Ans

Independant Event:-

If the occurance of one event is not effecting occurance of other event, they will be independant.

For example head & tail. head effect occuran of tail so they are dependant event.

→ In class if we have two events A & B i.e A represent no. of student who fail & B represent who will passed.
As they both not depend on each other so A & B are Independant event.

$$\boxed{P(A/B) = P(A)}$$
$$\boxed{P(B/A) = P(B)}$$

$$P(A \cap B) = P(A)\,P(B)$$
If A & B are Independant.

Example # 6.26:-

Two events A & B are such that $P(A) = 1/4$, $P(A/B) = 1/2$ & $P(B/A) = 2/3$.

i. Are A & B Independant events?
ii. Are A & B mutually exclusive event?

(iii) Find $P(A \cap B)$ & $P(B)$.

Solution:-

(i) Are A & B are Independant = ?

2

For Independant the following condition should be satisfied.

$P(A/B) = P(A)$.

Since $P(A/B) = 1/2$ & $P(A) = 1/4$.

Since $1/2 \neq 1/4$.

So They are not Independant event.

(ii) Are A & B mutually exclusive?

2

For mutually exclusive.

$P(A/B) = 0$. But here $P(A/B) = 1/2$.

So not mutually exclusive.

As $P(A/B) = \frac{P(A \cap B)}{P(B)} = 0$.

(iii) $P(A \cap B) = ?$

$P(A \cap B) = P(A) \cdot P(B/A)$

$= 1/4 \cdot (2/3) = 2/6 = 1/6$

$P(B) = ?$ As; $P(A \cap B) = P(B) \cdot P(A/B)$

$= P(A \cap B) / P(A/B) = \frac{1/6}{1/2}$

$\boxed{P(B) = 1/3}$

Indin.

Example #6-27:-

Two fair dice, one red & one green, are thrown. Let A denote the event that red die shows an even number & B, the event that the green die shows a 5 or 6. Show that A & B are Independant.

Sol:- As for two events to be Independant.

$$P(A \cap B) = P(A)\, P(B) \longrightarrow ①.$$

Let A represent event that red die show even number.

$$A = \{(1,3),(1,5),(1,1),(2,2),(2,4),(2,6),(3,1),(3,3),(3,5),(4,2)$$
$$(4,4),(4,6),(5,1),(5,3),(5,5),(6,2),(6,4),(6,6)\}.$$

$n(A) = 18.$

But as two dics are thrown so;

$n(S) = 6^2 = 36.$

$$P(A) = \frac{n(A)}{n(S)} = \frac{18}{36} \Rightarrow P(A) = 1/2.$$

Let B represent an event that greed die shows a 5 or 6.

$$B = \{(5,1),(5,2),(5,3)(5,4),(5,6),(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}$$

$n(B) = 12.$

$P(B) = \frac{n(B)}{n(S)}$

$= \frac{12}{36} = 1/3.$

$n(A \cap B) = 6.$

$P(A \cap B) = \frac{6}{36} = 1/6$

so;

eq (1) $\Rightarrow$ $P(A \cap B) = P(A) P(B)$

If Independant above eq^n will satisfy.

$\frac{1}{6} = \frac{1}{2} \cdot (1/3)$

$\boxed{1/6 = 1/6}$ so A & B are Independant event.

& $P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{1/6}{1/3} = 1/2 = P(A)$

$P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{1/6}{1/2} = 1/3 = P(B)$.

So A & B are Independant event (proved)

Example #16.28
Let A be the event that family has children of both sexes A & B be the event that family has at most one boy. If a family is known to have (i) Three children, then show that A & B are Independant events, (ii) Four Children, then show A & B are dependant.

Solution:- ① Three children.

As we have two possible out comes so;

$n(S) = 2^3 = 8$. $n(S) = \{BBB, BBG, BGB, GGG, GGB, GBG, GBB, BGG\}$.

Let A represent event contain both sexes children.

$A = \{BBG, BGB, GGB, GBG, GBB, BGG\}$.

$n(A) = 6$.

Let B represent event contain at most one boy.

$B = \{BGG, GGB, GBG, GGG\}$.

$n(B) = 4$.

As for A & B to be Independant;

$P(A \cap B) = P(A) \cdot P(B) \longrightarrow$ Ⓐ

So, $P(A) = \frac{n(A)}{n(S)} = \frac{6}{8} = 3/4$.

$P(B) = n(B)/n(S) = 4/8 = 1/2$.

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$= 1 - P(A \cup B)$ $\quad \because$ Complement law

$= 1 - [P(A) + P(B) - P(A \cap B)]$ $\quad \because$ Addition law.

$= 1 - P(A) - P(B) + P(A \cap B)$

As Independant so, $P(A \cap B) = P(A)P(B)$

$= 1 - P(A) - P(B) + P(A)\,P(B)$

$=$ ~~$P(B)$~~

$= 1(1 - P(A)) - P(B)(1 - P(A))$

$= (1 - P(A))(1 - P(B))$

$= P(\bar{A}).\ P(\bar{B})$

(Proved!)

Example # 6.29:-

Two Cards are drawn from well shuffled ordinary deck of 52 cards Find the probability that both aces if the first card is i. replaced (ii) Without replaced.

Solution:- As it is the case of Independant bcz selection of ace is Independant on selection of other aces.

so,

$$P(A \cap B) = P(A)\,P(B) = \frac{n(A)}{n(S)} \cdot \frac{n(B)}{n(S)}$$

$$= \frac{4}{52} \cdot \frac{4}{52} = \frac{1}{(13)^2}$$

$$= \frac{16}{(52)^2} = \frac{1}{169}$$

As we have 4 aces in deck of 52 cards.

so;

$n(A) = 4$.

(ii) <u>Without replaced:-</u>

$$P(A \cap B) = P(A)\,P(B)$$

$$= \frac{n(A)}{n(S)} \cdot \frac{n(B)}{n(S)}$$

$$= \frac{4}{52} \cdot \frac{3}{51} = \frac{1}{221} = P(A \cap B)$$

$$\boxed{P(A \cap B) = \frac{1}{221}}$$ Ans.

Example # 6.30:-

A pair of fair dice are thrown twice. What is probability of getting total of 5 &

Example # 6.31:-

Probability that the man will be alive in 25 years with is 3/5 & The probability of his wife will be alive is 2/3. Find the probability that (i) both will be alive (ii) Only the man will be alive (iii) Only the wife will be alive. (iv) At least one will be alive (v) neither will be alive in 25 years.

Solution:- As the events are Independent.

Let A event represent Probability of man.

B event represent probability of wife.

(i) Both will be alive

$P(A \cap B) = P(A) \cdot P(B)$ As Independent

$= 3/5 \cdot 2/3 \Rightarrow \boxed{2/3 = P(A \cap B)}$

(ii) Only man will be alive: we need.

$P(A \cap \bar{B}) = P(A) \cdot P(\bar{B})$

$= P(A) \cdot (1 - P(B))$

$= 3/5 (1 - 2/3) \Rightarrow 3/5 \left(\frac{3-2}{3}\right)$

$= 3/5 \left(\frac{1}{3}\right)$

$\boxed{P(A \cap \bar{B}) = 1/5}$

$n(A \cap B) = 3$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{3}{8}$$

eq (A) =)

$$\frac{3}{8} = \frac{3}{4} \times \frac{1}{2}$$

$$\boxed{\frac{3}{8} = \frac{3}{8}}$$ Proved!!

So A & B are Independant events.

(ii) Four children:-

If four children so,

S = { BBBB, BBBG, BBGG, BBGB, BGGG, BGBB, GBBB, GGBB, GGGB, GGGG, BGBG, GBBG, BGGB, GGBG, BBGG, GBGB }.

$n(S) = 2^4 = 16.$

Let A event represent both sexes.

A = { BBBG, BBGB, BGBB, GBBB, GGBB, GGGB, GBGB, BBGG, BGBG, GBBG, BGGG, GBGG, GGBG, BGGB }.

$n(A) = 14$

Let B event represent at most one boy.

$B = \{bggg, gbgg, ggbg, gggb, gggg\}$.

$n(B) = 5$.

As A & B are dependant if satisfy the following equation.

$$P(A \cap B) \neq P(A)P(B) \longrightarrow ①.$$

$$L.H.S = P(A \cap B)$$

$$= \frac{n(A \cap B)}{n(S)}$$

$$= \frac{4}{16} = 1/4.$$

$$R.H.S = P(A).P(B)$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{14}{16} = \frac{7}{8}.$$

$$P(B) = \frac{5}{16}.$$

$$So; = P(A) \cdot P(B).$$

$$= \frac{7}{8} \cdot 5/16.$$

$$= \frac{35}{128}$$

As ~~ = L.H.S $\neq$ R.H.S.
So A & B are dependant events.

Theorams:-

Theoram 6.8

If A & B are Independant event proved that

$P(A \cap B) = P(A)\, P(B)$

Proved:- As $P(A \cap B) = P(A) P(B)$.

Consider L.H.S.

$$P(A \cap B) = P(B)\, P(A/B).$$

As for Independant event $P(A/B) = P(A)$

So;

$$\boxed{P(A \cap B) = P(B)\, P(A)}$$

(proved)

Theoram 6.9

If A & B are Independant events in sample

space S Then proved

i- $\bar{A}, B$ are Independant (ii) $A, \bar{B}$ are Independant.

(iii) $\bar{A}$ & $\bar{B}$ are Independant.

Solution:- Prove

For $\bar{A}$ & B to be Independ;

$$P(\bar{A} \cap B) = P(\bar{A}).\, P(B) \longrightarrow (A).$$

$$L.H.S = P(\bar{A} \cap B)$$

As;

$$P(\bar{A} \cap B) = P(B) - P(A \cap B).$$ (we proved this in 2nd lect.

$$= P(B) - P(A).\, P(B)$$

$$= (1 - P(A))\, P(B).$$

$\therefore (1 - P(A)) = P(\bar{A})$

$= P(\bar{A}) P(B)$

(Proved)

(ii) A & $\bar{B}$ are Independant.

Proof:- we have to proof.

$P(A \cap \bar{B}) = P(A) P(\bar{B}) \longrightarrow$ (B).

$L.H.S = P(A \cap \bar{B})$

Since $\therefore P(A \cap \bar{B}) = P(A) - P(A \cap B)$

$= P(A) - P(A).(P(B))$

$= P(A)(1 - P(B))$

$\because P(\bar{B}) = 1 - P(B)$

$L.H.S = P(A).P(\bar{B})$

$L.H.S = R.H.S$ (Proved)

(iii) $P(\bar{A} \cap \bar{B})$. i.e to show $\bar{A}$ & $\bar{B}$ are Independant.

Proof:- For $\bar{A}$ & $\bar{B}$ to be Independant

$P(\bar{A} \cap \bar{B}) = P(\bar{A}).P(\bar{B})$

$L.H.S = P(\bar{A} \cap \bar{B})$

$P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B})$ $\because$ Demorgan Theoram.

(iii) Only his wife will be alive.

we need $P(\bar{A} \cap B)$.

Since $P(\bar{A} \cap B) = P(\bar{A}).P(B)$

$= (1 - P(A))\ P(B)$

$= (1 - 3/5)(2/3)$

$= \left(\frac{5-3}{5}\right)(2/3) = \left(\frac{2}{5}\right)(2/3)$

$\boxed{P(\bar{A} \cap B) = 4/15}$

iv) At least one will alive

we need $P(A \cup B)$.

Since $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

But $P(A \cap B) = P(A)\,P(B)$ As Independent

so; $P(A \cup B) = P(A) + P(B) - P(A)\,P(B)$.

$= 3/5 + 2/3 - 3/5(2/3) = 3/5 + \frac{2}{3} - 2/5$

$= \frac{9+10-6}{15} \Rightarrow \boxed{13/15 = P(A \cup B)}$

(v) Neither will be alive.

$P(\bar{A} \cap \bar{B}) = P(\bar{A})\,P(\bar{B})$.

$= (1 - P(A))(1 - P(B))$

$= (1 - 3/5)(1 - 2/3) = \left(\frac{5-3}{5}\right)\left(\frac{3-2}{3}\right)$

$\boxed{P(\bar{A} \cap \bar{B}) = 2/15}$ Ans

Baye's Theoram :-

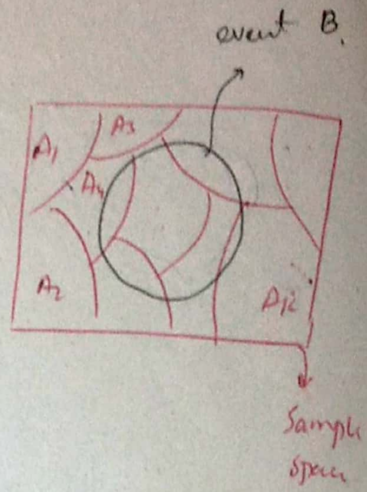
It says.

If we have K mutually exhaustive events

Than

we can write as;

$$S = \bigcup_{i=1}^{k} A_i.$$



As A_i events are mutually exclusive & represent given sample space

$$A_i \cap A_j = \phi \qquad i \neq j$$

let consider another event B. So to find what is probability of B that it either it comes from A_1, A_2 or any A_i can be calculated using Baye's theoram.

$$P(A_i/B) = \frac{P(A_i)\,P(B/A_i)}{\sum_{i=1}^{k} P(A_i)\,P(B/A_i)}$$

Proof :- As we can write;

$$B = B \cap S.$$

But $S = \bigcup_{i=1}^{k} A_i$

So; $B = B \cap (\bigcup_{i=1}^{k} A_i).$

$$B = B \cap (A_1 \cup A_2 \cup A_3 \cup \ldots A_k)$$

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$= (B\cap A_1)\cup(B\cap A_2)\cup(B\cap A_3)\ \ldots\ \cup(B\cap A_k).$

$$B = \sum_{i=1}^{k} P(B\cap A_i)$$

Apply probability.

$$P(B) = \sum_{i=1}^{K} P(B\cap A_i)$$

Using multiplication rule

$$P(B) = \sum_{i=1}^{K} P(A_i)\, P(B/A_i) \longrightarrow (A)$$

As;

$$P(A_i/B) = \frac{P(A_i\cap B)}{P(B)}$$

$$P(A_i/B) = \cancel{P(B)}\ \frac{P(A_i)\, P(B/A_i)}{P(B).} \longrightarrow (B)$$

By putting in eq (B) → Put eq (A) in eq (B)

$$\boxed{P(A_i/B) = \frac{P(A_i)\, P(B/A_i)}{\sum_{i=1}^{k} P(A_i)\, P(B/A_i)}}$$

(**Proved**)

Example #6.34:-

In a bolt factor machine A, B & C manufacture 25, 35 & 40 % of total output respectively. Of their output 5, 4, 2 % respectively are defective bolts. A bolt is selected & found to be defective. What is probability that bolt came from machine A.? B? C?

Solution:- $P(A) = 0.25$

$P(B) = 0.35$.

$P(C) = 0.4$.

The Baye's theoram is used when sum of probability are 1. for e.g $0.25 + 0.35 + 0.4 = 1$.

That is events are mutually exclusive

Bolts (defective) made by machine A.

$P(D/A) = 0.05$, $P(D/B) = 0.04$, $P(D/C) = 0.02$.

So probability of bolts came from machine A.

$$P(A/D) = \frac{P(A)\ P(D/A)}{P(A)\ P(D/A) + P(B)\ P(D/B) + P(C)\ P(D/C)}$$

$$P(A/D) = \frac{0.25 * 0.05}{0.25 * 0.05 + 0.35 * 0.04 + 0.4 * 0.02}$$

$$= \frac{0.0125}{0.0345}$$

$$\boxed{P(A/D) = 0.362}$$

$$P(B/D) = \frac{P(B)\,P(D/B)}{P(A)\,P(D/A) + P(B)\,P(D/B) + P(C)\,P(D/C)}$$

$$= \frac{0.35 * 0.04}{0.25 * 0.05 + 0.35 * 0.04 + 0.4 * 0.02}$$

$$\boxed{P(B/D) = 0.406}$$

$$P(C/D) = \frac{P(C)\,P(D/C)}{P(A)\,P(D/A) + P(B)\,P(D/B) + P(C)\,P(D/C)}$$

$$= \frac{0.4 * 0.02}{0.25 * 0.05 + 0.35 \times 0.04 + 0.4 * 0.02}$$

$$\boxed{P(C/D) = 0.232}$$ Ans 2.

Example # 6.35

An urn contains four balls which are known to be either (i) All white or (ii) two white & two black. A ball is drawn at random and is found to be white. What is probability that the balls are white.

Sol:— Let A be the event all are white & B be the event that 2 white & two black.

White or
4 2W
2B.

$P(A) = P(B) = 1/2$ as both have same chances of selection.

Let C is event that ball drawn in white.

So;

$$P(A/C) = \frac{P(A)\,P(C/A)}{P(A)P(C/A) + P(B)\,P(C/B)}$$

$$= \frac{1/2\left({}^4C_1/{}^4C_1\right)}{1/2\left({}^4C_1/{}^4C_1\right) + \frac{1}{2}\left({}^2C_1/{}^4C_1\right)}$$

$$= \frac{1/2\,(1)}{1/2(1)+1/2\,(1/2)} = 1/2 \times 4/3 = \boxed{2/3 = P(A/C)}$$

$$P(B/C) = \frac{P(B)\,P(C/B)}{P(A)P(C/A) + P(B)P(C/B)} = \frac{1/2\;(1/2)}{1/2(1) + 1/2(1/2)}$$

$P(B/C) = 1/3.$

As $P(A/C)$ is more so Balls is taken from 4 white balls.