MOS - II

Lecture # 14

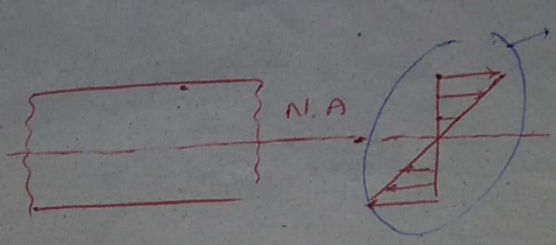
Stresses In Curved Beam:-

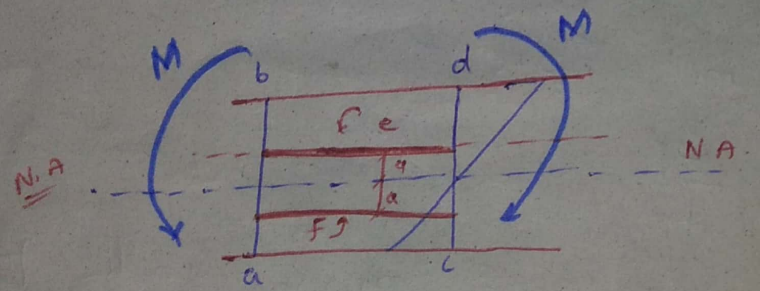
There are many case in which we have curved beam, the beam is not straight So the flexural stress distribution is not linear which is linear as calculated for straight beams.

→ The stresses and neutral axis determination is found slightly different from straight member.

Comparison:-

Straight Beam:-





Let consider a straight member (beam) In which moment are applied as shown.

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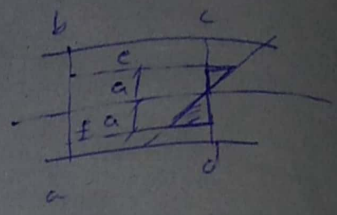
Let consider fibre e & f which are at distance a from neutral axis.

After moment is applied,

The deformation about fibre e & f will be same & can be found from similarity of triangles.

∴ section plane ↑ or remain plane after deformation.

As e & f fibre are at equidistant from N.A so the elongation in fibre e will be equal to contraction in fibre f.



So plane deformation:-

$$\Delta e = \Delta f$$

To find strain:-

As strain in fibre e ⇒

$$\epsilon_e = \frac{\Delta e}{L_o}$$

& strain in fibre f.

$$\epsilon_f = \frac{\Delta f}{L_o}$$

As $\Delta e = \Delta f$. & original length for both cases is also same.

So $\boxed{\epsilon_e = \epsilon_f}$

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From Hooke's law $\sigma = E\epsilon$ ... multiply ϵ with b/s.

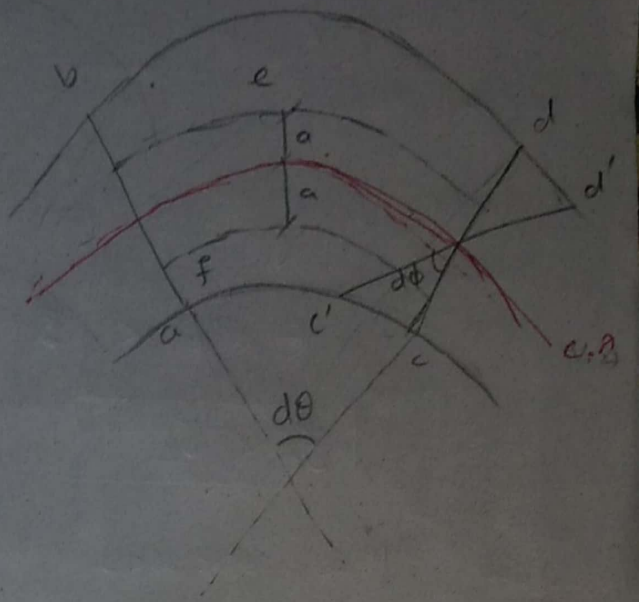
So, $\epsilon \sigma_e = E\epsilon_f$

$$\boxed{\sigma_e = \sigma_f}$$

So stresses will also same in fibre e & f.

Curved Beam:-

Let consider a curved beam on which moment is applied on both end. Now let the beam deform such that its plane section remain plane.



Let consider fibre e & f which is at a distance "a" from N.A.

∴ Assume same assumption, the plane section remain plane before bending & after bending as body is rigid.

The fibre e is elongated by amount Δe & fibre f is ~~elongated~~ contracted by amount f. i.e which are same;

So $\boxed{\Delta e = \Delta f}$

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$\epsilon_e = \frac{\Delta e}{L_e}$ & $\epsilon_f = \frac{\Delta f}{L_f}$

$\Delta e = \Delta f$

But length of exterior fibre i.e L_e is larger as compare to length of interior fibre i.e L_f. So strain in fibre e is small as strain in fibre f.

$$\epsilon_e < \epsilon_f.$$

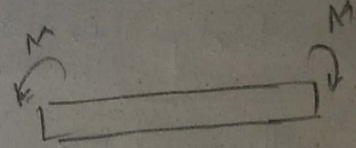
Multiply E on both sides

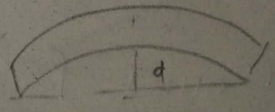
$$E\epsilon_e < E\epsilon_f$$

$$\sigma_e < \sigma_f$$

So the stresses in exterior fibre for curved beam is more as compare to stresses in interior fibre.

Note:- When a straight beam is subjected to moment from ends than even though it also curve. but we cannot say it is curved beam bcz its deflection is negligible as compared to length of beam.





$d << L$.

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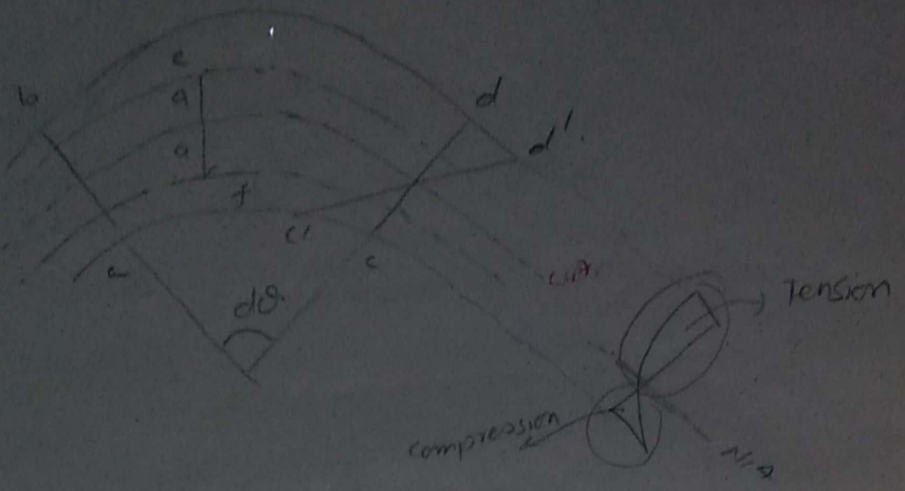
So, The stress profile for curved beam can be represented as.



stresses in inner fibre will be more than stresses in outer fibre.

In order to satisfy the equilibrium condition the neutral axis move toward the inner surface of curve beam.

For curved beam:-

1. ⇒ Stresses & strains are non linearly distributed along thickness.
2. → Stresses & strain are larger in inner fibre as compare to outer surface.
3. → The N.A even if it is symmatrical bending will not consider with C.A.

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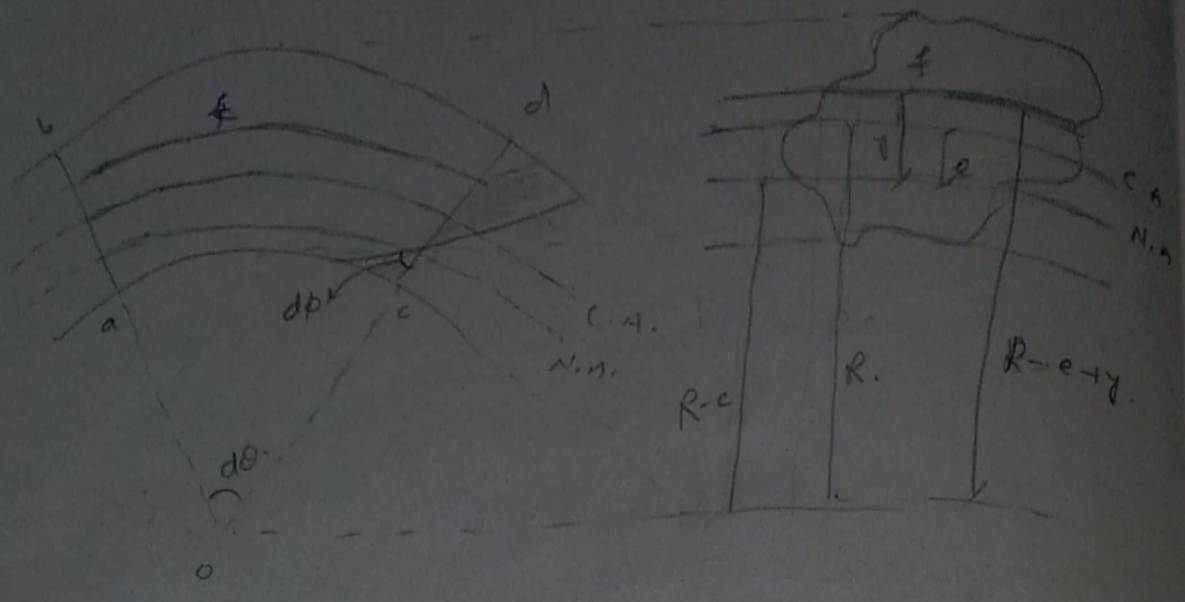
To derive Equations:-

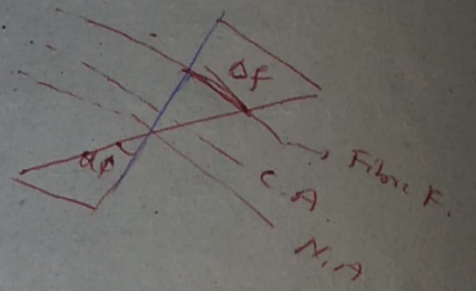
Consider a curved beam of any generic shape as shown.

Note :- $y = R - e + y$.



Let the fibre f is located at distance y^R from o.

After deformation, the fibre f is elongated by amount Δf.



So As

$$s = r\theta.$$

So here.

$$\Delta f = y\, d\phi.$$

y is distance (I) from N.A.

So strain become

$$\epsilon_f = \frac{\Delta f}{L_o} = \frac{y\, d\phi}{(R_o - e + y)\, d\theta}$$

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$$\epsilon_f = \frac{d\phi}{d\theta}\,\frac{y}{R-e+y}$$

strain in fibre

stress will be;

Multiply with E

$$\sigma_f = E\,\frac{d\phi}{d\theta}\,\frac{y}{R-e+y} \longrightarrow (1)$$

For particular cross section $\Sigma F = 0$.

W.K the total force acting on fibre of at differential element is; dF.

$$dF = \sigma\, dA$$

As $\Sigma F = 0$

$$dF = 0.$$

$$\sigma\, dA = 0$$

Integrate on b/s.

$$\int \sigma\, dA = 0.$$

$$\Rightarrow \int E\,\frac{d\phi}{d\theta}\,\frac{y}{R-e+y}\,dA = 0.$$

$$E\,\frac{d\phi}{d\theta}\int \frac{y}{y}\,dA = 0. \longrightarrow$$

either $E\,\frac{d\phi}{d\theta}$ is zero which is not possible as constant.

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So; $$\int \frac{y}{R-e+y} dA = 0 \quad \rightarrow (A)$$

As $v = R - e + y$

$$y = v - R + e$$

$$\text{Eq}(A) \Rightarrow \int \frac{v-(R+e)}{v} dA = 0$$

$$= \int \left(\frac{v}{v} - \frac{(R+e)}{v}\right) dA = 0$$

$$= \int 1\, dA - (R+e)\int \frac{1}{v} dA = 0$$

$$\boxed{e = R - \frac{A}{\int \frac{1}{v} dA}}$$

So the centroidal axis & neutral axis at distance e, which can be calculated by above formula.

Condition #2

Sum of Internal moment = External moment

$$\Sigma M = M$$

As $dF = \sigma dA$

$$dF \times y = M$$

$$\sigma dA \times y = M$$

By Integrating.

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$$\int \frac{\epsilon\, d\phi}{d\theta} \cdot \frac{y^2}{R-e+y}\, dA = M \quad \text{putting } \hat{r} \text{ value}$$

$\hookrightarrow$ (B).

Since

$$\frac{\epsilon\, d\phi}{d\theta} \int \frac{y^2}{R-e+y}\, dA = M \longrightarrow (B).$$

Since we can write;

$$y = y + (R-e) - (R-e)$$

So; $\because$ (B) $\Rightarrow$

$$\epsilon \frac{d\phi}{d\theta} \int \frac{y \cdot y}{(R-e+y)}\, dA = M.$$

$$\epsilon \frac{d\phi}{d\theta} \int \frac{y(y+(R-e)-(R-e))}{(R-e+y)}\, dA$$

$$\epsilon \frac{d\phi}{d\theta} \left[\int \frac{y(R-e+y)}{(R-e+y)} dA - \int \frac{(R-e)y}{(R-e+y)} dA \right]$$

$$\epsilon \frac{d\phi}{d\theta} \left[\int y\, dA - \int \frac{(R-e)\, y}{(R-e+y)}\, dA \right] = M \quad \hookrightarrow (C).$$

$\therefore$ from eq (A).

$$\int \frac{y\, dA}{R-e+y} = 0$$

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$$\frac{E\,d\phi}{d\theta}\left[\int y\,dA - (R-c)\int \frac{y\,dA}{y+R-c}\right] = M$$

$$\frac{E\,d\phi}{d\theta} \times \int y\,dA = 0.M.$$

$$\frac{E\,d\phi}{d\theta} \int y\,dA = M \qquad \therefore \int y\,dA = Ae$$

where we have e as the fibre distance is measure from N.A.

So; $\int y\,dA = Ae$

$\int y\,dA$ = First moment of Inertia

$$= \boxed{\frac{E\,d\phi}{d\theta} Ae = M.}$$

Or

$$\frac{E\,d\phi}{d\theta} = \frac{M}{Ae} \rightarrow (D).$$

∴ Bending moment is for cross section not for volume.

Actually it is for volume.

to eq (1) =>

$$\delta_f = \frac{E\,d\phi}{d\theta}\,\frac{y}{v}$$

Put eq D in above eqn.

$$\boxed{\delta_f = \frac{M}{Ae}\,\frac{y}{v}}$$

This is stress eqn for curved beam.

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As for complex shape structure the Integration is not easy. So some people (scientists) doed some research & performed different experiments on straight & curved beam. They found a relation b/w them i.e

$$\sigma_{f\,out}^{curved} = K_{out} * \sigma_f^{straight}$$

For Inner fibre

$$\sigma_{f\,in}^{curved} = K_{in} * \sigma_f^{straight}$$

Find the σ_f (bending moment) by $\frac{My}{I}$.
Then go to table & multiply that K value to get corresponding σ_f for curved beam.

As stresses in inner fibre > σ_{outer}

So; $K_{in} > K_{out}$.

These K_{in} & K_{out} are given for different R/c.

where R = Radius of Curvature of Centroidal axis & C is the distance of inner surface from Centroidal axis.

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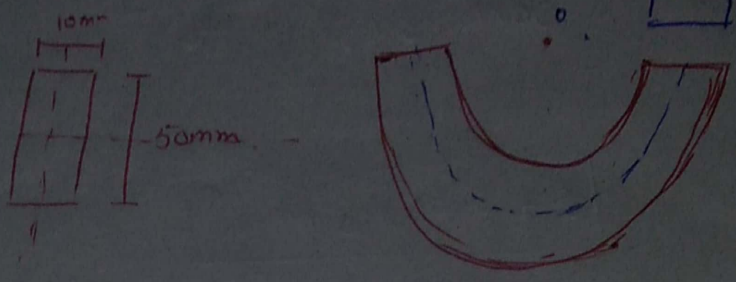
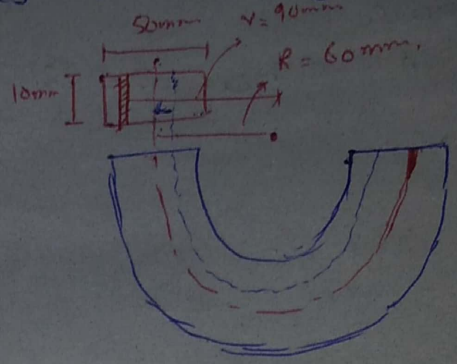
Pb#1340 A sharply curved beam of rectangular section is 10mm thick by 50mm deep. If the radius of curvature R = 60mm, Compute the stress in term of bending moment M at a point 2mm from the outer surface

Figure:-



Solution:-

to find stresses in term of Bending moment



First find location of N.A from C.A.

$$e = R - \frac{A}{\int \frac{1}{v} dA}$$

$$= 60 - \frac{50 \times 10}{10 \int \frac{1}{v} dv} = 60 - \frac{50}{\int_{35}^{85} \frac{1}{v} dv}$$

$$= 60 - \frac{50}{(\ln v)\Big|_{35}^{85}} \Rightarrow 60 - \frac{50}{\left(\ln \frac{85}{35}\right)}$$

$$= 60 - 56.887 \Rightarrow$$

$$\boxed{e = 3.65 mm}$$

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As; $\sigma = \frac{M}{a A e} * \frac{y}{v}$

By putting values.

$\sigma = \frac{M}{50 \times 10 \times 3.65} * \frac{(3.65+30)}{90}$.

$\boxed{\sigma = 2.05 \times 10^{-4} M}$

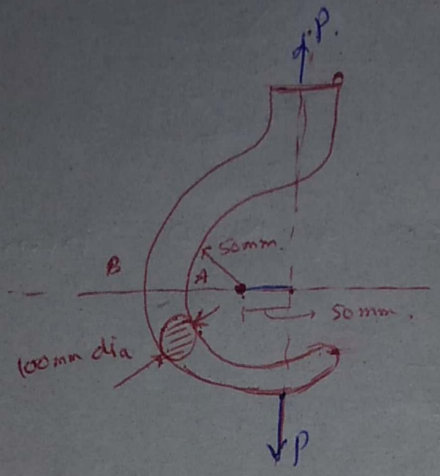
Pb # 1341

Given data. $\sigma = 120 MPa$.

(a) Find P when

(b) $\sigma_B = ?$

Figure:-



Solution:- Draw FBD of bottom section.

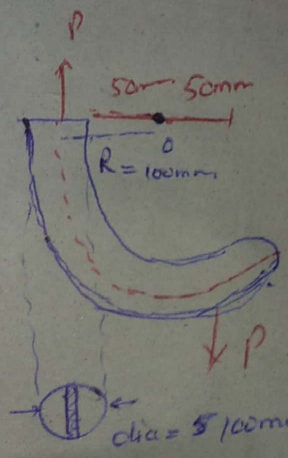
As; $R = \cancel{150mm}$

50

$\{ c = \cancel{100}mm$.

$R/c = 150/100$.

$= 1.5$.



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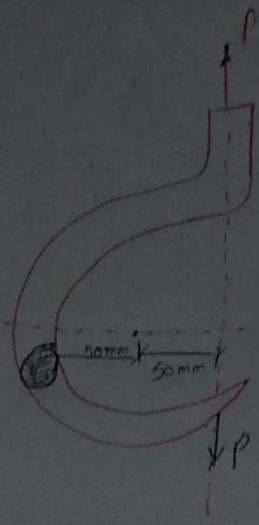
Pb 7.134

(a) Determine the max load P that may be supported without exceeding a stress of 120MPa

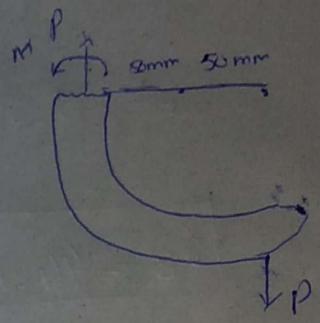
(b) What stress can exist at B?

Figure:-



Solution:-

(a) Draw FBD of lower portion.



As max stress occur at inside surface so,

$$-\sigma^A = -\frac{P}{A} - \left(\frac{My}{I}\right) K_{in} \rightarrow (A)$$

To find value of K_{in} & K_{out}

As $\frac{R}{c} = \frac{100}{2} = 2$

From table $K_{in} = 1.62$ & $K_{out} = 0.71$.

By putting values.

eq (A) $\Rightarrow$

$$-120 = \frac{-P}{\pi(50)^2} - \frac{P(150)(50)(4)(1.62)}{\pi(50)^4}$$

$$-120 = -\frac{P}{7850} - \frac{243}{98125} P.$$

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$$-120 = -1.2738\times10^{-3}P - 2.476\times10^{-3}P$$

$$P = \frac{-120}{2.6\times10^{-3}}$$

$$\boxed{P = 46.09 \text{ kips}}$$

(b) To find $\sigma_B = ?$

As;

$$\sigma_B = \frac{-P}{A} + \left(\frac{My}{I}\right) K_{out}$$

$$\sigma_B = -\frac{46.09\times10^{3}}{7850} + \frac{46.09\times10^{3}\times150\times50\times0.71\times4}{\pi(50)^4}$$

$$= -5.8718P + 50.0237P$$

$$\boxed{\sigma_B = 44.15P_{si}} \quad \text{Ans}$$

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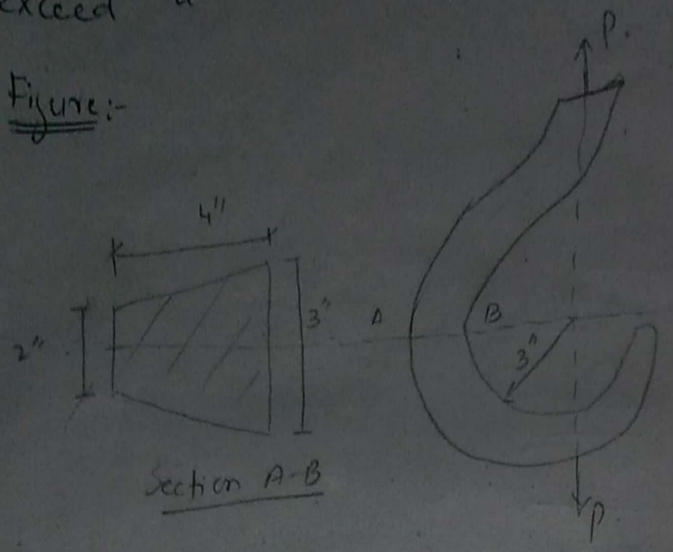
Pb # 13.14:-

A crane hooke has a cross section that is approximated by the trapozidal section shown in fig. What is the maximum load P that can will not exceed a stress of 18 Ksi?

Figure:-



Section A-B

$\therefore A = \frac{1}{2}(b_1+b_2)*h$

$= \frac{1}{2}(2+3)*(4)$

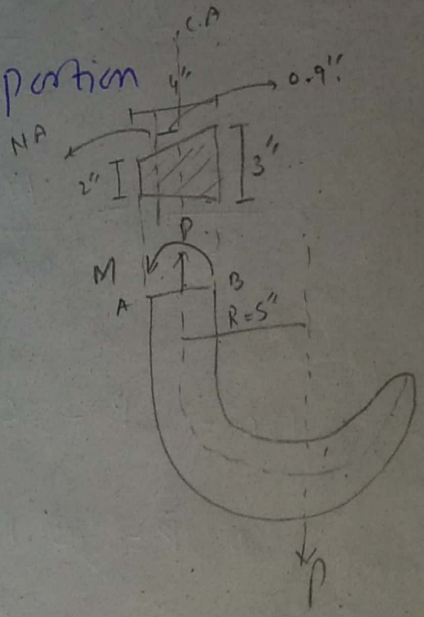
$A = 10''$

$b = 2''$

Sol:- As we know that;

$$\sigma = P/A + \frac{M}{Ae}\frac{y}{v} \longrightarrow (1)$$

Consider the bottom portion for FBD.



First find e=?

As

$$e = R - \frac{A}{\int \frac{1}{v}\,dA}$$

$$= R - \frac{A}{\int \frac{1}{v}\,b\,dv}$$

$$= R - \frac{A}{b\int \frac{1}{v}\,dv} \longrightarrow (2)$$

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$R = 5''$

$A = 10''$

$b = 2''$

Eq ② $\Rightarrow$ $e = 5'' - \frac{10}{2\int_3^7 \frac{1}{v}\,dv}$

$= 5 - \frac{10}{2(\ln 7/3)}$

$= 5 - 5$

$\boxed{e = -0.9''}$

Eq ① $\Rightarrow$ $\sigma_A = -\frac{P}{A} + \frac{M y_A}{Ae(v)}$ $\quad v = 3+4 = 7''$

$18 = \frac{P}{10} - \frac{P(5)(1.1)}{10(0.9)(7)}$

$180 = P - 0.873P$

$180 P = 0.126P \Rightarrow \boxed{P = 1.417 \text{ Kips}}$

or $\boxed{P = 1417 \text{ lb}}$

Similarly; $\sigma_B = \frac{P}{A} + \frac{M y_B}{Ae v}$

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$$\delta_D = +18 = \frac{P}{10} + \frac{P \times 2.9}{10(0.9)(3)}$$

$$180 = P + 1.074P.$$

$$2.074P = 180.$$

$$\boxed{P = 86.785 \text{ lb}}$$

$$\boxed{P = 0.086 \text{ kips}}$$

So the largest value of P is;

$$\boxed{P = 1417 \text{ lb}}$$ Ans

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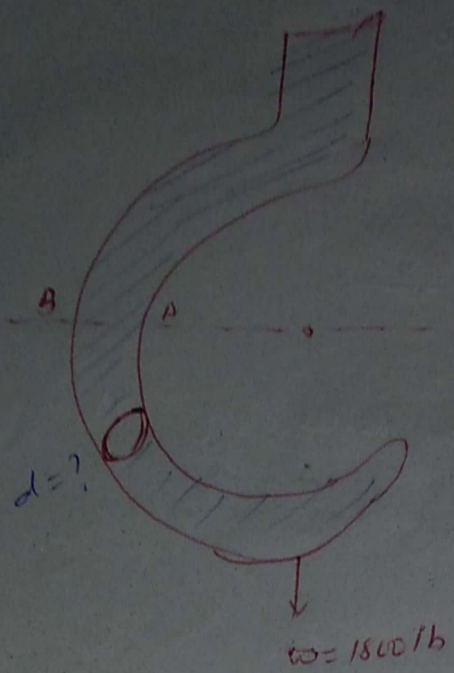
Pb # 1343 Determine the diameter of round steel rod that is to be used as a hook to lift 1800lb load acting through centre of curvature of axis of hook. Assume that R/c = 4 & max stress permitted is 20 ksi.



Given data;

$W = 1800\ lb$

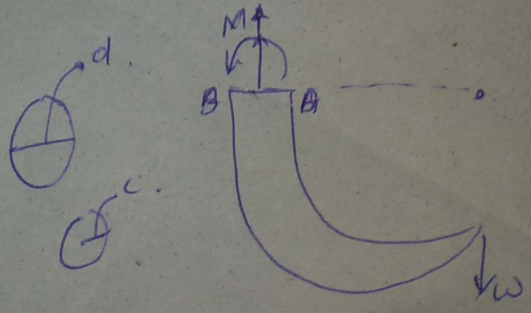
$R/c = 4$.

$\sigma_{max} = 20\ ksi$

Required $d = ?$

Solution :-

Consider the FBD of lower portion



As we know.

$d = 2c$ ——→ (1)

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As maximum stress occurs at A since inner fibre i.e A

$$\sigma^A = -\frac{\omega}{A} - \left(\frac{Mc}{I}\right) K_{in} \quad \rightarrow (1)$$

$\omega = 1800$ lb

$A = \pi c^2$

$M = \omega \times R$

$I = b\,\frac{\pi c^2}{4}$

As $R/c = 4$

From table $K_{in} = 1.23$

But $R = 4c$

Putting values in (1).

$$\sigma^A = -\frac{1800}{\pi c^2} - \left(\frac{1800 * R * c * 4}{\pi c^4}\right) * 1.33$$

$$-20 \times 10^3 = -\frac{1800}{\pi c^2} - \frac{1800 \times 4c * c * 4}{\pi c^4} * 1.33$$

$$c = \sqrt{\frac{((1.23) * 4 \times 4 + 1) * 1800}{\pi * 20 \times 10^3}}$$

$$\boxed{c = 0.22''}$$

As $D = 2c$

$D = 2(0.77)$

$\boxed{D = 1.54 \text{ in}}$ Ans

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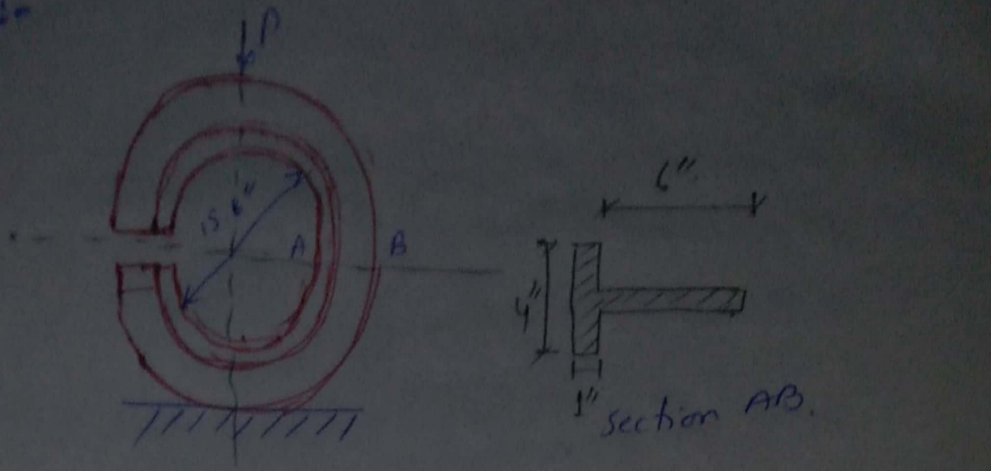
Pb#1345:- The cross section of a rod is T section as shown. The inside diameter of the ring is 15.6". Determine the value of P that will cause max. shear of 18 ksi.

Figure:-



Given data;

$D_i = 15.6"$.

$\tau_{max} = 18$ ksi

Required = ?

P = ?

Solution:-

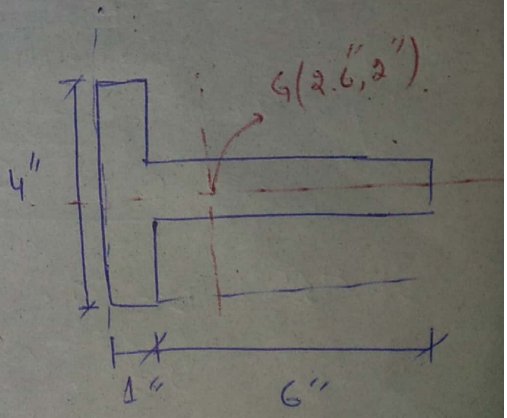
First we have to find I = ?

For I we need centroid.

$\bar{y} = 2"$.

$\bar{x} = \dfrac{4*1*\frac{1}{2} + 6*1*4.}{4+6.}$

$\boxed{\bar{x} = 2.6"}$



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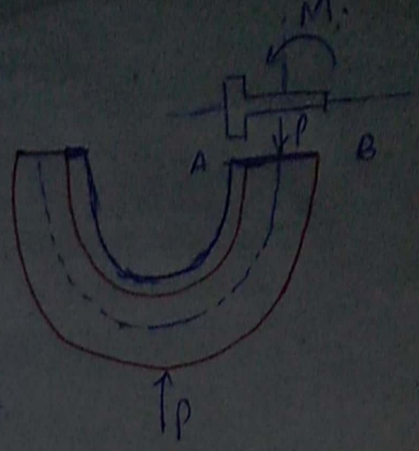
$I_y = \sum \left(\frac{bh^3}{12} + Ad^2\right)$



$I_y = \frac{4\times(1)^3}{12} + 4\times1\times(2.6-0.5)^2$

$+ \frac{(6)^3\times(1)}{12} + (6\times1)(3-2.6)^2$

$\boxed{I_y = 47.3 in^4}$

As we know; to moment is produce about y-axis from fig.

$$\delta^A = \frac{P}{A} + \left(\frac{My\,c}{Iy}\right) Kin \longrightarrow (A)$$

$$\delta^B = \frac{P}{A} - \left(\frac{My\,c}{Iy}\right) Kout \longrightarrow (B).$$

$(A) \Rightarrow \delta^A = \frac{P}{A} + \left(\frac{My\,c}{Iy}\right) Kin.$

$\delta^A = 18\times10^3 psi$

$A = 6\times1 + 4\times1 = 10 in^2$

$M = P*10.4.$

$c = 2.6$

For $R/t = \frac{7.8+2.6}{2.6}$

$R/t = 4$

From table

$Kin = 1.25$

$Kout = 0.86$

By putting values.

$18\times10^3 = \frac{P}{10} + \frac{P*10.4*(2.6)*1.25}{47.3}$

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$$P = \frac{18 \times 10^3}{0.1 + \frac{10.4 \times 2.6 \times 1.25}{47.3}}$$

$$\boxed{P = 22.27 \text{ kips}}$$

$$\{\text{(B)} \Rightarrow \sigma_B = \frac{P}{A} - \left(\frac{My}{I}\right)_{bout}$$

$$-18 \times 10^3 = \frac{P}{10} - \frac{10.4P \times 4.4 \times 0.56}{47.3}$$

$$P = -\frac{18 \times 10^3}{0.1 - \frac{10.4 \times 4.4 \times 0.56}{47.3}}$$

$$\boxed{P = 24.6 \text{ kips}}$$

So the safe value of P is;

$$\boxed{P_{safe} = 22.3 \text{ kips}} \quad \text{Ans}$$

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