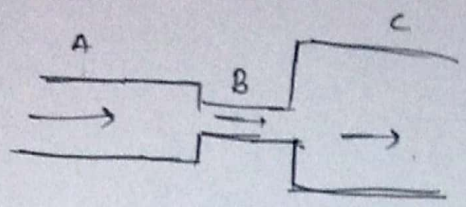
Equation of Continuity Problems.

Problem # 01



Given data.

$R_A = 3cm. = 3 \times 10^{-2} m.$

$V_A = 8 m/s.$

$R_B = 1 cm. = 1 \times 10^{-2} m$

$R_C = 5cm. = 5 \times 10^{-2} m.$

Required data V_B & $V_C = ?$

Solution:-

A_3 from eqn of Continuity.

$A_3 V_3 = A_1 V_1 = A_2 V_2 = Q.$

So; Consider $A_A V_A = A_B V_B.$ $\because A = \pi R^2.$

$$V_B = \frac{A_A V_A}{A_B} = \frac{\pi R_A^2 (V_A)}{\pi R_B^2}$$

$$V_B = \frac{(3 \times 10^{-2})^2 \pi (8)}{(1 \times 10^{-2})^2} \Rightarrow \boxed{V_B = 72 m/s}$$

To find $V_C = ?$ $V_C = \frac{A_A V_A}{A_C} \Rightarrow V_C = \frac{(3 \times 10^{-2})^2 (8)}{(5 \times 10^{-2})^2}$

$$\boxed{V_C = 2.88 m/s} \text{ Ans}$$

Pb #02

Given data

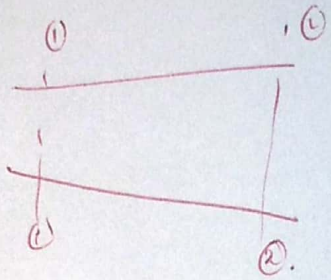
$D_1 = 10cm = 10 \times 10^{-2} m$

$D_2 = 15cm = 15 \times 10^{-2} m$

$V_1 = 5 m/s.$

$V_2 = ?$

Figure:-



Solution:-

As we know that

$$A_1 V_1 = A_2 V_2$$

$$V_2 = \frac{A_1 V_1}{A_2} \qquad \because A = \frac{\pi}{4} D^2$$

$$= \frac{\frac{\pi}{4} D_1^2 (V_1)}{\frac{\pi}{4} (D_2)^2} \Rightarrow \frac{(10 \times 10^{-2})^2 (5)}{(15 \times 10^{-2})^2}$$

$$\boxed{V_2 = 2.22 m/s} \text{ Ans}$$

Problem #03

Given data;

$D_1 = 30cm = 30 \times 10^{-2} m$.

$D_2 = 20cm = 20 \times 10^{-2} m$

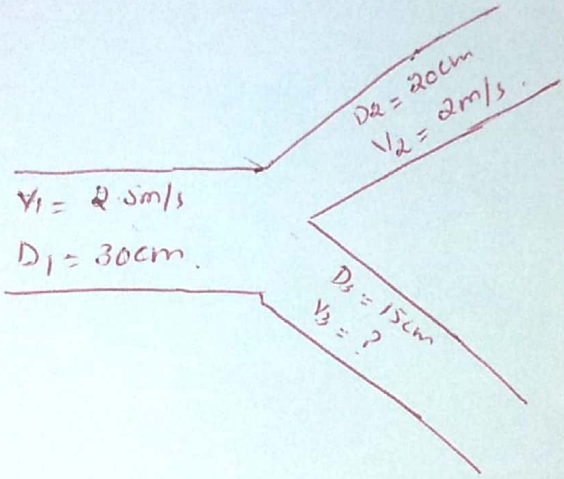
$D_3 = 15cm = 15 \times 10^{-2} m$.

$V_1 = 2.5 m/s$

(a) Find discharge in pipe = $Q = ?$

Find $V_3 = ?$ if $V_2 = 2 m/s$.

Figure:-



Solution:- To find discharge.

$$Q_1 = A_1 V_1 \Rightarrow \frac{\pi}{4} D_1^2 V_1 \quad \because A = \frac{\pi}{4} D^2$$

$$Q_1 = \frac{(3.14)}{4} * (30 \times 10^{-2})^2 * 2.5$$

$$\boxed{Q_1 = 0.1767 \, m^3/sec}$$

$$Q_2 = A_2 V_2 \Rightarrow \frac{\pi D_2^2}{4} * (2) \Rightarrow \frac{3.14}{4} * (20 \times 10^{-2})^2 * (2)$$

$$Q_2 = 0.0628 \, m^3/sec$$

To find $Q_3 = ?$

Since

$$Q_1 = Q_2 + Q_3.$$

$$Q_3 = Q_1 - Q_2.$$

$$\boxed{Q_3 = 0.1139 \ m^3/sec.}$$

To find $V_3 = ?$

As $Q_3 = A_3 V_3$

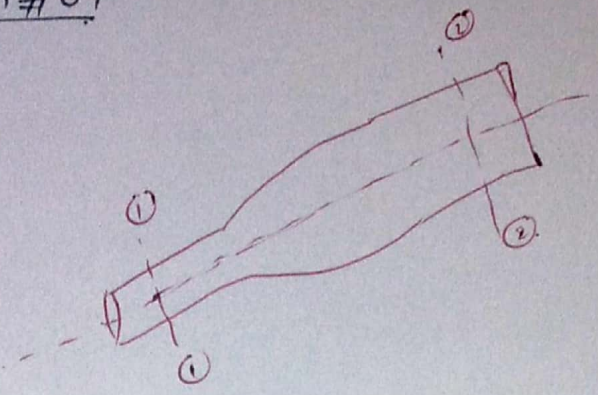
$$V_3 = \frac{Q_3}{A_3}.$$

$$= \frac{Q_3}{\pi/4 (D_3)^2}.$$

$$= \frac{0.1139}{\left(\frac{3.14}{4}\right)(15 \times 10^{-2})^2}.$$

$$\boxed{V_3 = 6.44 m/s} \ \text{Ans}$$

Problem #04



Given data;

$D_1 = 50mm = 50 \times 10^{-3} m.$

$D_2 = 100mm = 100 \times 10^{-3} m.$

$V_1 = 8 m/s$

Calculate;

(a) $V_2 = ?$

(b) Volume flow rate $= Q = ?$

(c) Weight flow rate $= ?$

(d) Mass flow rate $= ?$

Solution:- (a) $V_2 = ?$

From equation of continuity,

$A_1 V_1 = A_2 V_2$.

$\therefore A = \frac{\pi}{4} D^2$.

$V_2 = \frac{A_1 V_1}{A_2} \Rightarrow \frac{\pi/4 \, D_1^2 V_1}{\pi/4 \, D_2^2}$.

$V_2 = \frac{(50 \times 10^{-3})^2 (8)}{(100 \times 10^{-3})^2} \Rightarrow \boxed{V_2 = 2 m/s}$

(b) Volume flow rate:-

$Q_1 = A_1 V_1 \Rightarrow \pi/4 \, D_1^2 \times V_1 \Rightarrow \frac{3.14 \times (50 \times 10^{-3})^2 (8)}{4}$.

$\boxed{Q_1 = 0.0157 \, m^3/sec}$

$Q_2 = A_2 V_2$.

$Q_2 = \frac{\pi}{4} D_2^2 V_2$.

$= \pi/4 (100 \times 10^{-3})^2 * (2)$.

$\boxed{Q_2 = 0.628 \ m^3/sec}$

(c) Weight flow rate :-

As $G = \gamma A V$.

For $G_1 = \gamma A_1 V_1$. $\because Q_1 = A_1 V_1$.

$G_1 = \gamma Q_1$. $\longrightarrow$ (1).

$\gamma = \rho g \Rightarrow 1000 * 9.81 = 9810 \ N/m^3$.

$\because$ (1) $\Rightarrow$ $G_1 = 9810 * 0.0157$

$\boxed{G_1 = 154 \ N/sec}$

$G_2 = \gamma A_2 V_2 = \gamma Q_2$.

$G_2 = 9810 * 0.628$.

$\boxed{G_2 = 6160.7 \ N/sec}$

(d) Mass flow rate:-

Ans;

$$M = \rho A V \longrightarrow (1).$$

For section (1) - (1)

$$M_1 = \rho A_1 V_1 .$$: for water $\rho = 1000\ kg/m^3$

$$M_1 = \rho Q_1$$ $\because A_1 V_1 = Q_1$.

$$M_1 = 1000 * 0.0157 .$$

$$\boxed{M_1 = 15.7\ kg/sec}$$

For section (2) - (2).

$$M_2 = \rho A_2 V_2 .$$
$$= \rho Q_2$$
$$= 1000 * 0.628 .$$

$$\boxed{M_2 = 628\ kg/sec}$$

Problem #05

Given data

$D_{AB} = 1.2m$

$V_{AB} = 3m/s$

$D_{BC} = 1.5m$

$D_{CD} = 0.8m$

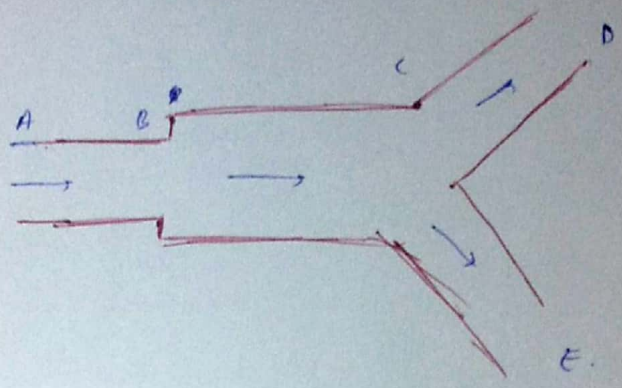
$V_{CE} = 2.5m/s$

$Q_{AB} = ?$

$V_{BC} = ?$

$V_{CD} = ?$

$D_{CE} = ?$



$\because Q_{CD} = \frac{1}{3} Q_{AB}$

Solution:-

$Q_{AB} = V_{AB} A_{AB}$. $\quad \because A = \pi/4 D^2$.

$= \frac{3.14}{4} (1.2)^2 * (3)$.

$$Q_{AB} = 3.4 \ m^3/sec$$

For $V_{BC} = ?$

As from eqn of Continuity;

$A_{AB} V_{AB} = A_{BC} V_{BC}$.

$V_{BC} = \frac{A_{AB} V_{AB}}{A_{BC}}$ $\quad \because A = \pi/4 D^2$.

$= \frac{\pi/4 D_{AB}^2 V_{AB}}{\pi/4 D_{BC}^2} \Rightarrow \frac{(1.2)^2 * (3)}{(1.5)^2}$.

$$V_{BC} = 1.92 m/s$$

As; $Q_{CD} = \frac{1}{3} Q_{AB}$

$Q_{CD} = \frac{1}{3} \times 3.4$.

$Q_{CD} = 1.133 \ m^3/sec.$

To find $V_{CD} = ?$

As $Q_{CD} = V_{CA} \ A_{CD}$.

$\therefore A = \frac{\pi}{4} D_{CD}$.

$V_{CD} = \frac{Q_{CD}}{A_{CD}}$.

$= \frac{1.133}{\pi/4 (0.8)^2}$.

$V_{CD} = 2.25 m/s$

$D_{CE} = ?$

As $Q_{AB} = Q_{CD} + Q_{CE}$. $\Rightarrow$ $Q_{CE} = Q_{AB} - Q_{CD}$.

$Q_{CE} = 3.4 - 1.133 \Rightarrow$ $Q_{CE} = 2.267 \ m^3/sec.$

As; $Q_{CE} = A_{CE} V_{CE} \Rightarrow$ $Q_{CE} = \pi/4 \ D_{CE}^2 \ V_{CE}$.

$D_{CE}^2 = \frac{Q_{CE} \times 4}{\pi \ V_{CE}} \Rightarrow$ $D_{CE} = \sqrt{\frac{4 \times (2.267)}{3.14 \times (2.5)}}$.

$D_{CE} = 1.07m$ Ans

Problem # 06

Given data

$Q = 10 m^3/hr$.

$D_1 = 100mm = 100 \times 10^{-3} m$

$D_2 = 80mm = 80 \times 10^{-3} m$

$V = ?$

$\because$ As 1 hr = 3600 sec

Solution:- For 100mm diameter pipe.

As $Q = AV$.

$$Q = \frac{\pi}{4} D_1^2 * V \Rightarrow V = \frac{4Q}{\pi D_1^2}$$

$$= \frac{4 * (10 \times \frac{1}{3600})}{3.14 (100 \times 10^{-3})^2}$$

$$V = 0.35 m/s$$

For 80mm diameter pipe:-

$$Q = AV \Rightarrow Q = \pi/4 D_2^2 * V.$$

$$V = \frac{4Q}{\pi D_2^2} = \frac{4 * (10 \times \frac{1}{3600})}{(3.14 \times 80 \times 10^{-3})^2}$$

$$V = 0.55 m/s$$ Ans