Probability & statistics

Lecture # 09

Event:-

"Any subset of sample space of any experiment

or "set of favourable outcomes is called event."

Explaination:-

Let consider experiment of tossing a coin so by doing we get ~~two~~ one results either it is a head or tail. So the events for this experiment can be written as;

$E_1 = \{H\}$, $E_2 = \{T\}$

$E_3 = S$

E3 shows sample space. It will be in event If we say statement like that if we toss a coin what is probability that so head or tail appear. So we mean whole sample space = $S = \{H, T\}$.

→ So called sure event. ↗ {Chances of happening is sure as either head come or tail come}

$E_4 = \phi$.

E4 represents empty set. As empty set is also subset of every set. So it also event of every experiment.

→ Also called Impossible event bcz chances of happening of that event is not possible.

→ If coin is toss what is probability that both head & tail appear?

Mutually Exclusive Event:-

"If the Intersection of two events are not possible or If two events cannot happen togeth these events are said to be mutually exclusive."

→ If there is no common between two set or experiment than they are said to be mutually exclusive event.

Mathematically:-

If A & B are two events.

Then $A \cap B = \phi$.

Example:-

As $E_1 = \{H\}$, $E_2 = \{T\}$

when toss a coin either head will occur or tail so they are mutually exclusive Sinc

$$\boxed{E_1 \cap E_2 = \phi}$$

If two coins are toss together:-

Then sample space is;

$$S = \{HH, HT, TH, TT\}$$

$$E_1 = \{HT\}, \quad E_2 = \{TH\}.$$

E_1 & E_2 are mutually exclusive as no Commo

In them - If one happen other cannot

$\boxed{E_1 \cap E_2 = \phi}$

let other event is,

$E_3 = \{HT, TT\}$.

E_1 & E_3 are not mutually exclusive.

since $\boxed{E_1 \cap E_3 = HT}$

Note :- If common exist so not mutually exclusive

& If no common exist so mutually exclusive.

→ **Equally likely Event :-**

If two events have same chances of happening than they are said to be equally likely event.

For example By tossing a coin both head & tail have qual chance of happening so E_1 & E_2 are equally like event.

$E_1 = \{H\}$, $E_2 = \{T\}$

$E_3 = \{HT, TT\}$, $E_4 = \{TH, HH\}$.

E_1 & E_4 are not equally likely bcz no. of element more in E_4 (Probability more in E_4 than E_1)

While E_3 & E_4 are equally likely.

Mutually Exhausted Event:-

Those mutually exclusive event which cover the whole sample space called mutually exhausted event.

→ If we have events let E_1 & E_2 If we take its union & their union show a sample space.

If example consider tossing a coin.

So Sample space is,

$$E \quad S = \{H, T\}.$$

events are $E_1 = \{H\}$, $E_2 = \{T\}$.

As E_1 & E_2 are mutually exclusive & thei union give sample space.

As $E_1 \cup E_2 = \{H, T\}$. So E_1 & E_2 a mutually exhausted event.

If two coin tossed together.

$$S = \{HH, TH, HT, TT\}$$

$$E_1 = \{HT, TH\}, \quad E_2 = \{HH, TT\}$$

So E_1 & E_2 are mutually exhausted event.

Complementary Event:-

If there are two events and they are mutually exhausted event (They can be more) but if two so they are compliment of each other.

→ For e.g $E_1 = \{H\}$, $E_2 = \{T\}$ E_1 & E_2 are mutually exhausted as $E_1 \cup E_2 = \{H, T\} = S$ so E_1 is complementary event of E_2 & E_2 is complementary event of E_1.

If A is event of occurance of something A' or A^c will be event of not occurance of that thing.

- For eg throwing a dice If E_1 is event of getting even number so complement of E_1 will be E_2 which is event of getting odd number.

→ E_1 & E_2 are mutually exhaustive also.
So E_1 & E_2 are complementary event.

Complement of any event let say A can be written as $\boxed{A^c \text{ or } \bar{A} = S / A}$

Element of that event if we subtract from sample space we get complement of that event.

Remember these:

A or B mean $A \cup B$.

A & B mean $A \cap B$.

not A mean A^c or $\bar{A}$ or A'.

A but not B = $A \cap B'$
(A happen but not B).

Definitions of Probability:-

(i) Classical definition:-

If A is subset of sample space.

So;

$$P(A) = \frac{\text{No. of favorable outcomes}}{\text{No. of total outcomes.}}$$

E.g Tossing a Coin.
Total out comes = 2
Probability of head = ?
So; favorable out come = 1

$$\boxed{P(A) = \frac{1}{2}}$$ This is probability of head appeared.

(ii) Relative Frequency Definition:- or (Posterior def)

If we have such experiment which are repeated for e.g if coin is tossed more times so we find its probability after 5 times

, 10 time, 100 times so than we can conclude from this that if this is probabi so probability at 1000 times will be finded by this definition.

Mathematically

$$P(A) = \lim_{n \to \infty} \frac{m}{n}.$$

where m is number of favourable outcomes & n is number of total outcomes.

(3) Axiomatic Definition of Probability :-

If a function defines these axioms that it will comes in definition of probalit

① $0 \leq P \leq 1$.

Probability cannot be -ve or less than 0 & greater than 1. It in between 0 & 1

② Probability of sure event must be 1.

$$P(S) = 1.$$

③ If A and B are mutually exclusive i.e $A \cap B = \phi$ then;

$$\boxed{P(A \cup B) = P(A) + P(B)}.$$

Note:- Both First & Secon
def. should statisfy
axioms.

iv) Probability of an Event:-

If A is subset of S
i.e $A \subseteq S$

Then $P(A) = \frac{n(A)}{n(S)}$

→ the difference b/w first & definition (Classical & here is that at first case we not need to make set for event. But In this case we need event in form set.

- For e.g observing in class who wearing black clothes. ~~to~~ So If we observe randomly so come in first definition & if we make a set of that observe ~~test~~ data & find its probability.

(V) Personalistic or Subjective Definition:-

- Here we consider real life problem from observing real life problem we than make a function & than find its probability. But that probability should satisfy the axiomatic definitions.

Properties of Probability:-

1. Probability of impossible event is always zero.
i.e $P(\phi) = 0$.

As empty set has no element i.e

$n(\phi) = 0$.

So if we divide both side by $n(s)$

$$\frac{n(\phi)}{n(s)} = \frac{0}{n(s)}$$

$$\boxed{P(\phi) = 0}$$

(ii) Complementation law:-

$$\boxed{P(A') = 1 - P(A)} \longrightarrow ①$$

As we know that

$A \cup \bar{A} = S$ & $A \cap A' = \phi$.
$\hookrightarrow ②$

Consider eq ② ⇒ $A \cup \bar{A} = S$.

Apply probability on both sides.

i.e $n(A \cup \bar{A}) = n(s)$

$$\frac{n(A \cup \bar{A})}{n(s)} = \frac{n(s)}{n(s)}$$

$$P(A \cup \bar{A}) = 1 \longrightarrow ③$$

Using third Axiom i.e $P(A \cup B) = P(A) + P(B)$

eq ③ ⇒ $P(A) + P(\bar{A}) = 1$.

& $\boxed{P(\bar{A}) = 1 - P(A)}$ Proved!

iii)

If A is subset of B, then P(A) will be less than probability of B.

i.e $A \leq B$ so $P(A) \leq P(B)$.

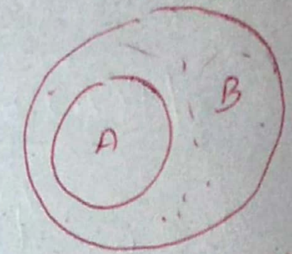
Proof:-

As A is subset of B. So number of elements in A will also less than or equal to B.

$n(A) \leq n(B)$

Divide by n(S) b/s.

$$\frac{n(A)}{n(S)} \leq \frac{n(B)}{n(S)}$$



$n(A) \leq n(B)$

$P(A) \leq P(B)$

$\boxed{P(A) \leq P(B)}$ (Proved)

$A \cap B'$.

iv) Probability that A will occur while B will not occur is Probability of A minus Probability of A Intersection B.

i.e $\boxed{P(A \cap B') = P(A) - P(A \cap B)}$

Since we know that

$A \cap \bar{B} = A / B$. ∴ / = difference

So,

$n(A\cap B') = n(A) - n(A\cap B)$ ∵ n = no. of element

Divide by n(s).

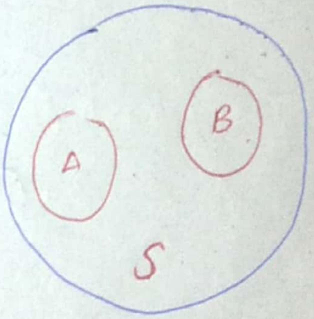
$$\frac{n(A\cap B')}{n(S)} = \frac{n(A)}{n(S)} - \frac{n(A\cap B)}{n(S)}$$

$$\boxed{P(A\cap B') = P(A) - P(A\cap B)}$$ Proved

Second method:-

As, $A = A\cap S$. A is subset when S is sample space.

But S can also written as;



$A = A\cap(B\cup\bar{B})$

$A = (A\cap B)\cup(A\cap\bar{B})$.

$n(A) = n(A\cap B)\cup n(A\cap\bar{B})$

Note!
This rule is apply when two events i.e A & B are mutually exclusive.

Divide by n(s)

$$\frac{n(A)}{n(S)} = \frac{n(A\cap B)}{n(S)} \cup \frac{n(A\cap\bar{B})}{n(S)}$$

As union mean +

$P(A) = P(A\cap B) + P(A\cap\bar{B})$

So, $\boxed{P(A\cap\bar{B}) = P(A) - P(A\cap B)}$.

(v) Addition law:-

If two events are not mutually exclusive

Then;

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Consider $S = \{1, 2, 3, 4, 5, 6\}$.

$E_1 = \{2, 3, 5\}$, $E_2 = \{1, 4, \overset{2,3}{}6\}$.

No. of elements in $P(E_1 \cup E_2)$

$6 =$

No. of element in E_1

$P(E_1)$

3

No. of el- in E_2

5.

No. of element in $P(E_1 \cap E_2)$.

2.

$6 = 3 + 5 - 2 \Rightarrow \boxed{6 = 6}$

Proof:-

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

Divide by $n(S)$

$$\frac{n(A \cup B)}{n(S)} = \frac{n(A)}{n(S)} + \frac{n(B)}{n(S)} - \frac{n(A \cap B)}{n(S)}$$

$$\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$

Proved!

For three events:-

$$P(A\cup B\cup C) = P(A)+P(B)+P(C) - P(A\cap B) - P(A\cap C) - P(B\cap C) - P(A\cap B\cap C). \hookrightarrow ①$$

Eq ① is for $\overset{not}{}$ mutually exclusive event.

Proof:-

Consider L.H.S $= P(A\cup B\cup C)$.

let $B\cup C = D$

$$P(A\cup D) = P(A) + P(D) - P(A\cap D).$$

But $D = B\cup C$.

$$P(A\cup B\cup C) = P(A) + P(B\cup C) - P(A\cap B\cup C)$$

$\therefore$ $A\cap B\cup C = (A\cap B)\cup(A\cap C)$

$$= P(A) + P(B) + P(C) - P(B\cap C) - P((A\cap B)\cup(A\cap C))$$

$$= P(A) + P(B) + P(C) - P(B\cap C) - [P(A\cap B) + P(A\cap C) - P(A\cap B\cap C)].$$

$$= P(A) + P(B) + P(C) - P(B\cap C) - P(A\cap B) - P(A\cap C) + P(A\cap B\cap C)$$

Re-arranging.

$$= P(A) + P(B) + P(C) - P(\overset{A}{B}\cap C) - P(A\cap B) - P(B\cap C) + P(A\cap B\cap C)$$

$\boxed{\text{Proved}}$

Pb #6.1 A club consist of four members. How many sample space when three officers; presidant secretary & treasurer to be choosen?

Solution:- As here the designation is mentioned and if we change the order it will effect designation So we will used Permutation.

$$^nP_r = \frac{n!}{(n-r)!}$$

As; Permutation ^nP_r.

$n = 4$ & $r = 3$.

$$^4P_3 = \frac{4!}{(4-3)!} = 4 \times 3 \times 2 \times 1 = 24 \text{ Ans}$$

Example #6.2

A three person comittee is to be formed from list of four person. How many sample points are associated with experiment?

Solution:- As here a commitle is form in which any three persons can be selected from 4 so order not effect. So used Combination

$n = 4$ & $r = 3$.

$$^nC_r = \frac{n!}{r!(n-r)!}$$

$$= \frac{4!}{3!(4-3)!} \Rightarrow \frac{4 \times \cancel{3 \times 2 \times 1}}{\cancel{3 \times 2 \times 1}} = 4 \text{ Ans}$$

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Example #6.3

How many sample points are in sample space when a person draw a hand of 5 cards from well shuffled ordinary deck of 52 cards.

Sol:- As here any five cards from deck can be choosen. so;

use Combination.

$$ {}^{n}C_{r} = \frac{n!}{r!\,(n-r)!} $$

$n = 52$, $r = 5$.

$$ = \frac{52!}{5!\,(52-5)!} $$

$$ = \frac{52 * 51 * 50 * 49 * \cancel{48 * 47!}}{5 \times 4 \times 3 \times 2 * 1 * \cancel{48 * 47!}} $$

$$ = 2598960 $$ Ans

Example #6.4:-

If a card is drawn from deck of 52 cards. Find the probability that

i- The card is red (ii) The card is diamond (iii) The card is 10.

Solution:- As we know that;

$$ P(A) = \frac{m}{n} = \frac{\text{No. of favourable outcomes}}{\text{Total no. of outcomes}} $$

(i) Since favourable out comes are = 26.

so;

$$P(A) = \frac{26}{52} = \frac{1}{2}.$$

(ii) Let B represent the card drawn is diamond.

Since m = 13.

n = 52.

So;

$$P(B) = \frac{13}{52} = \frac{1}{4}$$

(iii) Let C represent the card drawn is 10. So

m = 4

& n = 52.

$$n(C) = \frac{m}{n} = \frac{4}{52} = \frac{1}{13}$$ Ans

Example #6.5

A fair coin is tossed three times. What is the probability that at least head appear?

Sol:-

As coin is tossed 3 times.

So; Total out comes $= 2^3$.

$= 8$.

S = {H H H, HTT, HTH, TTH, THT, HHT, TTH, TTT}

Since at least head occur. So for that we have

$A = \{HHH, HTH, HHT, TTH, THT, HTT, HHT\}$

$n(A) = 7$

$n(S) = 8$

So Probability $= \frac{n(A)}{n(S)} = 7/8$ $\underline{\underline{Ans}}$

Example # 6.6:-

If two fair dice are thrown? what is probability of getting (i) a double six (ii) Sum of 8 or more dots.

Solution:-

As two dices are thrown.

so; Sample space will be;

$$S = \begin{Bmatrix} (1,1), (1,2), \cdots (1,6) \\ (2,1), (2,2), \cdots (2,6) \\ \vdots \\ (6,1), (6,2), \cdots (6,6) \end{Bmatrix}$$

$n(S) = 6^2 = 36$

let A denote double six

so; $A = \{(6,6)\}$ i.e $n(A) = \underline{1}$

$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{36}$$

Sum of 8 or more dots.

Let B represent That;

$B = \{(2,6), (3,6), (4,6), (5,6), (6,6).$
$(3,5), (4,5), (6,5)$
$(4,4), (5,4), (6,4),$
$(5,3), (6,3), (5,5), (3,5)\}$

$n(B) = 15.$

$P(B) = \frac{n(B)}{n(S)} = \frac{15}{36} = 5/12.$ Ans 2.

Example #6.7 :-

Six white balls & four black balls which are Indistinguishable apart from other are placed in bag. If six balls are taken from bag Find the probability of Their being three white & Three black

Sol:- First find $n(S) = ?$

As Total balls = 10.

No. of balls selected = Six = 6.

So as any 6 balls selected So use Combinat

$$n(S) = {}^{10}C_6 = \frac{10!}{6!(10-6)!} = 210.$$

Now let A represent favourable out coms.

$$So;\ n(A) = {}^{6}C_3 \times {}^{4}C_3 \Rightarrow \frac{6!}{3!(6-3)!} \times \frac{4!}{3!(4-3)!}$$

$$n(A) = 80.$$

So $P(A) = \frac{n(A)}{n(S)} = \frac{80}{210} = \frac{8}{21}$ Ans

Example #6.8

An employeer wishes to hire three people from group of 15 applicants, 8 men and 7 women, all of whom are equally classified to fill the position. If he selects the three at random, What is probability that (i) All three are (will) be men, (ii) at least one will be woman?

Solution:-

As we have total number of applicant = 15

& selected (favorable) ~~ever~~ applicant = 3

So; $n(S) = {}^{15}C_3$ ∴ As order is not important any 3 So i.e why we used combination.

$$n(S) = \frac{15!}{3!(15-3)!}$$

$$= \frac{15!}{3! \times 12!} = \frac{\overset{5}{\cancel{15}} \times \overset{7}{\cancel{14}} \times 13 \times \cancel{12!}}{\underset{1}{\cancel{3}} \times \underset{1}{\cancel{2}} \times \cancel{12!}}$$

$$\boxed{n(S) = 455}$$

(i) All three are men.

Let A represent event which represent all men

$n(A) = {}^{8}C_3$

$$= \frac{8!}{3!(8-3)!} = \frac{8 \times 7 \times 6 \times 5!}{3 \times 2 (5!)} = 56$$

Since we know that

$$P(A) = \frac{n(A)}{n(S)}$$

$$= \frac{56}{455}$$

$$\boxed{P(A) = 8/65}$$

(ii) At least one will be woman.

At least one women will be selected so we can also select two & 3 women

So let B represent event of at least one woman.

So $n(B) = \binom{7}{1}\binom{8}{2} + \binom{7}{2}\binom{8}{1} + \binom{7}{3}\binom{8}{0}$

$$= {}^{7}C_1 * {}^{8}C_2 + {}^{7}C_2 * {}^{8}C_1 + {}^{7}C_3.$$

$$n(B) = \frac{7!}{1!(7-1)!} * \frac{8!}{2!(8-2)!} + \frac{7!}{2!(7-2)!} * \frac{8!}{2!(8-1)!} + \frac{7!}{3!(7-3)!}$$

$$= \frac{7 \times 6!}{6!} * \frac{8 \times 7 \times 6!}{2 \times (6!)} + \frac{7 \times 6 \times 5!}{2 \times 5!} * \frac{8 \times 7!}{1 \times (7!)} + \frac{7 \times 6 \times 5 \times 4!}{3 \times 2 \times 1 (4!)}$$

$n(B) = 196 + 168 + 35.$

$n(B) = 399$

So probability of event B will be;

$$P(B) = \frac{n(B)}{n(S)}.$$

$$= \frac{399}{455}$$

$$P(B) = \frac{57}{65}$$

$$\boxed{P(B) = 0.87}$$

Example #6.9 Four items are taken at random from a box of 12 items & inspected. The box is rejected if more than 1 item is found to be faulty. If there are 3 faulty items in box. Find the probability that box is accepted.

Sol:- As propability of box to be selected is,

$$P(A) = \frac{n(A)}{n(S)} \rightarrow (A)$$

Let A represent the event that the box is to be accepted.

$$\text{so; } n(S) = {}^{12}C_4 = \frac{12!}{4! \times (12-4)!} = \frac{12 \times 11 \times 10 \times \cancel{9 \times 8} \times 9 \times 8!}{4 \times 3 \times 2 \times 1\,(8!)}.$$

$$\boxed{n(S) = 495}.$$

Now to find

$n(A) = ?$

$n(A) = \{$No faulty or at least one faulty$\}$.

$A = 4N$ or $1F$ & $3N$.

As given that there are 3 faulty item & remaining 9 are non-faulty. So.

$$n(A) = {}^9C_4 + {}^3C_1 * {}^9C_3.$$

$$= \frac{9!}{4!(9-4)!} + \frac{3!}{1!(3-1)!} * \frac{9!}{3!(9-3)!}.$$

$$= \frac{9 * \overset{2}{\cancel{8}} * 7 * \overset{3}{\cancel{6}} \times \cancel{5!}}{\underset{1}{\cancel{4}} \times \underset{1}{\cancel{3}} \times \cancel{2} \times (\cancel{5!})} + \frac{3 \times \cancel{2!}}{\cancel{2!}} * \frac{\overset{3}{\cancel{9}} \times \overset{4}{\cancel{8}} \times 7 \times \cancel{6!}}{\underset{1}{\cancel{3}} \times \underset{1}{\cancel{2}}(\cancel{6!})}.$$

$$= 126 + 252.$$

$$n(A) = 378.$$

& Ⓐ ⇒ $P(A) = \frac{n(A)}{n(S)}$

$$= \frac{378}{495}$$

$$\boxed{P(A) = 0.76}$$ Ans

Pb# 6.10 A coin is tossed 4 times in succession. What is probability at least one head occur?

Sol:- For at least one head occur.

Remember this

$$P(\text{at least one head}) = 1 - P(\text{no head occur})^n$$

where n represent no. of repeatition of exp.

So; $P(\text{at least one head}) = 1 - (P(t))^n \rightarrow Ⓐ$

where $P(t)$ is probability of tail (only occuring.

Since we have $P(t) = 1/2$. As $\{TTTT\}$ (Favorable).

As $n = 4$, 4 times repeated.

So; $\{$Ⓐ $\Rightarrow$

$$P(\text{at least one head}) = 1 - \left(\frac{1}{2}\right)^4$$

$$= 1 - \frac{1}{16}$$

$$= \frac{15}{16}$$

So probability of at least one head occur is $15/16$. Ans

Pb#6.11 A coin is baised so that the probability that it falls showing tail is $3/4$.

(a) Find the probability of obtaining at least one head when the coin is tossed five times

(b) How many times the coim be tossed so that probability of obtaining at least one head is greater than 0.98?

Sol:- ⓐ Probability of obtaining at least one head

As for at least one head probability is;

$$P(\text{at least one head occur}) = 1 - P(\text{No head occur})^n$$

$$= 1 - (P(\text{Tail}))^n$$

As $P(\text{Tail}) = 3/4$ & $n = 5$.

So; $P(\text{at least one head}) = 1 - (3/4)^5$

$$= 1 - 0.237$$

$$\boxed{P(\text{At least one head}) = 0.2676}$$

ⓑ How many times $= n = ?$

It can be written as;

$$P(\text{at least one head}) \geq 0.98 \longrightarrow Ⓐ.$$

But $P(\text{at least one head}) = 1 - P(\text{no head})^n$

$$= 1 - P(\text{Tail})^n.$$

ξⒶ ⇒

$$1 - \left(\tfrac{3}{4}\right)^n \geq 0.98.$$

$$1 - 0.98 \geq \left(\tfrac{3}{4}\right)^n.$$

$$0.02 \geq \left(\tfrac{3}{4}\right)^n.$$

Apply log on b/s.

$$\log(0.02) \geq \log\left(\tfrac{3}{4}\right)^n.$$

$$-1.698 \geq n \log\left(\tfrac{3}{4}\right).$$

$$-1.698 \geq n(-0.1249)$$

$$n \geq 13.6$$

As divide by '-' so sign of '≥' will be change.

or $\boxed{n = 14}$

When a coin is tossed 14 times, the probability of obtaining at least one head will be greater than 0.98.

Pb # 6.12 If one card is selected at random from deck of 52 cards (playing), what is probability that the card is club or a face card or both.

Sol:- To find probability of selecting both club face card is;

$\underline{P(A \cup B) = ?}$ which $P(A \cup B) = P(A) + P(B) - P(A \cap B)$, $\hookrightarrow$ Ⓐ.

where let A represent Probability of selecting club card.

$n(S) = 52$

& $n(A) = 13$.

so; $P(A) = \frac{n(A)}{n(S)}$

$$\boxed{P(A) = \frac{13}{52}}$$

let B represent Face card probability.

$P(B) = ?$

$n(B) = 12$

$n(S) = 52$.

$P(B) = \frac{n(B)}{n(S)}$

$$\boxed{P(B) = \frac{12}{56}}$$

Now As not mutually exclusive;

$n(A \cap B) = 3$

$n(S) = 52$.

so $P(A \cap B) = \frac{n(A \cap B)}{n(S)}$

$P(A \cap B) = \frac{3}{52}$

eq Ⓐ → $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{13}{52} + \frac{12}{52} - \frac{3}{52} = \frac{13+12-3}{52} = \frac{22}{52}$.

$\boxed{P(A \cup B) = 22/26}$ Ans

Example # 6.13 An Integer is choosen from the first 200 positive Integers randomly. What is the probability that the integer chosen is divisible by 6 or by 8?

Sol:- Let A represent the event in which the integers divisible by 6 are count & let B represent an event in which the Integers divisible by 8 are count.

So sample space;

$$S = \{1, 2, 3, \cdots 200\}.$$

$$n(S) = 200.$$

$$n(A) = \left\lceil \frac{200}{6} \right\rceil = \lceil 33.33 \rceil = 33.\cancel{3}$$

$$n(B) = \left\lceil \frac{200}{8} \right\rceil = 25$$

As ~~these~~ we have given Integer so there is chance that interger i.e 200 is not mutually i.e there will be chance that event divisible by & both 6 & 8.

$2 \mid 6, 8$
$4 \mid 3, 4$
$3 \mid 3, 1$
$ \mid 1, 1$

So for this; $n(A \cap B) = \left\lceil \frac{200}{24} \right\rceil = \lceil 8.33 \rceil$.

$$n(A \cap B) = 8.$$

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As we know that for non mutually exclusive event.

$$P(A\cup B)= P(A)+P(B)-P(A\cap B) \longrightarrow ①.$$

$$P(A)=\frac{33}{200} \; , P(B)=\frac{25}{200} \; , \; P(A\cap B)=\frac{8}{200}.$$

eq ① ⇒ which is probability of A or B. is;

$$P(A\cup B)=\frac{33}{200}+\frac{25}{200}-\frac{8}{200}.$$

$$=\frac{33+25-8}{200} \Rightarrow P(A\cup B)=\frac{50}{200}=\frac{1}{4}$$

$$\boxed{P(A\cup B)= 1/4}$$ Ans

Example #6.14

A pair of dice is thrown. Find the probability of getting a total of either 5 or 11.

Solution:- To find probability of getting either 5 or 11. We need two events i.e A which represent probability of getting 5 & B which represent probability of getting 11. so probability of getting either 5 or 11 is;

$$P(A\cup B) = P(A) + P(B) - P(A\cap B) \longrightarrow ①.$$

To find $P(A) = ?$

As, $P(A) = \frac{n(A)}{n(S)}$

∵ $n(S)$ is total outcomes.

which is $6^2 = 36$.

$\Rightarrow n(S) = 36$.

$n(A) = 4$.

As $A = \{(1,4), (2,3), (3,2)$
$(4,1)\}$.

$P(A) = \frac{4}{36} = \frac{1}{9}$

$B = \{(5,6), (6,5)\}$.

$P(B) = \frac{n(B)}{n(S)} = \frac{2}{36}$.

$= \frac{1}{18}$.

As mutually exclusive event b/c both canno occur together.

so; $P(A \cap B) = \phi$.

Eq(1) $\Rightarrow$ $P(A \cup B) = P(A) + P(B)$

$= \frac{1}{9} + \frac{1}{18}$.

$= \frac{2+1}{18} = \frac{3}{18} = \frac{1}{16}$

$\boxed{P(A \cup B) = 1/16}$ Ans

Example #6.15:-

Three horses A, B and C are in race; A is twice as likely to win as B and B is twice as likely to win as C. What is probability that A or B wins.

Sol:- Let P(A) represent probability of horse A win.

& P(B) represent probability of horse B win.

P(C) represent probability of horse C win.

Let $n\cancel{P}(C) = p$.

So according to condition,

$$n\cancel{P}(B) = 2\overset{n}{\cancel{P}}(C) = 2P.$$

$$\& \quad \overset{n}{\cancel{P}}(A) = \overset{2\ n}{\cancel{2P}}(B) = 2[2(\overset{n}{\cancel{P}}(C))] = 4P.$$

As to find probability of A or B win.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

As ~~not~~ mutually exclusive i.e If A win, B not and vice versa. So;

$$P(A \cap B) = 0.$$

$n(S) = p + 2p + 4p$

$n(S) = 7p.$

$$P(A \cup B) = P(A) + P(B).$$

$p \rightarrow n$

$$= \frac{n(A)}{n(S)} + \frac{n(B)}{n(S)} \Rightarrow \frac{\cancel{2}4}{7} + \frac{2}{7}$$

$$\frac{4+2}{7} = \boxed{6/7 = P(A \cup B)} \text{ Ans.}$$

Example # 6.16

A card is drawn at random from deck as ordinary playing card what is probability that is diamond, or face or a king?

Sol:- As $n(S) = 52$ (Total cards)

Let A for diamond which are 13.

So, $P(A) = \frac{n(A)}{n(S)} = \frac{13}{52}$.

Let B for face; $P(B) = \frac{n(B)}{n(S)} = \frac{12}{52}$.

& C for king; $P(C) = \frac{n(C)}{n(S)} = \frac{4}{52}$.

Since we know that; $\rightarrow$ Ⓐ.

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{3}{52}$. which are both diamond & face.

$P(A \cap C) = \frac{n(A \cap C)}{n(S)} = 1/52$. Both diamond & king.

$P(B \cap C) = \frac{n(B \cap C)}{n(S)} = 4/52$ Both ~~d~~ face & king.

$P(A \cap B \cap C) = \frac{n(A \cap B \cap C)}{n(S)} = \frac{1}{52}$. which is diamond, face & king.

eq Ⓐ $\Rightarrow$

$P(A \cup B \cup C)$

$$= \frac{13}{52} + \frac{12}{52} + \frac{4}{52} - \frac{3}{52} - \frac{1}{52} - 4/52 + 1/52$$

$$\frac{13+12+4-3-4}{52}$$

$P(A \cup B \cup C) = 22/52 = 0.423$ Ans