Q5:- $yy' + 36x = 0.$

Sol:- $y\frac{dy}{dx} = -36.$

$y\,dy = -36x\,dx.$

$\int y\,dy = -36\int x\,dx.$

$\frac{y^2}{2} = -36\frac{x^2}{2} + c.$

$\boxed{y^2 = -36x^2 + c}$ Ans

Q9:- $xy' = y^2 + y.$

Sol:- $x\frac{dy}{dx} = y^2 + y.$

$\frac{1}{y^2+y}\,dy = \frac{1}{x}\,dx.$

$\frac{1}{y(y+1)}\,dy = \frac{1}{x}\,dx \longrightarrow (1)$

Take $\int$ on b/s.

$$\int \frac{1}{y(y+1)}\,dy = \ln x + c \longrightarrow (1)$$

$$\frac{1}{y(y+1)} = \frac{A}{y} + \frac{B}{(y+1)} \longrightarrow (B)$$

$$\frac{1}{\cancel{y(y+1)}} = \frac{A(y+1)+B(y)}{\cancel{y(y+1)}}$$

$$1 = A(y+1) + By \longrightarrow (A)$$

Put $y=0$.

$$\boxed{A = 1}$$

Put $y = -1$

$$\boxed{B = -1}$$

Put in eq (B).

$$\frac{1}{y(y+1)} = \frac{1}{y} + \frac{(-1)}{y+1}$$

Taking $\int dy$ on b/s.

$$\int \frac{1}{y(y+1)}\,dy = \int \frac{1}{y}\,dy + \int \frac{-1}{y+1}$$

$= \ln y - \int \frac{1}{y+1} dy.$

$\int \frac{1}{y(y+1)} dy = \ln y - \ln|y+1| . \rightarrow ①$

Put value in eq ①

$\ln y - \ln|y+1| = \ln x + c$

$\ln\left|\frac{y}{y+1}\right| = \ln x + \ln c.$

$\ln\left|\frac{y}{y+1}\right| = \ln x/c.$

Taking Antilog on b/s.

$$\boxed{y/y+1 = x/c}$$

$y = (y+1)\left(\frac{x}{c}\right)$ Ans

Q5:- $yy' + 36x = 0$.

Sol:- $y\frac{dy}{dx} + 36x = 0 \longrightarrow ①$.

let $u = y/x \Rightarrow y = ux$.

$\frac{dy}{dx} = u'x + u$

By Substitution.

By putting values in eq ①

$ux(u'x+u) + 36x = 0$.

$x(u(u'x+u) + 36) = 0$.

$u(u'x+u) + 36 = 0$.

$u'x + u = -\frac{36}{u}$.

$u'x = -\frac{36}{u} - u$.

$\frac{du}{dx}x = \frac{-36-u^2}{u}$.

$\frac{u}{-36-u^2}du = \frac{1}{x}dx$.

taking $\int$ on b/s.

$\frac{1}{-1}\int \frac{u}{u^2+36}du = \int \frac{1}{x}dx$.

$-\frac{1}{2}\int \frac{2u}{u^2+36}du = \ln x + \ln c$

$-\frac{1}{2}\ln(u^2+36x) = \ln xc$.

$-\frac{1}{2}\ \cancel{\ln}(u^2+36x) = \ln x\,c$

$-\frac{1}{2}(u^2+36x) = x\,c.$

$u^2+36x = -2xc.$

$u^2 = -36x - 2xc.$

$(y/x)^2 = -36x - 2xc.$

$\boxed{y^2 = -36x^3 + c.}$

$2xc = \text{constant}$

Ans

2.

Q9:- $xy' = y^2 + y$ ∴ Set $y/x = u$.

Solution:- $x\frac{dy}{dx} = y^2 + y \longrightarrow ①$

By Substitution:-

let $u = y/x \Rightarrow y = ux$.

$\frac{dy}{dx} = u'x + u$.

Put values in eq ①.

$\cancel{x}(u'x + u) = (ux)^2 + ux$.

$u'x + \cancel{u} = u^2x + \cancel{u}$

$\frac{du}{dx}\cancel{x} = u^2\cancel{x}$.

$\frac{1}{u^2}du = dx$.

Taking $\int$ on b/s.

$\int\frac{1}{u^2}du = \int dx$.

$-\frac{1}{u} = x + c$

By putting value $u = y/x$.

$-\frac{1}{y/x} = x + c$.

$-x = y(x+c)$.

$\boxed{y = \frac{-x}{x+c}}$ Ans

Q13:- $y' \cosh^2 x = \sin^2 y$, $y(0) = \frac{1}{2}\pi$.

Sol:- $\frac{dy}{dx} \cosh^2 x = \sin^2 y$.

$$\frac{1}{\sin^2 y} dy = \frac{1}{\cosh^2 x} dx$$

$$\operatorname{cosec}^2 y \, dy = \operatorname{sech}^2 x \, dx$$

∵ Taking ∫ on b/s.

$$\int \operatorname{cosec}^2 y \, dy = \int \operatorname{sech}^2 x$$

$$-\cot y = \tanh x + c \longrightarrow ①$$

To find specific solution;

$y(0) = \frac{1}{2}\pi$.

$$-\cancel{\cot\left(\tfrac{1}{2}\pi\right)}^{0} = \cancel{\tanh}^{0}(0) + c$$

$$\boxed{c = 0}$$

eq ① ⇒ $\boxed{-\cot y = \tanh x}$ Ans

Specific solution.

or $\boxed{y = -\cot^{-1}(\tanh x)}$

Q17:- $xy' = y + 3x^4 \cos^2(y/x)$, $y(1) = 0$.

$\therefore$ (Set $y/x = u$)

Sol:- $xy' = y + 3x^4 \cos^2(y/x)$.

$$x\frac{dy}{dx} = y + 3x^4 \cos^2(y/x) \longrightarrow ①$$

Let $u = y/x \Rightarrow y = ux$.

$$\frac{dy}{dx} = u'x + u.$$

Put values in eq ①.

$$x(u'x + u) = ux + 3x^{\cancel{4}\,3} \cos^2(ux/\cancel{x})$$

$$u'x + \cancel{u} = \cancel{u} + 3x^3 \cos^2 u.$$

$$u'\cancel{x} = 3x^{\cancel{3}\,2} \cos^2 u.$$

$$u' = 3x^2 \cos^2 u.$$

$$\frac{du}{dx} = 3x^2 \cos^2 u.$$

$$\frac{1}{\cos^2 u} du = 3x^2 dx \Rightarrow \sec^2 u \, du = 3x^2 dx.$$

Taking $\int$ on b/s.

$$\int \sec^2 u \, du = 3\int x^2 dx$$

$$\tan u = x^3 + c \rightarrow ② \qquad y(1) = 0.$$

$$\tan(y/x) = x^3 + c.$$

$$0 = 1 + c \Rightarrow \boxed{c = -1}$$

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$$\tan(y/x) = x^3 - 1$$

$$y/x = \tan^{-1}(x^3 - 1)$$

$$\boxed{y = x\tan^{-1}(x^3 - 1)}$$ Ans 2