EXCERCISE 1.3

2. $y^3 y' + x^3 = 0$

Sol:- $y^3 \frac{dy}{dx} = -x^3$

$y^3 dy = -x^3 dx$

$\int y^3 dy = -\int x^3 dx \Rightarrow \frac{y^4}{4} = -\frac{x^4}{4} + c$

$y^4 = -x^4 + c/4$

$\boxed{y^4 = -x^4 + c_1}$

By Subtitution:-

Put $\frac{y}{x} = u \Rightarrow y = ux$

$(ux)^3(u'x + u) = -x^3$

$u^3 \cancel{x^3}(u'x + u) = -\cancel{x}^3$

$u^3 u'x + u^4 = -1$

$\frac{dy}{dx} = u'x + u$

By putting values in eq (A)

$u'x + u = -\left(\frac{1}{u^3}\right)$

$u'x = -\frac{1}{u^3} - u \Rightarrow u'x = \frac{-1 - u^4}{u^3}$

$\ln(1 + u^4)x^4 = c$

$(1 + u^4)x^4 = c$ $\quad \because e^c = c$

$(1 + y^4/x^4)x^4 = c$

$x^4 + y^4 = c$

$\boxed{y^4 = -x^4 + c}$ Ans

dx

$\frac{dy}{dx} = \frac{-x^3}{y^3} \rightarrow (1)$

Let $u = y/x$.

$y = ux$.

$\frac{u^3}{1+u^4} = -\frac{1}{x} dx$

$\frac{1}{4}\int \frac{4u^3}{1+u^4} = -\int \frac{1}{x} dx$

$\frac{1}{4}\ln(1 + u^4) = -\ln x + c$

$\ln(1 + u^4) + \ln x^4 = c$

$\ln(1 + u^4)x^4 = c$

3 $y' = \sec^2 y.$

Sol:- $\frac{dy}{dx} = \sec^2 y.$
2

$$\int \frac{1}{\sec^2 y}\, dy = \int dx. \Rightarrow \int \cos^2 y\, dy = x + c$$

$$\cancel{\tfrac{k}{x}} \cdot \frac{1}{2y} + \frac{1}{4}\sin 2y. = x + c.$$

$$\boxed{\frac{1}{2y} + \frac{1}{4}\sin 2y = x + c,}$$

Ans
2

(4) $y' \sin 2\pi x = \pi y \cos 2\pi x$
2

Sol:-
2

$$\frac{dy}{dx}\sin 2\pi x = \pi y \cos 2\pi x.$$

$$\frac{1}{y}\, dy = \frac{\cos 2\pi x}{\sin 2\pi x}\, dx.$$

$$\int \frac{1}{y}\, dy = \int \cot 2\pi x\, dx.$$

$$\ln y = \ln|\sin x| + c.$$

$$\boxed{y = \sin x + c}$$

Ans
2

(5) $yy' + 36x = 0$.

Sol:- $y\dfrac{dy}{dx} = -36x$.

$y\,dy = -36x\,dx \Rightarrow \int y\,dy = -36\int x\,dx$.

$\dfrac{y^2}{2} = -36\left(\dfrac{x^2}{2} + c\right)$.

$\dfrac{y^2}{2} = -36\dfrac{x^2}{2} - 36c$. $\quad c_1 = -36c$.

$y^2 = -36x^2 + c \Rightarrow \boxed{y^2 + 36x^2 = c}$ Ans

(6) $y' = e^{2x-1}y^2$.

Sol:- $\dfrac{dy}{dx} = e^{2x-1}y^2$.

$\dfrac{1}{y^2}dy = e^{2x-1}dx \Rightarrow \int \dfrac{1}{y^2}dy = \int e^{2x-1}dx$.

$-\dfrac{1}{y} = \dfrac{1}{2}e^{2x-1} + c. \Rightarrow \dfrac{1}{y} = -\dfrac{1}{2}e^{2x-1} + c$

$$\boxed{y = \frac{2}{(e^{2x-1} + c)}}$$ Ans

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Q7:- $xy' = y + 2x^3 \sin^2(y/x)$. (Set $u = y/x$)

Solution:-

$$x\frac{dy}{dx} = y + 2x^3 \sin^2(y/x) \quad \text{(Not seperable)}$$

$$x\frac{dy}{dx} = y + 2x^3 \sin^2(y/x) \longrightarrow (1)$$

Put $y = ux$ i.e $u = y/x$.

$y' = u'x + u$.

eq(1) =)

$$x(u'x + u) = ux + 2x^3 \sin^2(ux/x)$$

$$x^2u' + \cancel{ux} = \cancel{ux} + 2x^3 \sin^2 u$$

$$\cancel{x^2}u' = 2x^{\cancel{3}} \sin^2 u$$

$$u' = 2x \sin^2 u$$

$$\frac{du}{dx} = 2x \sin^2 u$$

$$\frac{1}{\sin^2 u} du = 2x\,dx \Rightarrow \int \operatorname{cosec}^2 u\,du = 2\int x\,dx$$

$$-\cot u = \frac{2x^2}{2} + c$$

$$-\cot u = x^2 + c \Rightarrow \boxed{-\cot(y/x) = x^2 + c}$$

Ans

& $y/x = \operatorname{arccot}(x^2 + c)$

$$\boxed{y = x\cot^{-1}(x^2 + c)}$$

Ans

Q8:- $y' = (y+4x)^2$ Set $y+4x = u$.

Sol:- $y' = (y+4x)^2$

$u = y+4x \Rightarrow y = u-4x$.

$\frac{dy}{dx} = u' - 4$.

$u' - 4 = u^2$.

$u' = u^2+4$

$\frac{du}{dx} = (u^2+(2)^2) \Rightarrow \frac{1}{u^2+2^2} du = dx$.

$\int \frac{1}{u^2+2^2} du = \int dx$.

$\therefore \int \frac{1}{1+u^2} du = \tan^{-1}(u) + c$.

$\frac{1}{2} \tan^{-1}\left(\frac{u}{2}\right) = x + c$.

$\tan^{-1}(u/2) = 2x + c$.

$u/2 = \tan(2x+c)$

$u = 2\tan(2x+c)$.

$y+4x = 2\tan(2x+c)$ But $u = y+4x$.

$\boxed{y = 2\tan(2x+c) - 4x}$ Ans

Q9:- $xy' = y^2+y$ Set $(y/x = 4)$.

Sol:- $x\frac{dy}{dx} = y^2+y$.

$$\frac{1}{y^2+y}\,dy = \frac{1}{x}\,dx$$

$$\int \frac{1}{y^2+y}\,dy = \int \frac{1}{x}\,dx \longrightarrow (A).$$

$$\frac{1}{y^2+y} = \frac{1}{y(y+1)} = \frac{A}{y} + \frac{B}{y+1}.$$

$$\frac{1}{y(\cancel{y+1})} = \frac{A(y+1)+B(y)}{y(\cancel{y+1})}.$$

$$1 = A(y+1) + By.$$

Put $y = -1$.

$\boxed{B = -1}$ Put $y = 0$ $\boxed{A = 1}$

$$\frac{1}{y(y+1)} = \frac{1}{y} - \frac{1}{y+1}.$$

$$\int \frac{1}{y(y+1)} = \int \frac{1}{y}\,dy - \int \frac{1}{y+1}\,dy.$$

$$= \ln y - \ln y+1.$$

$$= \ln\left(\frac{y}{y+1}\right).$$

using eq (A)

$$\ln\left(\frac{y}{y+1}\right) = \ln x + c.$$

$$\frac{y}{y+1} = x + c.$$

$$\frac{y+1}{y} = \frac{1}{x+c}.$$

$$\frac{y}{y} + \frac{1}{y} = \frac{1}{x+c}.$$

$$1 + \frac{1}{y} = \frac{1}{x+c}.$$

$$\frac{1}{y} = \frac{1}{x+c} - 1.$$

$$y = \left(\frac{1}{x+c} - 1\right)^{-1} \quad \text{Ans}$$

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$Q9_2$:- $xy' = y^2 + y \longrightarrow ①$

Let $u = y/x \Rightarrow y = ux$.

$y' = u'x + u$.

Put in eq ①

$x(u'x + u) = (ux)^2 + ux$.

$\cancel{x^2}u' + \cancel{ux} = u^2\cancel{x^2} + \cancel{ux}$

$\frac{du}{dx} = u^2 \Rightarrow \int \frac{1}{u^2} du = \int dx$

$-\frac{1}{u} = x + c$.

$\frac{1}{u} = -x + c$.

$u = \frac{1}{-x+c}$.

$y/x \cdot \cancel{y/x} = \frac{1}{-x+c}$

$$\boxed{y = \frac{x}{-x+c}}$$ Ans

Q10:- $xy' = x + y$ Set $y/x = u$.

Sol:- $x\frac{dy}{dx} = x + y$.

let $u = y/x$.

$y = ux$.

$y' = u'x + u$.

$x(u'x + u) = x + ux$.

$u'x^2 + \cancel{ux} = x + \cancel{ux}$

$u'x = 1$.

$\frac{du}{dx} = \frac{1}{x}$. $\Rightarrow \int du = \int \frac{1}{x} dx$

$u = \ln x + c$.

$y/x = \ln x + c$

$\boxed{y = x\ln x + cx}$ Ans

Q = Solve IVP :-

11. $xy' + y = 0$, $y(4) = 6$.

Sol:- $x\frac{dy}{dx} + y = 0$.

$x\, dy/dx = -y$

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$$\int \frac{1}{y}\,dy = \int -\frac{1}{x}\,dx$$

$$\ln y = -\ln x + c.$$

$$\ln y + \ln x = c.$$

$$\ln(yx) = c.$$

$$yx = e^{c} \Rightarrow \boxed{xy = c.}$$

By putting Initial Condition.

$$y(4) = 6.$$

$$6(4) = c.$$

$$\boxed{24 = c}$$

$$xy = 24 \Rightarrow \boxed{y = 24/x} \text{ Ans}$$

12. $y' = 1 + 4y^2.$ $\quad y(1) = 0.$

Sol:- $\frac{dy}{dx} = 1 + 4y^2.$

$$\frac{1}{1+(2y)^2}\,dy = dx.$$

$$\int \frac{1}{1+(2y)^2}\,dy = \int x\,dx. \longrightarrow ①$$

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let $u = 2y$.

$$\frac{du}{dy} = 2.$$

$$du = 2\,dy. \Rightarrow dy = \frac{1}{2} du.$$

By putting values in eq ①

$$\frac{1}{2}\int \frac{1\, du}{1+u^2} = x + c.$$

$$\frac{1}{2}\tan^{-1}(u) = x + c.$$

$$\tan^{-1}(u) = 2(x + c).$$

$$u = \tan(2(x+c)).$$

$\because u = 2y.$

$$2y = \tan(2(x+c))$$

$$y = \frac{1}{2}\tan(2(x+c)) \rightarrow (A)$$

Initial condition.

$y(1) = 0.$

$$0 = \frac{1}{2}\tan(2(x+c)).$$

$$\tan^{-1}(0) = 2(x+c).$$

$$0 = 2x + 2c$$

$$0 = 2((1) + c)$$

$$\boxed{c = -1}$$

eq (A) $\Rightarrow$ $\boxed{y = \frac{1}{2}\tan(2(x-1))}$ Ans.

Q13:- $y' \cosh^2 x = \sin^2 y$. $\quad y(0) = \frac{1}{2}\pi$.

Sol:- $\frac{dy}{dx} \cosh^2 x = \sin^2 y$.

$$\frac{1}{\sin^2 y} dy = \frac{1}{\cosh^2 x} dx.$$

$$\int \operatorname{cosec}^2 y \, dy = \int \operatorname{sech}^2 x \, dx.$$

$$-\cot y = \tanh x + c. \longrightarrow (A)$$

Applying condition $(y)(0) = \frac{1}{2}\pi$.

$$-\cot\left(\tfrac{1}{2}\pi\right) = \tanh(0) + c.$$

$$0 = 0 + c \Rightarrow \boxed{c = 0}$$

& (A) $\Rightarrow$

$$-\cot y = \tanh x + 0.$$

$$-\cot y = \tanh x.$$

$$\cot y = -\tanh x.$$

$$\boxed{y = \cot^{-1}(-\tanh x)}$$

Ans

Q14:- $\frac{dr}{dt} = -2tr.$ $r(0) = r_0.$

Sol:- $\frac{dr}{dt} = -2tr.$

$\frac{1}{r} dr = -2t\,dt.$

$\int \frac{1}{r} dr = -2 \int t\,dt.$

$\ln r = -2\frac{t^2}{2} + 2c$

$\ln r = -t^2 + c.$

$r = e^{-t^2} + e^c \Rightarrow r = e^{-t^2} + c. \longrightarrow$ (A).

Applying Initial Condition.

$r_0 = e^{-0} + c \Rightarrow c = r_0 - 1$

$\therefore$ (A) $\Rightarrow$ $\boxed{r = e^{-t^2} + r_0 - 1.}$ Ans

Q15:- $y' = -4x/y$ $y(2) = 3.$

Sol:- $\frac{dy}{dx} = -4x/y.$

$y\,dy = -4x\,dx. \Rightarrow \int y\,dy = -4\int x\,dx.$

$y^2/2 = -\frac{4x^2}{2} + c$

$y^2 = -4x^2 + c \longrightarrow (1)$

$y(2) = 3$

$(3)^2 = -4(2)^2 + c$

$9 = -16 + c$

$\boxed{c = 25}$

eq (1) $\Rightarrow$ $\boxed{y^2 = -4x^2 + 25}$ Ans

Q16:- $y' = (x+y-2)^2$, $y(0) = 2$ Set $u = x+y-2$.

Sol:- $\frac{dy}{dx} = (x+y-2)^2$

let $x+y-2 = u$

$1 + \frac{dy}{dx} = u'$

$dy/dx = u' - 1$

$u' - 1 = u^2$

$\frac{du}{dx} = u^2 + 1 \Rightarrow \int \frac{1}{u^2+1} du = \int dx$

$\tan^{-1}(u) = x + c$

$u = \tan(x+c)$

$x+y-2 = \tan(x+c) \longrightarrow (A)$

$\tan^{-1}(x+y-2) = x+c$

Apply Condition ($y(0) = 2$

$\tan^{-1}(0+2-2) = 0+c$

$\boxed{c=0}$

$\&\ (A) \Rightarrow$

$x+y-2 = \tan(x)$

$\boxed{y = \tan x - x + 2}$ Ans 2

Q17:- $xy' = y + 3x^4\cos^2(y/x) \quad y(1)=0 \qquad$ Set $y/x = u$

Sol:-

$x\frac{dy}{dx} = y + 3x^4\cos^2(y/x)$

let $u = y/x \Rightarrow y = ux$

$\frac{dy}{dx} = u'x + u$

$x(u'x+u) = ux + 3x^4\cos^2(u)$

$u'x^2 + \cancel{xu} = \cancel{ux} + 3x^4\cos^2(u)$

$u' = 3x^2\cos^2 u$

$\frac{du}{dx} = 3x^2\cos^2 u$

$$\int \frac{1}{\cos^2 u}\, du = \int 3x^2\, dx.$$

case

$$\int \sec^2 u\, du = 3\int x^2\, dx.$$

$$\tan^{u} = x^3 + c.$$

$$\tan u = x^3 + c \Rightarrow u = \tan^{-1}(x^3 + c).$$

$$y/x = \tan^{-1}(x^3 + c).$$

$$y/x = \tan^{-1}(x^3 + c) \longrightarrow (A).$$

To find $c = ?$

$$\tan(y/x) = x^3 + c.$$

Apply Initial condition i.e $y(1) = 0$.

$$0 = 1 + c \Rightarrow \boxed{c = -1}$$

eq (A) $\Rightarrow$

$$y/x = \tan^{-1}(x^3 - 1)$$

$$\boxed{y = x \tan^{-1}(x^3 - 1)}$$ Ans