ODEs, Integrating Factor.

1. $2xy\,dx + x^2dy = 0$.

Sol:- As general solution is;

$M\,dx + N\,dy = 0$.

By comparing.

$M = 2xy$ $\qquad N = x^2$.

$M_y = 2x$ $\qquad N_x = 2x$

As $\boxed{M_y = N_x}$

So the given equation is exact.

To find solution;

$\frac{\partial u}{\partial x} = M$.

$\partial u = M\,\partial x$.

$\partial u = 2xy\,\partial x$.

$\int \partial u = \int 2xy\,\partial x$

$u = x^2y + c(y)$. $\longrightarrow$ (1)

Diff w.r.t y.

$\frac{du}{dy} = x^2 + c'(y)$. $\qquad \frac{du}{dy} = N$.

$N = x^2 + c'(y)$.

$x^2 = x^2 + c'(y)$

$\boxed{c'(y) = 0}$

$\int c'(y) = \int 0 \, d.$

$c(y) = c.$

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Eq (1) =>

$u = x^2 y + c.$

$u = x^2 y + c$

As $u =$ Constant $= C_1.$

$C_1 = x^2 y + c.$

$C_1 - c = x^2 y.$

$\boxed{c = x^2 y}$ Ans

2. $x^3 dx + y^3 dy = 0 \rightarrow (1)$

Sol:- As general ~~solti~~ equation is,

$M dx + N dy = 0 \rightarrow (2)$

By comparing;

$M = x^3, \quad N = y^3.$

$M_y = 0 \quad N_x = 0.$

As $M_y = N_x$ So the given eqn is exact.

To find solution.

As we know that;

$$\frac{\partial u}{\partial x} = M.$$

$$\partial u = x^3 \, \partial x.$$

$$u = \int x^3 \, \partial x.$$

$$u = \frac{x^4}{4} + c(y) \longrightarrow ①$$

To find $c(y) = ?$

Diff eq ① w.r.t y.

$$\frac{\partial u}{\partial y} = 0 + c'(y).$$

$$c'(y) = \frac{\partial u}{\partial y} = N = y^3.$$

$$c'(y) = N = y^3.$$

$$c'(y) = y^3.$$

$$\int c'(y) = \int y^3 dy.$$

$$c(y) = y^4/4$$

By putting values in eq ①.

$$u = x^4/4 + y^4/4. \qquad \text{As } u = \text{Const}.$$

$$4c = x^4 + y^4 \Rightarrow \boxed{x^4 + y^4 = c}$$ Ans

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Q3:- $\sin x \cos y \, dx + \cos x \sin y \, dy = 0$.

Sol:- By comparing we get.

$M = \sin x \cos y$. & $N = \cos x \sin y$.

$M_y = -\sin x . \sin y$. $N_x = -\sin x \sin y$.

As $\boxed{N_x = M_y.}$

so the given eqn is exact.

to find solution.

We know.

$$\frac{\partial u}{\partial x} = M.$$

$$\partial u = + \sin x \cos y \, \partial x.$$

$$\int \partial u = \cos y \int \sin x \, \partial x$$

$$u = \cos y(-\cos x) + c(y).$$

$$u = -\cos x . \cos y + c(y) \longrightarrow (A)$$

To find (A).

$$\frac{du}{\partial y} = -\cos x(-\sin y) + c'(y). \qquad \frac{du}{\partial y} = N.$$

$$N = \sin y \cos x + c'(y). \qquad N = + \sin y \cos x$$

$$\cancel{\sin y \cos x} = \cancel{\sin y \cos x} + c'(y).$$

$$c'(y) = 0.$$

$\int c'(y) = \int 0.$

$c(y) = c.$

Eq (A) =)

$u = -\cos x \cos y + c.$

But $u = \text{constant} = c_1$

$c_1 = -\cos x \cos y + c.$

$c = -\cos x \cos y.$

$\boxed{y = \cos^{-1}\left(\frac{c}{\cos x}\right)}$ Ans

Q4:- $e^{3\theta}(dr + 3r\,d\theta) = 0.$

Sol:- $e^{3\theta} dr + e^{3\theta} 3r\,d\theta = 0.$

By comparing we get with gen. eqn.

$\theta = M = e^{3\theta}.$ $N = e^{3\theta} 3r.$

$M_\theta = 3e^{3\theta}$ $N_r = 3e^{3\theta}.$

As $\boxed{M_\theta = N_r}$

so the given eq. is Exact.

To find general solution.
We know that,

$\frac{\partial u}{\partial r} = M.$

$\partial u = e^{3\theta} \cdot \partial r.$

$u = \int e^{3\theta} \partial r.$

$u = e^{3\theta} \int \partial r.$

$u = e^{3\theta}(r) + c(\theta) \longrightarrow ①.$

To find $c(\theta) = ?$

Diff w.r.t θ.

$$\frac{\partial u}{\partial \theta} = \frac{\partial}{\partial \theta}\left(e^{3\theta}(r) + c'(\theta)\right).$$

$\frac{\partial u}{\partial \theta} = N.$

$N = 3e^{3\theta}(r).$

$N = 3r\, e^{3\theta} + c'(\theta).$

$\cancel{3re^{3\theta}} = \cancel{3re^{3\theta}} + c'(\theta)$

$c'(\theta) = 0.$

$\int c'(\theta) = \int 0.$

$c'(\theta) = c \longrightarrow ②.$

$\&$ ① $\Rightarrow$

$u = e^{3\theta}(r) + c.$

But $u = \text{constant} = c_1.$

$c_1 = e^{3\theta}(r) + c.$

$c_1 - c = e^{3\theta}(r).$

$\because c_1 - c = c.$

$$\boxed{c = e^{3\theta}(r)}$$

Ans

$Q5$:- $(x^2+y^2)\,dx - 2xy\,dy = 0 \longrightarrow (A)$

Sol:- By Comparing, We get.

$P = M = x^2+y^2 \qquad Q = N = -2xy.$

$P_y = M_y = 2y \qquad Q_x = N_x = -2y.$

As $\boxed{M_y \neq N_x}$

So not exact.

to find Integrating factor we know that;

$$I.F = e^{\int \frac{P_y - Q_x}{Q}\,dx}$$

$$I.F = e^{\int \left(\frac{2y+2y}{-2xy}\right)dx} \Rightarrow = e^{\int \frac{4y}{-2xy}\,dx}.$$

$$= e^{-2\int \frac{1}{x}\,dx}. \Rightarrow F = e^{-2\ln x}.$$

$$F = \cancel{e}^{\ln x^{-2}} \Rightarrow \boxed{F = x^{-2}}$$

Multiply with eq (A).

$$x^{-2}(x^2+y^2)\,dx - 2xy \cdot x^{-2}\,dy = 0.$$

$$(1+x^{-2}y^2)\,dx - 2x^{-1}y\,dy = 0.$$

Now; $M = 1+x^{-2}y^2 \qquad N = -2x^{-1}y.$

$$M_y = x^{-2}(2y)$$

$$= \frac{2y}{x^2}$$

$$N_x = \frac{2y}{x^2}.$$

As $\boxed{M_y = N_x}$ Now exact.

To find solution;

As we know that;

$$\frac{\partial u}{\partial x} = M \Rightarrow \partial u = \left(1 + \frac{y^2}{x^2}\right)\partial x.$$

$$\int \partial u = \int \left(1 + y^2/x^2\right) \partial x.$$

$$u = x - \frac{y^2}{x} + c(y) \longrightarrow (A).$$

To find $c(y) = ?$

Diff eq (A) w.r.t y.

$$\frac{\partial u}{\partial y} = N.$$

$$\frac{\partial u}{\partial y} = 0 - \frac{2y}{x} + c'(y).$$

$$N = -2y/x + c'(y).$$

$$N = -2x^{-1}(y)$$
$$= -\frac{2y}{x}.$$

$$-\frac{\cancel{2y}}{x} = -\frac{\cancel{2y}}{x} + c'(y)$$

$$c'(y) = 0.$$

$$\int c'(y) = \int 0.$$

$$c(y) = c.$$

eq (A) $\Rightarrow$

$$u = x - y^2/x + c.$$

But $u = \text{Constant} = c_1$

$$c_1 = x - y^2/x + c.$$

$$c_1 - c = x - y^2/x.$$

$$c = x - y^2/x$$

$$c = \frac{x^2 - y^2}{x} \Rightarrow x^2 - y^2 = cx.$$

$$\boxed{y^2 = x^2 - cx}$$

Ans

Q6:- $3(y+1)\,dx = 2x\,dy$; $(y+1)x^{-4}$.

Sol:- As Integrating factor i.e $(y+1)x^{-4}$ is given

so multiply with given eqn.

$$3(y+1)(y+1)x^{-4}dx - 2x(y+1)x^{-4}dy = 0.$$

$$3(y+1)^2x^{-4}dx - 2x^{-3}(y+1)dy = 0.$$

~~Mx~~

$M = 3(y+1)^2x^{-4}$. $N = -2x^{-3}(y+1)$.

$M_y = 3(2)(y+1)x^{-4}$. $N_x = 6x^{-4}(y+1)$.

$M_y = 6(y+1)x^{-4}$. $N_x = 6(y+1)x^{-4}$.

As $\boxed{M_y = N_x}$

So given eqn is exact

to find solution;

As; we know that;

$$\frac{\partial u}{\partial x} = M.$$

$$\partial u = 3(y+1)^2x^{-4}\partial x.$$

$$\int \partial u = 3(y+1)^2 \int x^{-4}\partial x$$

$$u = \cancel{3}(y+1)^2\frac{x^{-3}}{-\cancel{3}} + \phi(y).$$

$u = -(y+1)^2 x^{-3} + c(y) \longrightarrow$ (A).

Diff w.r.t y.

$$\frac{\partial u}{\partial y} = -2(y+1)x^{-3} + c'(y).$$

$\frac{\partial u}{\partial y} = N.$

$$N = -2(y+1)x^{-3} + c'(y)$$

But $N = -2x^{-3}(y+1).$

$$-2\cancel{(y+1)x^{-3}} = -2\cancel{(y+1)x^{-3}} + c'(y)$$

$$c'(y) = 0.$$

$$\int c'(y) = \int 0 \, dc.$$

$$\boxed{c(y) = c.}$$

$\therefore$ (A) $\Rightarrow$

$$u = -(y+1)^2 x^{-3} + c.$$

But $u =$ Constant $= c_1$.

$$c_1 = -(y+1)^2 x^{-3} + c.$$

$$c_1 - c = -(y+1)^2 x^{-3}.$$

$$c = -(y+1)^2 x^{-3} \Rightarrow -(y+1)^2 = \cancel{x^{-3}} \, c/x^{-3}.$$

$$\boxed{(y+1)^2 = -cx^3.}$$

Ans

Q7:- $2x \tan y \, dx + \sec^2 y \, dy = 0$.

Sol:- By comparing.

$P = M = 2x \tan y$. $Q = N = \sec^2 y$.

$P_y = M_y = 2x \sec^2 y$. $Q_x = N_x = 0$.

As $M_y \neq N_x$

So not exact.

Now find I.F.

$$I.F = e^{\int \frac{P_y - Q_x}{Q} dx}.$$

$$= e^{\int \frac{2x \cancel{\sec^2 y} - 0 \, dx}{\cancel{\sec^2 y}}} \Rightarrow e^{\int 2x \, dx}$$

$$\boxed{I.F = e^{x^2}}$$

Multiply with given equation.

$2x \tan y . e^{x^2} dx + \sec^2 y \, e^{x^2} dy = 0$.

$P = M = 2x \tan y . e^{x^2}$. $N = \sec^2 y \, e^{x^2}$,

$M_y = 2x e^{x^2} \sec^2 y$. $N_x = \sec^2 y \, e^{x^2} (2x)$

As $\boxed{M_y = N_x}$ so exact.

to find solution;

As we know that,

$\partial u / \partial x = M$.

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$$\partial u = 2x \tan y \, e^{x^2} dx .$$

$$\int \partial u = \tan y \int 2x \, e^{x^2} dx.$$

$$u = \tan y \, e^{x^2} + c(y) \longrightarrow (A).$$

Diff w.r.t y.

$$\frac{\partial u}{\partial y} = \sec^2 y \, e^{x^2} + c'(y).$$

$$\frac{\partial u}{\partial y} = N = \sec^2 y \, e^{x^2} + c'(y) \qquad N = \sec^2 y \, e^{x^2}$$

$$\cancel{\sec^2 y \, e^{x^2}} = \cancel{\sec^2 y \, e^{x^2}} + c'(y)$$

$$c'(y) = 0.$$

$$\int c'(y) = \int 0 \, dc.$$

$$\boxed{c(y) = c}$$

Eq (A) $\Rightarrow$ $$u = \tan y \, e^{x^2} + c.$$

But $u = \text{constant} = c_1$.

$$c_1 - c = \tan y \, e^{x^2}.$$

$$c = \tan y \, e^{x^2} \Rightarrow \tan y = c/e^{x^2}.$$

$$\boxed{y = \tan^{-1}\left(c/e^{x^2}\right) \text{ Ans}}.$$

Q8:- $e^x(\cos y\,dx - \sin y\,dy) = 0.$

Sol:- $e^x \cos y\,dx - e^x \sin y\,dy = 0 \longrightarrow (1)$

$M = e^x \cos y \qquad N = -e^x \sin y.$

$M_y = -e^x \sin y \qquad N_x = -e^x \sin y.$

As $\boxed{M_y = N_x}$

so The Given eqn is differential.

to find general eqn.

As $\frac{\partial u}{\partial x} = M.$

$\partial u = e^x \cos y\,\partial x.$

$\int \partial u = \int e^x \cos y\,\partial x. \Rightarrow u = \cos y \int e^x dx.$

$u = \cos y\, e^x + c(y). \longrightarrow (A).$

To find $c(y) = ?$

As; Diff eq (A).

$\frac{\partial u}{\partial y} = -\sin y\, e^x + c'(y).$

$N = -e^x \cancel{\sin y} = -\cancel{\sin y}\, e^x + c'(y).$

$\left.\begin{matrix} \\ \\ \end{matrix}\right\} \frac{\partial u}{\partial y} = N.$ & $N = -e^x \sin y.$

$c'(y) = 0.$

$\int c'(y) = \int 0 \Rightarrow c(y) = c.$

& (A) $\rightarrow$

$u = \cos y\, e^x + c.$

But $u = \text{Constant} = c_1.$

$c_1 = \cos y\, e^x + c_0 \qquad c_1 - c = C.$

$\cos y\, e^x = c$

$\cos y = c/e^x \Rightarrow \boxed{y = \cos^{-1}(c/e^x)}$

Ans

Q9:- $e^{2x}(2\cos y\, dx - \sin y\, dy) = 0$; $y(0) = 0$.

Sol:- $2e^{2x}\cos y\, dx - e^{2x}\sin y\, dy = 0$.

$M = 2e^{2x}\cos y$. $\qquad N = -e^{2x}\sin y$.

$M_y = -2e^{2x}\sin y$. $\qquad N_x = -2e^{2x}\sin y$

As $\boxed{M_y = N_x}$ so differential eqn is exact.

To find general sol.

$\frac{\partial u}{\partial x} = M = \quad \partial u = 2e^{2x}\cos y$.

$u = 2\cos y \int e^{2x}\partial x$.

$u = 2\cos y\left(\frac{e^{2x}}{2}\right) + c(y) \Rightarrow u = \cos y\, e^{2x} + c(y)$. — (A)

$\frac{\partial u}{\partial y} = N = -\sin y\, e^{2x} + c'(y)$.

$\cancel{-\sin y\, e^{2x}} = \cancel{-\sin y\, e^{2x}} + c'(y) \Rightarrow c'(y) = 0$.

$\int c'(y) = \int 0\, dc \Rightarrow c(y) = c$.

& (A) $\Rightarrow$

$u = \cos y\, e^{2x} + c$.

But $u = \text{Constant} = c_1$.

$c_1 = \cos y\, e^{2x} + c_0 \Rightarrow \quad \cos y\, e^{2x} = c$.

$\cos y = c/e^{2x}$. $\qquad$ Ans

$y = (c/e^{2x})\cos^{-1} \Rightarrow \boxed{y = \cos^{-1}(c/e^{2x})}$

For Initial Condition ($y(0) = 0$).

$0 = \cos^{-1}(c/e^0)$

$\boxed{1 = c}$

& $\boxed{y = \cos^{-1}(1/e^{2x})}$

Ans

Q_{10}:- $y\,dx + [y + \tan(x+y)]\,dy = 0$, $\cos(x+y)$.

Sol:- As I.F $= \cos(x+y)$ so multiply with given eqn.

$y\cos(x+y)\,dx + \cos(x+y)[y + \tan(x+y)]\,dy = 0.$

$M = y\cos(x+y)$.

$N = \cos(x+y)\,(y + \tan(x+y))$

$M_y = \cos(x+y) + y(-\sin(x+y))$.

~~$N_x = \sec^2(x+y)$~~.

$M_y = \cos(x+y) - y\sin(x+y)$.

$N = \cos(x+y)(y + \tan(x+y))$.

$N = \cos(x+y)\,y + \cancel{\cos(x+y)}\cdot\frac{\sin(x+y)}{\cancel{\cos(x+y)}}$

$N = \cos(x+y)\,y + \sin(x+y)$.

$N_x = y(-\sin(x+y)) + \cos(x+y)$.

$N_x = \cos(x+y) - \sin(x+y)\,y$.

As $\boxed{M_y = N_x}$

So exact eqn.

To find solution;

$\frac{\partial u}{\partial x} = M$.

$\partial u = y\cos(x+y)\,\partial x \Rightarrow u = y\int\cos(x+y)\,\partial x$.

$u = y\sin(x+y) + c(y). \longrightarrow (1)$.

$\frac{\partial u}{\partial y} = \sin(x+y) + y\cos(x+y) + c'(y)$.

But $\frac{\partial u}{\partial y} = N$.

$N = \sin(x+y) + y\cos(x+y) + c'(y)$

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$$\cancel{\cos(x+y)y} + \cancel{\sin(x+y)} = \cancel{\sin(x+y)} + \cancel{\cos(x+y)y} + c'(y) \quad \Big\{ N = \cos(x+y)y + \sin(x+y)$$

$$c'(y) = 0.$$

$$\int c'(y) = \int 0 \, dc$$

$$c(y) = c.$$

$\{ 4 \Rightarrow$

$$u = y\sin(x+y) + c.$$

But $u = \text{constant} = c_1.$

$$c_1 = y\sin(x+y) + c.$$

$$c_1 - c = c.$$

$$c = y\sin(x+y).$$

$$\boxed{y = \frac{c}{\sin(x+y)}}$$

Ans

Q11:- $2\cosh x \cos y \, dx = \sinh x \sin y \, dy.$

Sol:- $2\cosh x \cos y \, dx - \sinh x \sin y \, dy = 0.$

$P = M = 2\cosh x \cos y.$ $\quad Q = N = -\sinh x \sin y.$

$P_y = M_y = -2\sin y \cosh x$ $\quad Q_x = N_x = -\cosh x \sin y.$

As $\boxed{M_y \neq N_x}$

so not exact.

To find Integrating factor.

$$I.F = e^{\int \frac{P_y - Q_x}{Q} dx}$$

$$I.F = e^{\int \frac{-2\sin y \cosh x + \cosh x \sin y}{-\sinh x \sin y} dx}$$

$$= e^{\int \frac{+3\sin y \cosh x}{+\sinh x \sin y} dx} = e^{3\int \frac{\cosh x}{\sinh x} dx}$$

$$= e^{3\int \coth x \, dx} \Rightarrow e^{3\ln|\sinh x|}$$

$$= e^{\ln(\sinh x)^3} \Rightarrow I.F = (\sinh x)^3$$

Multiply with given Eq.

$$2\cosh x \cos y \sinh x \, dx - \sinh^2 x \sin y \, dy = 0.$$

$$\sin 2hx \cos y \, dx - \sinh^2 x \sin y \, dy = 0.$$

$M = \sin 2hx \cos y.$ $\qquad$ $N = -\sinh^2 x \sin y.$

$M_y = \sin 2hx(-\sin y).$ $\qquad$ $N_x = -2\sinh x . \cosh x . \sin y.$

$M_y = -\sin y \sin 2hx.$ $\qquad$ $N_x = -\sin y . \sin 2hx.$

$\boxed{M_y = N_x}$

Now exact.

To find solution.

$$\frac{\partial u}{\partial x} = M \Rightarrow \int \partial u = \int \sin 2hx \cos y \, \partial x.$$

$$u = \cos y \int \sin 2hx$$

$$u = \frac{\cos y . \cos 2hx}{2} + c(y).$$

$$u = \frac{1}{2} \cos y \cos 2hx + c(y). \longrightarrow A.$$

Diff w.r.t y.

$$\frac{\partial u}{\partial y} = -\frac{1}{2}\cos 2hx(\sin y) + c'(y).$$

$$N = -\frac{1}{2}\cos 2hx \sin y + c'(y).$$

$\because N = -\sinh^2 x \sin y.$

$$-\sinh^2 x \sin y = -\frac{1}{2}\cos 2hx \sin y + c'(y).$$

$$c'(y) = -\sinh^2 x \sin y + \frac{1}{2}\cos 2hx \sin y.$$

$$\int c'(y) = -\int \sinh^2 x \sin y \, dy + \frac{1}{2}\int \cos 2hx \sin y \, dy$$

$$c(y) = -\sinh^2 x \int \sin y \, dy + \frac{1}{2} \cos 2hx \int \sin y \, dy$$

$$= +\sinh^2 x \cos y \mp \frac{1}{2} \cos 2hx \cos y.$$

$$c(y) = \sinh^2 x \cos y - \frac{1}{2} \cos 2hx \cos y. \quad \hookrightarrow (2).$$

$\{ (A) \Rightarrow$

$$u = \frac{1}{2} \cos y \cancel{\cos 2hx} + \sinh^2 x \sin y \mp \frac{1}{2} \cancel{\cosh x} \cos y.$$

$$u = +\sinh^2 x \sin y.$$

But $u = \text{constant} = c_1$.

$$c_1 = \sinh^2 x \sin y.$$

$$\sin y = \frac{c_1}{\sinh^2 x}$$

$$\boxed{y = \sin^{-1}\left(\frac{c_1}{\sinh^2 x}\right)} \text{ Ans}$$

Q12:- $(2xy\,dx + dy)\,e^{x^2} = 0$, $y(0) = 2$.

Sol:- $e^{x^2} 2xy\,dx + e^{x^2} dy = 0$.

$M = e^{x^2} 2xy$. $N = e^{x^2}$.

$M_y = e^{x^2}(2x)$ $N_x = e^{x^2}(2x)$.

$M_y = 2xe^{x^2}$ $N_x = 2xe^{x^2}$.

As $\boxed{M_y = N_x}$

so exact eqn.

to find sol.

$\frac{\partial u}{\partial x} = M \Rightarrow \partial u = e^{x^2} 2xy\,\partial x$

$u = y\int e^{x^2} 2x\,\partial x$. $\because \int e^{f(x)} f'(x)\,dx = e^{f(x)}$

$u = ye^{x^2} + c(y)$. $\longrightarrow$ (1).

Diff w.r.t y.

$\frac{\partial u}{\partial y} = e^{x^2} + c'(y)$. But $\frac{\partial u}{\partial y} = N = e^{x^2}$.

$\cancel{e^{x^2}} = \cancel{e^{x^2}} + c'(y)$.

$c'(y) = 0 \Rightarrow \int c'(y) = \int 0\,dc \Rightarrow C(y) = C$

& (1) $\Rightarrow$ $u = ye^{x^2} + c$. But $u = \text{Constant} = C_1$

$C_1 = ye^{x^2} + c \Rightarrow \boxed{C = ye^{x^2}}$

or $\boxed{y = C/e^{x^2}}$ Ans

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Q13:- $e^{-y}dx + e^{-x}(-e^{-y}+1)dy = 0$ $\quad F = e^{x+y}$.

Sol:- As $F = e^{x+y}$. So multiply with eq.

$$e^{-y}.e^{x+y}dx + e^{-x}(-e^{-y}+1)e^{x+y}dy = 0.$$

$$e^{x}dx + (-e^{-x-y}+e^{-x})e^{x+y}dy = 0.$$

$$e^{x}dx + (-1+e^{y})dy = 0.$$

So; $\quad M = e^{x} \quad$ & $\quad N = -1+e^{y}$,

$M_y = 0 \qquad N_x = 0$.

so $\boxed{M_y = N_x}$ So the given eq is exact.

The gen. eqn is;

$$\frac{\partial u}{\partial x} = M \Rightarrow \int \partial u = \int e^{x}\partial x.$$

$$u = e^{x} + c(y). \quad \text{↳①}. \quad \text{Diff w.r.t } y.$$

$$\frac{\partial u}{\partial y} = 0 + c'(y). \qquad N = \frac{\partial u}{\partial y} = -1+e^{y}.$$

$$c'(y) = N = -1+e^{y}.$$

$$c'(y) = -1+e^{y}$$

$$\int c'(y) = \int(-1+e^{y})dy$$

$$c(y) = -y + e^{y} + C_1.$$

Put in eq ① => $\quad u = e^{x} + (-y) + e^{y} + C_1$.

But $u =$ Constant.

$$c = e^{x} - y + e^{y} + C_1. \qquad C_1 - C = C$$

$$\boxed{c = e^{x} + e^{y} - y} \text{ Ans}$$

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Q14:- $(a+1)y\,dx + (b+1)x\,dy = 0 \qquad y(1) = 1.$

Sol:- As $F = x^a y^b$ is I.F So. $\qquad F = x^a y^b$

$$x^a y^b (a+1) y\,dx + x^a y^b (b+1) x\,dy = 0.$$

$$x^a (a+1) y^{b+1} dx + y^b (b+1) x^{a+1} dy = 0.$$

$M = x^a (a+1) y^{b+1} \qquad N = y^b (b+1) x^{a+1}.$

$M_y = x^a (a+1)(b+1) y^b \qquad N_x = y^b (b+1)(a+1) x^a.$

As $\boxed{M_y = N_x}$

so exact.

For solution:-

$$\frac{\partial u}{\partial x} = M \Rightarrow \partial u = x^a (a+1) y^{b+1} \partial x.$$

$$u = (a+1) y^{b+1} \int x^a \partial x.$$

$$u = \cancel{(a+1)} y^{b+1} \frac{x^{a+1}}{\cancel{(a+1)}} + c(y).$$

$$u = y^{b+1} . x^{a+1} + c(y). \rightarrow (1).$$

To find $c(y) = ?$

$$\frac{\partial u}{\partial y} = (b+1) y^b . x^{a+1} . + c'(y). \qquad \text{As } \frac{\partial u}{\partial y} = N.$$

$$N = (b+1) y^b x^{a+1} + c'(y)$$

But $N = y^b (b+1) x^{a+1}.$

$$(b+1) \cancel{y^b x^{a+1}} = (b+1) \cancel{y^b x^{a+1}} + c'(y).$$

$$c'(y) = 0.$$

$$\int c'(y) = \int 0 \, dc$$

$$\boxed{c(y) = c}$$

eq (A) $\Rightarrow$

$$u = y^{b+1} x^{a+1} + c.$$

But $\quad u = \text{constant} = c_1.$

$$c_1 = y^{b+1} \cdot x^{a+1} \cdot + c. \qquad\qquad c_1 - c = c.$$

$$c = y^{b+1} \cdot x^{a+1}.$$

Apply Initial condition. $\qquad y(1) = 1.$

$$c = \cancel{y}\, (1)^{b+1} \cdot (1)^{1+1}.$$

$$\boxed{c = 1}$$

So specific solution is;

$$\cancel{y}\ \boxed{1 = y^{b+1} x^{a+1}} \text{ Ans}$$