Excercise 1.5

Find the general solution if Initial condition also find the particular solution & graph or sketch it.

Q1:- $e^{-\ln x} = 1/x$.

Sol:- (i) $e^{-1\ln x} = e^{\ln x^{-1}} = e^{\ln x^{-1}}$

$= x^{-1} = 1/x$ (proved)

(2) → $e^{-\ln(\sec x)} = \cos x$.

Sol:- $e^{-\ln \sec x} \Rightarrow e^{\ln(\sec x)^{-1}}$.

$= (\sec x)^{-1} = \dfrac{1}{\sec x} = \cos x$ (Proved)

Q3:- $y' - y = 5.2$.

Sol:- $y' - y = 5.2$ is linear in y as it is of form of $y' + P(x)y = Q(x)$.

Here $P(x) = -1$, $Q(x) = 5.2$.

As the general solution is;

$$y = \frac{1}{F}\left[\int FQ\,dx + c\right] \rightarrow (1).$$

First find $F = ?$

$F = e^{\int P(x)\,dx}$.

By putting values

$F = e^{\int -1\, dx}$

$F = e^{-x}$

So eq (i) $\Rightarrow$

$$y = \frac{1}{e^{-x}}\left[\int \left[e^{-x}(5.2)\right]dx + c\right]$$

$$y = \frac{1}{e^{-x}}\left[-5.2\, e^{-x} + c\right]$$

$$y = \frac{1}{e^{-x}}\left(-5.2\, e^{-x}\right) + e^{x} c$$

$$\boxed{y = -5.2 + ce^{x}}$$ Ans

Q. $y' = 2y - 4x$

Sol:- $y' - 2y = -4x$

The given eqn is linear in y as it is written in form of

$$y' + P(x)\,y = Q(x)$$

Here $P = -2$, $Q(x) = -4x$

As we know that,

$$y = \frac{1}{F}\left[\int Q(x)\, F\, dx + c\right] \rightarrow (1)$$

First find $F = ?$

$F = e^{\int P(x)\,dx}$

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$F = e^{\int -2\,dx}$

$= e^{-2x}$

eq(1) =)

$$y = \frac{1}{e^{-2x}}\left[\int e^{-2x}(-4x)\,dx + c\right]$$

$$y = \frac{1}{e^{-2x}}\left[-4\int e^{-2x}x\,dx + c\right] \longrightarrow (A)$$

$$\int \underset{II}{e^{-2x}}\,\underset{I}{x}\,dx = \frac{x\,e^{-2x}}{-2} - \int\left(\frac{e^{-2x}}{-2}(1)\right)dx$$

$$= -\frac{1}{2}x e^{-2x} + \frac{1}{2}\int e^{-2x}\,dx$$

$$= -\frac{1}{2}x e^{-2x} - \frac{1}{4}e^{-2x} + c$$

using eq (A)

$$y = \frac{1}{e^{-2x}}\left[-4\left(-\frac{1}{2}x e^{-2x} - \frac{1}{4}e^{-2x}\right) + c\right]$$

$$y = \frac{1}{\cancel{e^{-2x}}}(-2)\cancel{e^{-2x}}(x) + \frac{\cancel{e^{-2x}}}{\cancel{e^{-2x}}} + \frac{c}{e^{-2x}}$$

$$\boxed{y = -2(x) + 1 + e^{2x}c} \text{ Ans}$$

(ii)

$y' + ky = e^{-kx}$.

Sol: The given eqn is linear in y as;

$y' + P(x)y = Q(x)$.

Here $P(x) = k$

& $Q(x) = e^{-kx}$.

As general solution is;

$$y = \frac{1}{F} \int F\, Q(x)\, dx + C \longrightarrow (1)$$

First find $F = ?$

$$F = e^{\int P(x)\, dx}.$$

$$= e^{\int k\, dx} \Rightarrow e^{kx}.$$

eq (1) $\Rightarrow$

$$= \frac{1}{e^{kx}} \int e^{kx} . e^{-kx} dx + c.$$

$$= \frac{1}{e^{kx}} \left[\int e^{0} dx + c \right].$$

$$= \frac{1}{e^{kx}} [x + c].$$

$$y = \frac{x}{e^{kx}} + c/e^{kx} \Rightarrow \boxed{e^{-kx}(x+c) = y}$$

Ans

Q6:- $y' + 2y = 4\cos 2x$; $y\left(\frac{1}{4}\pi\right) = 3.$

Sol:- $y' + 2y = 4\cos 2x.$

$y' + 2y = 4\cos 2x$, is linear in y.

By comparing

$P(x) = 2, \quad Q(x) = 4\cos 2x.$

As general solution is;

$$y = \frac{1}{F}\int F\,Q(x)\,dx + c \longrightarrow (A).$$

To find $F = ?$

$$F = e^{\int P(x)\,dx}.$$

$$F = e^{\int 2\,dx} \Rightarrow F = e^{2x}.$$

Using eq (A) $\Rightarrow$

$$y = \frac{1}{e^{2x}}\int e^{2x}(4\cos 2x)\,dx + c$$

$$= \frac{1}{e^{2x}}\, 4\int e^{2x}\cos 2x\,dx + c.$$

$I = \int e^{2x}\cos 2x\,d$
II I.

$I = \frac{\cos 2x\,(e^{2x})}{2} + 2\int \frac{e^{2x}}{2}\sin 2x\,(\cancel{2})$

$I = \frac{\cos 2x\,e^{2x}}{2} + \int e^{2x}\sin 2x\,dx.$
II · I

$I = \frac{\cos 2x\,e^{2x}}{2} + \frac{e^{2x}\sin 2x}{2} - \int \frac{e^{2x}}{\cancel{2}}\cos 2x\,(2)\,dx$

$I = \frac{\cos 2x\,e^{2x}}{2} + \frac{e^{2x}\sin 2x}{2} - \int e^{2x}\cos 2x\,dx$

$I = \cos 2x\,e^{2x} + e^{2x}\sin 2x.$

$$y = \frac{1}{e^{2x}}\, 4\left[\cos 2x\, e^{2x} + e^{2x}\sin 2x\right] + c$$

$$y = \frac{4\cos 2x\,\cancel{e^{2x}}}{\cancel{e^{2x}}} + \frac{4\cancel{e^{2x}}\sin 2x}{\cancel{e^{2x}}} + \frac{c}{e^{2x}}$$

$$\boxed{y = 4\cos 2x + 4\sin 2x + c} \text{ Ans.}$$

Q7:- $xy' = 2y + x^3 e^x$.

Sol:-

$$xy' - 2y = x^3 e^x.$$

~~Here P(x)~~

Divide by x.

$$y' - \frac{2y}{x} = x^2 e^x.$$

Here $P(x) = -2/x$ & $Q(x) = x^2 e^x$.

The general solution is;

$$y = \frac{1}{F}\left[\int F Q(x)\, dx + c\right] \longrightarrow (1).$$

First find $F = ?$

$$F = e^{\int P(x)\, dx} = e^{\int -2/x\, dx}.$$

$$F = e^{-2\ln x} \Rightarrow \boxed{x^{-2} = F} \text{ or } \boxed{F = 1/x^2}$$

& (1) $\Rightarrow$ $$y = \frac{1}{1/x^2}\left[\int \frac{1}{\cancel{x^2}} \cancel{x^2} e^x\, dx + c\right].$$

$$y = x^2 e^x + c x^2.$$

$$\boxed{y = x^2(e^x + c)}$$ Ans

Q8:- $y' + y\tan x = e^{-0.01x}\cos x.$ $y(0) = 0$

Sol:- is linear in y.

Here $P(x) = \tan x$, $Q(x) = e^{-0.01}\cos x$.

As general solution is;

$$y = \frac{1}{F}\int FQ(x)\,dx + c \rightarrow ①$$

Frist find $F = ?$

$$F = e^{\int P(x)dx} \Rightarrow e^{\int \tan x\,dx}.$$

$$= e^{-\ln\cos x} \Rightarrow e^{\ln(\cos x)^{-1}} \Rightarrow \frac{1}{\cos x}. = F.$$

$\&$ ① $\Rightarrow$ $$\frac{1}{1/\cos x}\int \frac{1}{\cancel{\cos x}}\cdot e^{-0.01x}\cancel{\cos x}\,dx + c.$$

$$= \cos x\left[\int e^{-0.01x}dx + c\right]$$

$$= \cos x\left[\left(\frac{e^{-0.01x}}{-0.01}\right) + c\right].$$

$$y = -100\cos x\, e^{-0.01} + \cos x\, c. \rightarrow Ⓐ$$

Apply Initial condition $y(0) = 0$.

$$0 = -100(1) + c \Rightarrow \boxed{c = 100}$$

$$\boxed{y = -100\cos x\, e^{-0.01} + \cos x\,(100)}$$ Ans

Q9:- $y' + y\sin x = e^{\cos x}$ $\quad y(0) = -2.5.$

Sol:- $y' + \sin x\, y = e^{\cos x}.$ is linear in y.

By Comparing;

$P(x) = \sin x.$

$Q(x) = e^{\cos x}.$

∴ The general solution is;

$$y = \frac{1}{F}\int F\,Q(x)\,dx + c \rightarrow (A).$$

$$F = e^{\int P(x)\,dx}.$$

$$F = e^{\int \sin x\,dx}$$

$$F = e^{-\cos x}$$

Using eq (A)

$$y = \frac{1}{e^{-\cos x}}\int e^{-\cos x}.e^{\cos x}\,dx + C.$$

$$y = \frac{1}{e^{-\cos x}}\left[\int e^{0}\,dx + c\right] \Rightarrow \frac{1}{e^{-\cos x}}\left[x + c.\right]$$

$$y = \frac{x}{e^{-\cos x}} + \frac{c}{e^{-\cos x}}.$$

$$\because \frac{c}{e^{-\cos x}} = c$$

$$y = \frac{x}{e^{-\cos x}} + C. \rightarrow (A).$$

Apply Initial condition i.e $y(0) = -2.5.$

$$-2.5 = \cancel{x}\,0 + c \Rightarrow \boxed{C = -2.5}$$

eq (A) $\Rightarrow$ $\boxed{y = x/e^{-\cos x} - 2.5}$ Ans

Q_{10}:- $y'\cos x + (3y-1)\sec x = 0$; $y\left(\frac{1}{4}\pi\right) = 4/3$.

Sol:- Divide by $\cos x$.

$$y' + (3y-1)\frac{1}{\cos^2 x} = 0.$$

$$y' + \frac{3y}{\cos^2 x} = \frac{1}{\cos^2 x}.$$

Here $P(x) = \frac{3}{\cos^2 x}$, $Q(x) = \frac{1}{\cos^2 x}$.

As general solution is;

$$y = \frac{1}{F}\int F\,Q(x)\,dx + c. \longrightarrow (A)$$

To find $F = ?$

$$F = e^{\int P(x)dx} \Rightarrow e^{\int \frac{3}{\cos^2 x}dx} = e^{3\int \sec^2 x\,dx}$$

$$F = e^{3\tan x}.$$

$$y = \frac{1}{e^{3\tan x}}\int e^{3\tan x}\left(\frac{1}{\cos^2 x}\right)dx + c.$$

$$y = \frac{1}{e^{3\tan x}}\int e^{3\tan x}\sec^2 x\,dx + c$$

$\because \int e^{f(x)} f'(x)dx = e^{f(x)} + c$

$$= \frac{1}{e^{3\tan x}}\cdot\left[e^{3\tan x} + c\right].$$

$$y = 1 + \frac{c}{e^{3\tan x}} \longrightarrow (A)$$

Applying Initial Conditions i.e $y\left(\frac{1}{4}\pi\right) = 4/3$.

$$\frac{4}{3} = 1 + \frac{c}{e^{3\tan(\pi/4)}}$$

$$4/3 = 1 + \frac{c}{e^3} \Rightarrow \frac{4}{3} = \frac{e^3 + c}{e^3}$$

$$\frac{4e^3}{3} = e^3 + c$$

$$\frac{4}{3}e^3 - e^3 = c \Rightarrow \frac{4e^3 - 3e^3}{3}$$

$$\frac{e^3}{3} = c \Rightarrow \boxed{c = 6.695}$$

$$\text{eq Ⓐ} \Rightarrow \boxed{y = 1 + \frac{c}{e^{3\tan x}}}$$

Ans

Q11:- $y' = (y-2)\cot x$

Sol:- $y' - y\cot x = -2\cot x$ is linear in y.

Here $P(x) = -\cot x$ & $Q(x) = -2\cot x$.

As general solution is;

$$y = \frac{1}{F}\left[\int F\, Q(x)\, dx + c\right] \rightarrow (A)$$

First find $F = ?$

$$F = e^{\int P(x)\, dx} \Rightarrow e^{\int -\cot x\, dx}$$

$$= e^{\int -(\cot x) dx} \Rightarrow e^{(-)(-) \ln \sin x} = e^{\ln \sin x}$$

$F = \sin x$.

& (A) $\Rightarrow$

$$y = \frac{1}{\sin x}\left[\int \sin x \cdot (-2\cot x)\, dx + c\right].$$

$$= \frac{1}{\sin x}\left[-2\int \cancel{\sin x} \cdot \frac{\cos x}{\cancel{\sin x}}\, dx + c\right].$$

$$= \frac{1}{\sin x}\left[-2\int \cos x + c.\right] \Rightarrow \frac{1}{\sin x} \cdot \left[-2\sin x + c\right]$$

$$\boxed{y = -2 + \frac{c}{\sin x}}$$

Ans

Q12:- $xy' + 4y = 8x^4$; $y(1) = 2$.

Sol:- Divide by x.

$$y' + \frac{4}{x}y = 8x^3.$$

Here $P(x) = 4/x$, $Q(x) = 8x^3$.

As general equation is;

$$y = \frac{1}{F}\left[\int F\,Q(x)\,dx + c\right] \longrightarrow (1).$$

First find $F = ?$

$$F = e^{\int P(x)\,dx} \Rightarrow e^{\int \frac{4}{x}\,dx}.$$

$$= e^{4\int \frac{1}{x}\,dx} \Rightarrow e^{4\ln x} \Rightarrow e^{\ln x^4}$$

$$\boxed{F = x^4}$$

eq(1) $\Rightarrow$ $$y = \frac{1}{x^4}\left[\int x^4 \cdot (8x^3)\,dx + c\right].$$

$$= \frac{1}{x^4}\left[8\int x^7\,dx + c\right] \Rightarrow \frac{1}{x^4}\left[\frac{8x^8}{8} + c\right].$$

$$y = \frac{1}{x^4}x^8 + \frac{c}{x^4} \Rightarrow \boxed{y = x^4 + c/x^4}$$

Apply Initial Condition. $y(1) = 2$.

$$2 = (1)^4 + c/(1)^4 \Rightarrow \boxed{c = 1}$$

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The particular solution is;

$$\boxed{y = x^4 + c/x^4}$$ Ans

Non linear ODEs

Q22:- $y' + y = y^2$ $\qquad y(0) = -1/3$.

Sol:-

$$y' = y^2 - y.$$

$$\frac{dy}{dx} = y(y-1)$$

$$\frac{1}{y(y-1)} dy = dx.$$

$$\int \frac{1}{y(y-1)} dy = \int dx. \longrightarrow ②$$

$$\int \frac{1}{y(y-1)} dy = ?$$

$$\frac{1}{y(y-1)} = \frac{A}{y} + \frac{B}{y-1}.$$

$$\frac{1}{\cancel{y(y-1)}} = \frac{A(y-1) + By}{\cancel{y(y-1)}}.$$

$$1 = A(y-1) + By. \longrightarrow Ⓐ.$$

Put $y = 1 \Rightarrow \boxed{B = 1}$

Put $y = 0$ $\boxed{A = -1}$

So;
$$\int \frac{1}{y(y-1)} dy = \int \frac{-1}{y} dy + \int \frac{1}{y-1} dy$$

$$= -\ln y + \ln|y-1|.$$

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$$\int \frac{1}{y(y-1)}\,dy = \frac{\ln|y-1|}{y}.$$

Put in eq ⓧ

$$\ln\frac{|y-1|}{y} = x + c.$$

$$\frac{y-1}{y} = e^{x} + e^{c} \qquad e^{c} = c.$$

$$\frac{y-1}{y} = e^{x} + c.$$

$$\frac{y}{y} - \frac{1}{y} = e^{x}+c \Rightarrow 1 - \frac{1}{y} = e^{x}+c.$$

$$-1/y = e^{x} - 1 + c. \qquad c-1=c.$$

$$-1/y = e^{x} + c.$$

$$1/y = -e^{x} + c.$$

$$y = \frac{1}{-e^{x}+c} \longrightarrow Ⓐ.$$

Apply Initial Condition $y(0) = -1/3.$

$$-\tfrac{1}{3} = \frac{1}{-e^{0}+c} \Rightarrow -3 = -1 + c.$$

$$\boxed{c = -2}$$

eq Ⓐ $\Rightarrow$ $\boxed{y = \frac{1}{e^{-x} - 2}}$ Ans

Q23:- $y' + xy = xy^{-1}$ $\quad y(0) = 3$.

Sol:- Here as Bernoulli eqn.

with $P(x) = x$, $Q(x) = x$ $\quad a = -1$

$v = y^{1-a}$ $\quad a = -1$ $\quad v' \pm (1-a)v = -x(1-a)$

As general eqn is;

$$v\,y = \frac{1}{F}\left[\int (1-a) F\, Q(x)\, dx + c\right] \hookrightarrow (A).$$

To find $F = ?$

$$F = e^{\int (1-a) p(x)\, dx} \Rightarrow F = e^{\int ((1)-(-1)) x\, dx}$$

$$F = e^{2\int x\, dx} \Rightarrow F = e^{x^2}.$$

$$\& \ (A) \Rightarrow v\,y = \frac{1}{e^{x^2}}\left[\int (1-(-1))\, e^{x^2} (x)\, dx + c\right].$$

$$v\,y = \frac{1}{e^{x^2}}\left[\int 2 e^{x^2} x\, dx + c\right].$$

$$v\,y = \frac{1}{e^{x^2}}\left[\int e^{x^2} 2x\, dx + c\right].$$

$$v = \frac{1}{e^{x^2}}\left(e^{2x} + c\right).$$

$$v\,y = 1 + \frac{c}{e^{2x}} \rightarrow (B).$$

Apply Initial Condition.

$$3 = 1 + c/e^{0} \Rightarrow \boxed{c = 2}$$

$v = y^{1-a}$

$v = y^{1-(-1)}$ $\quad \& \ (B) \Rightarrow$ $\boxed{y^2 = 1 + \frac{2}{e^{2x}}}$ Ans

$v = y^2$

Q24:- $y' + y = -x/y$.

Sol:- $y' + y = -x/y \Rightarrow y' + y = -x y^{-1}$ (Bernoulli ODE)

$\Rightarrow$

Here $P(x) = 1$, $Q(x) = -x$ $a = -1$

$V = y^{(1-a)}$ & $a = -1$. So $V' + (1-a)V = -x(1-a)$

The general solution is;

$$V = \frac{1}{F} \int F^{(1-a)} Q(x)\, dx + c \longrightarrow ①$$

First to find $F = ?$

$$F = e^{\int (1-a) P(x)\, dx}$$

$$F = e^{\int (1-(-1)) 1\, dx} \Rightarrow e^{2\int dx} \Rightarrow e^{2x}$$

& ① $\Rightarrow$

$$V = \frac{1}{e^{2x}} \int e^{2x} (1-(-1)) (-x)\, dx + C$$

$$V = \frac{1}{e^{2x}} - \int e^{2x}\, 2x\, dx + c$$

$$V = \frac{1}{e^{2x}} \left[-(e^{2x}) + c \right]$$

$$V = \frac{1}{\cancel{e^{2x}}} (-\cancel{e^{2x}}) + \frac{c}{e^{2x}}$$

$\because V = y^{1-a}$

$V = y^{1-(-1)}$

$\boxed{V = y^2}$

$$\boxed{y^2 = -1 + \frac{c}{e^{2x}}}$$ Ans

25. $y' = 3.2y - 10y^2$ (Not linear in y Bernoulli eq)

Sol: $y' - 3.2y = -10y^2$

$P(x) = -3.2$, $a = 2$. $v = y^{1-a}$; $Q(x) = -10$.

As $$v = \frac{1}{F}\int (1-a)Q(x)F\,dx + c \longrightarrow (A).$$

$$F = e^{\int (1-a)P(x)\,dx} \Rightarrow e^{\int (1-a)(-3.2)dx} = e^{\int (1-2)3.2\,dx}$$

$$\boxed{F = e^{+3.2x}.}$$

$$v = \frac{1}{e^{3.2x}}\left[\int (1-2)e^{3.2x}(-10)dx + c\right].$$

$$= \frac{1}{e^{3.2x}}\left[10\int e^{3.2x}dx + c\right]$$

$$= \frac{1}{e^{3.2x}}\left[10\frac{e^{3.2x}}{3.2} + c\right].$$

$$v = \frac{10\cancel{e^{3.2x}}}{\cancel{e^{3.2x}}(3.2)} + \frac{c}{e^{3.2x}}.$$

$$v = \frac{10}{3.2} + \frac{c}{e^{3.2x}}$$

As $v = y^{1-a}$, $v = y^{1-2}$. $v = y^{-1}$.

$$y^{-1} = \frac{10}{3.2} + \frac{c}{e^{3.2x}}.$$

$$\boxed{y = \frac{3.2}{10} + \frac{e^{3.2x}}{c}}$$ Ans

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Q26:- $y' = \frac{\tan y}{(x-1)}$ $\quad y(0) = \frac{1}{2}\pi$.

Sol:-

$$\frac{dy}{dx} = \frac{\tan y}{x-1}.$$

$$\frac{1}{\tan y} dy = \frac{1}{x-1} dx$$

By Integrating.

$$\int \frac{1}{\tan y} dy = \int \frac{1}{x-1} dx.$$

$$\int \frac{\cos y}{\sin y} dy = \ln|x-1| + c.$$

$$\ln|\sin y| = \ln|x-1| + c$$

$$\sin y = x-1 + e^c$$

$$\sin y = x-1+c. \longrightarrow (A) \qquad e^c = c.$$

Applying Initial Condition. $y(0) = \frac{1}{2}\pi$.

$$\sin \pi/2 = 0-1+c \Rightarrow c = 1+1 = \boxed{c=2}$$

eq (A) $\Rightarrow$ $\sin y = x-1+2$.

$$\boxed{\sin y = x+1} \Rightarrow \boxed{y = \sin^{-1}(x+1)}$$ Ans

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Q27:- $y' = \frac{1}{6e^y - 2x}$.

Sol:-

$$y' = \frac{1}{6e^y - 2x} \Rightarrow \frac{dy}{dx} = \frac{1}{6e^y - 2x}$$

$$\frac{dx}{dy} = 6e^y - 2x.$$

$$\frac{dx}{dy} + 2x = 6e^y$$ linear in y

$P(y) = 2, \quad Q(y) = 6e^y.$

As general eqn is.

$$x = \frac{1}{F}\int F\,Q(y)\,dy + c \longrightarrow (A).$$

$$F = e^{\int P(y)\,dy} \Rightarrow e^{\int 2dy} \Rightarrow \boxed{F = e^{2y}}$$

& (A) $\Rightarrow$

$$x = \frac{1}{e^{2y}}\int e^{2y}.6e^y\,dy + c.$$

$$x = \frac{1}{e^{2y}}\,6\int e^{3y}\,dy + c$$

$$x = \frac{1}{e^{2y}}\left[6\frac{e^{3y}}{3} + c\right].$$

$$= e^{-2y}\left[2e^{3y} + c\right].$$

$$\boxed{x = 2e^y + c}$$ Ans

Q28:- $2xyy' + (x-1)y^2 = x^2 e^x$ set $(y^2 = z)$.

Sol:- $2xyy' + (x-1)y^2 = x^2 e^x$.

Divide by $2x$.

$$yy' + \frac{(y)^2(x-1)}{2x} = \frac{x e^x}{2} \rightarrow (1)$$

let $y^2 = z$.

$$\frac{dz}{dy} = 2yy'$$

$$\frac{dz}{dy} = 2y\,y'$$

$$y \cdot y' = \frac{1}{2}\frac{dz}{dy}$$

By putting values.

$$\cancel{2x}\left(\frac{1}{\cancel{2}}\frac{dz}{dy}\right) + \frac{(x-1)z}{\cancel{2}x} = \frac{x e^x}{\cancel{2}}$$

$$\frac{dz}{dy} + \frac{(x-1)z}{x} = x e^x$$

$$z' + \frac{(x-1)}{x} z = x e^x$$ is linear in z

$P(x) = \frac{x-1}{x}$ $Q(x) = xe^x$.

The general solution is;

$$y = \frac{1}{F}\int F\,Q(x)\,dx + c \rightarrow (A).$$

to find $F = ?$

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$F = \int_e P(x)dx$

$F = \int_e \left(\frac{x-1}{x}\right)dx$

$F = \int_e \frac{x dx}{x} - \int \frac{1}{x} dx \Rightarrow e^{x} \cdot e^{-\ln x} \cdot \Rightarrow e^{x} \cdot e^{\ln x^{-1}}$

$F = e^{x} \cdot x^{-1} \Rightarrow \boxed{Fx = \frac{e^{x}}{x}}$

eq (A) $\Rightarrow$

$y = \frac{1}{e^x/x} \int e^x/x \cdot x e^x dx + c$

$= \frac{x}{e^x}\left[\int e^{2x} dx + c\right]$

$= x/e^x \cdot \left(\frac{e^{2x}}{2} + c\right)$

$\boxed{y = \frac{x}{2e^x} + \frac{xc}{e^x}}$ Ans.