## Ex = 2.2

$\boxed{y''+ay'+by=0.}$ → Remember

**Q Find the general solution.**

1. $y''-0.25y=0.$

Sol:- $y''-0.25y=0 \rightarrow (1).$

Let the solution of eq (1) be.

$y=e^{\lambda x}. \rightarrow (2)$

to find $\lambda=?$

from eq (1) we have $a=0,\ b=-0.25.$

Using formula i.e.

$$\lambda=\frac{-a\pm\sqrt{a^2-4b}}{2}$$

$$=\frac{0\pm\sqrt{0-4(-0.25)}}{2}$$

$\lambda=\pm 1/2 \Rightarrow \boxed{\lambda=1/2}$ or $\boxed{\lambda=-1/2}$

So, eq (2) ⇒ $y_1=e^{1/2x}$ and $y_2=e^{-1/2x}$

So the general solution is;

$$\boxed{y=c_1e^{\frac{1}{2}x}+c_2e^{-1/2x}}$$ Ans

Q2:- $y''+36y=0.$

Sol:- $y''+36y=0 \rightarrow (1)$

Here $a=0,\ b=36.$

The solution of eq (1) is;

$y=e^{\lambda x}\rightarrow (2)$

To find $\lambda = ?$

$$\lambda = \frac{-a \pm \sqrt{a^2 - 4b}}{2a}$$

$$\lambda = \frac{0 \pm \sqrt{0 - 4(36)}}{2}$$

$$\lambda = \frac{0 \pm \sqrt{-12^2}}{2} \Rightarrow \frac{\pm 12i}{2}$$

$\lambda = +6i$ or $\lambda = -6i$.

So;

eq (2) $\Rightarrow$

$y_1 = e^{6i}$ & $y_2 = e^{-6i}$.

So

general solution will be;

$y = c_1 e^{6i} + c_2 e^{-6i}$

or

$$\boxed{y = c_1 e^{\lambda} \cos 6ix + c_2 e^{\lambda} \sin(6i)x}$$

Ans 2

(3) $y'' + 4y' + 2.5y = 0$.

Sol:- $y'' + 4y' + 2.5y = 0 \longrightarrow (1)$

Here $a = 4$, $b = 2.5$.

The solution is; $y = e^{\lambda x} \longrightarrow (1)$

To find $\lambda = ?$

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$$\lambda = \frac{-a \pm \sqrt{a^2 - 4b}}{2}$$

$$\lambda = \frac{-4 \pm \sqrt{16 - 4(2.5)}}{2}$$

$$= \frac{-4 \pm \sqrt{6}}{2}$$

$$= \frac{-4 \pm 2.45}{2}$$

$\lambda_1 = \frac{-4 + 2.45}{2}$ & $\lambda_2 = \frac{-4 - 2.45}{2}$

$\lambda_1 = -0.775$. $\lambda_2 = -3.225$.

eq (2) $\Rightarrow$ $y_1 = e^{-0.775x}$ & $y_2 = e^{-3.225x}$.

So. General Solution will be;

$$\boxed{y = c_1 e^{-0.775x} + c_2 e^{-3.225x}}$$ Ans

Q5:- $y'' + 2\pi y' + \pi^2 y = 0$.

Sol:- $y'' + 2\pi y' + \pi^2 y = 0$. $\longrightarrow$ (1).

Here $a = 2\pi$, $b = \pi^2$.

Let the Solution of eq (1) be

$y = e^{\lambda x} \longrightarrow$ (2)

to find "$\lambda$" use formula

$$\lambda = \frac{-a \pm \sqrt{a^2 - 4b}}{2}$$

$$\lambda = \frac{-2\lambda \pm \sqrt{(2\lambda)^2 - 4(\lambda^2)}}{2}$$

$$= \frac{-2\lambda \pm \sqrt{0}}{2} \Rightarrow \boxed{\lambda = -\lambda}$$

$$y_1 = e^{-\lambda x}.$$

To find other $\lambda = ?$

As we know that;

$$y_2 = u y_1 \longrightarrow (A)$$

$$u = \int U \, dx.$$

$$U = \frac{1}{y_1^2} e^{\int -p(x) dx}.$$

$$\rightarrow p(x) = 2\lambda.$$

$$= \frac{1}{(e^{-\lambda x})^2} e^{\int -2\lambda \, dx}.$$

$$U = \frac{1}{\cancel{e^{-2\lambda x}}} \cancel{e^{-2\lambda x}} \Rightarrow U = 1$$

(3) $$u = \int U \, dx.$$

$$u = \int 1 \, dx \Rightarrow u = x + \cancel{c}.$$

$$\text{eq}(A) \Rightarrow \quad y_2 = x e^{-\lambda x}.$$

So

General Solution is;

$$y = c_1 e^{-\lambda x} + c_2 e^{-\lambda x}(x) \quad \text{Ans}$$

Q4:- $y'' + 4y' + (\pi^2+4)y = 0.$

Sol:- $y'' + 4y' + (\pi^2+4)y = 0 \rightarrow (1)$

Since; $y'' + ay' + by = 0 \rightarrow (2)$
By comparing (1) & (2)

$a = 4, \quad b = \pi^2+4.$

Let the solution for eq (1) be
$y = e^{\lambda x} \rightarrow (1).$

But

$\lambda = -\dfrac{a \pm \sqrt{a^2-4b}}{2}$ ∵ By putting values.

$\lambda = \dfrac{-4 \pm \sqrt{(4)^2-4(\pi^2+4)}}{2}$

$= \dfrac{-4 \pm \sqrt{\cancel{16}-4\pi^2-\cancel{16}}}{2}$

$\lambda = \dfrac{-4 \pm \sqrt{-4\pi^2}}{2} \Rightarrow \dfrac{-4 \pm \pi(2)\sqrt{-1}}{2}.$

$\lambda = \dfrac{+\cancel{2}(-2 \pm \pi\sqrt{-1})}{\cancel{2}} \Rightarrow -2 \pm \pi i.$

$\boxed{\lambda_1 = -2+\pi i}$ ; $\boxed{\lambda_2 = -2-\pi i}$

So, $y_1 = e^{(-2+\pi i)x}$ & $y_2 = e^{(-2-\pi i)x}$

or $\boxed{y_1 = e^{-2x} \cos \pi x.}$ $\boxed{y_2 = e^{-2x} \sin \pi x}$

$y = (e^{-2x} \cos \pi x)c_1 + c_2(e^{-2x} \sin \pi x).$ Ans

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Q8:- $y'' + y' + 3.25y = 0.$

Sol:- $y'' + y' + 3.25y = 0 \longrightarrow ①$

Since; $y'' + ay' + by = 0 \longrightarrow ②$

By comparing coefficient of ① & ②

$a = 1, \quad b = 3.25$

Since; $y = e^{\lambda x} \longrightarrow ③$

to find $\lambda = ?$

$$\lambda = \frac{-a \pm \sqrt{a^2 - 4b}}{2}$$

$$\lambda = \frac{-1 \pm \sqrt{1 - 4(3.25)}}{2}$$

$$= \frac{-1 \pm \sqrt{-12}}{2} \Rightarrow \frac{-1 \pm 2\sqrt{3}i}{2}$$

$$\lambda_1 = \frac{-1 + 2\sqrt{3}i}{2}, \quad \lambda_2 = \frac{-1 - 2\sqrt{3}i}{2}$$

$$\lambda_1 = \frac{-1}{2} + \sqrt{3}i, \quad \lambda_2 = \frac{-1}{2} - \sqrt{3}i$$

So eq ①;

$$y_1 = e^{(-1/2 + \sqrt{3}i)x} \quad ; \quad y_2 = e^{(-1/2 - \sqrt{3}i)x}$$

$$y_1 = e^{-1/2 x + \sqrt{3}ix} \quad ; \quad y_2 = e^{-1/2 - \sqrt{3}ix}.$$

$$\boxed{y_1 = e^{-1/2 x} \cos \sqrt{3}x} \quad ; \quad \boxed{y_2 = e^{-1/2 x} \sin \sqrt{3}x}$$

Ans
Z.

Q6:- $10y'' - 32y' + 25.6y = 0.$

Sol:-

$$10y'' - 32y' + 25.6y = 0.$$

Divide by 10

$$y'' - 3.2y' + 2.56y = 0 \longrightarrow ①$$

$a = -3.2 \qquad b = 2.56$

The solution of eq ① is;

$$y = e^{\lambda x} \longrightarrow (ii)$$

To find $\lambda = ?$

$$\lambda = \frac{-a \pm \sqrt{a^2 - 4b}}{2}$$

$$\lambda = \frac{-(-3.2) \pm \sqrt{(-3.2)^2 - 4(2.56)}}{2}$$

$$= \frac{3.2 \pm \sqrt{10.24 - 10.24}}{2}$$

$$\lambda = \frac{3.2}{2} \Rightarrow \boxed{\lambda = 1.6}$$

So one of solution is $y_1 = e^{1.6x}$

to find other solution;

$$y_2 = u\,y_1 \longrightarrow ②$$

$$u = \int U \, dx \longrightarrow ③.$$

To find $U = \frac{1}{y_1^2} e^{\int -p(x)dx}$

$$U = \frac{1}{(e^{1.6x})^2} e^{-\int -3.2\,dx}$$

$P(x) = -3.2$

$$= \frac{1}{\cancel{e^{3.2x}}} \cancel{e^{+3.2x}}$$

$$U = 1$$

$$u = \int U\,dx$$

$$u = x$$

Put in eq ②.

$$y_2 = x e^{1.6x}$$

So general solution is;

$$\boxed{y = c_1 (e^{1.6x}) + x e^{1.6x} c_2}$$ Ans

Q7:- $y'' + 1.25y' = 0$.

Sol:- $y'' + 1.25y' = 0 \longrightarrow ①$

Here $a = 1.25$, $b = 0$.

The solution of eq ① is:

$$y = e^{\lambda x} \longrightarrow ②$$

To find $\lambda = ?$ use the following formula

$$\lambda = \frac{-a \pm \sqrt{a^2 - 4b}}{2}$$

$$\lambda = \frac{-1.25 \pm \sqrt{(1.25)^2 - 0}}{2}$$

$$\lambda = \frac{-1.25 \pm 1.25}{2}$$

$$\lambda_1 = \frac{-1.25 + 1.25}{2}, \quad \lambda_1 = \frac{-1.25 - 1.25}{2}$$

$$\lambda_1 = 0, \quad \lambda_2 = -1.25$$

Eq ②.

$$y_1 = e^{+\lambda_1 x}, \quad y_2 = e^{+\lambda_2 x}$$

$$y_1 = e^{0x}$$

$$y_1 = 1 \qquad y_2 = e^{-1.25x}$$

The General eqn is

$$y = c_1 y_1 + c_2 y_2$$

$$y = 1(c_1) + c_2 (e^{-1.25x})$$

$$\boxed{y = c_1 + e^{-1.25x}}$$ Ans

Q9:- $y'' + 1.75y' - 0.5y = 0$.

Sol:- $y'' + 1.75y' - 0.5y = 0 \longrightarrow (1)$

$a = 1.75 \qquad b = -0.5$.

The root of the given ODE is;

$y = e^{+\lambda x} \longrightarrow (1)$.

to find $\lambda = ?$

$\lambda = \dfrac{-a \pm \sqrt{a^2 - 4b}}{2}$

$\lambda = \dfrac{-1.75 \pm \sqrt{(1.75)^2 - 4(-0.5)}}{2}$

$\lambda = \dfrac{-1.75 \pm \sqrt{3.0625 + 2}}{2} \Rightarrow \lambda = \dfrac{-1.75 \pm \sqrt{5.0625}}{2}$

$\lambda = \dfrac{-1.75 \pm 2.25}{2}$

$\lambda_1 = \dfrac{-1.75 + 2.25}{2}, \qquad \lambda_2 = \dfrac{-1.75 - 2.25}{2}$

$\lambda_1 = 0.25. \qquad \lambda_2 = -2$

eq (1) $\Rightarrow$ $y_1 = e^{0.25x}, \quad y_2 = e^{-2x}$.

So the general equation is;

$y = c_1 y_1 + c_2 y_2$

$\boxed{y = c_1 (e^{0.25x}) + c_2 e^{-2x}}$ Ans

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Q10:- $100y'' + 240y' + (196\pi^2 + 144)y = 0$.

Sol.- Divide by 100.

$$y'' + 2.4y' + \left(\frac{196\pi^2 + 144}{100}\right)y = 0 \rightarrow (1)$$

$$y'' + 2.4y' + (1.96\pi^2 + 1.44)y = 0.$$

Here $a = 2.4$ $b = (1.96\pi + 1.44)$.

The root of eq (1) is;

$$y = e^{\lambda x} \rightarrow (1).$$

To find $\lambda = ?$ use formula.

$$\lambda = \frac{-a \pm \sqrt{a^2 - 4b}}{2}$$

$$= \frac{-2.4 \pm \sqrt{(2.4)^2 - 4(1.96\pi + 1.44)}}{2}.$$

$$\lambda = \frac{-2.4 \pm \sqrt{5.76 - 7.84 - 5.76}}{2}$$

$$\lambda = \frac{-2.4 + 2.8i}{2} \Rightarrow \lambda_1 = \frac{-2.4 + 2.8i}{2}$$

or $\lambda_2 = \frac{-2.4 - 2.8i}{2}$

$\boxed{\lambda_1 = -1.2 + 2.4i}$ or $\boxed{\lambda_2 = -1.2 - 2.4i}$

$y_1 = e^{(-1.2+2.4i)x}$ or $y_2 = e^{(-1.2-2.4i)x}$.

So general solution will be

$$y = c_1\left(e^{(-1.2+2.4i)x}\right) + c_2\left(e^{(-1.2-2.4i)x}\right)$$

or $y = c_1 e^{-1.2x} \cos(2.4i)x + c_2 e^{-1.2x} e^{-1.2x} \sin(2.4i)x$

Ans

Q11:- $4y'' - 4y' - 3y = 0.$

Sol:- Divide by 4.

$$y'' - y' - \frac{3}{4}y = 0 \longrightarrow (1)$$

$$a = -1 \quad , \quad b = -3/4.$$

The root of given eqn is;

$$y = e^{\lambda x} \longrightarrow (2)$$

to find $\lambda = ?$

$$\lambda = \frac{-a \pm \sqrt{a^2 - 4b}}{2}.$$

$$\lambda = \frac{+1 \pm \sqrt{1 - 4(-3/4)}}{2} \Rightarrow \frac{1 \pm \sqrt{1+3}}{2}.$$

$$= \frac{1 \pm 2}{2} \quad , \Rightarrow \lambda_1 = \frac{1+2}{2} \quad , \text{ or } \quad \lambda_2 = \frac{1-2}{2}.$$

$$\boxed{\lambda_1 = 1.5} \quad , \quad \boxed{\lambda_2 = -0.5}$$

eq (2) $\Rightarrow$

$$y_1 = e^{1.5x} \quad \text{or} \quad y = e^{1.5x}.$$

The General eqn is;

$$y = c_1 y_1 + c_2 y_2.$$

$$= c_1(e^{1.5x}) + c_2(e^{-0.5x}) \quad \underline{\text{Ans}}$$

Q13:- $9y'' - 30y' + 25y = 0.$

Sol:- Divide by 9

$$y'' - \frac{10}{3}y' + \frac{25}{9}y = 0.$$

$$a = -\frac{10}{3} \quad , \quad b = \frac{25}{9}$$

The root of given system is;

$y = e^{\lambda x} \rightarrow (1).$

To find $\lambda = ?$

$\lambda = \frac{a \pm \sqrt{a^2 - 4b}}{2}$

$\lambda = \frac{10/3 \pm \sqrt{(10/3)^2 - 4(25/9)}}{2}$

$= \frac{10/3 \pm \sqrt{100/9 - 100/9}}{2}$

$\lambda = \frac{10}{6}\,\frac{5}{3}$ $\boxed{\lambda = 1.67}$

$y_1 = e^{1.67x}$

To find other solution;

$u = ?$ i.e $y_2 = ?$

$y_2 = u y_1 \longrightarrow (2).$

$u = \int U dx \longrightarrow (3)$

$U = \frac{1}{y_1^2} e^{\int -P(x) dx}$

$U = \frac{1}{(e^{1.67x})^2} e^{\int +10/3\, dx}$

$= \frac{1}{e^{10/3 x}} e^{10/3 x}$ $U = 1$

By (3) $u = \int U dx = \Rightarrow$ $u = \int dx$

$u = x.$

So eq ② ⇒ $y_2 = u y_1$

$y_2 = x e^{1.67x}$

So the general solution is;

$y = c_1 (e^{1.67x}) + c_2 x e^{1.67x}$ Ans

Q14:-

$y'' + 2k^2 y' + k^4 y = 0.$

Sol:- Given that;

$y'' + 2k^2 y' + k^4 y = 0. \rightarrow ①$

$a = 2k^2, \quad b = k^4$

Since one of the solution of eq ① is;

$y = e^{\lambda x} \rightarrow ②$

To find $\lambda = ?$

$\lambda = \frac{-a \pm \sqrt{a^2 - 4b}}{2}$

$= \frac{-2k^2 \pm \sqrt{(2k^2)^2 - 4(k^4)}}{2} \Rightarrow \frac{-2k^2 \pm \sqrt{4k^4 - 4k^4}}{2}$

$\lambda = \frac{-2k^2}{2} \Rightarrow \boxed{\lambda = -k^2}$

Since one solution is; $y_1 = e^{-k^2 x}$ so other solution will be; $y_2 = x e^{-k^2 x}$

So general solution will be;

$y = c_1 y_1 + c_2 y_2$

$y = c_1 (e^{-k^2 x}) + c_2 (x e^{-k^2 x})$ Ans

Q15:- $y'' + 0.54y' + (0.0729 + \pi)y = 0$

Sol:- $y'' + 0.54y' + (0.0729 + \pi)y = 0. \longrightarrow (1)$

Here $a = 0.54$, $b = (0.0729 + \pi)$

The solution of eq (1) is;

$y = e^{\lambda x} \longrightarrow (2)$

where $\lambda = \dfrac{-a \pm \sqrt{a^2 - 4b}}{2}$

$\lambda = \dfrac{-0.54 \pm \sqrt{(0.54)^2 - 4(0.0729 + \pi)}}{2}$

$= \dfrac{-0.54 \pm \sqrt{0.2916 - 0.2916 - 4\pi}}{2}$

$= \dfrac{-0.54 \pm \sqrt{-4\pi}}{2} \Rightarrow \dfrac{-0.54 \pm 2i\sqrt{\pi}}{2}$

$\lambda_1 = -0.27 + i\sqrt{\pi}$ or $\lambda_2 = -0.27 - i\sqrt{\pi}$

So, eq (2) $\Rightarrow$

$y_1 = e^{(-0.27 + i\sqrt{\pi})x}$ or $y_2 = e^{(-0.27 + i\sqrt{\pi})x}$

So the general solution will be;

$y = c_1 y_1 + c_2 y_2$

$= c_1 (e^{-0.27 + i\sqrt{\pi}x}) + c_2 (e^{(-0.27 - i\sqrt{\pi})x})$

$= c_1 e^{-0.27} \cos i\sqrt{\pi}x + c_2 (e^{-0.27} \sin(-i\sqrt{\pi})x)$ Ans.

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Find an ODE:-

Q 17:- $e^{-\sqrt{2}x}$, $xe^{-\sqrt{2}x}$.

Sol:- As the general ODE is,

$$y'' + ay' + by = 0 \rightarrow (1)$$

Here $y = e^{-\sqrt{2}x}$

$y' = e^{-\sqrt{2}x}(-\sqrt{2})$

$y'' = 2e^{-\sqrt{2}x}$.

Put values in eq (1)

$$2e^{-\sqrt{2}x} + a(-\sqrt{2}e^{-\sqrt{2}x}) + b(e^{-\sqrt{2}x}) = 0$$

$$2e^{-\sqrt{2}x} - \sqrt{2}ae^{-\sqrt{2}x} + be^{-\sqrt{2}x} = 0 \rightarrow (2)$$

Multiply $x$ with eq (2) & subtract from (3).

$$2xe^{-\sqrt{2}x} - \sqrt{2}xae^{-\sqrt{2}x} + bxe^{-\sqrt{2}x} = 0 \rightarrow (A)$$

$y = xe^{-\sqrt{2}x}$.

$y' = x(-\sqrt{2})e^{-\sqrt{2}x} + e^{-\sqrt{2}x}$

$y'' = -\sqrt{2}[e^{-\sqrt{2}x} + x\sqrt{2}e^{-\sqrt{2}x}] + e^{-\sqrt{2}x}$

$y'' = -\sqrt{2}e^{-\sqrt{2}x} + 2xe^{-\sqrt{2}x} + e^{-\sqrt{2}x}$

$y'' = -\sqrt{2}e^{-\sqrt{2}x} + \cancel{3}e^{-\sqrt{2}x} + 2xe^{-\sqrt{2}x}$

Put values in eq (1).

$$2xe^{-\sqrt{2}x} - \sqrt{2}e^{-\sqrt{2}x} + \cancel{3}e^{-\sqrt{2}x} + a[(-\sqrt{2})xe^{-\sqrt{2}x} + e^{-\sqrt{2}x}] + b(xe^{-\sqrt{2}x})$$

$-\cancel{2\sqrt{2}}$

$\cancel{+6e^{-\sqrt{2}x} + a[-\sqrt{2}e^{-\sqrt{2}x}x + e^{-\sqrt{2}x}]}$

$\cancel{+bxe^{-\sqrt{2}x}}$. $\rightarrow (3)$

$$2xe^{-\sqrt{2}x} - \sqrt{2}e^{-\sqrt{2}x} + e^{-\sqrt{2}x} + a[-\sqrt{2}xe^{-\sqrt{2}x} + e^{-\sqrt{2}x}] + bxe^{-\sqrt{2}x}) \rightarrow (1)$$

$$\cancel{2xe^{-\sqrt{2}x}} - \sqrt{2}xae^{-\sqrt{2}x} + \cancel{bxe^{-\sqrt{2}x}} = 0.$$

$$\overset{-}{+}\ \cancel{2xe^{-\sqrt{2}x}} + a[-\sqrt{2}xe^{-\sqrt{2}x} + e^{-\sqrt{2}x}] + \cancel{bxe^{-\sqrt{2}x}} + e^{-\sqrt{2}x} - \sqrt{2}e^{-\sqrt{2}x} = 0.$$

$$-\cancel{\sqrt{2}x}ae^{-\sqrt{2}x} + \cancel{\sqrt{2}x}e^{-\sqrt{2}x}a - e^{-\sqrt{2}x}a - e^{-\sqrt{2}x} - \sqrt{2}e^{-\sqrt{2}x} = 0$$

$$e^{-\sqrt{2}x}a - e^{-\sqrt{2}x} - \sqrt{2}e^{-\sqrt{2}x} = 0$$

$$e^{-\sqrt{2}x}(a-1-\sqrt{2})=0$$

$$a-1-\sqrt{2}=0.$$

$$\boxed{a=1+\sqrt{2}}$$

Put in eq (4).

$$2xe^{-\sqrt{2}x}-\sqrt{2}xe^{-\sqrt{2}x}(1+\sqrt{2})]+bxe^{-\sqrt{2}x}=0$$

$$\cancel{2xe^{-\sqrt{2}x}}-\sqrt{2}xe^{-\sqrt{2}x}-\cancel{2xe^{-\sqrt{2}x}}+bxe^{-\sqrt{2}x}=0$$

$$-\sqrt{2}xe^{-\sqrt{2}x}+bxe^{-\sqrt{2}x}=0.$$

$$e^{-\sqrt{2}x}x(-\sqrt{2}+b)=0.$$

$$b-\sqrt{2}=0$$

$$\boxed{b=\sqrt{2}}$$

Put values in eq (1) to get ODE.

$$\boxed{y''+(1+\sqrt{2})y'+\sqrt{2}y=0}$$ Ans.

Q18:- $\cos 2\pi x$, $\sin 2\pi x$.

Sol:-

The general solution will be;

$$y'' + ay' + by = 0 \longrightarrow (1)$$

Now to find $a$ & $b = ?$

As $y_1 = \cos 2\pi x$ & $y_2 = \sin 2\pi x$

$y' = -\sin 2\pi x\,(2\pi)$ $\qquad$ $y' = \cos 2\pi x\,(2\pi)$

$y'' = -4\pi^2 \cos 2\pi x$ $\qquad$ $y'' = -\sin 2\pi x\,(4\pi^2)$

By putting values in eq (1)

$$-4\pi^2 \cos 2\pi x + a(-\sin 2\pi x(2\pi)) + b(\cos 2\pi x) = 0.$$

$$-4\pi^2 \cos 2\pi x - 2\pi a \sin 2\pi x + b\cos 2\pi x = 0 \longrightarrow (2).$$

Now for other solution:

$$-\sin(2\pi x)4\pi^2 + a(2\pi \cos 2\pi x) + b(\sin 2\pi x) = 0.$$

$$-4\pi \sin(2\pi x) + 2\pi a \cos 2\pi x + b\sin 2\pi x = 0. \hookrightarrow (3)$$

Multiply $\cos 2\pi x$ with eq (2) & $\sin 2\pi x$ with (3) & then add

$$-4\pi^2 \cos^2 2\pi x - \cancel{2\pi a \sin 2\pi x \cos 2\pi x} + b\cos^2 2\pi x = 0.$$

$$-4\pi \sin^2(2\pi x) + \cancel{2\pi a \sin 2\pi x \cos 2\pi x} + b\sin^2 2\pi x = 0$$

$$-4\pi^2\underbrace{(\cos^2 2\pi x + \sin^2(2\pi x))}_{1} + b\underbrace{(\cos^2 2\pi x + \sin^2 2\pi x)}_{1} = 0$$

$$-4\pi^2 + b = 0$$

$$\boxed{b = 4\pi^2}$$

Put in eq (2)

$$-4\pi^2 \cancel{\cos 2\pi x} - 2\pi a \sin 2\pi x + 4\pi^2 \cancel{\cos 2\pi x} = 0.$$

$\boxed{a = 0}$ & $2\pi a \sin 2\pi x = 0$

$$a \sin 2\pi x = 0$$

$$\boxed{a = 0}$$

Put in eq ① General ODE.

$\boxed{y'' + 4\pi^2 y = 0}$ Ans

Q19:- $e^{(-1+i\sqrt{2})x}$ , $e^{(-1-i\sqrt{2})x}$.

Sol:- Let the general ODE is;

$y'' + ay' + by = 0 \longrightarrow ①$

We have to find a & b.

As $y = e^{(-1+i\sqrt{2})x}$. also $y = e^{(-1-i\sqrt{2})x}$.

$y' = (-1+i\sqrt{2})\, e^{(-1+i\sqrt{2})x}$ ; $y' = (-1-i\sqrt{2})\, e^{(-1-i\sqrt{2})x}$

$y'' = (-1+i\sqrt{2})(-1+i\sqrt{2})\, e^{(-1+i\sqrt{2})x}$

$y'' = [(-1) + (i\sqrt{2})]^2\, e^{(-1+i\sqrt{2})x}$.

$y'' = (-1-i\sqrt{2})^2\, e^{(-1-i\sqrt{2})x}$.

$= (-1)^2 + (i\sqrt{2})^2 - 2(-1)i\sqrt{2}\, e^{(-1-i\sqrt{2})x}$

$= 1 - 2 + 2i\sqrt{2}\, e^{(-1-i\sqrt{2})x}$

$y'' = (-1)^2 + (i\sqrt{2})^2 + 2(-1)(i\sqrt{2})\, e^{(-1+i\sqrt{2})x}$.

$y'' = -1 + 2i\sqrt{2}\, e^{(-1-i\sqrt{2})x}$.

$y'' = 1 - 2 - 2i\sqrt{2}\, e^{-1+i\sqrt{2}x}$.

$y'' = -1 - 2\sqrt{2}\, i\, e^{-1+i\sqrt{2}x}$

By putting values in eq ①

eq ① ⇒

$y'' + ay' + by = 0$

$$-1 - 2\sqrt{2}i \, e^{(-1+i\sqrt{2})x} + a(-1+i\sqrt{2}) e^{(-1+i\sqrt{2})x} + b e^{(-1+i\sqrt{2})x} = 0 \longrightarrow ②$$

Similarly for other.

$$-1 + 2\sqrt{2}i \, e^{(-1-i\sqrt{2})x} + a(-1-i\sqrt{2}) e^{(-1-i\sqrt{2})x} + b e^{(-1-i\sqrt{2})x} = 0.$$

③

Multiply $e^{(-1-i\sqrt{2})x}$ with ② & $(-1-i\sqrt{2})$ & $e^{(-1+i\sqrt{2})x}$ with ③ & $(-1+i\sqrt{2})$ & then subtract

$$-1 - 2\sqrt{2}i \, e^{(-1+i\sqrt{2})x} \cdot e^{(-1-i\sqrt{2})x} + a(-1+i\sqrt{2}$$

Solve the Initial Value Problems.

Q21:- $y'' + 9y = 0, \quad y(0) = 0.2, \quad y'(0) = -1.5.$

Sol:- $y'' + 9y = 0 \rightarrow (1)$.

let $u = y'$

$u' = y''$ But $u' = \frac{du}{dx} = \frac{du}{dy} \cdot \frac{dy}{dx}$.

& (1) $\Rightarrow$ $y'' = u' = \frac{du}{dy} \cdot u$

$\frac{du}{dy} \cdot u + 9y = 0$

$u\,du = -9y\,dy$.

$\int u\,du = -9\int y\,dy$.

$\frac{u^2}{\cancel{2}} du = -\frac{9y^2}{\cancel{2}} + \frac{c}{\cancel{2}}$

$u^2 = -9y^2 + c$. Put $u = y'$

$y'^2 = -9y^2 + c$.

$y'^2 = -9y^2 + c. \Rightarrow$ By Apply Initial cond

& $\boxed{0.2 = c}$

$y'^2 = -9y^2 + 0.2$

$y' \cdot y' = -9y \cdot y + 0.2$.

$\int \frac{y'}{y} \cdot \frac{y'}{y} = \int -9 + \int \frac{0.2}{y^2}$

$\ln y + \ln y$.

or $\ln(y+y). = -9y + \frac{(0.2)\ (-1)}{y} + c$

$\ln(2y) = -9y - \frac{0.2}{y} + c$.

Solve Initial value Problems.

Q21:- $y'' + 9y = 0$ , $y(0) = 0.2$ , $y'(0) = -1.5$.

Sol:-

Let $u = y'$

than $y'' = u'$.

But $u' = \frac{du}{dx}$.

But putting values.

$\frac{du}{dx} = \frac{du}{dy} \cdot \frac{dy}{dx}$

$u\frac{du}{dy} + 9y = 0$

$y'' = \frac{du}{dx} = u \cdot \frac{du}{dy}$

$u\,du = -9y\,dy$.

By Integration;

$\int u\,du = -9\int y\,dy \Rightarrow \frac{u^2}{\cancel{2}} = -\frac{9}{\cancel{2}}y^2 + \frac{c}{2}$

$u^2 = -9y^2 + c$.

$u = y'$

$y'^2 = -9y^2 + c$.

$y' = (-9y^2 + c)^{1/2}$.

$\frac{dy}{dx} = (-9y^2 + c)^{1/2}$,

$\frac{dy}{dx} = (-9y^2 + c)^{1/2}$

By putting Initial Condition;

$\int \frac{1}{-9y^2 + c}\,dy = \int dx$.

$\int \frac{1}{(-(3y)^2 + (c)^2)^{1/2}}\,dy = x + c$

$-\frac{1}{3y}\tan^{-1}\frac{(3y)}{3} = x + c$

$\tan^{-1}$

By putting values in eq (A).

$$\boxed{y'' + 1.7y' - 11.18y = 0.}$$ Ans.

Q16

Find an ODE. eq

$y'' + ay' + by = 0. \rightarrow$ (A)

(16) $e^{2.6x}$, $e^{-4.3x}$

$y = e^{2.6x}$ $y = e^{-4.3x}$

$y' = 2.6\,e^{2.6x}$ $y' = -4.3\,e^{-4.3x}$

$y'' = (2.6)^2 e^{2.6x}$ $y'' = (-4.3)^2 e^{-4.3x}$

Put in eq (A).

$(2.6)^2 e^{2.6x} + a(2.6e^{2.6x}) + b(e^{2.6x}) = 0.$

$6.76\,e^{2.6x} + 2.6e^{2.6x}a + \cancel{2.6} \; e^{2.6x}\, b = 0.$

$6.76 + 2.6a + b = 0. \longrightarrow$ (1)

Now put other values.

$18.49\,e^{-4.3x} + a(-4.3e^{-4.3x}) + b(e^{-4.3x}) = 0$

$18.49 - 4.3a + b = 0 \longrightarrow$ (2)

Subtract (2) from (1).

$6.76 + 2.6a + b = 0$

$\underline{-18.49 \mp 4.3a \; -b = 0}$

$6.9(a) = 11.73 \rightarrow \boxed{a = 1.7}$

Put in eq (2).

$18.49 - 4.3(1.7) + b = 0.$

$\boxed{b = -11.18}$

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$Q_{12}$:- $y'' + 9y' + 20y = 0.$

Sol:- $y'' + 9y' + 20y = 0 \rightarrow (1)$

As;

$y'' + ay' + by = 0 \rightarrow (2)$

By comparing co-efficients of $y'$ & $y$.

$a = 9, \quad b = 20.$

Since solution is;

$y = e^{\lambda x} \longrightarrow (3)$

To find $\lambda = ?$

As $\lambda = \dfrac{-a \pm \sqrt{a^2 - 4b}}{2}$

$\lambda = \dfrac{-9 \pm \sqrt{9 - 4(20)}}{2} \Rightarrow \dfrac{-9 \pm \sqrt{9 - 80}}{2}$

$\lambda = \dfrac{-9 \pm \sqrt{-71}}{2}$

$\lambda = \dfrac{-9 \pm 8.34i}{2}$

$= -4.5 \pm 4.17i$

or $\boxed{\lambda_1 = -4.5 + 4.17i}$ or $\boxed{\lambda_2 = -4.5 - 4.17i}$

eq (3) $\Rightarrow$

$y_1 = e^{-4.5 + 4.17i}$

$\boxed{y_1 = e^{-4.5x} \cos 4.17x}$ $\boxed{y_2 = e^{-4.5x} \sin(4.17x).}$

Ans

**Q20:-** $e^{-3.1x}\cos 2.1x \,,\ e^{-3.1x}\sin 2.1x$

Sol:- $y = e^{-3.1x}\cos 2.1x$

$y' = e^{-3.1x}(2.1)(-\sin 2.1x) + \cos 2.1x(-3.1)e^{-3.1x}$

$y' = -2.1\,e^{-3.1x}(\sin 2.1x) - 3.1\,e^{-3.1x}(\cos 2.1x)$

$y'' = (-2.1)(-3.1)\,e^{-3.1x}(\sin 2.1)x - (+2.1)^2 e^{-3.1}\cos 2.1x$

$- \left[-(3.1)^2 e^{-3.1x}\cos 2.1x - (3.1)(2.1)e^{-3.1}\sin 2.1x\right]$

$= 6.51\,e^{-3.1x}\sin 2.1x - 4.41\,e^{-3.1}\cos 2.1x + 9.61\,e^{-3.1x}\cos 2.1x + 6.51\,e^{-3.1}\sin 2.1x$

$y'' = 13.02\,e^{-3.1}\sin 2.1x - 14.02\,e^{-3.1x}\cos 2.1x$

$y = e^{-3.1x}\sin 2.1x$

$y' = e^{-3.1x}\cos 2.1x\,(2.1) + (-3.1)\,e^{-3.1x}\sin 2.1x$

$y' = 2.1\,e^{-3.1x}\cos 2.1x - 3.1\,e^{-3.1x}\sin 2.1x$

$y'' = -(2.1)^2 e^{-3.1x}\sin 2.1x + (2.1)(-3.1)e^{-3.1x}\cos 2.1x$

$- \left[(3.1)^2 e^{-3.1x}\sin 2.1x - 3.1(2.1)e^{-3.1x}\cos 2.1x\right]$

$y'' = -4.41\,e^{-3.1x}\sin 2.1x - \cancel{6.51\,e^{-3.1}\cos 2.1x} + 9.61\,e^{-3x}\sin 2.1x + \cancel{6.51\,e^{-3.1x}\cos 2.1x}$

$y'' = 5.2\,e^{-3.1x}\sin 2.1x$

We know that;

$$y'' + ay' + by = 0$$

$$y'' + ay' + by = 0 \qquad \text{By putting values}$$

$$13.02e^{-3.1x}\sin 2.1x - 14.02e^{-3.1x}\cos 2.1 + a\left(-2.1e^{-3.1x}\sin 2.1x - 3.1e^{-3.1x}\cos 2.1x\right) + b\left(e^{-3.1x}\cos 2.1x\right) = 0.$$

↳①

Similarly;

$$5.2e^{-3.1x}\sin 2.1x + a\left(-3.1e^{-3.1x}\sin 2.1x + 2.1e^{-3.1x}\cos 2.1x\right) + b\left(e^{-3.1x}\sin 2.1x\right) = 0$$

Q22:- Solve the IVP. Check your answer satisfies ODE as well as Initial Condition.

⇒ the ODE in Pb. 4 $y(1/2) = 1$, $y'(1/2) = -2$.

Sol:- As Pb. 4 is;

$$y'' + 4y' + (\pi^2+4) y = 0.$$

As we get.

$\lambda_1 = -2 + \pi i$ & $\lambda_2 = -2 - \pi i$.

$y_1 = e^{-2+\pi i}$ $y_2 = e^{-2-\pi i}$

$y_1 = e^{-2x} \cos \pi i$ $y_2 = e^{-2x} \sin(-\pi i)$.

$y_1 = e^{-2x} \cos \pi i$ $y_2 = -e^{-2x} \sin(\pi i)$.

As we can write;

$y_1 c_1 + y_2 c_2 = y.$

$$y = c_1 (e^{-2x} \cos \pi i) + c_2 (-e^{-2x} \sin(\pi i)) \rightarrow (1)$$

By applying Initial Condition.

$$1 = c_1 (e^{-2(1/2)} \cos \pi i) + c_2 (-e^{-2(1/2)} \sin \pi i)$$

$$1 = c_1 e^{-1} \cos \pi i + c_2 (-e^{-1} \sin \pi i) \rightarrow (11).$$

Eq (1) ⇒ $y = c_1 (e^{-2x} \cos \pi i) + c_2 (-e^{-2x} \sin \pi i)$

$$y' = c_1 (-2 e^{-2x} \cos \pi i) + c_2 (+2 e^{-2x} \sin \pi i)$$

$$y' = -2 c_1 (e^{-2x} \cos \pi i) + 2 c_2 (e^{-2x} \sin \pi i) \rightarrow (2).$$

Apply condition,

$$-2 = -2c_1(e^{-2(1/2)}\cos\pi i) + 2c_2(e^{-2(1/2)}\sin\pi i)$$

$$-2 = -2c_1(e^{-1}\cos\pi i) + 2c_2(e^{-1}\sin\pi i) \longrightarrow (3).$$

Multiply 2 with eq (2) & add with (3).

$$2 = 2c_1 e^{-1}\cos\pi i + 2c_2 e^{-1}\sin\pi i$$

$$-2 = -2c_1 e^{-1}\cos\pi i + 2c_2 e^{-1}\sin\pi i$$

---

$$0 = 4c_2 e^{-1}\sin\pi i$$

$$\boxed{c_2 = 0}$$

Put in eq (2)

$$1 = c_1 e^{-1}\cos\pi i.$$

$$\boxed{c_1 = \frac{e}{\cos\pi i}}$$

Put values in eq (1)

$$y = c_1(e^{-2x}\cos\pi i) + c_2(e^{-2x}\sin\pi i)$$

$$y = \frac{e}{\cancel{\cos\pi i}} \cdot e^{-2x}\cancel{\cos\pi i} + 0.$$

$$= e \cdot e^{-2x}.$$

$$\boxed{y = e^{1-2x}}$$ Ans

Q2:- The ODE in Pb # 4 , $y(\frac{1}{2}) = 1$ , $y'(\frac{1}{2}) = -2$.

Sol:- As ODE in pb is;

$$y'' + 4y' + (\pi^2 + 4)y = 0.$$

As one of solutions are

$$y_1 = e^{-2x}\cos\pi x \quad \& \quad y_2 = e^{-2x}\sin\pi x.$$

So;

$$y = c_1(e^{-2x}\cos\pi x) + c_2 e^{-2x}\sin\pi x \longrightarrow (1).$$

By apply Initial Condition;

$$1 = c_1 e^{-2(1/2)}\cancel{\cos\pi(\tfrac{1}{2})}^{0} + c_2 e^{-2(1/2)}\cancel{\sin\pi(1/2)}^{1}.$$

$$1 = c_2(1) \Rightarrow c_2 = 1.$$

Now differentiate eq (1).

$$y' = c_1[e^{-2x}(-2)\cos\pi x - 2e^{-2x}\sin\pi x(\pi)]$$

$$+ c_2[e^{-2x}\cos\pi x(\pi) - 2e^{-2x}\sin\pi x]$$

Apply condition.

$$-2 = c_1[e^{-2(1/2)}(-2)\cancel{\cos\pi(\tfrac{1}{2})}^{0} - e^{-2(1/2)}\cdot\sin\pi(\tfrac{1}{2})\pi]$$

$$+ c_2[e^{-2(1/2)}\cancel{\cos\pi(\tfrac{1}{2})}^{0}\pi - 2e^{-2(1/2)}\sin\pi(\tfrac{1}{2})]$$

$$-2 = \pi e^{-1}c_1 + c_2(-2e^{-1}) \Rightarrow -2 = \pi e^{-1}c_1 + c_2(-2e^{-1})$$

$$-2 = \frac{\pi}{e}C_1 - \frac{2C_2}{e} \Rightarrow -2e = \pi C_1 - 2C_2$$

Put $C_2 = 1$.

$$-2e = \pi C_1 - 2 \Rightarrow 2 - 2e = \pi C_1.$$

$$\boxed{C_1 = \frac{2-2e}{\pi}}$$

Use eq ①.

$$y = \frac{2-2e}{\pi}\left(e^{-2x}\cos\pi x\right) + 1\, e^{-2x}\sin\pi x.$$

$$\boxed{y = \frac{2-2e}{\pi}\left(e^{-2x}\cos\pi x\right) + e^{-2x}\sin\pi x}$$

Ans

Q23 :- $y'' - 3y' - 4y = 0$, $y(0) = 2$, $y'(0) = 1$.

Sol:- As $y'' - 3y' - 4y = 0$ ① is general ODE.

Here $a = -3$, $b = -4$.

To find solution;
let one of general solution is;

$$y = e^{\lambda x} \longrightarrow ②.$$

To find $\lambda = ?$

$$\lambda = \frac{-a \pm \sqrt{a^2 - 4b}}{2}$$ By putting values.

$$\lambda = \frac{+3 \pm \sqrt{(-3)^2 - 4(4)}}{2} \Rightarrow \frac{3 \pm \sqrt{25}}{2}.$$

$\lambda_1 = \frac{3+5}{2}$ or $\lambda_2 = \frac{3-5}{2}$

$\boxed{\lambda_1 = 4}$ or $\boxed{\lambda_2 = -1}$

So the Solutions will be;

$y_1 = e^{4x}$ or $y_2 = e^{-x}$.

So the general eqn of ODE in Term of y can be written as;

$y = c_1 y_1 + c_2 y_2.$

$y = c_1 e^{4x} + c_2 e^{-x} \longrightarrow (2)$

Applying Initial Condition. $y(0) = 2$

$2 = c_1 e^0 + c_2 e^0$

$2 = c_1 + c_2 \rightarrow (3).$

Differentiate eq (2).

$y' = c_1 e^{4x}(4) - c_2 e^{-x}.$

Applying Condition (initial) i.e $y'(0) = 1$

$1 = 4c_1 e^0 - c_2 e^0 \Rightarrow 1 = 4c_1 - c_2 \rightarrow (4).$

Add eq (3) & (4),

$$\begin{array}{r} 1 = 4c_1 - \cancel{c_2} \\ 2 = c_1 + \cancel{c_2} \\ \hline 5c_1 = 3 \end{array} \Rightarrow \boxed{c_1 = 3/5}$$

Put in eq (3) $\Rightarrow$ $2 = c_1 + c_2 \Rightarrow c_2 = 2 - 3/5$

$c_2 = \frac{10-3}{5} = \boxed{7/5 = c_2}$

Put values in eq (2)

$\boxed{y = 3/5 e^{4x} + 7/5 e^{-x}}$ Ans

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Q26:- $y'' - k^2 y = 0 \quad (k \neq 0)$, $y(0) = 1$, $y'(0) = 1$.

Sol:- Given ODE is;

$$y'' - k^2 y = 0 \longrightarrow (1)$$

Here $a = 0$, $b = -k^2$.

The general solution is;

$$y = e^{\lambda x}.$$

To find $\lambda = ?$

$$\lambda = \frac{-a \pm \sqrt{a^2 - 4b}}{2}.$$

$$\lambda = \frac{0 \pm \sqrt{-4(-k^2)}}{2} \Rightarrow \lambda = \frac{2k}{2} \Rightarrow \boxed{\lambda = k}$$

$y_1 = e^{kx}$ & $y_2 = x e^{kx}$.

The general solution of given ODE is;

$$y = c_1 e^{kx} + c_2 x e^{kx} \longrightarrow (2)$$

Apply Initial Condition.

$$1 = c_1 e^0 + c_2(0) e^0.$$

$$\boxed{c_1 = 1}$$

Now differentiate eq (2).

$$y' = c_1 k e^{kx} + c_2 [x k e^{kx} + e^{kx}].$$

Applying Initial Conditions

$$1 = c_1 k e^0 + c_2 [0 \cancel{(k)} e^0 + e^0]$$

$$1 = c_1 k + c_2 \Rightarrow \text{As } c_1 = 1 \Rightarrow \boxed{c_2 = 1 - k}$$

in eq (1) $\Rightarrow$ $\boxed{y = (1) e^{kx} + (1-k) e^{kx} (x)}$ Ans.

Q24:- $y'' - 2y' - 3y = 0$ , $y(-1) = e$ , $y'(-1) = -e/4$.

Sol:- As given that;

$$y'' - 2y' - 3y = 0.$$

As;

$$y'' + ay' + by = 0$$

By comparing we get.

$$a = -2, \quad b = -3,$$

Solution of given ODE is

$$y = e^{+\lambda x} \rightarrow ①.$$

to find $\lambda = ?$

$$\lambda = \frac{-a \pm \sqrt{a^2 - 4b}}{2}$$

$$\lambda = \frac{+2 \pm \sqrt{4 - 4(-3)}}{2} \Rightarrow \frac{2 \pm \sqrt{16}}{2} = \frac{2 \pm 4}{2}$$

$$\lambda_1 = \frac{2+4}{2} \quad ; \quad \lambda_2 = \frac{2-4}{2}.$$

$$\boxed{\lambda_1 = 3} \quad \quad \lambda_2 = \frac{-2}{2}$$

$$\boxed{\lambda_2 = -1}$$

So; Solutions are.

$$y_1 = e^{3x} \quad \& \quad y_2 = e^{-x}$$

We can write it as;

$$y = c_1 y_1 + c_2 y_2.$$

$$y = c_1(e^{3x}) + c_2(e^{x}) \rightarrow ②.$$

Apply condition;

$$e = c_1(e^{3(-1)}) + c_2(e^{-(-1)})$$

$$e = c_1 e^{-3} + c_2 e \longrightarrow (3)$$

Now eq (2) =)

$$y' = 3c_1 e^{3x} - e^{-x} c_2$$

Apply condition.

$$-e/4 = 3c_1 e^{3(-1)} - e^{1} c_2$$

$$-e/4 = 3c_1 e^{-3} - e c_2 \longrightarrow (4)$$

Add eq (3) & (4).

$$e = c_1 e^{-3} + c_2 \cancel{e}$$

$$-e/4 = 3c_1 e^{-3} - e\cancel{c_2}$$

$$e - e/4 = 3c_1 e^{-3} + c_1 e^{-3}$$

$$\frac{4e - e}{4} = 4c_1 e^{-3} \Rightarrow \frac{3e}{4} = 4c_1 e^{-3}$$

$$c_1 = \frac{3}{16}\frac{e}{e^{-3}} \Rightarrow \frac{3}{16} e^{4}$$

eq (3) =)

$$\cancel{e} = \frac{3}{16} e^{4^3}(e^{-3x}) + c_2 \cancel{e}$$

$$1 = \frac{3}{16} e^{3} \cdot e^{-3} + c_2 \Rightarrow c_2 = 1 - 3/16$$

$$c_2 = \frac{16-3}{16} = \boxed{\frac{13}{16} = c_2}$$

Us. eq (2):-

$$y = c_1 e^{3x} + c_2 e^{-x}.$$

$$y = \frac{3}{16} e^{4} \cdot e^{3x} + \frac{13}{16} e^{-x}.$$

$$y = \frac{1}{16}\left(3e^{4+3x} + 13e^{-x}\right) \text{ Ans}$$

Q25:- $y'' - y = 0$ $y(0) = 2$, $y'(0) = -2$.

Sol:- $y'' - y = 0 \longrightarrow (1).$

Here $a = 0$, $b = -1$.

The general eqn for ODE is;

$$y = e^{\lambda x} \longrightarrow (2).$$

First find $\lambda = ?$

$$\lambda = \frac{-a \pm \sqrt{a^2 - 4b}}{2}$$ By putting values.

$$\lambda = \frac{0 \pm \sqrt{-4(-1)}}{2}$$

$$\lambda = \frac{2}{2} = 1 \qquad \lambda_1 = 1.$$

if $\lambda$ = one value than with second general solution just multiply $x$.

$\lambda'_1 = 1$ &

So

$y_1 = e^{x}$ & $y_2 = x e^{x}$.

The general solution will be;

$$y = c_1 y_1 + c_2 y_2.$$

$$y = c_1 e^{x} + c_2 x e^{x} \longrightarrow (3)$$

Applying Condition (initial) i.e $y(0) = 2$

$2 = c_1 e^0 + \cancel{0 c_2 e^0}$

$\boxed{c_1 = 2}$

Now diff eq (3)

$y' = c_1 e^x + c_2 [e^x + xe^x]$.

$= c_1 e^x + c_2 e^x + c_2 x e^x$. Applying Initial Condition

$-2 = c_1 e^0 + c_2 e^0 + c_2 (0) e^0$.

$-2 = c_1 + c_2 + 0$

Put $c_1 = 2$.

$-2 = 2 + c_2$.

$\boxed{c_2 = -4}$

use these values in eq (3) to get general solution

$\boxed{y = 2e^x - 4x e^x}$

Ans 2

Q27:- The ODE in pb 5.

$y(0) = 4.5$ , $y'(0) = +4.5\pi - 1 = 13.137$

Sol:- As from pb # 5 the general solution is;

$$y = c_1 e^{-\pi x} + c_2 x e^{-\pi x} \longrightarrow (1)$$

By applying Initial Condition.

$$4.5 = c_1 e^0 + 0 c_2 e^0$$

$$\boxed{c_1 = 4.5}$$

Diff eq (1)

$$y' = c_1(-\pi)e^{-\pi x} + c_2\left[-\pi x e^{-\pi x} + e^{-\pi x}\right]$$

Applying initial Condition.

$$13.137 = c_1(-\pi)e^0 + c_2(0 + e^0)$$

$$13.137 = -\pi c_1 + c_2$$

$\because c_1 = 4.5$

$$13.137 = -3.14(4.5) + c_2$$

$$c_2 = 13.137 + 14.13$$

$$\boxed{c_2 = 27.267}$$

So the general solution will be;

$$\boxed{y = 4.5 e^{-\pi x} + 27.267 x e^{-\pi x}}$$ Ans

Q 29:- $6y'' - y' - y = 0$ $\quad y(0) = 0, \quad y'(0) = 1.25.$

Sol.- Divide "6".

$$y'' - \frac{1}{6}y' - \frac{1}{6}y = 0 \longrightarrow (1).$$

From eq (1).

$$a = -1/6, \quad b = -1/6$$

The general solution is;

$$y = e^{\lambda x} \longrightarrow (2).$$

to find $\lambda = ?$

$$\lambda = \frac{-a \pm \sqrt{a^2 - 4b}}{2}$$

By putting values.

$$\lambda = \frac{1/6 \pm \sqrt{(1/6)^2 - 4(-1/6)}}{2}$$

$$= \frac{\frac{1}{6} \pm \sqrt{\frac{1}{36} + 2/3}}{2} \Rightarrow \frac{\frac{1}{6} \pm \sqrt{\frac{1+24}{36}}}{2}$$

$$\cancel{= \frac{\frac{1}{6} \pm \frac{1}{6}\sqrt{+23}}{2}} \Rightarrow \cancel{\frac{\frac{1}{6}(1+\sqrt{+23})}{2}}$$

$$\cancel{= \frac{1 \pm \sqrt{23}i}{12}}$$

$$\cancel{\lambda_1 = \frac{1+4.8i}{12}} \quad \text{or} \quad \lambda_2 = \frac{1-4.8i}{12}$$

$$y_1 = e^{(1/12 + \frac{4.8}{12}i)x} \quad \text{and} \quad y_2 = e^{(1/12 - \frac{4.8}{12}i)x}.$$

$$\lambda = \frac{\frac{1}{6} \pm \sqrt{\frac{25}{36}}}{2} = \frac{\frac{1}{6} \pm \frac{5}{6}}{2}$$

$$= \frac{\frac{1}{6} \pm \frac{5}{6}}{2} = \frac{\frac{1 \pm 5}{6}}{2}$$

$$\lambda = \cancel{\tfrac{1}{2}} \quad \frac{\frac{1}{6} \pm \frac{5}{6}}{2} \Rightarrow$$

$$\lambda_1 = \frac{\frac{1+5}{6}}{2}$$

$$\lambda_1 = \frac{6/6}{2}$$

$$\boxed{\lambda = 1/2}$$

$$\lambda_2 = \frac{\frac{1}{6} - 5/6}{2}$$

$$= \frac{1-5}{6}/2$$

$$= -4/6 / 2$$

$$= -2/3/2$$

$$\boxed{\lambda_2 = -1/3}$$

$$y_1 = e^{1/2 x} \quad \& \quad y_2 = e^{-1/3 x}$$

So general solution will be;

$$y = c_1 e^{1/2 x} + c_2 e^{-1/3 x} \rightarrow (3)$$

Applying Initial condition i.e $y(0) = 0$.

$$0 = c_1 e^0 + c_2^0 .$$

$$0 = c_1 + c_2 \rightarrow (4) .$$

Diff eq (3).

$$y' = c_1 \frac{1}{2} e^{1/2 x} - 1/3 e^{-1/3 x} c_2 .$$

Now apply Initial condition

$$1.25 = c_1 \frac{1}{2} e^0 - \frac{1}{3} e^0 c_2.$$

$$1.25 = \frac{1}{2} c_1 - \frac{1}{3} c_2 \longrightarrow 5$$

Multiply $\frac{1}{3}$ with eq 4 & add with 5

$$0 = \frac{1}{3} c_1 + \cancel{\frac{1}{3} c_2}$$

$$1.25 = \frac{1}{2} c_1 - \cancel{\frac{1}{3} c_2}$$

$$1.25 = \frac{1}{3} c_1 + \frac{1}{2} c_1.$$

$$1.25 = \frac{2c_1 + 3c_1}{6} \Rightarrow 7.5 = 2c_1 + 3c_1$$

$$\boxed{c_1 = 1.5}$$

Put in eq 4.

$$0 = 1.5 + c_2 \Rightarrow \boxed{c_2 = -1.5}$$

By putting values in general eqn.

$$\boxed{y = 1.5 e^{1/2 x} - 1.5 e^{-1/3 x}.}$$

Ans 2