Higher order Non homogenous ODE.

Ex 3.3

Q1:- $y''' - 3y'' + 3y' - y = e^x - x - 1$.

Sol- As given that,

$$y''' - 3y'' + 3y' - y = e^x - x - 1 \longrightarrow ①$$

Sol of eq ① is;

$$y = y_c + y_p \longrightarrow ②.$$

For $y_c = ?$

Consider

$$y''' - 3y'' + 3y' - y = 0 \longrightarrow ③$$

is linear in y so solution is;

$$y = e^{\lambda x}$$

$$y' = \lambda e^{\lambda x}, \quad y'' = \lambda^2 e^{\lambda x}, \quad y''' = \lambda^3 e^{\lambda x}.$$

$$\lambda^3 e^{\lambda x} - 3\lambda^2 e^{\lambda x} + 3\lambda e^{\lambda x} - e^{\lambda x} = 0.$$

$$\lambda^3 - 3\lambda^2 + 3\lambda - 1 = 0.$$

$$\lambda^3 - (1)^3 - 3\lambda^2 + 3\lambda = 0.$$

$$(\lambda - 1)(\lambda^2 + \lambda + 1) - 3\lambda(\lambda - 1) = 0$$

$$(\lambda - 1)[\lambda^2 + \lambda + 1 - 3\lambda] = 0$$

$$(\lambda - 1)[\lambda^2 - 2\lambda + 1] = 0$$

$$(\lambda - 1)(\lambda - 1)^2 = 0.$$

so; $\boxed{\lambda_1 = 1}$, $\boxed{\lambda_2 = 1}$ & $\boxed{\lambda_3 = 1}$

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Basis $= \langle e^{x}, xe^{x}, x^2 e^{\lambda x} \rangle$.

G. Solution of y_c is,

$$\boxed{y_c = c_1 e^{x} + c_2 e^{x} x + c_3 x^2 e^{\lambda x}} \rightarrow (A)$$

Now to find $y_p = ?$

$$y_p = x^3 a e^{x} - a_0 - a_1 x.$$

$$y_p' = a x^3 e^{x} + a e^{x}(3x^2) - a_1$$

$$y_p'' = a x^3 e^{x} + 3x^2 a e^{x} + a e^{x}(3x^2) + 6x a e^{x} + 0$$

$$= a x^3 e^{x} + 6x^2 a e^{x} + 6x a e^{x}.$$

$$y_p''' = a x^3 e^{x} + 3x^2 a e^{x} + 6x^2 a e^{x} + 12x a e^{x} + 6x a e^{x} + 6a e^{x}.$$

$$y_p''' = a x^3 e^{x} + \cancel{6}9 x^2 a e^{x} + 18 x a e^{x} + 6 a e^{x}.$$

Put values in eq (1).

$$a x^3 e^{x} + \cancel{6}9 x^2 a e^{x} + 18 x a e^{x} + 6 a e^{x} - 3(a x^3 e^{x} + 6x^2 a e^{x} + 6 x a e^{x})$$

$$+ 3(a x^3 e^{x} + 3x^2 a e^{x} + a_1) - x^3 a e^{x} - a_0 - a_1 x = e^{x} - 1 - x$$

By comparing co-efficient of e^{x}, x & x^0.

~~e// / a x^3 + 6x^2 a + 18 a x + 6a + 3ax^3 - 18x^2 a - 18 x a + 3 a x^3 + 9 x^2 a e^x~~
~~+3a - x^3 a = 1~~

~~+13x^2 a~~

e^x: $\cancel{9x^3} + 9ax^2 + 18xa + 6a - 3ax^3 - 18ax^2 - 18ax + 3ax^3$
$+ 9ax^2 - \cancel{x^3 a} = 1$

$6a = 1$

$\boxed{a = 1/6}$

x: $-a_1 = -1$

$\boxed{a_1 = 1}$

x^0: $3a_1 - a_0 = -1$

$-a = -3 - 1$

$\boxed{a = 4}$

So the general solution will be;

$y = y_c + y_p$

$$\boxed{y = c_1 e^x + c_2 x e^x + c_3 x^2 e^x + \frac{1}{6} x^3 e^x + x + 4}$$

Ans

Q2:- $y''' + 2y'' - y - 2y = 1 - 4x^3$.

Sol:- As given that;

$$y''' + 2y'' - y' - 2y = 1 - 4x^3 \longrightarrow (1)$$

The general sol of eq.(1) is;

$$y = y_c + y_p \longrightarrow (2)$$

For $y_c = ?$

Consider $y''' + 2y'' - y' - 2y = 0 \longrightarrow (3)$

$$y = e^{\lambda x} \longrightarrow (4)$$

Diff eq (4) & put in eq (3).

eq (3) =)

$$\lambda^3 e^{\lambda x} + 2\lambda^2 e^{\lambda x} - \lambda e^{\lambda x} - 2e^{\lambda x} = 0.$$

$$\lambda^3 + 2\lambda^2 - \lambda - 2 = 0.$$

$$\lambda^2(\lambda + 2) - 1(\lambda + 2) = 0.$$

$$(\lambda^2 - 1)(\lambda + 2) = 0.$$

$$(\lambda + 1)(\lambda - 1)(\lambda + 2) = 0.$$

$$\lambda = 1, -1, -2.$$

So; Basis $= \langle e^{x}, e^{-x}, e^{-2x} \rangle$

So y_c will be,

$$y_c = c_1 e^{x} + c_2 e^{-x} + c_3 e^{-2x}$$

Now to find $y_p = ?$

For y_p

Consider

$$y_p = -a_0 - a_1 x - a_2 x^2 - a_3 x^3$$

$$y_p' = -a_1 - 2a_2 x - 3a_3 x^2$$

$$y_p'' = -2a_2 - 6a_3 x$$

$$y_p''' = -6a_3$$

Using eq ① By putting values

$$-6a_3 + 2(-2a_2 - 6a_3 x) - (-a_1 - 2a_2 x - 3a_3 x^2)$$
$$-2(-a_0 - a_1 x - a_2 x^2 - a_3 x^3) = 1 - 4x^3$$

By comparing co-efficient of x, x^3, x^2 & x^0

x^3: $+2a_3 = -4$

$$\boxed{a_3 = -2}$$

x^0: $-6a_3 - 4a_2 + a_1 + 2a_0 = 1$

$-4a_2 + a_1 + 2a_0 = -11 \longrightarrow$ ④ $\quad \because a_3 = -2$

x: $-12a_3 + 2a_2 + 2a_1 = 0$

$2a_2 + 2a_1 = 12(-2)$

$a_2 + a_1 = -12 \longrightarrow$ ⑤

x^2: $3a_3 + 2a_2 = 0 \Rightarrow 3(-2) + 2a_2 = 0 \Rightarrow \boxed{a_2 = 3}$

eq ⑤ ⇒ $a_1 + a_2 = -12.$

$a_2 = 3.$

$\therefore$

$\boxed{a_1 = -15}$

eq ④ ⇒

$-4a_2 + a_1 + 2a_0 = -11$

$-4(3) - 15 + 2a_0 = -11.$

$-12 - 15 + 2a_0 = -11$

$2a_0 = -11 + 27$

$\boxed{a_0 = 8}$

So,

$y_p = -8 + 15x - 3x^2 + 3x^3.$

So the genral solution of higher order non-homogenous ODE is;

$y = y_c + y_p$

$$\boxed{y = c_1 e^{x} + c_2 e^{-x} + c_3 e^{-2x} + (-8 + 15x - 3x^2 + 3x^3)}$$

Ans.

Q3:- $(D^4 + 5D^2 + 4I)y = 3.5\sinh 2x$.

Sol:- As given that;

$(D^4 + 5D^2 + 4I)y = 3.5\sinh 2x$.

$y^{iv} + 5y'' + 4y = 3.5\sinh 2x \rightarrow ①$

As gen solution of eq ① is;

$y = y_c + y_p \longrightarrow (A)$.

$y_c = ?$

Consider $y^{iv} + 5y'' + 4y = 0 \rightarrow ②$

Since $y = e^{\lambda x}$.

So diff & put their values in eq ②

$\lambda^4 e^{\lambda x} + 5(\lambda^2 e^{\lambda x}) + 4e^{\lambda x} = 0$

$\lambda^4 + 5\lambda^2 + 4 = 0$.

$\lambda^4 + 4\lambda^2 + \lambda^2 + 4 = 0$

$\lambda^2(\lambda^2 + 4) + 1(\lambda^2 + 4) = 0$.

$\lambda^2 = -1$ or $\lambda^2 = -4$

$\boxed{\lambda = \pm i}$ & $\boxed{\lambda = \pm 2i}$

Basis $= \langle e^{ix}, e^{-ix}, e^{2ix}, e^{-2ix} \rangle$.

So $\boxed{y_c = c_1 \cos x + c_2 \sin x + c_3 \cos 2x + c_4 \sin 2x}$

To find $y_p = ?$

Let;
$$y_p = a\cosh 2x + b\sinh 2x.$$

$$y_p' = 2a\sinh 2x + 2b\cosh 2x$$

$$y_p'' = 4a\cosh 2x + 4b\sinh 2x.$$

$$y_p''' = 8a\sinh 2x + 8b\cosh 2x$$

$$y_p^{iv} = 16a\cosh 2x + 16b\sinh 2x.$$

By putting values in eq ①.

$$16a\cosh 2x + 16b\sinh 2x + 5(4a\cosh 2x + 4b\sinh 2x) + 4(a\cosh 2x + b\sinh 2x) = \overset{35}{\cancel{a\cosh 2x + 3}}\sinh 2x$$

By comparing co-efficient of $\cosh 2x$ & $\sinh 2x$.

$\cosh 2x$: $16a + 20a + 4a = 0.$

$$\boxed{a = 0}$$

$\sinh 2x$: $16b + 20b + 4b = 3.5.$

$$\boxed{b = 7/80}$$

So; $y_p = \overset{(35)}{7/80}\sinh 2x \Rightarrow y_p = \frac{49}{160}\sin 2x$

So; general sol is;

eq (A) ⇒ $\boxed{y = c_4\cos x + c_1\sin x + c_3\cos 2x + c_4\sin 2x + \frac{49}{160}\sin 2hx}$

Ans.

Q4.- $(D^3 + 3D^2 - 5D - 39I)y = -300\cos x$.

Sol:- $y''' + 3y'' - 5y' - 39y = -300\cos x$ → ①

For $y_c = ?$

$$y''' + 3y'' - 5y' - 39y = 0.$$

$$y = e^{\lambda x}.$$

$$\lambda^3 e^{\lambda x} + 3\lambda^2 e^{\lambda x} - 5\lambda e^{\lambda x} - 39 e^{\lambda x} = 0.$$

$$\lambda^3 + 3\lambda^2 - 5\lambda - 39 = 0.$$

$\lambda = 3$ is a factor.

$$\begin{array}{r|l} & \lambda^2 + 6\lambda + 13 \\ \lambda - 3 & \lambda^3 + 3\lambda^2 - 5\lambda - 39 \\ & \underline{\pm\cancel{\lambda^3} \mp 3\lambda^2} \\ & \cancel{6\lambda^2} - 5\lambda - 39 \\ & \underline{\pm \cancel{6\lambda^2} \mp 18\lambda} \\ & 13\lambda - 39 \\ & \underline{13\lambda - 39} \\ & \times \end{array}$$

$$(\lambda - 3)(\lambda^2 + 6\lambda + 13) = 0.$$

$$(\lambda - 3)(\lambda^2 + 6\lambda + 13) = 0. \qquad \boxed{\lambda = 3}$$

For $\lambda^2 + 6\lambda + 13 = 0$.

Use Quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

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$$\lambda = \frac{-6 \pm \sqrt{36-52}}{2}$$

$$= \frac{-6 \pm \sqrt{-16}}{2}$$

$$= \frac{-6 \pm 4i}{2}$$

$\boxed{\lambda_1 = -3+2i}$; $\boxed{\lambda_2 = -3-2i}$

$$y_c = c_1 e^{3x} + c_2 e^{-3+2i} + c_3 e^{-3-2i}$$

$$y_c = c_1 e^{3x} + c_2 e^{-3x} \cos 2x + c_3 e^{-3x} \sin 2x.$$

Now to find $y_p = ?$

$$y_p = A \cos x + B \sin x$$

$$y' = -A \sin x + B \cos x$$

$$y'' = -A \cos x - B \sin x.$$

$$y''' = A \sin x - B \cos x.$$

eq ① =)

$$A \sin x - B \cos x + 3(-A \cos x - B \sin x) - 5(-A \sin x + B \cos x)$$
$$- 39(A \cos x + B \sin x) = -300 \cos x$$

Comparing Co-effs.

$\cos x$:

$$-B - 3A - 5B - 39A = -300.$$

$$-6B - 42A = -300.$$

$$\boxed{6B + 42A = 300} \rightarrow ①.$$

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~~Cos : Asinx~~

Sinx : $A - 3B + 5A - 39B = 0$.

$+4A - 42B = 0. \longrightarrow (3)$

Subtract eq (3) from (2)

$\cancel{-4A - 42B = 0}$

$\cancel{-6B - 42B =}$

$\cancel{+4A - 42B = 0}$

$-42A - 6B = 300$

$\Rightarrow B + 7A = 50 \longrightarrow (4)$

$28A - 294B = 0$

$\underline{\overset{-}{+} 28A \underset{-}{+} 4B = 300}$

$298B = 300.$

$\boxed{B = 1}$

eq (4) $\Rightarrow$ $1 + 7A = 50.$

$A = 7$

$y_P = 7\cos x + \sin x.$

So,

$y = C_1 e^{3x} + C_2 e^{-3x} \cos 2x + C_3 e^{-3x} \sin 2x$

Ans

2

Q8:- $y^{iv} - 5y'' + 4y = 10e^{-3x} \to (A)$ $y(0) = 1, y'(0) = 0$

Sol:- First find y_c = ?

$y^{iv} - 5y'' + 4y = 0 \longrightarrow (1)$

$y = e^{\lambda x}$

$\lambda^4 e^{\lambda x} - 5\lambda^2 e^{\lambda x} + 4e^{\lambda x} = 0$

$\lambda^4 - 5\lambda^2 + 4 = 0$

$\lambda^4 - 4\lambda^2 - \lambda^2 + 4 = 0$

$\lambda^2(\lambda^2 - 4) - 1(\lambda^2 - 4) = 0$

$(\lambda^2 - 1)(\lambda^2 - 4) = 0$

$\lambda^2 - 1 = 0$

$\lambda = \pm 1$

$\lambda^2 - 4 = 0$

$\lambda = \pm 2$

$y_c = c_1 e^{x} + c_2 e^{-x} + c_3 e^{2x} + c_4 e^{-2x}$

To find y_p = ?

$y_p = a e^{-3x} \longrightarrow (2)$

$y_p' = a(-3)e^{-3x}$

$y_p'' = 9a e^{-3x}$

$y_p''' = -27a e^{-3x}$

$y_p^{iv} = 81 e^{-3x} a$

eq (A) =)

$81ae^{-3x} - 5(9)a e^{-3x} + 4ae^{-3x} = 10e^{-3x}$

$81a - 45a + 4a = 10$

$a = 10/40$

$\boxed{a = 1/4}$

eq $y_p = \frac{1}{4}e^{-3x}$

So the gen. sol.

$y = c_1e^{x} + c_2e^{-x} + c_3e^{2x} + c_4e^{-2x} + \frac{1}{4}e^{-3x}$ (B).

Apply initial condition

$1 = c_1 + c_2 + c_3 + c_4 + 1/4$

$3/4 = c_1 + c_2 + c_3 + c_4 \longrightarrow$ (3).

eq (B) ⇒ $y' = c_1e^{x} - c_2e^{-x} + 2c_3e^{2x} - 2c_4e^{-2x} - \frac{3}{4}e^{-3x} \longrightarrow$ (C)

Apply condition.

$y'(0) = 0$

$0 = c_1 - c_2 + 2c_3 - 2c_4 - 3/4$

$3/4 = c_1 - c_2 + 2c_3 - 2c_4 \longrightarrow$ (4)

$y'' = c_1e^{x} + c_2e^{-x} + 4c_3e^{2x} + 4c_4e^{-2x} + \frac{9}{4}e^{-3x} \longrightarrow$ (B)

Apply initial condition.

$y''(0) = 0$.

$0 = c_1 + c_2 + 4c_3 + 4c_4 + 9/4$

$-9/4 = c_1 + c_2 + 4c_3 + 4c_4 \rightarrow (5)$.

& D =>

$y''' = c_1 e^{x} - c_2 e^{-x} + 8e^{2x} c_3 - 8c_4 e^{-2x} - \frac{27}{4} e^{-2x}$

Apply Initial condition.

$0 = c_1 - c_2 + 8c_3 - 8c_4 - \frac{27}{4}$

$\frac{27}{4} = c_1 - c_2 + 8c_3 - 8c_4 \rightarrow (6)$

Add & (3) & (4).

$3/4 = c_1 + \cancel{c_2} + c_3 + c_4$

$3/4 = c_1 - \cancel{c_2} + 2c_3 - 2c_4$

$2(3/4) = 2c_1 + 3c_3 - c_4 \rightarrow (7)$.

Add & (6) & (5).

$-9/4 = c_1 + \cancel{c_2} + 4c_3 + 4c_4$

$27/4 = c_1 - \cancel{c_2} + 8c_3 - 8c_4$

$\frac{27-9}{4} = 2c_1 + 12c_3 - 4c_4$

$17/4 = 2c_1 + 12c_3 - 4c_4 \rightarrow (8)$

Subtract q 7 from 8

$\frac{17}{4} = 2c_1 + 12c_3 - 4c_4$

$\frac{3}{2} = \cancel{2c_1} + 3c_3 - c_4$

$\frac{17-6}{4} = 9c_3 - 3c_4$

$\frac{11}{4} = 9c_3 - 3c_4 \longrightarrow 9$

Add q 6 & 4

$-\frac{27}{4} = c_1 - \cancel{c_2} + 8c_3 - 8c_4$

$\frac{3}{4} = c_1 + \cancel{c_2} + c_3 + c_4$

$\frac{-27+3}{4} = 2c_1 + 9c_3 - 7c_4$

$-6 = 2c_1 + 9c_3 - 7c_4 \longrightarrow 10$

Add 5 & 4

$-\frac{9}{4} = c_1 + \cancel{c_2} + 4c_3 + 4c_4$

$\frac{3}{4} = c_1 - \cancel{c_2} + 2c_3 - 2c_4$

$-\frac{3}{2} = 2c_1 + 6c_3 + 2c_4 \longrightarrow 11$

Sub q 11 from 10.

$-6 = \cancel{2c_1} + 9c_3 - 7c_4$

$-\frac{3}{2} = \cancel{2c_1} + 6c_3 - 2c_4$

$\frac{-12+3}{2} = 3c_3 - 5c_4$

$-9/2 = 3c_3 - 5c_4 \longrightarrow$ (12)

eq (9) $\Rightarrow$ $\frac{33}{4} = 27c_3 - 9c_4$.

eq (11) $\Rightarrow$ $-\frac{81}{2} = 27c_3 - 45c_4$

$\frac{33}{4} - \frac{81}{2} = 36c_4$.

$\frac{33 - 162}{4} = 36c_4$.

$c_4 = \cancel{32.25 \times 4}$.

$c_4 = 0.9$.

$c_3 = 0$.

eq (10) $\Rightarrow$ $c_1 = 0.3$

eq (7) $\Rightarrow$ $\cancel{6/4 = 0.6 - 0.9}$.

eq (4) $\Rightarrow$ $3/4 = 0.3 + c_2 + 0. + 0.9$.

$c_2 = 0.45$

So particular soln. is;

$$y = 0.3e^{x} + 0.45e^{-x} - 2(0.9)e^{-2x} + \frac{1}{4}e^{-3x}$$

Ans

$c_2 = -c_3$ let $c_3 = r$, where $r \in \mathbb{R}$.

$\boxed{c_2 = -r}$ if $c_3 = r$.

$$\boxed{y = 2c_1 + (-r)e^{2x} + re^{-2x} + \cos x - 2\sin x.}$$

where $r \in \mathbb{R}$.

$y_p = \cos x - 2\sin x.$

so,

eq (2) $\Rightarrow$ $y = C_1 e^{0x} + C_2 e^{+2x} + C_3 e^{-2x} + a\cos x + b(-2)\sin x.$

$y = C_1 + C_2 e^{2x} + C_3 e^{-2x} + a\cos x - 2b\sin x.$

$y(0) = 3$

$3 = C_1 + C_2 + C_3 + 1$

$2 = C_1 + C_2 + C_3 \longrightarrow$ (A).

$y' = 2C_2 e^{2x} - 2C_3 e^{-2x} - \sin x - 2\cos x.$

$y'(0) = -2.$

$-2 = 2C_2 - 2C_3 - 2$

$1 = -C_2 + C_3 + 1$

$0 = -C_2 + C_3 \longrightarrow$ (B)

$y'' = 4C_2 e^{2x} + 4C_3 e^{-2x} - \cos x.$

$y''(0) = -1.$

$-1 = 4C_2 + 4C_3 - 1$

$= \boxed{C_2 + C_3 = 0} \longrightarrow$ (C)

eq (A) $\Rightarrow$ $2 = C_1 + 0 \Rightarrow C_1 = 2$

Q13:- $(D^3 - 4D)y = 10\cos x + 5\sin x$ $y(0) = 3$.

$y'(0) = -2$, $y''(0) = -1$.

Sol:- $y''' - 4y' = 10\cos x + 5\sin x \longrightarrow (1)$.

G. solution of (1) is;

$y = y_c + y_p \longrightarrow (2)$

$y_c = ?$ $y''' - 4y' = 0. \longrightarrow (3)$.

Consider $y = e^{\lambda x}$.

$\Rightarrow$ (3) $\Rightarrow$ $\lambda^3 e^{\lambda x} - 4\lambda . e^{\lambda x} = 0$.

$\lambda^3 - 4\lambda = 0$.

$\lambda(\lambda^2 - 4) = 0$

$\boxed{\lambda = 0}$, $\boxed{\lambda = \pm 2}$

C_F Basis $= \langle e^{0x}, e^{2x}, e^{-2x} \rangle$.

So; $y_c = c_1 e^{0x} + c_2 e^{2x} + c_3 e^{-2x}$.

To find $y_p = ?$

$y_p = a\cos x + b\sin x$.

$y_p' = -a\sin x + b\cos x$, $y_p'' = -a\cos x - b\sin x$

$y_p''' = a\sin x \mp b\cos x$.

$\Rightarrow$ (1) $\Rightarrow$ $a\sin x - b\cos x - 4(-a\sin x + b\cos x) = 10\cos x + 5\sin x$.

$5a\sin x - 5b\cos x = 10\cos x + 5\sin x$.

$\cos x$: $-5b = 10 \Rightarrow b = -2$

$\sin x$: $5a = 5 \Rightarrow \boxed{a = 1}$

To find $y_p = ?$

$$y_p = a e^{2x}.$$

$$y' = 2ae^{2x}, \quad y'' = 4ae^{2x}, \quad y''' = 8ae^{2x}.$$

& (1) $\Rightarrow$

$$\cancel{8a}e^{2x} - \cancel{2(4ae^{2x})} - 9(\cancel{2ae^{2x}}) + \cancel{18ae^{2x}} = e^{2x}.$$

$$0 = e^{2x}.$$

$$a = 0$$

Q12 :- $(D^3 - 2D^2 - 9D + 18I)y = e^{2x}$, $y(0) = 4.5$,
$y'(0) = 5.8$, $y''(0) = 17.2$.

Sol:- $y''' - 2y'' - 9y' + 18y = e^{2x}$ $\longrightarrow$ ①.

The general solution is;

$y = y_c + y_p$ $\longrightarrow$ ②

For $y_c = ?$

$y''' - 2y'' - 9y' + 18y = 0$ $\longrightarrow$ ③

$y = e^{\lambda x}$.

③ ⇒ $\lambda^3 e^{\lambda x} - 2\lambda^2 e^{\lambda x} - 9\lambda e^{\lambda x} + 18 e^{\lambda x} = 0$.

$\lambda^3 - 2\lambda^2 - 9\lambda + 18 = 0$.

$\lambda^2(\lambda - 2) - 9(\lambda - 2) = 0$.

$(\lambda^2 - 9)(\lambda - 2) = 0$.

$\boxed{\lambda = 2}$ $\boxed{\lambda = \pm 3}$

Basis $= \langle e^{2x}, e^{+3x}, e^{-3x} \rangle$.

$y_c = c_1 e^{2x} + c_2 e^{3x} + e^{-3x} c_3$

Q_{11}:- $(D^3 + D^2 - 2D)y = 4e^{-2x}/\cos x$.

$y(0) = 0.4, \quad y'(0) = -0.4, \quad y''(0) = -0.4$.

Sol:-

$$y''' + y'' - 2y' = \frac{4e^{-2x}}{\cos x} \rightarrow ①$$

The general sol is;

$$y = y_c + y_p \rightarrow ②$$

To find $y_c = ?$
Consider $y = e^{\lambda x}$.

problem in its y_p

w_i. get zero

$u_i = \int \frac{w_i}{w} dx$
$\neq 0$.

$\{ \quad y''' + y'' - 2y' = 0$.

$$\lambda^3 e^{\lambda x} + \lambda^2 e^{\lambda x} - 2\lambda e^{\lambda x} = 0.$$

$$\lambda^3 + \lambda^2 - 2\lambda = 0.$$

$$\lambda(\lambda^2 + \lambda - 2) = 0.$$

$\lambda = 0$

$$\lambda^2 + \lambda - 2 = 0.$$

$$\lambda^2 + 2\lambda - \lambda - 2 = 0.$$

$$\lambda(\lambda+2) - 1(\lambda+2) = 0$$

$$(\lambda - 1)(\lambda + 2) = 0$$

$\lambda = 1$ & $\lambda = -2$.

So the

y_c = Basis $= \langle e^{0x}, e^{x}, e^{-2x} \rangle$

$$y_c = c_1 e^{0} + c_2 e^{x} + c_3 e^{-2x}.$$

Apply Conditions

$y(1) = 1$

$1 = c_1 + 1$

$\boxed{c_1 = 0}$

$$y' = c_1 + x\left(\frac{1}{x}\right)c_2 + \left(2x\ln x + x^2\left(\frac{1}{x}\right)\right)c_3 + 2x$$

$$y' = c_1 + c_2 + (2x\ln x + x)c_3 + 2x$$

$y'(0) = 3$

$3 = 0 + c_2 + 0 + 0$

$\boxed{c_2 = 3}$

$$y'' = \left(2x\left(\frac{1}{x}\right) + 2\ln x\right)c_3 + 2$$

$y''(1) = 14$

$14 = (2 + 0)c_3 + 2$

$c_3 = 12/2 \Rightarrow \boxed{c_3 = 6}$

So, the required solution will be,

$$\boxed{y = 3x\ln x + 6x^2\ln x}$$ Ans

$y_c = c_1 x + c_2 x \ln x + c_3 x^2 \ln x.$

To find $y_p = ?$

$y_p = a_0 + a_1 x + a_2 x^2.$

$y_p' = a_1 + 2a_2 x$

$y_p'' = 2a_2$

Put in eq ①.

$x^3(2a_2) + x(a_1 + 2a_2 x) - a_0 - a_1 x - a_2 x^2 = x^2.$

$2a_2 x^3 + \cancel{a_1 x} + 2a_2 x^2 - a_0 - \cancel{a_1 x} - a_2 x^2 = x^2.$

By comparing co-efficient of x^3, x^2, x & x^0.

$x^3: 0.$

$a_2 = 0$

$x^2: \quad 2a_2 x^2 - a_2 x^2 = 1$

$a_2 = 1$

$x: \quad a_1 - a_1 = 0$

$a_1 = 0$

$x^0: \quad \boxed{a_0 = 0}$

$y_p = 1x^2 \Rightarrow \boxed{y_p = x^2}$

eq ① $\Rightarrow \quad y = c_1 x + x \ln x \, c_2 + x^2 \ln x \, c_3 + x^2.$

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Q10:- $x^3y''' + xy' - y = x^2$, $y(1) = 1$, $y'(0) = 3$
$y''(1) = 14$

Sol:- $x^3y''' + xy' - y = x^2 \longrightarrow ①$

Cauchy Euler;

Consider $y = y_c + y_p \longrightarrow ②$

For y_c = ? consider $x^3y''' + xy' - y = 0 \longrightarrow ③$

$y = x^m$

$y' = mx^{m-1}$, $y'' = m(m-1)x^{m-2}$, $y''' = m(m-1)(m-2)x^{m-3}$

③ ⇒

$x^3(m(m-1)(m-2)x^{m-3}) + x(mx^{m-1}) - x^m = 0$

$m(m-1)(m-2) + m - 1 = 0$

$(m-1)[m(m-2) + 1] = 0$

$m = 1$

$m(m-2) + 1 = 0$

$m^2 - 2m + 1 = 0$

$m^2 - m - m + 1 = 0$

$m(m-1) - 1(m-1) = 0$

$m = 1$, $m = 1$

Basis = $\langle x, x\ln x, x^2\ln x \rangle$

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eq ① ⇒ $1 = c_1 + c_4$

$c_4 = 1 + 1/3$

$\boxed{c_4 = 4/3}$

Consider eq ③ & ④

$\cancel{x} = 2c_2 + \cancel{c_3} + \cancel{x}$

$0 = -8c_2 - \cancel{c_3} - 16$

$-6_8 c_2 = 16$

$\boxed{c_2 = -8/3}$

eq ③ ⇒ $2c_2 + c_3 = 0$

$2(-8/3) + c_3 = 0$

$\boxed{c_3 = 16/3}$

So the general solution can be written as.

$y = -\frac{1}{3}\cos 3x + \frac{4}{3}\sin 3x - \frac{8}{3}\cos x + \frac{16}{3}\sin x + \frac{1}{2}\sin 4x$

Ans

eq (a) $\Rightarrow$ $y = y_c + y_p$.

$$\boxed{y = c_1\cos 2x + c_2 \sin 2x + c_3 \sin x + c_4 \cos x + \tfrac{1}{2}\sin 4x.}$$

To find c_1, c_2, c_3 & c_4 apply Initial condition,

$y(0) = 1$

$1 = c_1 + c_4 \longrightarrow (1).$

$y' = -2c_1 \sin 2x + 2c_2 \cos 2x + c_3 \cos x - c_4 \sin x + 2\cos 4x.$

$y'(0) = 2.$

$2 = 2c_2 + c_3 + 2 \longrightarrow (2).$

$y'' = -4c_1 \cos 2x - 4c_2 \sin 2x - c_3 \sin x - c_4 \cos x - 4 \sin 4x.$

$y''(0) = 0.$

$0 = -4c_1 - c_4 \longrightarrow (3).$

$y''' = 8c_1 \sin 2x - 8c_2 \cos 2x - c_3 \cos x + c_4 \sin x - 16 \cos 4x.$

$y'''(0) = 0.$

$0 = -8c_2 - c_3 - 16 \longrightarrow (4).$

eq (1) & (3) $\Rightarrow$

$0 = -4c_1 - c_4.$

$1 = c_1 + c_4$

$-3c_1 = 1 \Rightarrow c_1 = -\tfrac{1}{3}$

$\lambda^2 = -4$, $\lambda^2 = -1$

$\boxed{\lambda = \pm 2i}$, $\boxed{\lambda = \pm 1i}$

Basis =

$$\langle e^{2ix}, e^{-2ix}, e^{ix}, e^{-ix} \rangle$$

$y_c = c_1 \cos 2x + c_2 \sin 2x + c_3 \cos x + c_4 \sin x.$

To find $y_p = ?$

$y_p = a\cos 4x + b \sin 4x.$

$y_p' = -4a\sin 4x + 4b\cos 4x.$

$y_p'' = -16a\cos 4x - 16 b\sin 4x.$

$y_p''' = 64 a \sin 4x - \cancel{1}64\, b\cos 4x.$

$y_p^{iv} = 256\, a \cos 4x + 256\, b \sin 4x.$

eq ① $\Rightarrow$.

$256a\cos 4x + 256 b\sin 4x + 5(-16a\cos 4x - 16 b\sin 4x)$

$+ 4(a\cos 4x + b\sin 4x) = 90 a\sin 4x.$

$256a\cos 4x + 256 b\sin 4x - 80a\cos 4x - 80 b\sin 4x + 4a\cos 4x$

$+ 4b\sin 4x = 90 a\sin 4x$

$180\, a\cos 4x + 180\, b\sin 4x = 90\, a\sin 4x.$

$\cos 4x$: $180a = 0.$

$a = 0$

$y_p = \frac{1}{2}\sin 4x$

$\sin 4x$: $180\, b = 90.$

$\boxed{b = 1/2}$

Q9:- $y^{iv}+5y''+4y=90\sin 4x$, $y(0)=1$, $y'(0)=2$,

$y''(0)=0$, $y'''(0)=0$.

Solution:-

$$y^{iv}+5y''+4y=90\sin 4x \longrightarrow ①$$

The general sol. of eq ① is;

$$y=y_c+y_p \longrightarrow ②$$

For $y_c=?$ Consider

$$y^{iv}+5y''+4y=0 \longrightarrow ③$$

As sol. of eq ③ is;

$$y=e^{\lambda x}$$

$y'=\lambda e^{\lambda x}$, $y''=\lambda^2 e^{\lambda x}$, $y'''=\lambda^3 e^{\lambda x}$, $y^{iv}=\lambda^4 e^{\lambda x}$.

eq ③ $\Rightarrow$

$\lambda^4 e^{\lambda x}+5\lambda^2 e^{\lambda x}+4e^{\lambda x}=0$. Divide by $e^{\lambda x}$.

$$\lambda^4+5\lambda^2+4=0.$$

Let $\lambda^2=y$.

So;
$$y^2+5y+4=0$$
$$y^2+4y+y+4=0$$
$$y(y+4)+1(y+4)=0 \Rightarrow (y+4)(y+1)=0.$$

$\boxed{y=-4}$, $\boxed{y=-1}$

Put $y=\lambda^2$.

So;

$y_c = \frac{4}{5}\cos x + \frac{2}{5}\sin x.$

So the general solution is;

$$\boxed{y = c_1 e^{-x} + c_2 e^{3.73x} + c_3 e^{0.27x}}$$ Ans

Checked

8-17

So, $y_c = c_1 e^{-x} + c_2 e^{3.73x} + c_3 e^{0.27x}$.

To find $y_p = ?$

$$y_p = a\cos x + b\sin x$$

$$y_p' = -a\sin x + b\cos x.$$

$$y_p'' = -a\cos x - b\sin x.$$

$$y_p''' = a\sin x - b\cos x$$

By putting values in eq ①.

$$a\sin x - b\cos x - 3(-a\cos x - b\sin x) + 3(-a\sin x + b\cos x) + a\cos x + b\sin x = 4\cos x.$$

$$a\sin x - b\cos x + 3a\cos x + 3b\sin x - 3a\sin x + 3b\cos x + a\cos x + b\sin x = 4\cos x.$$

$$-2a\sin x + 2b\cos x + 4a\cos x + 4b\sin x = 4\cos x.$$

Comparing co-efficient of $\cos x$ & $\sin x$,

$\cos x$: $2b + 4a = 4$.

$b + 2a = 2$

$\sin x$: $-2a + 4b = 0$.

$a = 2b$.

$b + 4b = 2 \Rightarrow$ $5b = 2 \Rightarrow$ $b = 2/5$.

$\boxed{a = 4/5}$

$y' = \lambda e^{\lambda x}$.

$y'' = \lambda^2 e^{\lambda x}$.

$y''' = \lambda^3 e^{\lambda x}$.

$\lambda^3 e^{\lambda x} - 3\lambda^2 e^{\lambda x} + 3\lambda e^{\lambda x} + e^{\lambda x} = 0$.

$\lambda^3 - 3\lambda^2 \cancel{+} 3\lambda + 1 = 0$

$\lambda^3 - 3\lambda^2 - 3\lambda + 1 = 0$.

$$
\begin{array}{r|l}
 & \lambda^2 - 4\lambda + 1 \\
\hline
(\lambda+1) & \lambda^3 - 3\lambda^2 - 3\lambda + 1 \\
 & \pm\lambda^3 \pm \lambda^2 \\
\hline
 & -4\lambda^2 - 3\lambda + 1 \\
 & -4\lambda^2 - 4\lambda \\
\hline
 & \lambda + 1 \\
 & \pm\lambda \pm 1 \\
\hline
 & \times
\end{array}
$$

so, $= (\lambda+1)(\lambda^2 - 4\lambda + 1) = 0$.

$\lambda + 1 = 0$

$\boxed{\lambda = -1}$

$\lambda^2 - 4\lambda + 1 = 0$.

$\boxed{\lambda = 3.73}$, $\boxed{\lambda = 0.27}$

By comparing co-efficients of $\sin x$ & $\cos x$,

$\sin x$: $-3a = 1$

$$\boxed{a = -1/3}$$

$\cos x$: $3b = 0$

$$\boxed{b = 0}$$

$y_p = -\frac{1}{3} \cos x.$

So the general solution is;

$$\boxed{y = c_1 + c_2 \cos 2x + c_3 \sin 2x - \frac{1}{3} \cos x.}$$

Ans 2

Q7:- $(D^3 - 3D^2 + 3D + I)y = 4\cos x.$

Sol:- $y''' - 3y'' + 3y' + y = 4\cos x \longrightarrow ①.$

As general solution of eq ① is;

$y = y_c + y_p \longrightarrow ②.$

For $y_c = ?$ $\quad y''' - 3y'' + 3y' + y = 0. \longrightarrow ③.$

As linear in y so.

$y = e^{\lambda x} \longrightarrow$ is solution of ③ eq ③ ⟹ $y''' + 4y' = 0.$

$\lambda^3 e^{\lambda x} + 4\lambda e^{\lambda x} = 0.$ Divide by $e^{\lambda x}$.

$\lambda^3 + 4\lambda = 0.$

$\lambda(\lambda^2+4) = 0.$

$\boxed{\lambda = 0}$, $\lambda^2 = -2$ ⟹ $\boxed{\lambda = \pm 2i}$

Basis $= \langle e^0, e^{2ix} + e^{-2ix} \rangle$.

$y_c = c_1 + c_2 \cos 2x + c_3 \sin 3x.$

To find $y_p = ?$

As;

$y_p = a\cos x + b\sin x.$

$y_p' = -a\sin x + b\cos x$

$y_p'' = -a\cos x - b\sin x.$

$y_p''' = a\sin x - b\cos x.$

By putting values in eq ①.

$a\sin x - b\cos x + 4(-a\sin x + b\cos x) = \sin x.$

$a\sin x - b\cos x - 4a\sin x + 4b\cos x = \sin x$

$-3a\sin x + 3b\cos x = \sin x.$

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To find $y_p = ?$

$$y_p = a_0 + a_1 x^{-1} + a_2 x^{-2}.$$

$$y_p' = a_1(-1)x^{-2} + a_2(-2)x^{-3}$$

$$y_p' = -a_1 x^{-2} - 2a_2 x^{-3}.$$

$$y_p'' = +2a_1 x^{-3} + 6a_2 x^{-4}.$$

$$y_p''' = -6a_1 x^{-4} - 24a_2 x^{-5}$$

Using these values in eq ①.

$$x^3(-6a_1 x^{-4} - 24a_2 x^{-5}) + 2x^2(2a_1 x^{-3} + 6a_2 x^{-4}) - x(-a_1 x^{-2} - 2a_2 x^{-3})$$

$$+ a_0 + a_1 x^{-1} + a_2 x^{-2} = x^{-2}.$$

$$-6a_1 x^{-1} - 24a_2 x^{-2} + 4a_1 x^{-1} + 12a_2 x^{-2} + a_1 x^{-1} + 2a_2 x^{-2} + a_0 + a_1 x^{-1}$$

$$+ a_2 x^{-2} = x^{-2}.$$

By comparing co-efficient of x^{-2}, x^{-1} & x^0.

x^{-2} : $-24a_2 + 12a_2 + 2a_2 + a_2 = 1.$

$$-9a_2 = 1.$$

$$\boxed{a_2 = -1/9}$$

x^{-1} : $-6a_1 + 4a_1 + a_1 + a_1 = 0.$

$$\boxed{a_1 = 0}$$

x^0 : $\boxed{a_0 = 0}$

so; $y_p = 0 + 0 - \frac{1}{9}x^{-2}$.

$$\boxed{y_p = -\frac{1}{9}x^{-2}}$$

So the general eqn of non ODE is;

$y = y_c + y_p$

$$\boxed{y = c_1 x + c_2 x^{-1} + c_3 x \ln x - \frac{1}{9}x^{-2}}$$

Ans

Q6:- $(D^3 + 4D)y = \sin x$.

Sol:- $y''' + 4y' = \sin x. \longrightarrow ①$.

The general sol. of eq ① is;

$y = y_c + y_p \longrightarrow ②$.

to find $y_c = ?$

As for y_c consider.

$y''' + 4y' = 0. \longrightarrow ③$

As linear in y so;

$y = e^{\lambda x}$

$y' = \lambda e^{\lambda x}$, $y'' = \lambda^2 e^{\lambda x}$, $y''' = \lambda^3 e^{\lambda x}$.

Q5:- $(x^3D^3 + 2x^2D^2 - xD + 1)y = x^{-2}$.

Sol:-
$$x^3y''' + 2x^2y'' - xy' + y = x^{-2} \longrightarrow ①.$$

is Euler Cauchy equation.

As general solution of eq ① is;

$$y = y_c + y_p \longrightarrow ②.$$

To find $y_c = ?$

Consider

$$x^3y''' + 2x^2y'' - xy' + y = 0$$

By trail;
$$y = x^m.$$

$$x^3 m(m-1)(m-2)x^{m-3} + 2x^2 m(m-1)x^{m-2} - x\,m\,x^{m-1} + x^m = 0.$$

$$m(m-1)(m-2) + 2m(m-1) - m + 1 = 0.$$

$$(m-1)\left[m(m-2) + 2m - 1\right] = 0.$$

$m - 1 = 0$ or $m(m-2) + 2m - 1 = 0.$

$\boxed{m = 1}$

$$m^2 - \cancel{2m} + \cancel{2m} - 1 = 0.$$

$$m^2 = 1$$

$$\boxed{m = \pm 1}$$

So $y_p = c_1x^1 + c_2x^{-1} + c_3 x \ln x.$