Ex # 4.3

Q1:- $y_1' = 2y_1 - y_2$

$y_2' = 3y_1 - 2y_2$

Sol:- Can be write as;

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}' = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}.$$

$$y' = Ay \longrightarrow ①$$

As linear in $y$.

so, $y = \underline{x} e^{\lambda t}$ be sol. of ①

$$y' = x e^{\lambda t} \lambda \longrightarrow ②$$

By comparing eq ① & ② we get.

$$x e^{\lambda t} \lambda = e^{\lambda t} A \underline{x}$$

$$(A - \lambda I)\underline{x} = 0 \longrightarrow Ⓐ.$$

To find $\lambda = ?$

Consider $\det(A - \lambda I) = 0$

$$\det\left(\begin{bmatrix} 2-\lambda & -1 \\ 3 & -2-\lambda \end{bmatrix}\right) = 0$$

$$(2-\lambda)(-2-\lambda) + 3 = 0.$$

$$-4 - 2\lambda + 2\lambda + \lambda^2 + 3 = 0$$

$$\lambda^2 - 1 = 0.$$

$\boxed{\lambda = \pm 1}$

Consider $(A - \lambda I)\underline{X} = 0$.

$$\begin{bmatrix} 2-\lambda & -1 \\ 3 & -2-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

$$(2-\lambda)x_1 - x_2 = 0. \rightarrow (B)$$

$$3x_1 + (-2-\lambda)x_2 = 0 \rightarrow (C)$$

For $\lambda = 1$.

$\Rightarrow (B) \Rightarrow$

$$x_1 - x_2 = 0.$$

$$\boxed{x_1 = x_2}$$

Let $x_2 = r$ where $r \in \mathbb{R}$.

$$x^{(1)} = \begin{bmatrix} r \\ r \end{bmatrix} = r \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

For $\lambda = -1$.

$\Rightarrow (B) \Rightarrow$ $3x_1 - x_2 = 0$

$$\boxed{x_1 = \tfrac{1}{3} x_2}$$ $x_2 = r,\ r \in \mathbb{R}$

$$x^{(2)} = \begin{bmatrix} \tfrac{1}{3} r \\ 1 r \end{bmatrix} = r \begin{bmatrix} \tfrac{1}{3} \\ 1 \end{bmatrix}.$$

So general sol. is;

$$\boxed{y = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{t} + c_2 \begin{bmatrix} \tfrac{1}{3} \\ 1 \end{bmatrix} e^{-t}}$$ Ans.

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Q3:- $y_1' = -2y_1 + \frac{3}{2} y_2$
$y_2' = -4y_1 + 3y_2$

Sol:- we can write;

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}' = \begin{bmatrix} -2 & 3/2 \\ -4 & 3 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

$$y' = Ay \longrightarrow (1)$$

As linear in $y$ so sol is;

$$y = \underline{x} e^{\lambda t}$$

$$y' = \underline{x} \lambda e^{\lambda t} \longrightarrow (2)$$

From comparing eq (1) & (2) we get

$$(A - \lambda I)\underline{x} = 0 \longrightarrow (A)$$

To find eigen values $\lambda = ?$

Consider
$$\det(A - \lambda I) = 0.$$

$$\det\left(\begin{bmatrix} -2-\lambda & 3/2 \\ -4 & 3-\lambda \end{bmatrix}\right) = 0.$$

$$(-2-\lambda)(3-\lambda) + 6 = 0$$

$$-\cancel{6} + 2\lambda - 3\lambda + \lambda^2 + \cancel{6} = 0.$$

$$\lambda^2 - \lambda = 0.$$

$$\lambda(\lambda - 1) = 0.$$

$$\boxed{\lambda = 0} \quad \text{or} \quad \boxed{\lambda = 1}$$

Consider

$$(A - \lambda I)\underline{x} = 0.$$

$$\begin{bmatrix} -2-\lambda & 3/2 \\ -4 & 3-\lambda \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

$$(-2-\lambda)x_1 + 3/_2\, x_2 = 0. \longrightarrow (B)$$

$$-4x_1 + (3-\lambda)x_2 = 0. \longrightarrow (C)$$

For $\lambda = 0$.

eq (B) $\Rightarrow$

$$-2x_1 + 3/_2\, x_2 = 0.$$

$$x_1 = 3/_4\, x_2 .$$ let $x_2 = r$, $r \in \mathbb{R}$.

$$x^{(1)} = \begin{bmatrix} 3/_4 r \\ r \end{bmatrix} = r\begin{bmatrix} 3/4 \\ 1 \end{bmatrix}.$$

For $\lambda = 1$.

$$-3x_1 + 3/_2\, x_2 = 0.$$

$$x_1 = 1/_2\, x_2$$ let $x_2 = r$, $r \in \mathbb{R}$.

then.

$$x^{(2)} = \begin{bmatrix} 1/_2 r \\ 1r \end{bmatrix} = r\begin{bmatrix} 1/_2 \\ 1 \end{bmatrix}.$$

So the general sol. can be.

$$y = c_1\begin{bmatrix} 3/4 \\ 1 \end{bmatrix}e^{0t} + c_2\begin{bmatrix} 1/_2 \\ 1 \end{bmatrix}e^{t}.$$

Ans

Q5:- $y_1' = 2y_1 + 5y_2$

$y_2' = 5y_1 + 12.5y_2$

Sol:- We can write as;

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}' = \begin{bmatrix} 2 & 5 \\ 5 & 12.5 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}.$$

$$y' = Ay \longrightarrow ①$$

is linear in $y$ so gen. sol is;

$$y = \underline{x} e^{\lambda t}.$$

$$y' = \lambda \underline{x} e^{\lambda t} \longrightarrow ②$$

From comparing ① & ② we get.

$$(A - \lambda I)\underline{x} = 0. \longrightarrow ③$$

To find $\lambda = ?$

Consider $\det(A - \lambda I) = 0.$

$$\det\left(\begin{bmatrix} 2-\lambda & 5 \\ 5 & 12.5-\lambda \end{bmatrix}\right) = 0.$$

$$(2-\lambda)(12.5-\lambda) - 25 = 0.$$

$$\cancel{25} + \lambda^2 - 12.5\lambda - 2\lambda - \cancel{25} = 0$$

$$\lambda^2 - 14.5\lambda = 0.$$

$$\lambda(\lambda - 14.5) = 0,$$

$$\boxed{\lambda = 0} \qquad \boxed{\lambda = 14.5}$$

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Consider eq ① $\Rightarrow$ $(A-\lambda I)\underline{X} = 0$

$$\begin{bmatrix} 2-\lambda & 5 \\ 5 & 12.5-\lambda \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

$$(2-\lambda)x_1 + 5x_2 = 0 \longrightarrow Ⓐ$$
$$5x_1 + (12.5-\lambda)x_2 = 0 \longrightarrow Ⓑ$$

For $\lambda = 0$.

eq Ⓐ $\Rightarrow$ $2x_1 + 5x_2 = 0 \Rightarrow x_1 = -5/2\, x_2$,

let $x_2 = r$ then $x_1 = -5/2\, r$ where $r \in \mathbb{R}$.

$$x^{(1)} = \begin{bmatrix} -5/2\, r \\ r \end{bmatrix} = x^{(1)} = r\begin{bmatrix} -5/2 \\ 1 \end{bmatrix}.$$

For $\lambda = 14.5$

eq Ⓑ $\Rightarrow$ $5x_1 + (12.5 - 14.5)x_2 = 0$.

$5x_1 - 2x_2 = 0$.

$x_1 = 2/5\, x_2$ let $x_2 = r$, $r \in \mathbb{R}$

$$x^{(2)} = \begin{bmatrix} 2/5\, r \\ r \end{bmatrix} = r\begin{bmatrix} 2/5 \\ 1 \end{bmatrix}.$$

So general sol can be.

$$y = c_1\begin{bmatrix} -5/2 \\ 1 \end{bmatrix}e^{0t} + c_2\begin{bmatrix} 2/5 \\ 1 \end{bmatrix}e^{14.5t} \quad \text{Ans}$$

Q4.- $y_1' = ay_2$

$y_2' = -ay_1 + ay_3$

$y_3' = -ay_2$

we can write as;

Sol:- $\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}' = \begin{bmatrix} 0 & a & 0 \\ -a & 0 & a \\ 0 & -a & 0 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$.

$y' = Ay \longrightarrow ①$

As linear in $y$.

so; $y = \underline{x}\, e^{\lambda t}$

$y' = \underline{x}\, \lambda e^{\lambda t} \longrightarrow ②$

from comparing ① & ②. we get

$(A - \lambda I)\underline{x} = 0$

To find $\lambda = ?$ Consider.

$\det(A - \lambda I) = 0$.

$$\det\left(\begin{bmatrix} 0 & a & 0 \\ -a & 0 & a \\ 0 & -a & 0 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}\right) = 0.$$

$$\det\left(\begin{bmatrix} -\lambda & a & 0 \\ -a & -\lambda & a \\ 0 & -a & -\lambda \end{bmatrix}\right) = 0.$$

$-\lambda(\lambda^2 + a^2) - a(\lambda a) = 0.$

$\lambda^3 - \lambda a^2 - \lambda a^2 = 0$

$\lambda^3 - 2\lambda a^2 = 0$

$\lambda(\lambda^2 - 2a^2) = 0.$

$\boxed{\lambda = 0}$ , $\lambda^2 = 2a^2 \Rightarrow \boxed{\lambda = \pm a\sqrt{2}}$

Consider eq ① =)

$(A - \lambda I)\underline{x} = 0$

$$\begin{bmatrix} -\lambda & a & 0 \\ -a & -\lambda & a \\ 0 & -a & -\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-\lambda x_1 + a x_2 = 0 \longrightarrow (1)$$

$$-a x_1 - \lambda x_2 + a x_3 = 0 \longrightarrow (2)$$

$$-a x_2 - \lambda x_3 = 0 \longrightarrow (3)$$

For $\lambda = 0$

eq (2) $\Rightarrow$ $-a x_1 + a x_3 = 0$.

$x_1 = x_3$      let $x_3 = r$.

$\boxed{x_1 = x_3 = r}$      so, $x^{(1)} = \begin{bmatrix} r \\ 0 \\ r \end{bmatrix} = r \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$.

eq (3) $\Rightarrow$ $\boxed{x_2 = 0}$

For $\lambda = \pm a\sqrt{2}$.

eq (1) $\Rightarrow$ $-a\sqrt{2} x_1 + a x_2 = 0 \Rightarrow x_1 = \frac{1}{\sqrt{2}} x_2$.

$\boxed{x_1 = \frac{1}{\sqrt{2}} x_2}$      let $x_2 = r$.

then; $\boxed{x_1 = \frac{1}{\sqrt{2}} r.}$

For $x_3 = ?$

eq (3) $\Rightarrow$ $-a x_2 - a\sqrt{2} x_3 = 0$.

$x_3 = -\frac{1}{\sqrt{2}} x_2 \Rightarrow \boxed{x_3 = -\frac{1}{\sqrt{2}} r}$

so; $x^{(2)} = \begin{bmatrix} \frac{1}{\sqrt{2}} r \\ r \\ -\frac{1}{\sqrt{2}} r \end{bmatrix} = x^{(2)} = r \begin{bmatrix} 1/\sqrt{2} \\ 1 \\ -1/\sqrt{2} \end{bmatrix}$.

For $\lambda = -a\sqrt{2}$.

eq ① ⇒ $a\sqrt{2}\,x_1 + a x_2 = 0$

$\boxed{x_1 = -\frac{1}{\sqrt{2}} x_2}$. let $\boxed{x_2 = r}$, $r \in \mathbb{R}$.

For $x_3 = ?$

eq ③ ⇒ $-a x_2 + a\sqrt{2}\,x_3 = 0$.

$\boxed{x_3 = \frac{1}{\sqrt{2}} x_2} \Rightarrow \boxed{x_3 = \frac{1}{\sqrt{2}} r}$

$$x^{(3)} = \begin{bmatrix} -1/\sqrt{2}\,r \\ r \\ 1/\sqrt{2}\,r \end{bmatrix} = r \begin{bmatrix} -1/\sqrt{2} \\ 1 \\ 1/\sqrt{2} \end{bmatrix}.$$

So the general solution is;

$$y = c_1 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} e^{0t} + c_2 \begin{bmatrix} 1/\sqrt{2} \\ 1 \\ -1/\sqrt{2} \end{bmatrix} e^{a\sqrt{2}t} + \begin{bmatrix} -1/\sqrt{2} \\ 1 \\ 1/\sqrt{2} \end{bmatrix} e^{-a\sqrt{2}t}.$$

Ans

# Initial Value Problem

**Q11:-** $y_1' = -\frac{5}{4}y_1 + \frac{9}{4}y_2$

$y_2' = -y_1 + 2y_2.$

$y_1(0) = -2, \quad y_2(0) = 0.$

**Sol:-** We can write as;

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}' = \begin{bmatrix} -5/4 & 9/4 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}.$$

$y' = Ay \longrightarrow ①$

As linear in $y$.

So, $y = \underline{x}\, e^{\lambda t}$

$y' = \underline{x}\,\lambda e^{\lambda t} \longrightarrow ②$

From ① & ② we set

$(A - \lambda I)\underline{x} = 0. \longrightarrow ④$

To find $\lambda = ?$

Consider $\det(A - \lambda I) = 0.$

$$\det\left(\begin{bmatrix} -5/4-\lambda & 9/4 \\ -1 & 2-\lambda \end{bmatrix}\right) = 0.$$

$\frac{-\cancel{10}+9}{4} = -\frac{1}{4}$

$(-5/4 - \lambda)(2 - \lambda) + 9/4 = 0.$

$-5/2 + 5/4\lambda - 2\lambda + \lambda^2 + 9/4 = 0$

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$\lambda^2 - 3/4\lambda - 1/4 = 0.$

$\boxed{\lambda_1 = 1}, \boxed{\lambda = -1/4}$

Consider; $(A - \lambda I)\underline{x} = 0.$

$$\begin{bmatrix} -5/4 - \lambda & 9/4 \\ -1 & 2-\lambda \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

$(-5/4 - \lambda)x_1 + 9/4 x_2 = 0. \longrightarrow (B)$

$-1 x_1 + (2-\lambda)x_2 = 0 \longrightarrow (C)$

For $\lambda = 1.$

eq (B) $\Rightarrow$ $(-5/4 - 1)x_1 + 9/4 x_2 = 0.$

$\left(\frac{-5-4}{4}\right)x_1 + 9/4 x_2 = 0.$

$\boxed{x_1 = x_2}$ $x_2 = r, \ r \in \mathbb{R}.$

$x^{(1)} = \begin{bmatrix} r \\ r \end{bmatrix} = r\begin{bmatrix} 1 \\ 1 \end{bmatrix}.$

For $\lambda = -1/4.$

eq (B) $\Rightarrow$ $(-5/4 + 1/4)x_1 + 9/4 x_2 = 0.$

$\frac{-5+1}{4} x_1 + 9/4 x_2 = 0.$

$\boxed{x_1 = 9/4 x_2}$

$x_1 = 9/4 \, r$ let $x_2 = r$, $r \in R$

$\therefore$

$$x^{(2)} = \begin{bmatrix} 9/4 \, r \\ r \end{bmatrix} \Rightarrow x^{(2)} = r \begin{bmatrix} 9/4 \\ 1 \end{bmatrix}$$

So The general sol. can be;

$$y = c_2 \begin{bmatrix} 9/4 \\ 1 \end{bmatrix} e^{-1/4 t} + c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{t}.$$

Apply Conditions i.e.

$y_1(0) = -2$, $y_2(0) = 0$.

$$y_1 = 9/4 \, e^{-1/4 t} c_2 + c_1 e^{t}.$$

$$y_2 = c_2 e^{1/4 t} + c_2 e^{t}.$$

$y_1(0) = -2$. & $y_2(0) = 0$.

$$-2 = 9/4 \, c_2 + \cancel{c_1}. \qquad 0 = c_2 + c_1.$$

$$\pm 0 = \pm c_2 \pm \cancel{c_1}$$

$9/4 - 1$

$\frac{9-4}{4} = 5/4$.

$$5/4 \, c_2 = -2$$

$$\boxed{c_2 = -8/5}$$

$$\boxed{c_1 = 8/5}$$

So,

$$\boxed{y = 8/5 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{t} + (-8/5) \begin{bmatrix} 9/4 \\ 1 \end{bmatrix} e^{-1/4 t}.}$$ Ans

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Q13:- $y_1' = 2y_2$ $\quad y_1(0) = 0$

$y_2' = 2y_1$. $\quad y_2(0) = 1$.

Sol:- $\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}' = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$.

$y' = Ay \longrightarrow$ ①.

is linear in $y$.

$y = \underline{X} e^{\lambda t}$

$y' = \underline{X} \lambda e^{\lambda t} \longrightarrow$ ②.

From comparing ① & ②, we get

$(A - \lambda I)\underline{\underline{X}} = 0 \longrightarrow$ Ⓐ.

For $\lambda = ?$

Consider

$\det(A - \lambda I) = 0$.

$\det\left(\begin{bmatrix} -\lambda & 2 \\ 2 & -\lambda \end{bmatrix}\right) = 0$.

$\lambda^2 - 4 = 0$.

$\boxed{\lambda = \pm 2}$

As $(A - \lambda I)\underline{X} = 0$.

$\begin{bmatrix} -\lambda & 2 \\ 2 & -\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

$-\lambda x_1 + 2x_2 = 0. \longrightarrow$ Ⓑ

$2x_1 - \lambda x_2 = 0. \longrightarrow$ Ⓒ.

For $\lambda = 2$. $\quad -2x_1 + 2x_2 = 0$. $\quad x^{(1)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

$\boxed{x_1 = x_2 = r}$

For $\lambda = -2$

$$+2x_1 + 2x_2 = 0,$$

$$\boxed{x_1 = -x_2}$$

$$x^{(2)} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}.$$

So general sol. is;

$$y = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{2t} + c_2 \begin{bmatrix} -1 \\ 1 \end{bmatrix} e^{-2t}.$$

$$y_1 = c_1 e^{2t} + (-c_2) e^{-2t}, \quad y_2 = c_1 e^{2t} + c_2 e^{-2t}.$$

Apply condition.

$$0 = c_1 + (-c_2) \Rightarrow 1 = c_1 + c_2$$

$$1 = c_1 + c_2$$

$$0 = c_1 - c_2 \Rightarrow 2c_1 = 1 \Rightarrow \boxed{c_1 = 1/2}$$

$$\Rightarrow c_1 + c_2 = 1 \Rightarrow \boxed{c_2 = 1/2}$$

So, Particular sol is;

$$\boxed{y = \frac{1}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{2t} + 1/2 \begin{bmatrix} -1 \\ 1 \end{bmatrix} e^{-2t}.}$$

Ans

Q15:- $y_1' = y_1 + 2y_1$. $\quad y_1(0) = 0.25$

$y_2' = 2y_1 + y_2 \quad y_2(0) = -0.25$.

Sol:- $\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}' = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$.

$y' = Ay \longrightarrow ①$

is linear in $y$.

$y = e^{\lambda t} \underline{x}$.

$y' = \underline{x} \lambda e^{\lambda t} \longrightarrow ②$

From comparing ① & ②. we get.

$(A - \lambda I) \underline{X} = 0. \longrightarrow Ⓐ$.

To find $\lambda = ?$

Consider

$\det(A - \lambda I) = 0$.

$\det\left(\begin{bmatrix} 1-\lambda & 2 \\ 2 & 1-\lambda \end{bmatrix}\right) = 0$.

$(1-\lambda)(1-\lambda) - 4 = 0$.

$\lambda^2 - \lambda - \lambda + 1 - 4 = 0$.

$\lambda^2 - 2\lambda - 3 = 0$.

$\lambda^2 - 3\lambda + \lambda - 3 = 0$.

$\lambda(\lambda - 3) + 1(\lambda - 3) = 0$.

$(\lambda - 3)(\lambda + 1) = 0$

$\boxed{\lambda = 3}, \boxed{\lambda = -1}$

Consider $(A-\lambda I)\underline{x}=0$.

$$\begin{bmatrix}1-\lambda & 2\\ 2 & 1-\lambda\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}.$$

$$(1-\lambda)x_1+2x_2=0 \longrightarrow (B)$$

$$2x_1+(1-\lambda)x_2=0. \longrightarrow (C)$$

For $\lambda=-1$

$(B) \Rightarrow$ $\quad +2x_1+2x_2=0$.

$$\boxed{x_1=-x_2}=r, \quad r\in\mathbb{R}.$$

$$x^{(1)}=\begin{bmatrix}-r\\ r\end{bmatrix}=r\begin{bmatrix}-1\\ 1\end{bmatrix}.$$

For $\lambda=3$.

$(B) \Rightarrow$ $\quad -2x_1+2x_2=0$

$$\boxed{x_1=x_2=r}, \; r\in\mathbb{R}$$

$$x^{(2)}=\begin{bmatrix}r\\ r\end{bmatrix}=r\begin{bmatrix}1\\ 1\end{bmatrix}.$$

So; $\quad y=c_1\begin{bmatrix}-1\\ 1\end{bmatrix}e^{-t}+c_2\begin{bmatrix}1\\ 1\end{bmatrix}e^{3t}$.

Apply condition but

$y_1=-c_1e^{-t}+c_2e^{3t}$, $\quad y_2=c_1e^{-t}+c_2e^{3t}$.

$y_1(0) = 0.25$

$y_2(0) = -0.25$

$0.25 = \cancel{\xi} - c_1 + c_2$

$-0.25 = c_1 + c_2$

Add both eqns.

$$\begin{aligned} 0.2\cancel{5} &= -\cancel{c_1} + c_2 \\ -0.\cancel{25} &= \cancel{c_1} + c_2 \end{aligned}$$

$$\boxed{c_2 = 0}$$

$c_1 = ?$

$0.25 = -c_1 + c_2$

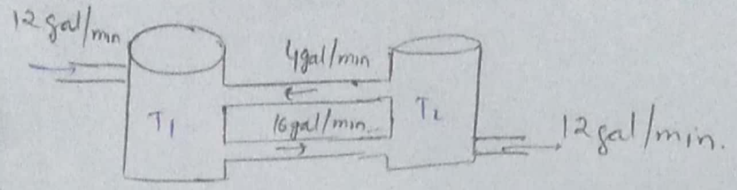
$$\boxed{c_1 = -0.25}$$

So the particular solution is;

$$\boxed{y_1 = +0.25\,e^{-t}} \quad \& \quad \boxed{y_2 = -0.25\,e^{-t}}$$

Ans

Q18




Sol:- $y_1' = -\frac{16}{200} y_1 + \frac{4}{200} y_2 + 0.12$.

$y_2' = \frac{16}{200} y_1 + \frac{(\cancel{-}4-12)}{200} y_2$.

So we can write;

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}' = \begin{bmatrix} -16/200 & 4/200 \\ 16/200 & -16/200 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}.$$

$y' = Ay. \longrightarrow (1)$

As linear in $y$;

so; $y = \underline{x} e^{\lambda t}$.

$y' = \underline{x} \lambda e^{\lambda t} \longrightarrow (2)$.

From comparing (1) & (2) we get.

$(A - \lambda I)\underline{x} = 0. \longrightarrow (4)$.

To find $\lambda = ?$

Consider $\det(A - \lambda I) = 0$.

$$\det\left(\begin{bmatrix} -16/200-\lambda & 4/200 \\ 16/200 & -16/200-\lambda \end{bmatrix}\right) = 0.$$

$$(-16/200 - \lambda)(-16/200 - \lambda) - \frac{64}{40000} = 0.$$

$$\frac{256}{40000} + \frac{16}{200}\lambda + \frac{16}{200}\lambda + \lambda^2 - \frac{64}{40000} = 0$$

$$\lambda^2 + \frac{16}{100}\lambda + \frac{3}{625} = 0.$$

$$\boxed{\lambda_1 = -1/25} \quad , \quad \boxed{\lambda_2 = -3/25}$$

Consider;

$$(A - \lambda I)\underline{x} = 0.$$

$$\begin{bmatrix} -16/200 - \lambda & 4/200 \\ 16/200 & -16/200 - \lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

$$\left(\frac{-16}{200} - \lambda\right)x_1 + 4/200\, x_2 = 0. \longrightarrow (B)$$

$$16/200\, x_1 + \left(\frac{-16}{200} - \lambda\right)x_2 = 0. \longrightarrow (C).$$

For $\lambda = -1/25$.

$$(-16/200 + 1/25)x_1 + 4/20\, x_2 = 0.$$

$$-1/25\, x_1 + 1/50\, x_2 = 0.$$

$$\boxed{x_1 = 1/2\, x_2}$$

let $x_2 = r$, $r \in \mathbb{R}$

$$x^{(1)} = \begin{bmatrix} 1/2 \\ 1 \end{bmatrix}.$$

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For $\lambda = -3/25$.

eq (B) $\Rightarrow$ $\left(-\frac{16}{200} + \frac{3}{25}\right) x_1 + \frac{4}{200} x_2 = 0.$

$\frac{1}{25} x_1 = \frac{-1}{250} x_2$

$x_1 = -\frac{1}{2} x_2.$ let $x_2 = r$, $r \in \mathbb{R}$

$x^{(2)} = \begin{bmatrix} -1/2 \\ 1 \end{bmatrix}.$

So, the general sol. is,

$$\boxed{y = c_1 \begin{bmatrix} 1/2 \\ 1 \end{bmatrix} e^{-1/25 t} + c_2 \begin{bmatrix} -1/2 \\ 1 \end{bmatrix} e^{-3/25 t}}$$

Ans