# Ex # 4.6

2. $y_1' = y_1 + y_2 + 10\cos t$.

$y_2' = 3y_1 - y_2 - 10\sin t$.

Sol:- $\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}' = \begin{bmatrix} 1 & 1 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} + \begin{bmatrix} 10 \\ -0 \end{bmatrix}\cos t + \begin{bmatrix} 0 \\ -10 \end{bmatrix}\sin t$

$$y' = Ay + g(t) \longrightarrow ①$$

To find eigen value = ?

$x \neq 0$.

$A\underline{x} = \lambda \underline{x}$.

$(A - \lambda I)\underline{x} = 0 \longrightarrow ②$

For $\lambda$ $\det(A - \lambda I) = 0$.

$$\det\left(\begin{bmatrix} 1 & 1 \\ 3 & -1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}\right) = 0.$$

$$\det\left(\begin{bmatrix} 1-\lambda & 1 \\ 3 & -1-\lambda \end{bmatrix}\right) = 0.$$

$(1-\lambda)(-1-\lambda) - 3 = 0$.

$-1 + \cancel{\lambda} - \cancel{\lambda} + \lambda^2 - 3 = 0$.

$\lambda^2 = 4 \Rightarrow \boxed{\lambda = \pm 2}$

So; To find eigen vector i.e $\underline{x} = ?$

consider eq ② =)

$(A - \lambda I)\underline{x} = 0$.

$$\left(\begin{bmatrix} 1 & 1 \\ 3 & -1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}\right)\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1-\lambda & 1 \\ 3 & -1-\lambda \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$(1-\lambda)x_1 + x_2 = 0 \longrightarrow (3)$

$3x_1 + (-1-\lambda)x_2 = 0. \longrightarrow (4).$

Put $\lambda = 2$

$-x_1 + x_2 = 0$

$x_1 = x_2 = r \in \mathbb{R}$

so;

$x^{(1)} = \begin{bmatrix} r \\ r \end{bmatrix} = r\begin{bmatrix} 1 \\ 1 \end{bmatrix}.$

For $\lambda = -2$.

& (3) $\Rightarrow$

$3x_1 + x_2 = 0$

$-x_2 = +x_1$

Let $x_1 = r$

$x_2 = -3r.$

$x^{(2)} = \begin{bmatrix} r \\ -3r \end{bmatrix} = r\begin{bmatrix} 1 \\ -3 \end{bmatrix}.$

so;

Solution (complementary)

$y_c = c_1\begin{bmatrix} 1 \\ 1 \end{bmatrix}e^{2t} + c_2\begin{bmatrix} 1 \\ -3 \end{bmatrix}e^{-2t} \longrightarrow (A)$

To find particular solution.

$y_p = A\cos t + B\sin t.$

$y_p = \begin{bmatrix} A_0 \\ A_1 \end{bmatrix}\cos t + \begin{bmatrix} B_0 \\ B_1 \end{bmatrix}\sin t.$

To find $A_0, A_1$ & $B_0, B_1$.

$y_p' = -\begin{bmatrix} A_0 \\ A_1 \end{bmatrix}\sin t + \begin{bmatrix} B_0 \\ B_1 \end{bmatrix}\cos t.$

eq ① $\Rightarrow$ $y' = AY + g(t)$.

$$-\begin{bmatrix} A_0 \\ A_1 \end{bmatrix}\sin t + \begin{bmatrix} B_0 \\ B_1 \end{bmatrix}\cos t = \begin{bmatrix} 1 & 1 \\ 3 & -1 \end{bmatrix}\left(\begin{bmatrix} A_0 \\ A_1 \end{bmatrix}\cos t + \begin{bmatrix} B_0 \\ B_1 \end{bmatrix}\sin t\right)$$

$$+ \begin{bmatrix} 10 \\ 0 \end{bmatrix}\cos t + \begin{bmatrix} 0 \\ -10 \end{bmatrix}\sin t.$$

By comparing co-efficient of sint & cost.

cost:
$$\begin{bmatrix} B_0 \\ B_1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} A_0 \\ A_1 \end{bmatrix} + \begin{bmatrix} 10 \\ 0 \end{bmatrix}$$

$B_0 = A_0 + A_1 + 10. \longrightarrow ⑤$

$B_1 = 3A_0 - A_1 + 0. \longrightarrow ⑥$.

sint:
$$-\begin{bmatrix} A_0 \\ A_1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} B_0 \\ B_1 \end{bmatrix} + \begin{bmatrix} 0 \\ -10 \end{bmatrix}$$

$-A_0 = B_0 + B_1 + 0. \longrightarrow ⑦$

$-A_1 = 3B_0 - B_1 - 10. \longrightarrow ⑧$.

eq ⑤ + eq ⑥ $\Rightarrow$ ~~$A_0 + A_1 +$~~

$B_0 + B_1 = 4A_0 + 10$.

put in eq ⑦.

$-A_0 = 4A_0 + 10$.

$-5A_0 = 10. \Rightarrow \boxed{A_0 = -2}$

eq ⑤ & ⑥ $\Rightarrow$ $B_0 = -2 + A_1 + 10$.

$\boxed{B_0 = A_1 + 8} \longrightarrow ⑪$

eq (8) $\Rightarrow$ $-A_1 = 3(A_1+8) - B_1 - 10.$

$$-A_1 = 3A_1 + (8)\times 3 - B_1 - 10.$$

$$-4A_1 = -B_1 + 14 \longrightarrow (9)$$

eq (6) $\Rightarrow$

$$B_1 = 3A_0 - A_1.$$

$$B_1 = -6 - A_1.$$

$$A_1 = -6 - B_1. \longrightarrow (10)$$

eq (10) - eq (9)

$$\begin{aligned} -4A_1 &= -\cancel{B_1} + 14 \\ \mp A_1 &= \mp \cancel{B_1} \mp 6 \\ \hline -5A_1 &= 20. \end{aligned}$$

$$\boxed{A_1 = -4}$$

eq (10) $\Rightarrow$

$$-B_1 = 6 - 4.$$

$$\boxed{B_1 = -2}$$

eq (11) $\Rightarrow$

$$B_0 = A_1 + 8.$$

$$B_0 = -4 + 8 \Rightarrow \boxed{B_0 = 4}$$

So;

$$y_p = \begin{bmatrix} -2 \\ -4 \end{bmatrix} \cos t + \begin{bmatrix} -2 \\ 4 \end{bmatrix} \sin t$$

So The G. solution is;

$$y = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{2t} + c_2 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-2t} + \begin{bmatrix} -2 \\ -4 \end{bmatrix} \cos t + \begin{bmatrix} -2 \\ 4 \end{bmatrix} \sin t$$

Ans

Q3:- $y_1' = y_2 + e^{2t}$.

$y_2' = y_1 - 3e^{2t}$.

Sol:- $\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}' = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} + \begin{bmatrix} 1 \\ -3 \end{bmatrix} e^{2t}$

$y' = Ay + g(t) \longrightarrow$ (A).

As; sol is $y = y_c + y_p \longrightarrow$ ($\lambda$).

To find $y_c = ?$

As; for linear;

$y = \underline{x} e^{\lambda t}$.

where $\underline{x}$ is eigen vector & $\lambda$ = eigen value

$y' = \underline{x} \lambda e^{\lambda t}$

so; we get.

$A\underline{x} = \lambda \underline{x}$.

$(A - \lambda I)\underline{x} = 0 \longrightarrow$ (B).

To find $\lambda = ?$

$x \neq 0$.

so;

$\det(A - \lambda I) = 0$

$\det\begin{pmatrix} -\lambda & 1 \\ 1 & -\lambda \end{pmatrix} = 0$.

$\lambda^2 - 1 = 0$

$\boxed{\lambda = \pm 1}$

Consider eq (B) for $\underline{x} = ?$

As; $(A - \lambda I)\underline{x} = 0$.

$\begin{bmatrix} -\lambda & 1 \\ 1 & -\lambda \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{matrix} 0 \\ 0 \end{matrix}$.

$-\lambda x_1 + x_2 = 0 \longrightarrow ①$

$x_1 + \lambda x_2 = 0 \longrightarrow ②.$

For $\lambda = 1$.

eq $Ⓐ^1 \Rightarrow \quad -x_1 + x_2 = 0$

$\boxed{x_2 = x_1}$

So let $x_1 = r$ where $r \in \mathbb{R}$.

So, $\quad x_1 = x_2 = r$.

$x^{(1)} = \begin{bmatrix} r \\ r \end{bmatrix} = r \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$

For $x = -1$.

eq ① $\Rightarrow \quad x_1 + x_2 = 0$.

$x_1 = -x_2$. then $x_2 = -r$ where $r \in \mathbb{R}$.

if $x_1 = r$

$x^{(2)} = \begin{bmatrix} r \\ -r \end{bmatrix} = r \begin{bmatrix} 1 \\ -1 \end{bmatrix}.$

So general solution is;
$(y_c)$

$y_c = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{t} + c_2 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-t} \longrightarrow ③.$

Now to find $y_p = ?$

As; $\quad y_p = a e^{2t}$

$y_p' = 2a e^{2t}$.

Put values in eq Ⓐ.

$$2ae^{2t} = \begin{bmatrix} 0 & 1 \\ 01 & 0 \end{bmatrix}\left(\begin{bmatrix} a_0 \\ a_1 \end{bmatrix} e^{2t}\right) + \begin{bmatrix} 1 \\ -3 \end{bmatrix} e^{2t}$$

$$2\begin{bmatrix} a_0 \\ a_1 \end{bmatrix} e^{2t} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} a_0 \\ a_1 \end{bmatrix} e^{2t} + \begin{bmatrix} 1 \\ -3 \end{bmatrix} e^{2t}$$

By comparing co-efficient of $e^{2t}$.

$e^{2t}$: $2a_0 = a_1 + 1$. $\Rightarrow a_1 - 2a_0 + 1 = 0 \longrightarrow (4)$

$2a_1 = a_0 - 3$. $\quad a_0 - 2a_1 - 3 = 0 \longrightarrow (5)$.

Multiply 2 with eq (4)

$$\cancel{2a_1} - 4a_0 + 2 = 0.$$
$$\cancel{-2a_1} + a_0 - 3 = 0$$

---

$$3a_0 = 1.$$
$$\boxed{a_0 = 1/3}$$

$a_1 = ?$

$2a_0 = a_1 + 1$.

$2(1/3) = a_1 + 1 \Rightarrow \quad a_1 = 2/3 - 1 \Rightarrow \frac{2-3}{3}$

$$\boxed{a_1 = -1/3}$$

So; $y_p = \begin{bmatrix} 1/3 \\ -1/3 \end{bmatrix} e^{2t}$.

So The general solution is;

eq (x) $\Rightarrow$

$y = y_c + y_p$.

$$\boxed{y = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{2t} + c_2 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-t} + \begin{bmatrix} 1/3 \\ -1/3 \end{bmatrix} e^{2t}}$$ Ans

Q4:- $y_1' = 4y_1 - 8y_2 + 2\cosh t$.
$y_2' = 2y_1 - 6y_2 + \cosh t + 2\sinh t$.

Sol:-
$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}' = \begin{bmatrix} 4 & -8 \\ 2 & -6 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \end{bmatrix}\cosh t + \begin{bmatrix} 0 \\ 2 \end{bmatrix}\sinh t.$$

As; $y' = Ay + g(t) \longrightarrow$ (i).

Since the general solution is;

$$y = y_c + y_p \longrightarrow \text{(ii)}.$$

For $y_p = ?$
Consider.
$$(A - \lambda I)\underline{X} = 0. \longrightarrow \text{(iii)}.$$

To find $\lambda = ?$
Consider $X \neq 0$.

Since $\det(A - \lambda I) = 0$.

$$\det\left(\begin{bmatrix} 4-\lambda & -8 \\ 2 & -6-\lambda \end{bmatrix}\right) = 0.$$

$$(4-\lambda)(-6-\lambda) + 16 = 0.$$

$$-24 + 6\lambda - 4\lambda + \lambda^2 + 16 = 0.$$

$$\lambda^2 + 2\lambda - 8 = 0.$$

$$\lambda^2 + 4\lambda - 2\lambda - 8 = 0.$$

$$\lambda(\lambda+4) - 2(\lambda+4) = 0.$$

$\lambda + 4 = 0$ or $\lambda - 2 = 0$.

$\boxed{\lambda = -4}$ $\boxed{\lambda = 2}$

To find $\underline{X} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = ?$

As; $\Rightarrow$ (iii) $\Rightarrow (A - \lambda I)\underline{X} = 0$.

$$\begin{bmatrix} 4-\lambda & -8 \\ 2 & -6-\lambda \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

$(4-\lambda)x_1 - 8x_2 = 0. \longrightarrow$ (A)

$2x_1 + (-6-\lambda)x_2 = 0. \longrightarrow$ (B).

Put $\lambda = -4$.

$8X_1 - 8x_2 = 0.$

$\boxed{x_1 = x_2}$

so let $x_1 = r = x_2$ where $r \in \mathbb{R}$.

$$\underline{x}^{(1)} = \begin{bmatrix} r \\ r \end{bmatrix} \Rightarrow r\begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

For $\lambda = 2$.

& (A) $\Rightarrow$ $2x_1 - 8x_2 = 0.$

$x_1 = 4x_2$

let $x_2 = r.$

$x_1 = 4r.$ when $r \in \mathbb{R}$.

$$x^{(2)} = \begin{bmatrix} 4r \\ r \end{bmatrix} = r\begin{bmatrix} 4 \\ 1 \end{bmatrix}.$$

So; $y_c = c_1\begin{bmatrix} 1 \\ 1 \end{bmatrix}e^{-4t} + c_2\begin{bmatrix} 4 \\ 1 \end{bmatrix}e^{2t}$

To find $y_p = ?$

Consider $y_p = a\cosh t + b\sinh t.$

$y_p' = a\sinh t + b\cosh t.$

& (1) $\Rightarrow$ $\begin{bmatrix} a_0 \\ a_1 \end{bmatrix}\sinh t + \begin{bmatrix} b_0 \\ b_1 \end{bmatrix}\cosh t = \begin{bmatrix} 4 & -8 \\ 2 & -6 \end{bmatrix}\left(\begin{bmatrix} a_0 \\ a_1 \end{bmatrix}\cosh t + \begin{bmatrix} b_0 \\ b_1 \end{bmatrix}\sinh t\right)$

$+ \begin{bmatrix} 2 \\ 1 \end{bmatrix}\cosh t + \begin{bmatrix} 0 \\ 2 \end{bmatrix}\sinh t.$

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cosht: $b_0 = 4a_0 - 8a_1 + 2 \Rightarrow a_0 - 2a_1 - \frac{b_0}{4} = -\frac{2}{4}$ → (a)

$b_1 = 2a_0 - 6a_1 + 1 \Rightarrow 2a_0 - 6a_1 + 0 - b_1 = -1$ → (b)

sinht: $a_0 = 4b_0 - 8b_1 + 0 \Rightarrow a_0 - 4b_0 - 8b_1 = 0$ → (c)

$a_1 = 2b_0 - 6b_1 + 2 \Rightarrow a_1 - 2b_0 - 6b_1 = -2$ → (d).

In matrix form;

$R_2 \leftrightarrow R_3$

$$\left[\begin{array}{cccc|c} 1 & -2 & -1/4 & 0 & -1/2 \\ 2 & -6 & 0 & -1 & -1 \\ 1 & +0 & -4 & -8 & 0 \\ 0 & 1 & -2 & -6 & -2 \end{array}\right] \Rightarrow \left[\begin{array}{cccc|c} 1 & -2 & -1/4 & 0 & -1/2 \\ 1 & 0 & -4 & -8 & 0 \\ 2 & -6 & 0 & -1 & -1 \\ 0 & 1 & -2 & -6 & -2 \end{array}\right]$$

$$\sim \left[\begin{array}{cccc|c} 1 & -2 & -1/4 & 0 & -1/2 \\ 0 & -4 & 1/4 & -1 & -1/2 \\ 0 & & & & \\ 0 & & & & \end{array}\right] \quad R_2 - R_1,\ R_3 - 2R_1$$

$$\sim \left[\begin{array}{cccc|c} 1 & -2 & -1/4 & 0 & -1/2 \\ 0 & 2 & -15/4 & -8 & 1/2 \\ 0 & -2 & 1/2 & -1 & 0 \\ 0 & 1 & -2 & -6 & -2 \end{array}\right] \quad R_2 - R_1,\ R_3 - 2R_1.$$

$$\sim \left[\begin{array}{cccc|c} 1 & -2 & -1/4 & 0 & -1/2 \\ 0 & 2 & -15/4 & -8 & 1/2 \\ 0 & 1 & -1/4 & 1/2 & 0 \\ 0 & 1 & -2 & -6 & -2 \end{array}\right] \quad R_3 \times -\frac{1}{2}.$$

$$\sim \left[\begin{array}{cccc|c} 1 & -2 & -1/4 & 0 & -1/2 \\ 0 & 2 & -15/4 & -8 & 1/2 \\ 0 & 0 & 29/4 & 15 & -1 \\ 0 & 0 & -7/4 & -\frac{113}{2} & -2 \end{array}\right] \quad R_4 - R_3,\ R_3 - 2R_2$$

We can write;

$-1 + 1/2 = \frac{-2+1}{2} = -1/2$

$-1+16 = 15$

$-4 + 1/4 = \frac{-16+1}{4} = -15/4$

$-1/4 + 15/2 = \frac{-1+30}{4} = 29/4$

$-6 - 1/2 = \frac{-12-1}{2} = \frac{-13}{2}$

$-2 + 1/4 = \frac{-8+1}{4} = -7/4$

$-2 + \frac{15}{2} = \frac{-4+15}{2} = 11/2$

$-6 + 8/6 = 610$

$-\frac{7}{4}b_0 - \frac{13}{4}b_1 = -2. \Rightarrow -7b_0 - 13b_1 = -8 \quad (\times \text{"4"})$

$\hookrightarrow$ ⓑ.

$29b_0 + 60b_1 = -4. \quad (\times \text{ by "4"}).$

$\frac{29}{4}b_0 + 15b_1 = -1.$

Multiply 60 with ① eq & 13 to 2nd eq.

$-420b_0 - 780b_1 = -480.$

$377b_0 + 780b_1 = -52$

$-43b_0 = 532.$

$\boxed{b_0 = 12.4}$

eq ⓑ $\Rightarrow$ $-7(12.4) - 13b_1 = -8.$

$\boxed{b_1 = 6}$

eq ⓓ $\Rightarrow$ $a_1 - \overset{2}{\cancel{6}}b_0 - 6b_1 = -2.$

$a_1 - 2(12.4) - 6(6) = -2$

$\boxed{a_1 = 58.8}$

eq ⓐ $\Rightarrow$ $a_0 - 2a_1 - \frac{b_0}{4} = -2/4.$

$a_0 - 2(58.8) - \frac{12.4}{4} = -1/2.$

$\boxed{a_0 = 120.2}$

So, gen eqn is;

$y = c_1\begin{bmatrix}1\\1\end{bmatrix}e^{-4t} + c_2\begin{bmatrix}4\\1\end{bmatrix}e^{2t} + \begin{bmatrix}120.2\\58.8\end{bmatrix}\cos 4t + \begin{bmatrix}12.4\\6\end{bmatrix}\sin 4t$. Ans

Q5:- $y_1' = 4y_1 + 3y_2 + t$

$y_2' = -2y_1 - y_2 - 2t$

Sol:-

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}' = \begin{bmatrix} 4 & +3 \\ -2 & -1 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} + \begin{bmatrix} 1 \\ -2 \end{bmatrix} t$$

$$y' = Ay + g(t) \longrightarrow (A)$$

Since gen. solution is;

$$y = y_c + y_p \longrightarrow (1)$$

To find $y_p = ?$

As; $(A - \lambda I)\underline{X} = 0$

To find $\lambda = ?$

Consider

$$\det(A - \lambda I) = 0$$

$$\det\left(\begin{bmatrix} 4 & 3 \\ -2 & -1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}\right) = 0$$

$$\det\left(\begin{bmatrix} 4-\lambda & 3 \\ -2 & -1-\lambda \end{bmatrix}\right) = 0$$

$$(4-\lambda)(-1-\lambda) + 6 = 0$$

$$-4 - 4\lambda + \lambda + \lambda^2 + 6 = 0$$

$$\lambda^2 - 3\lambda + 2 = 0$$

$$\lambda^2 - 2\lambda - \lambda + 2 = 0$$

$$\lambda(\lambda - 2) - 1(\lambda - 2) = 0$$

$$\boxed{\lambda = 2} \;\&\; \boxed{\lambda = 1}$$

To find $\underline{X} = ?$

Consider $(A - \lambda I)\underline{X} = ?$

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$$\begin{bmatrix} 4-\lambda & 3 \\ -2 & -1-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

$$(4-\lambda)x_1 + 3x_2 = 0. \longrightarrow ②$$

$$-2x_1 + (-1-\lambda)x_2 = 0. \longrightarrow ③.$$

For $\lambda = 1$.

eq ② $\Rightarrow$ $3x_1 + 3x_2 = 0.$

$x_1 = -x_2$.

let $x_1 = r$ than

$x_2 = -r$ where $r \in \mathbb{R}$.

$$x^{(1)} = \begin{bmatrix} r \\ -r \end{bmatrix} = r\begin{bmatrix} 1 \\ -1 \end{bmatrix}.$$

For $\lambda = 2$

$2x_1 + 3x_2 = 0.$

$x_1 = -3/2\, x_2.$

let $x_2 = r$ where $r \in \mathbb{R}$.

than

$x_1 = -3/2\, r.$

$$x^{(2)} = \begin{bmatrix} -3/2\, r \\ r \end{bmatrix} = r\begin{bmatrix} -3/2 \\ 1 \end{bmatrix}.$$

So the $y_c = c_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{t} + c_2 \begin{bmatrix} -3/2 \\ 1 \end{bmatrix} e^{2t}.$

<u>To find $y_p = ?$</u>

$y_p = a_0 + b t.$

$y_p' = b.$

Put in eq Ⓐ.

$$\begin{bmatrix} b_0 \\ b_1 \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ -2 & -1 \end{bmatrix}\left(\begin{bmatrix} a_0 \\ a_1 \end{bmatrix} + \begin{bmatrix} b_0 \\ b_1 \end{bmatrix} t\right) + \begin{bmatrix} 1 \\ -2 \end{bmatrix} t.$$

By comparing Co-efficient of $t$.

$t$: $\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 4b_0 + 3b_1 \\ -2b_0 + (-1)b_1 \end{bmatrix} + \begin{bmatrix} 1 \\ -2 \end{bmatrix}.$

$0 = 4b_0 + 3b_1 + 1$

$0 = -2b_0 - b_1 - 2$ (x by 2)

$0 = 4b_0 - 2b_1 - 4$

$0 = 4b_0 + 3b_1 + 1$

$-5b_1 - 5 = 0$

$b_1 = -1$

$0 = 4(b_0) + 3(-1) + 1$

$\boxed{b_0 = +1/2}$

$x$:

$b_0 = 4a_0 + 3a_1 \longrightarrow$ ⓔ

$b_1 = -2a_0 - a_1 \Rightarrow$ Multiply by 2

$2b_1 = -4a_0 - 2a_1$

$b_0 = 4a_0 + 3a_1$

$a_1 = 2b_1 + b_0$

$a_1 = 2(1/2) + (-1)$ By putting values

$\boxed{a_1 = 0}$

eq ⓔ $\Rightarrow$ $b_0 = 4a_0 + 3a_1$

$\cancel{3b_0} = \cancel{4a_0}$

$b_0 = 4a_0$

$a_0 = 1/2 \times 4 \Rightarrow \boxed{a_0 = 1/8}$

So the general solution is;

$$\boxed{y = c_1\begin{bmatrix}1\\-1\end{bmatrix}e^{t} + c_2\begin{bmatrix}-3/2\\1\end{bmatrix}e^{2t} + \begin{bmatrix}1/8\\0\end{bmatrix} + \begin{bmatrix}1/2\\-1\end{bmatrix}t}$$

Ans

Q6:- $y_1' = 4y_2$

$y_2' = 4y_1 - 16t^2 + 2.$

Sol:- $\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}' = \begin{bmatrix} 0 & 4 \\ 4 & 0 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} + \begin{bmatrix} 0 \\ -16 \end{bmatrix}t^2 + \begin{bmatrix} 0 \\ 2 \end{bmatrix}.$

$y' = AY + g(t) \longrightarrow (1).$

As the general solution is;

$y = y_c + y_p \longrightarrow (2).$

To find $y_c = ?$

Consider $(A - \lambda I)\underline{X} = 0. \longrightarrow (A)$

To find eigen values i.e

$\lambda = ?$

Consider $X \neq 0$ for this;

$\det(A - \lambda I) = 0.$

$\det\left(\begin{bmatrix} -\lambda & 4 \\ 4 & -\lambda \end{bmatrix}\right) = 0.$

$\lambda^2 - 16 = 0.$

$\boxed{\lambda = \pm 4}$

To find eigen vector consider.

$(A - \lambda I)\underline{X} = 0.$

$\left(\begin{bmatrix} 0 & 4 \\ 4 & 0 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}\right)\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0.$

$$\begin{bmatrix} -\lambda & 4 \\ 4 & -\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

$$-\lambda x_1 + 4x_2 = 0.$$

$$4x_1 - \lambda x_2 = 0.$$

For $\lambda = 4$

$$-4x_1 + 4x_2 = 0.$$

$x_1 = x_2 = r$ where $r \in \mathbb{R}$.

$$x^{(1)} = \begin{bmatrix} r \\ r \end{bmatrix} = r \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

For $\lambda = -4$.

$$4x_1 + 4x_2 = 0.$$

$x_1 = -x_2$ than $x_2 = +r$.

let $x_1 = -r$.

$$x^{(2)} = \begin{bmatrix} -r \\ r \end{bmatrix} = r \begin{bmatrix} -1 \\ 1 \end{bmatrix}.$$

So the $y_c = ?$

$$\boxed{y_c = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{4t} + c_2 \begin{bmatrix} -1 \\ 1 \end{bmatrix} e^{-4t}}$$

To find $y_p = ?$

$$y_p = a + bt + ct^2$$

$y_p' = b + 2c\,t$

Put values in eq ①.

$$\begin{bmatrix} b_0 \\ b_1 \end{bmatrix} + 2\begin{bmatrix} c_0 \\ c_1 \end{bmatrix} t = \begin{bmatrix} 0 & 4 \\ 4 & 0 \end{bmatrix}\left(\begin{bmatrix} a_0 \\ a_1 \end{bmatrix} + \begin{bmatrix} b_0 \\ b_1 \end{bmatrix} t + \begin{bmatrix} c_0 \\ c_1 \end{bmatrix} t^2\right) + \begin{bmatrix} 0 \\ -16 \end{bmatrix} t^2 + \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

By comparing coefficient of $t^2, t$ & $t^0$

$t^0$: $b_0 = 4a_1 + 0 \Rightarrow \boxed{a_1 = 0}$

$b_1 = 4a_0 + 2 \Rightarrow 1 + (-2) = 4a_0 \Rightarrow \boxed{a_0 = -1/4}$

$t^1$: $2c_0 = 4b_1 \Rightarrow \boxed{c_0 = 2b_1}$

$\boxed{c_1 = 2b_0}$

$t^2$: $0 = 4c_1 + 0 \Rightarrow c_1 = 0$

$0 = 4c_0 - 16 \Rightarrow \boxed{c_0 = 4}$

$\boxed{b_0 = 0}$

$\boxed{b_1 = 1}$

So, $y_p = a + bt + ct^2$

$$= \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} + \begin{bmatrix} b_0 \\ b_1 \end{bmatrix} t + \begin{bmatrix} c_0 \\ c_1 \end{bmatrix} t^2$$

$$y_p = \begin{bmatrix} 0 \\ -1/4 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} t + \begin{bmatrix} 2 \\ 0 \end{bmatrix} t^2$$

So the general solution becomes

$$y = c_1\begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{4t} + \begin{bmatrix} -1 \\ 1 \end{bmatrix} e^{-4t} + \begin{bmatrix} 0 \\ -1/4 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} t + \begin{bmatrix} 2 \\ 0 \end{bmatrix} t^2$$

Ans

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Q7:- $y_1' = -y_1 - y_2 + 5t + 5.$

$y_2' = -4y_1 + 2y_2 + 3e^{-t} - 15t - 2.$

Sol:-

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}' = \begin{bmatrix} -1 & -1 \\ -4 & 2 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 5 \\ -15 \end{bmatrix} t + \begin{bmatrix} 5 \\ -2 \end{bmatrix} + \begin{bmatrix} 0 \\ 3 \end{bmatrix} e^{-t}}_{g(t)} \to (A)$$

$y' = Ay + g(t) \to (1).$

As general solution of eq (1) is

$y = y_c + y_p \longrightarrow (1).$

<u>To find $y_c = ?$</u>

Consider

$(A - \lambda I)\underline{X} = 0.$

To find eigenvalue ?

Consider $\underline{X} \neq 0$

$\det(A - \lambda I) = 0.$

$$\det\left(\begin{bmatrix} -1-\lambda & -1 \\ -4 & 2-\lambda \end{bmatrix}\right) = 0.$$

$(-1-\lambda)(2-\lambda) - 4 = 0.$

$-2 + \lambda - 2\lambda + \lambda^2 - 4 = 0.$

$\lambda^2 - \lambda - 6 = 0.$

$\lambda^2 - 3\lambda + 2\lambda - 6 = 0.$

$\lambda(\lambda - 3) + 2(\lambda - 3) = 0.$

$\boxed{\lambda = -2}$ or $\boxed{\lambda = 3}$

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To find eigen vectors $\underline{X} = ?$

Consider

$$(A - \lambda I)\underline{X} = 0.$$

$$\left(\begin{bmatrix} -1 & -1 \\ -4 & -2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}\right)\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

$$\begin{bmatrix} -1-\lambda & -1 \\ -4 & -2-\lambda \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

$$(-1-\lambda)x_1 - x_2 = 0. \longrightarrow (B)$$

$$-4x_1 + (-2-\lambda)x_2 = 0. \longrightarrow (C).$$

For $\lambda = -2.$

$\{ (B) \Rightarrow$

$$x_1 - x_2 = 0.$$

$$\boxed{x_1 = x_2} \quad x_1 = x_2 = r \quad \forall\ r \in \mathbb{R}.$$

$$x^{(1)} = \begin{bmatrix} r \\ r \end{bmatrix} = r\begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

For $\lambda = 3.$

$\{ (B) \Rightarrow$

$$-4x_1 - x_2 = 0.$$

$$x_1 = -\frac{1}{4}x_2.$$

let $x_2 = r$ where $r \in \mathbb{R}$.

Then $x_1 = -\frac{1}{4}r.$

so, $$X^{(2)} = \begin{bmatrix} r \\ -\frac{1}{4}r \end{bmatrix} = r\begin{bmatrix} 1 \\ -\frac{1}{4} \end{bmatrix}.$$

so $$y_c = c_1\begin{bmatrix} 1 \\ 1 \end{bmatrix}e^{-2t} + c_2\begin{bmatrix} 1 \\ -\frac{1}{4} \end{bmatrix}e^{3t}.$$

To find $y_p = ?$

$$y_p = a_0 e^{-t} + b_0 + ct .$$

$$y_p' = -a_0 e^{-t} + c.$$

Using eq (1) $\Rightarrow$.

$$-\begin{bmatrix}a_0\\a_1\end{bmatrix}e^{-t} + \begin{bmatrix}c_0\\c_1\end{bmatrix} = \begin{bmatrix}-1 & -1\\-4 & 2\end{bmatrix}\left(\begin{bmatrix}a_0\\a_1\end{bmatrix}e^{-t} + \begin{bmatrix}b_0\\b_1\end{bmatrix} + \begin{bmatrix}c_0\\c_1\end{bmatrix}t\right)$$
$$+ \begin{bmatrix}8\\-15\end{bmatrix}t + \begin{bmatrix}8\\-2\end{bmatrix} + \begin{bmatrix}0\\3\end{bmatrix}e^{-t} .$$

By Comparing co-efficients.

$$e^{-t} : \quad -\begin{bmatrix}a_0\\a_1\end{bmatrix} = \begin{bmatrix}-1 & -1\\-4 & 2\end{bmatrix}\begin{bmatrix}a_0\\a_1\end{bmatrix} + \begin{bmatrix}0\\3\end{bmatrix}.$$

$$-a_0 = -a_0 - a_1 + 0 \Rightarrow \boxed{a_1 = 0}$$
$$-a_1 = -4a_0 + 2a_1 + 3 \Rightarrow \boxed{a_0 = 3/4}$$

$$t : \quad \begin{bmatrix}0\\0\end{bmatrix} = \begin{bmatrix}-1 & -1\\-4 & 2\end{bmatrix}\begin{bmatrix}c_0\\c_1\end{bmatrix} + \begin{bmatrix}8\\-15\end{bmatrix}.$$

$$\begin{bmatrix}0\\0\end{bmatrix} \Rightarrow \begin{matrix} 0 = -c_0 - c_1 + 8 \Rightarrow & 0 = -2c_0 - \cancel{2c_1} + \cancel{16} \\ 0 = -4c_0 + 2c - 15 \Rightarrow & 0 = -4c_0 + \cancel{2c_1} - \cancel{15} \\ & \overline{0 = -6c_0 + 1} \end{matrix}$$

$$\boxed{c_0 = 1/6}$$

$$0 = -1/6 - c_1 + 8 \Rightarrow \frac{-1+48}{6} \Rightarrow \boxed{c_1 = 47/6}$$

$t^0$: $\begin{bmatrix} c_0 \\ c_1 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ -4 & 2 \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \end{bmatrix} + \begin{bmatrix} 5 \\ -2 \end{bmatrix}$.

$c_0 = -b_0 - b_1 + 5 \Rightarrow 2c_0 = -2b_0 - \cancel{2b_1} + 10$

&

$c_1 = -4b_0 + 2b_1 - 2 \Rightarrow c_1 = -4b_0 + \cancel{2b_1} - 2$

$2c_0 + c_1 = -6b_0 + 8$.

$2c_0 + c_1 = -6b_0 + 8$.

$2\left(\frac{1}{6}\right) + \frac{47}{6} = -6b_0 + 8$

$\frac{2+47}{6} = -6b_0 + 8$

$\frac{49}{6} - 8 = -6b_0 \Rightarrow \frac{49-48}{6} = -6b_0$

$\boxed{b_0 = -\frac{1}{36}}$

$\therefore c_1 = -4\left(-\frac{1}{36}\right) + 2b_1 - 2$.

$\frac{47}{6} - \frac{1}{9} + 2 = 2b_1 \Rightarrow \frac{423 - 6 + 108}{54} = 2b_1$

$\boxed{b_1 = 4.86}$ $\boxed{y_p = \begin{bmatrix} 0 \\ 3/4 \end{bmatrix} e^{-t} + \begin{bmatrix} -1/36 \\ 4.86 \end{bmatrix} + \begin{bmatrix} 1/6 \\ 47/6 \end{bmatrix} t}$

As $y = y_c + y_p$ By putting values we get;

$y = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{-2t} + c_2 \begin{bmatrix} 1 \\ -1/4 \end{bmatrix} e^{3t} + \begin{bmatrix} 0 \\ 3/4 \end{bmatrix} e^{-t} + \begin{bmatrix} -1/36 \\ 4.86 \end{bmatrix} + \begin{bmatrix} 1/6 \\ 47/6 \end{bmatrix} t$

Ans