Ex # 5.1

Q10:- $y'' - y' + xy = 0.$

Sol:- $y'' - y' + xy = 0 \longrightarrow (1).$

Let $y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots \longrightarrow (A)$

be the solution of (1).

$y' = a_1 + 2a_2 x + 3a_3 x^2 + 4a_4 x^3 + \cdots$

$y'' = 2a_2 + 6a_3 x + 12a_4 x^2 + 20a_5 x^3 + \cdots$

By putting values in eq (1).

$[2a_2 + 6a_3 x + 12a_4 x^2 + 20a_5 x^3 + \cdots] - [a_1 + 2a_2 x + 3a_3 x^2$

$+ 4a_4 x^3 + \cdots] + x[a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots] = 0.$

By comparing co-efficient.

x^0: $2a_2 - a_1 = 0.$

$2a_2 = a_1$

$\boxed{a_2 = a_1/2!}$

x^1: $+6a_3 - 2a_2 + a_0 = 0.$

$+6a_3 = 2a_2 - a_0.$

$+6a_3 = a_1 - a_0.$

$a_3 = \frac{-a_0 + a_1}{6!} = a_3 = \frac{-a_0 + a_1}{3!}$

x^2: $12a_4 - 3a_3 + a_1 = 0$

$$12a_4 - 3\left(\frac{-a_0+a_1}{3!}\right) + a_1 = 0$$

$$12a_4 - \left(\frac{a_0 + a_0}{2}\right) + a_1 = 0$$

$$12a_4 = \frac{-a_0 + a_1 \cancel{+} a_1}{2} \Rightarrow 12a_4 = \frac{-a_0 + a_1 \cancel{+} 2a_1}{2}$$

$$a_4 = \frac{-a_0 - a_1}{12 \times 2}$$

$$\boxed{a_4 = \frac{-a_0 - a_1}{24}} \quad ? \quad a_4 = \frac{-a_0 - a_1}{4!}$$

$\S$ ① $\Rightarrow$

$$y = a_0 + a_1 x + \frac{a_1 x^2}{2!} + \left(\frac{a_1 - a_0}{3!}\right)x^3 + \left(\frac{-a_0 - a_1}{4!}\right)x^4 + \cdots$$

$$y = \left(a_0 \cancel{+} \frac{a_1 x^3}{3!} + \cdots\right) + a_1\left(x + \frac{x^2}{2!} \cancel{+} \frac{x^3}{3!} + \cdots\right)$$

Ans

Q11:- $y'' + y' + x^2 y = 0$.

Sol:- $y'' + y' + x^2 y = 0 \longrightarrow$ ①.

Let

$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots \longrightarrow$ ②

be the solution of ①

Then $y' = a_1 + 2a_2 x + 3a_3 x^2 + 4a_4 x^3 + \cdots$

$y'' = 2a_2 + 6a_3x + 12a_4x^2 + 20a_5x^3 + \cdots$

eq (1) =)

$$2a_2 + 6a_3x + 12a_4x^2 + 20a_5x^3 + \cdots + a_1 + 2a_2x + 3a_3x^2 + 4a_4x^3 + \cdots$$

$$+ x^2(a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots) = 0.$$

By comparing co-efficient

x^0: $2a_2 + a_1 = 0$

$$\boxed{a_2 = -\frac{1}{2}a_1} \Rightarrow -\frac{1}{2!}a_1$$

x^1: $6a_3 + 2a_2 = 0.$

$$6a_3 = -2a_2.$$

$$6a_3 = -2\left(\frac{1}{2}a_1\right).$$

$$a_3 = \frac{1}{6}a_1 = \frac{1}{3!}a_1$$

x^2: $12a_4 + 3a_3 + a_0 = 0.$

$$12a_4 + 3\left(\frac{1a_1}{6_2}\right) + a_0 = 0$$

$$12a_4 = -\frac{a_1}{2} - a_0$$

$$a_4 = -\frac{a_1}{4!} - \frac{a_0}{12}$$

x^3: $20a_5 + 4a_4 + a_3 = 0.$

$$20a_5 + 4\left(\frac{-a_1}{24} - \frac{a_0}{12}\right) + \frac{1}{3!}a_1 = 0.$$

$$20a_5 + 4\left(-\frac{a_1}{24} - \frac{a_0}{12}\right) + \frac{1}{3!}a_1 = 0$$

$$20a_5 + \left(-\frac{a_1}{6} - \frac{a_0}{3}\right) + \frac{1}{6}a_1 = 0$$

$$20a_5 = \frac{a_0}{3}$$

$$a_5 = \frac{a_0}{60}$$

$\{$ ② $\Rightarrow$

$$y = a_0 + a_1 x - \frac{1}{2!}a_1 x^2 + \frac{1}{3!}a_1 x^3 + \left(-\frac{a_1}{4!} - \frac{a_0}{12}\right)x^4 + \cdots$$

$$y = \left(a_0 - \frac{a_0}{12}x^4 + \cdots\right) + a_1\left(x - \frac{1}{2!}x^2 + \frac{1}{3!}x^3 - \frac{1}{4!}x^4 + \cdots\right)$$

Ans

Q12. $(1-x^2)y'' - 12xy' + 2y = 0$

Sol:- $(1-x^2)y'' - 12xy' + 2y = 0 \longrightarrow ①$

let

$$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5 + \cdots \longrightarrow ①$$

be solution of ①

Then

$$y' = a_1 + 2a_2 x + 3a_3 x^2 + 4a_4 x^3 + 5a_5 x^4 + \cdots$$

$$y'' = 2a_2 + 6a_3 x + 12a_4 x^2 + 20a_5 x^3 + \cdots$$

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eq (1) $\Rightarrow$

$$(1-x^2)(2a_2+6a_3x+12a_4x^2+20a_5x^3+\cdots)$$
$$-12(a_1+2a_2x+3a_3x^2+4a_4x^3+5a_5x^4+\cdots)+2(a_0+a_1x+a_2x^2$$
$$+a_3x^3+a_4x^4+\cdots)=0$$

Comparing co-efficients of .

x^0 : $2a_2-12a_1+2a_0=0$

$a_2=6a_1-a_0 \Rightarrow \boxed{a_2=6a_1-a_0}$

x^1 : $6a_3-24a_2+2a_1=0$

$6a_3-24(6a_1-a_0)+2a_1=0$

$6a_3-144a_1+24a_0+2a_1=0$

$6a_3-142a_1+24a_0=0$

$6a_3=142a_1-24a_0$

$$\boxed{a_3=\frac{71}{3}a_1-4a_0}$$

x^2 : $12a_4\cancel{x^2}-2a_2\cancel{x^2}-12\times3a_3\cancel{x^2}+2a_2\cancel{x^2}=0$

$12a_4\cancel{x^2}-\cancel{2a_2}\cancel{x^2}-36a_3\cancel{x^2}+\cancel{2a_2}=0$

$12a_4=36a_3 \Rightarrow a_4=3a_3$

$$\boxed{a_4=71a_1-12a_0}$$

$eq(1) \Rightarrow$

$$y = a_0 + a_1 x + (6a_1 - a_0)x^2 + \left(\frac{71}{3}a_1 - 4a_0\right)x^3 + (71a_1 - 12a_0)x^4 + \text{------}$$

$$y = (a_0 - a_0x^2 - 4a_0x^3 - 12a_0x^4 + \text{-----}) + a_1\left(x + 6x^2 + \frac{71}{3}x^3 + 71x^4 + \text{---}\right)$$

Ans

Q13:- $y'' + (1+x^2)y = 0.$

Sol:- $y'' + (1+x^2)y = 0 \longrightarrow (1).$

let

$$y = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + \text{------} \longrightarrow (2)$$

be the solution of (1).

$$y' = a_1 + 2a_2x + 3a_3x^2 + 4a_4x^3 + 5a_5x^4 + \text{------}$$

& $y'' = 2a_2 + 6a_3x + 12a_4x^2 + 20a_5x^3 + \text{------}$

$eq(1) \Rightarrow$

$$2a_2 + 6a_3x + 12a_4x^2 + 20a_5x^3 + \text{---} + (1+x^2)\left[a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + \text{---}\right] = 0$$

By Comparing Co-efficients.

x^0: $2a_2 + a_0 = 0$

$$a_2 = -\frac{a_0}{2}$$

x^1 : $6a_3 + a_1 = 0$

$6a_3 = -a_1.$

$$\boxed{a_3 = -\frac{1}{6}a_1.}$$

x^2 : $12a_4 + a_2 + a_0 = 0$

$12a_4 - \frac{a_0}{2} + a_0 = 0$

$12a_4 = -\frac{a_0}{2}$

$$\boxed{a_4 = -\frac{a_0}{24}}$$

x^3 : $20a_5 + a_3 + a_1 = 0$

$2a_5 + (-\frac{1}{6}a_1) + a_1 = 0.$

$20a_5 \frac{-1a_1 + 6a_1}{6} = 0$

$20a_5 + \frac{5a_1}{6} = 0.$

$a_5 = \frac{-5a_1}{20 \times 6} \Rightarrow \boxed{a_5 = -\frac{a_1}{24}}$

$\&(1) \Rightarrow$

$$y = a_0 + a_1 x + \left(-\frac{a_0}{2}\right)x^2 - \frac{1}{6}a_1 x^3 - \frac{a_0}{24}x^4 - \frac{a_1}{24}x^5 + \cdots$$

$$y = a_0\left(1 - \frac{x^2}{2!} - \frac{x^4}{4!} + \cdots\right) + a_1\left(x - \frac{x^3}{3!} - \frac{x^5}{4!} + \cdots\right)$$

Ans

Q14 $y'' - 4xy' + (4x^2 - 2)y = 0$.

Sol: $y'' - 4xy' + (4x^2 - 2)y = 0 \longrightarrow ①$

let

$$y = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5 + \cdots \rightarrow ②$$

be the solution of ①.

so,

$$y' = a_1 + 2a_2x + 3a_3x^2 + 4a_4x^3 + 5a_5x^4 + \cdots$$

$$y'' = 2a_2 + 6a_3x + 12a_4x^2 + 20a_5x^3 + \cdots$$

eq ① $\Rightarrow$

$$2a_2 + 6a_3x + 12a_4x^2 + 20a_5x^3 + \cdots + (-4x)(a_1 + 2a_2x + 3a_3x^2 + 4a_4x^3 + 5a_5x^4 + \cdots) + (4x^2 - 2)(a_0 + a_1x + a_2x^2 + \cdots) = 0$$

By comparing coefficient

x^0: $2a_2 - 2a_0 = 0$

$$\boxed{a_2 = a_0}$$

x^1: $6a_3 - 4a_1 - 2a_1 = 0$

$6a_3 - 6a_1 = 0$.

$6a_3 = 6a_0$

$$\boxed{a_3 = a_0}$$

x^2: $12a_4 - 8a_2 + 4a_0 - 2a_2 = 0$

$12a_4 - 8(a_0) + (4a_0) - 2a_0 = 0$

$12a_4 = 6a_0$

$$\boxed{a_4 = \frac{1}{2}a_0}$$

$\therefore$ (A) $\Rightarrow$

$y = a_0 + a_1 x + a_0 x^2 + a_0 x^3 + \frac{1}{2} a_0 x^4 + \text{---}$

$y = a_0\left(1 + x^2 + x^3 + \frac{1}{2}x^4 + \text{---}\right) + a_1(x + \text{---})$

Ans

Q14: $y'' - 4xy' + (4x^2 - 2)y = 0$.

Sol:- $y'' - 4xy' + (4x^2 - 2)y = 0 \longrightarrow (1)$.

Let

$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \text{---}$ be solution of (1)

then, $y' = a_1 + 2a_2 x + 3a_3 x^2 + 4a_4 x^3 + \text{---}$

$y'' = 2a_2 + 6a_3 x + 12a_4 x^2 + 20a_5 x^3 + \text{---}$

$\therefore$ (1) $\Rightarrow$

$(2a_2 + 6a_3 x + 12a_4 x^2 + 20a_5 x^3 + \text{---}) - 4x(a_1 + 2a_2 x + 3a_3 x^2 + 4a_4 x^3 + \text{---})$
$+ (4x^2 - 2)(a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \text{---}) = 0$

By comparing coefficients.

x^0: $2a_2 - 2a_0 = 0$

$2a_2 = 2a_0 \Rightarrow \boxed{a_2 = a_0}$

x^1: $6a_3 - 4a_1 - 2a_1 = 0$.

$\boxed{a_3 = a_1}$

x^2: $12a_4 - 8a_2 + 4a_0 - 2a_2 = 0$.

$12a_4 - 8(a_0) + 4(a_0) + 2(a_0) = 0$.

$12a_4 = 6a_0$

$\boxed{a_4 = \frac{1}{2} a_0}$

x^3: $20a_5 + 4a_1 - 12a_3 - 2a_3 = 0$

$20a_5 + 4a_1 - 14a_1 = 0$

$\boxed{a_5 = \frac{1}{2} a_1}$

Eq(1) $\Rightarrow$. $y = a_0 + a_1 x + a_0 x^2 + a_1 x^3 + \frac{1}{2} a_0 x^4 + \frac{1}{2} a_1 x^5 + \text{---}$

$y = a_0\left(1 + x^2 + \frac{1}{2}x^4 + \text{--}\right) + a_1\left(x + x^3 + \frac{1}{2}x^5 + \text{---}\right)$

Ans