Ex#6.1

Q1:- $2t+8$.

Sol:- Apply lablace transformation.

$$\mathcal{L}\{2t+8\} = 2\mathcal{L}\{t\} + 8\mathcal{L}\{1\}.$$

$$= 2\int_0^\infty e^{-st}.t\,dt + 8\int_0^\infty e^{-st}dt.$$

$$= 2\left[\left(t\frac{e^{-st}}{-s}\right)\Big|_0^\infty - \int_0^\infty \frac{e^{-st}}{-s}dt\right] + 8\left(\frac{e^{-st}}{-s}\right)\Big|_0^\infty$$

$$= 2\left[0-0+\frac{1}{s}\left(\frac{e^{-st}}{-s}\right)\Big|_0^\infty\right] + 8\left[0-(-1/s)\right]$$

$$= 2\left[\left(\frac{1}{+s^2}\right)\right] + 8\left[1/s\right].$$

$$= 2/s^2 + 8/s \quad \underline{\text{Ans}}.$$

$\underline{\underline{Q_3}}$:- $\cos 2\pi t$.

$\underline{\underline{Sol}}$:- Apply Laplace condition.

As;

$$\mathcal{L}\{f(t)\} = \mathcal{L}\{\cos 2\pi t\}. \qquad \therefore \cos at = \frac{e^{at}+e^{-at}}{2}.$$

$\cancel{\mathcal{L}\{e^{at}\}}$

$$\mathcal{L}\{\cos 2\pi t\} = \mathcal{L}\left\{\frac{e^{2\pi t}+e^{-2\pi t}}{2}\right\}.$$

$$= \frac{1}{2}\mathcal{L}\{e^{2\pi t}\} + \frac{1}{2}\mathcal{L}\{e^{-2\pi t}\}.$$

$$= \frac{1}{2}\left(\int_0^\infty e^{-st}.e^{2\pi t}dt\right) + \frac{1}{2}\left(\int_0^\infty e^{-st}.e^{-2\pi t}dt\right)..$$

$$= \frac{1}{2}\int_0^\infty e^{-(s-2\pi)t}dt + \frac{1}{2}\int_0^\infty e^{-(s+2\pi)t}dt.$$

$$= \frac{1}{2}\left(\frac{e^{-(s-2\pi)t}}{-(s-2\pi)}\right)\Big|_0^\infty + \frac{1}{2}\left(\frac{e^{-(s+2\pi)t}}{-(s+2\pi)}\right)\Big|_0^\infty.$$

$$= \frac{1}{2}\left(0-\left(-\left(\frac{1}{s-2\pi}\right)\right)\right) + \frac{1}{2}\left(0-\left(\frac{1}{-(s+2\pi)}\right)\right).$$

$$= \frac{1}{2}\left(\frac{1}{s-2\pi}\right) + \frac{1}{2}\left(\frac{1}{s+2\pi}\right).$$

$$= \frac{s+\cancel{2\pi}+s-\cancel{2\pi}}{2(s^2-(2\pi)^2)} = \frac{\cancel{2}s}{\cancel{2}(s^2-4\pi^2)} = \frac{s}{s^2-4\pi^2}$$

Q2:- $(a-bt)^2$.

Sol:- $a^2 - 2abt + b^2t^2$.

Apply $\mathcal{L}$. T.

$\mathcal{L}(a^2) - 2\mathcal{L}(abt) + \mathcal{L}(b^2t^2)$.

$= a^2\mathcal{L}\{1\} - 2ab\,\mathcal{L}\{t\} + b^2\mathcal{L}\{t^2\}$.

$\because \mathcal{L}(t^n) = \frac{n!}{s^{n+1}}$.

$= \frac{a^2}{s} + -\frac{2ab}{s^2} + \frac{b^2}{s^3}$. Ans

Q5:- $e^{3t}\sinh t$.

Sol:- $e^{3t}\sinh t$.

Apply $\mathcal{L}$. T.

$\mathcal{L}\{e^{3t}\sinh t\}$.

Apply shift property.

$\mathcal{L}\{\sinh t\}_{s \to s-a}$.

$= \left(\frac{a}{s^2-a^2}\right)_{s \to s-a}$.

$= \frac{3}{(s-3)^2 - 3} = \frac{3}{s^2-6s+9-9}$.

$= \frac{3}{s^2-6s}$ Ans

A: $\mathcal{L}\{e^{at}\sinh \omega t\} = \frac{a}{(s-a)^2 - a\omega^2}$.

$\mathcal{L}\{e^{3t}\sinh t\}$

$= \left(\frac{a}{s^2-(1)^2}\right)_{s \to s-a}$

$= \left(\frac{a}{s^2-1}\right)_{s \to s-a}$

$= \frac{3}{(s-3)^2-1} = \frac{3}{s^2+9-6s-1}$

$= \frac{3}{s^2-6s+8} = \frac{3}{s^2-4s-2s+8}$

$= \frac{3}{s(s-4)-2(s-4)}$

$= \frac{3}{(s-4)(s-2)}$

$= \frac{3}{(s-4)(s-2)}$

$a=3$

Ans

Q_4:- $\cos^2 \omega t$.

Sol:- $\cos^2 \omega t$.

As;

$$\cos^2 \omega t = \frac{1+\cos 2\omega t}{2}.$$

$$= \frac{1+\cos 2\omega t}{2} = \frac{1}{2} + \frac{\cos 2\omega t}{2}.$$

Apply L.T.

$$= \frac{1}{2}\mathcal{L}\{t^0\} + \frac{1}{2}\mathcal{L}\{\cos 2\omega t\}.$$

$\because \mathcal{L}\{\cos at\} = \frac{s}{s^2+a^2}$

$$= \frac{1}{2}\left(\frac{1}{s}\right) + \frac{1}{2}\left(\frac{s}{s^2+(2\omega)^2}\right).$$

$$= \frac{1}{2s} + \frac{1}{2}\left(\frac{s}{s^2+4\omega^2}\right) \text{ Ans}$$

Q_6:- $e^{-t}\sinh 4t$.

Sol:- Apply L.T.

$\mathcal{L}\{e^{-t}\sinh 4t\}$. $\because$ Apply shift property.

$\mathcal{L}\{\sinh 4t\}_{s \to s-a}$. $\mathcal{L}\{\sinh at\} = \frac{a}{s^2-a^2}$.

Here $a = 4t$.

$$\left(\frac{(4t)}{s^2-(4t)^2}\right)_{s \to s-a}.$$

$$= \frac{4}{(s+1)^2-16} = \frac{4}{s^2+1+2s-16} = \frac{4}{s^2+2s-15} = \boxed{\frac{4}{(s+5)(s-3)}}$$

Ans

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Q8:- $1.5 \sin(3t - \pi/2)$.

Sol:- As
$$\sin(\alpha - \beta) = \sin\alpha \cos\beta - \cos\alpha \sin\beta.$$

$$= 1.5\left[\sin 3t \cdot \overset{0}{\cancel{\cos \pi/2}} - \cos 3t \cdot \sin \pi/2\right).$$

$$= 1.5\left(-\cos 3t \cdot (1)\right).$$

$$= -1.5 \cos 3t.$$

Apply L.T.

$$= \mathcal{L}\{-1.5 \cos 3t\}.$$

$$= -1.5 \mathcal{L}\{\cos 3t\}.$$

As $\mathcal{L}\{\cos at\} = \dfrac{s}{s^2 + a^2}$.

Here $a = 3$

$$= -1.5 \frac{s}{s^2 + 9}$$

$$= \frac{-1.5 s}{s^2 + 9} \quad \text{Ans}$$

Q7:- $\cos(\omega t+\theta)$.

Sol:-

As $\cos(\alpha+\beta) = \cos\alpha \cdot \cos\beta - \sin\alpha \cdot \sin\beta$.

$\cos(\omega t+\theta) = \cos(\omega t) \cdot \cos\theta - \sin\omega t \sin\theta$.

Apply L.T.

$= \cos\theta \; \mathcal{L}\{\cos\omega t\} - \sin\theta \; \mathcal{L}\{\sin\omega t\}$.

(Apply $\mathcal{L}$ on variable)

$\because \mathcal{L} \sin at = \frac{a}{s^2+a^2}$.

$\because \mathcal{L} \cos at = \frac{s}{s^2+a^2}$.

$= \cos\theta \left(\frac{s}{s^2+\omega^2}\right) - \sin\theta \left(\frac{a}{s^2+\omega^2}\right)$

Ans

Q 25:- $\dfrac{0.2s+1.4}{s^2+1.96}$

Solution:-

The above eqn can be written as;

$$\frac{0.2s+1.4}{s^2+1.96} = \frac{0.2s}{s^2+1.96} + \frac{1.4}{s^2+1.96}.$$

Apply Inverse Laplace transformation.

$$= \mathcal{L}^{-1}\left\{\frac{0.2s}{s^2+1.96}\right\} + \mathcal{L}^{-1}\left\{\frac{1.4}{s^2+1.96}\right\}.$$

$$= 0.2\,\mathcal{L}^{-1}\left\{\frac{s}{s^2+1.96}\right\} + \mathcal{L}^{-1}\left\{\frac{1.4}{s^2+1.96}\right\}.$$

$$= 0.2\,\mathcal{L}^{-1}\left\{\frac{s}{s^2+(1.4)^2}\right\} + \mathcal{L}^{-1}\left\{\frac{1.4}{s^2+(1.4)^2}\right\}.$$

$\hookrightarrow$ ①.

Since we know that. (from table).

$\mathcal{L}\{\cos at\} = \dfrac{s}{s^2+a^2}$. & $\mathcal{L}\{\sin at\} = \dfrac{a}{s^2+a^2}$.

eq ① ⇒ $= 0.2\,\mathcal{L}^{-1}\{\mathcal{L}\cos at\} + \mathcal{L}^{-1}\{\mathcal{L}\sin at\}$.

∴ where $a = 1.4$.

$= 0.2\cos 1.4t + \sin 1.4t$

Ans

Q 26:- $\dfrac{5s+1}{s^2-25}$

Sol:- for simplyfying In order to make it applicable for formula we can write it as;

$$= \frac{5s+1}{s^2-25} = \frac{5s}{s^2-25} + \frac{1}{s^2-25}.$$

Apply Laplace (Inverse) transformation.

$$= \mathcal{L}^{-1}\left\{\frac{5s}{s^2-25}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{s^2-25}\right\}.$$

$$= 5\,\mathcal{L}^{-1}\left\{\frac{s}{s^2-25}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{s^2-25}\right\}.$$

or

$$= 5\,\mathcal{L}^{-1}\left\{\frac{s}{s^2-(5)^2}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{s^2-(5)^2}\right\}. \quad \text{(1)}$$

Since we know that (from table) (should be memorized)

$\mathcal{L}\cosh at = \dfrac{s}{s^2-a^2}$ & $\mathcal{L}\sinh at = \dfrac{a}{s^2-a^2}$.

& (1) ⇒

$$= 5\,\mathcal{L}^{-1}\{\mathcal{L}\cosh 5t\} + \frac{1}{5}\mathcal{L}^{-1}\{\mathcal{L}\sinh 5t\}$$

As a=5. & for second term multiply & divide by 5. to equilized with formula

$$= 5\cosh 5t + \frac{1}{5}\sinh 5t$$ Ans

Q27:-

$$\frac{s}{L^2 s^2 + \frac{1}{4}\pi^2}$$

Sol:- Given that

$$= \frac{s}{L^2 s^2 + \frac{1}{4}\pi^2}$$

On Simplifying In order to make it ready to use for formula.

$$= \frac{s}{L^2\left(s^2 + \frac{1\times\pi^2}{4L^2}\right)}$$

Apply Inverse Laplace transformation.

$$= \mathcal{L}^{-1}\left\{\frac{1 * s}{L^2\left(s^2+\frac{\pi^2}{4L^2}\right)}\right\}$$

$$= \frac{1}{L^2}\,\mathcal{L}^{-1}\left\{\frac{s}{s^2+\frac{\pi^2}{4L^2}}\right\} \longrightarrow (1)$$

Since $\frac{\pi^2}{4L^2}$ is constant so let $a = \frac{\pi^2}{2L}$.

Than as we know that.

$$\mathcal{L}\cos at = \frac{s}{s^2+a^2}$$

so $q(1) \Rightarrow$, $\frac{1}{L^2}\,\mathcal{L}^{-1}\{\mathcal{L}\cos at\}$

$\because a = \frac{\pi^2}{4L}$.

$$= \frac{1}{L^2}\left\{\cos\frac{\pi^2}{4L}t\right\}$$ Ans.

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Q 29:- $\frac{2}{s^4} - \frac{48}{s^6}$.

Sol:- As $\mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}$.

As $\mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}$

or $\mathcal{L}^{-1}\left\{\frac{n!}{s^{n+1}}\right\} = t^n$.

So; $\frac{1}{n!}\mathcal{L}\{t^n\} = \frac{1}{s^{n+1}}$

or $\mathcal{L}^{-1}\left\{\frac{1}{s^{n+1}}\right\} = \frac{t^n}{n!}$

So; Apply Inverse L.T.

$2\mathcal{L}^{-1}\left\{\frac{1}{s^4}\right\} - 48\,\mathcal{L}^{-1}\left\{\frac{1}{s^6}\right\}$.

$= 2\frac{t^3}{3!} - 48\frac{t^5}{5!}$.

$= \frac{\cancel{2}t^3}{3\times\cancel{2}} - \frac{\overset{2}{\cancel{48}}\,t^5}{5\times\cancel{4}\times\cancel{3}\times\cancel{2}}$

$= \frac{t^3}{3} - \frac{2t^5}{5}$ Ans.

Q30:-

$$\frac{4s+32}{s^2-16}$$

Sol:-

$$\frac{4s+32}{s^2-(4)^2}$$

$$\frac{4s}{s^2-16} + \frac{32}{s^2-16}$$

$$= 4\,\frac{s}{s^2-(4)^2} + 8\cdot\frac{4}{s^2-16 \text{ or } (4)^2} \rightarrow (1)$$

Since $\mathcal{L}^{-1}\frac{s}{s^2-a^2} = \cosh at$ & $\mathcal{L}^{-1}\left(\frac{a}{s^2-a^2}\right) = \sinh at$

Here $a = 4$

$$= 4\cosh 4t + 8\sinh 4t$$

so;

$$= 4\cosh 4t + 8\sinh 4t$$

Ans

Q31:-

$$\frac{-s+11}{s^2-2s-3}$$

Sol:-

$$\frac{-s+11}{s^2-2s-3} = \frac{-s+11}{s^2-3s+s-3}$$

$$= \frac{-s+11}{s(s-3)+1(s-3)} = \frac{-s+11}{(s-3)(s+1)}$$

Using P.F

$$\frac{-s+11}{(s-3)(s+1)} = \frac{A}{(s-3)} + \frac{B}{(s+1)} \rightarrow Ⓐ$$

$$-s+11 = A(s+1) + B(s-3) \rightarrow ①$$

Put $s=-1$.

$$\cancel{-12}\ 12 = B(-4) = B = -3$$

Put $s=3$.

$$8 = A(4) \Rightarrow A = 2$$

using $L^{-1}\left\{\frac{1}{s+a}\right\} = e^{-at}$

eq Ⓐ $\Rightarrow$

$$\frac{-s+11}{(s-3)(s+1)} = \frac{2}{s-3} - \frac{3}{s+1}$$

Apply Inv L.T.

$$= 2L^{-1}\left\{\frac{1}{s-3}\right\} - 3L^{-1}\left\{\frac{1}{s+1}\right\}$$

$$\boxed{= 2e^{3t} - 3e^{-t}}$$ Ans.

Q32.- $\frac{1}{(s+a)(s+b)}$

Sol:-

$$\frac{1}{(s+a)(s+b)}$$

Using Partial fraction.

$$\frac{1}{(s+a)(s+b)} = \frac{1}{b-a}\left[\frac{1}{s+a} - \frac{1}{s+b}\right].$$

Apply Inverse L.T

$$= \left(\frac{1}{b-a}\right)\left\{L^{-1}\left(\frac{1}{s+a}\right) - L^{-1}\left\{\frac{1}{s+b}\right\}\right)$$

$$= \frac{1}{b-a}\left[L^{-1}\left(\frac{1}{s+a}\right) - L^{-1}\left(\frac{1}{s+b}\right)\right].$$

As $L^{-1}\left(\frac{1}{s}\right) = t^{0}$.

$$L^{-1}\left(\frac{1}{s+a}\right) = e^{-at}.$$

$$= \frac{1}{b-a}\left(e^{-at} - e^{-bt}\right).$$

$$= \boxed{\frac{e^{-at} - e^{-bt}}{(b-a)}}$$ Ans

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Pb # 34 $Ke^{-at}\cos\omega t$.

Sol:- $Ke^{-at}\cos\omega t$

Apply L.T.

$$K\,\mathcal{L}\{e^{-at}\cos\omega t\}.$$

Apply shift property

$\because$ As $\mathcal{L}\{\cos at\} = \frac{s}{s^2+a^2}$.

$$K\,\mathcal{L}\{\cos\omega t\}_{s\to s-a}.$$

$$\Rightarrow K\left(\frac{s}{s^2+\omega^2}\right)_{s\to s-a}.$$

$a = -1$.

$s-(-1) = s+1$.

$$= K\frac{(s+a)}{(s+1)^2+\omega^2}$$ Ans

Q 33 t^3e^{-2t}

Sol:- t^3e^{-2t}.

Apply $\mathcal{L}$.T.

$$\mathcal{L}\{t^3e^{-2t}\}.$$

Apply shift property. $\because \mathcal{L}(t^3) = \frac{3!}{s^4}$.

$$\mathcal{L}\{t^3\}_{s\to s-a}.$$

$$= \left(\frac{3!}{s^4}\right)_{s\to s-a}.$$

$$= \frac{3!}{(s+2)^4}$$ Ans

Q35:- $2e^{-1/2t}\sin 4\pi t$.

Sol:- Apply $\mathcal{L}$ tran.

$$2\mathcal{L}\{e^{-1/2t}\sin 4\pi t\}.$$

Apply shift property.

i.e

$$\mathcal{L}\{e^{at}\sin at\}_{s\to s-a} = \left(\frac{a}{s^2+a}\right)_{s\to s-a}.$$

$\because \mathcal{L}\{\sin at\} = \frac{a}{s^2+a^2}$

Here $a = 4\pi$.

$$= 2\,\mathcal{L}\{\sin 4\pi t\}_{s\to s-a}.$$

$$= \left(\frac{2(4\pi)}{s^2+(4\pi)^2}\right)_{s\to s-a}.$$

$$= \frac{8\pi}{(s+0.5)^2+16\pi^2} = \frac{8\pi}{s^2+\frac{1}{4}+1/2+16\pi^2}.$$

$$= \frac{8\pi}{\frac{4s^2+4+64\pi^2}{4}} = \frac{32\pi}{4s^2+4+64\pi^2}$$

Ans

Q36:- $\sinh t \cos t$.

Sol:- As $\sinh t = \dfrac{e^{t} - e^{-t}}{2}$.

$= \dfrac{e^{t} - e^{-t}}{2} \cos t$

$= \dfrac{1}{2} \left[e^{t} \cos t - e^{-t} \cos t \right]$.

apply L. T.

$= \dfrac{1}{2} \left[\mathcal{L}\{e^{t} \cos t\} - \mathcal{L}\{e^{-t} \cos t\} \right]$.

Apply shift property.

$= \dfrac{1}{2} \left\{ \mathcal{L}\{\cos t\}_{s \to s-a} - \mathcal{L}\{\cos t\}_{s \to s-a} \right\}$.

$= \dfrac{1}{2} \left[\dfrac{s}{s^2 + a^2} - \dfrac{s}{s^2 + a^2} \right]$. Put $a = 1$ & -1.

$= \dfrac{1}{2} \left[\dfrac{s-a}{(s-a)^2 + a^2} - \dfrac{s-a}{(s-a)^2 + a^2} \right]$.

$= \dfrac{1}{2} \left[\dfrac{s-1}{(s-1)^2 + a^2} + \dfrac{s-(-1)}{(s-(-1))^2 + a^2} \right]$

$= \dfrac{1}{2} \left[\dfrac{s-1}{(s-1)^2 + a^2} + \dfrac{s+1}{(s+1)^2 + a^2} \right]$ Ans

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Q37:-

$$\frac{2\pi}{(s+\pi)^3}$$

Sol:- As we know

$$\mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}$$

$$\mathcal{L}^{-1}\left\{\frac{n!}{s^{n+1}}\right\} = t^n.$$

$\frac{1}{2!}\mathcal{L}\{e^{-at}t^2\} = \left(\frac{1}{s^2}\right)_{s \to s+a}$

$= \frac{1}{2(s+a)^3}$

For e.s

$$\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} = t^0.$$

But here $\mathcal{L}^{-1}\left\{\frac{1}{s^3}\right\} = \frac{t^2}{2!}$

As $\mathcal{L}^{-1}\left\{\frac{1}{(s+\pi)^3}\right\} = \frac{e^{-\pi t} \cdot t^2}{2!}$

$\Downarrow$

Using this

$$\mathcal{L}^{-1}\left\{\frac{1}{(s+a)^n}\right\} = e^{-at} \cdot \frac{t^{n-1}}{(n-1)!}$$

So, Apply Inverse Laplace trans.

$$\mathcal{L}^{-1}\left(\frac{2\pi}{(s+\pi)^3}\right) = 2\pi\,\mathcal{L}^{-1}\left(\frac{1}{(s+\pi)^3}\right)$$

$$= 2\pi \frac{e^{-\pi t} \cdot t^2}{2!} \Rightarrow \pi e^{-\pi t} t^2 \text{ Ans}$$

Q35:- $\dfrac{6}{(s+1)^3}$

Sol:- $\dfrac{6}{(s+1)^3}$

$\dfrac{6}{(s+1)^3}$

$6\,\mathcal{L}^{-1}\left\{\dfrac{1}{(s+1)^3}\right\} = \dfrac{1}{2!}\,e^{-t}\,t^2$

As; $\mathcal{L}\{t^n\} = \dfrac{n!}{s^{n+1}}$

~~$\mathcal{L}\{s^3\}$~~

$\mathcal{L}\{t^3\} = \dfrac{3!}{s^4}$

$\mathcal{L}\{t^2\} = \dfrac{2!}{s^3}$

So, $\mathcal{L}^{-1}\left\{\dfrac{1}{s^3}\right\} = \dfrac{t^2}{2!}$

But here $\dfrac{1}{s+a}$ case so multiply e^{-at}.

$= \cancel{e^{-at}\,\mathcal{L}\left\{\dfrac{1}{\ }\right.}$

Apply Inverse Laplace Trans.

$\mathcal{L}^{-1}\left\{\dfrac{6}{(s+1)^3}\right\} = 6\,\mathcal{L}^{-1}\left\{\dfrac{1}{(s+1)^3}\right\}$

$= 6\,e^{-t}\,\dfrac{t^2}{2!}$

$= 3e^{-t}\,t^2$

Ans

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Q39:- $\dfrac{90}{(s+\sqrt{3})^6}$

Sol:- $\dfrac{90}{(s+\sqrt{3})^6}$

$\dfrac{90}{(s+\sqrt{3})^6}$

As $\mathcal{L}^{-1}\left\{\dfrac{1}{s+a}\right\} = e^{-at}(1)$.

t^5

$-\sqrt{3}t$ e

at $\dfrac{1}{5!}$

Since

$$\mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} = t^0.$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s^3}\right\} = \frac{t^2}{\cancel{3}!}.$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s^6}\right\} = \frac{t^5}{6!}$$

But As here.

$$\mathcal{L}^{-1}\left\{\frac{1}{(s+a)^6}\right\} = \frac{e^{-at}\ t^5}{6!} - \cancel{t}.$$

But using this rule. Here $a=\sqrt{3}$.

$$\mathcal{L}^{-1}\left\{\frac{90}{(s+\sqrt{3})^6}\right\} = 90\,\mathcal{L}^{-1}\left\{\cancel{90}\frac{1}{(s+\sqrt{3})^6}\right\}.$$

$$= \frac{\cancel{90}\ e^{-\sqrt{3}t}\ t^5}{\cancel{5}\times 4\times \cancel{3}\times \cancel{2}} = \frac{3}{4}\,e^{-\sqrt{3}t}\ t^5$$

Ans

Q40:-

$$\frac{4}{s^2-2s-3}$$

Sol:- $\frac{4}{s^2-3s+s-3} = \frac{4}{s(s-3)+1(s-3)}$

$= \frac{4}{(s-3)(s+1)}$

By using Partial fraction.

$$\frac{4}{(s-3)(s+1)} = \frac{A}{s-3} + \frac{B}{s+1} \rightarrow (A)$$

$$\frac{4}{\cancel{(s-3)(s+1)}} = \frac{A(s+1)+B(s-3)}{\cancel{(s-3)(s+1)}}$$

$4 = A(s+1) + B(s-3)$

Put $s=-1$

$B = -1$

Put $s = 3$

$\boxed{A = 1}$

$\because \mathcal{L}^{-1}\left\{\frac{1}{s}\right\} = 1$

$\mathcal{L}^{-1}\left\{\frac{1}{s+a}\right\} = e^{-at}(1)$

$\mathcal{L}^{-1}\left\{\frac{1}{s+(-a)}\right\} = e^{-(-a)t}(1)$

$= e^{at}$

eq (A) $\Rightarrow$ $= \frac{1}{s-3} + \frac{(-1)}{s+1}$

Apply Inverse Lap. Transformation.

$= \mathcal{L}^{-1}\left\{\frac{1}{s-3}\right\} + \mathcal{L}^{-1}\left\{\frac{(-1)}{s+1}\right\}$

$= e^{3t} - (e^{-t}) \Rightarrow \boxed{e^{3t} - e^{-t}}$ Ans

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Q41:-

$$\frac{\pi}{s^2+4s\pi+3\pi^2}$$

Sol:-

$$\frac{\pi}{s^2+3s\pi+1s\pi+3\pi^2} = \frac{\pi}{s(s+3\pi)+\pi(s+3\pi)}$$

$$= \frac{\pi}{(s+3\pi)(s+\pi)}$$

$e^{-3\pi^{-1}}$

$$= \frac{\pi}{s+3\pi} + \frac{1}{s+\pi}$$

As;

$$\mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}$$

$$\mathcal{L}\{t\} = \frac{1}{s^2}, \quad \mathcal{L}\{1\} = \frac{1}{s}, \quad \mathcal{L}\{t^2\} = \frac{2}{s^3}$$

As; $\mathcal{L}^{-1}\left\{\frac{n!}{s+a}\right\} = e^{-at}\{t^a\}$

So; Apply Inverse Laplace transformation.

$$= \mathcal{L}^{-1}\left(\frac{\pi}{s+3\pi}\right) + \mathcal{L}^{-1}\left\{\frac{1}{s+\pi}\right\}$$

$$= \pi\,\mathcal{L}^{-1}\left(\frac{1}{s+3\pi}\right) + \mathcal{L}^{-1}\left(\frac{1}{s+\pi}\right)$$

$\boxed{a = 3\pi}$

$$= \pi e^{-\overset{3\pi}{a}t}(1) + e^{-\overset{\pi}{a}t}$$

$$= \pi e^{-\overset{3\pi}{a}t} + e^{-\overset{\pi}{a}t} \rightarrow \boxed{(\pi+1)e^{-\overset{2\pi}{a}t}}$$

$\pi e^{-3\pi t} + e^{-\pi t}$

Ans

Q42

$$\frac{a_0}{s+1} + \frac{a_1}{(s+1)^2} + \frac{a_2}{(s+1)^3} \rightarrow ①.$$

Sol:- As we know from table.

$$\mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}$$

so; $\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} = t^0 = 1.$

$\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\} = t^1.$

$\mathcal{L}^{-1}\left\{\frac{1}{s^3}\right\} = t^2.$ { so on.

But here $\mathcal{L}^{-1}\left\{\frac{1}{s+a}\right\} = e^{-at}\{t^0\}.$

$\mathcal{L}^{-1}\left\{\frac{1}{(s+1)^2}\right\} = e^{-at}\{t^1\}$

$\mathcal{L}^{-1}\left\{\frac{1}{(s+1)^3}\right\} = e^{-at}\left\{\frac{t^2}{2}\right\}.$

So eq ① =)

Apply Inverse Laplace trans.

$$= a_0 \mathcal{L}^{-1}\left(\frac{1}{s+1}\right) + a_1 \mathcal{L}^{-1}\left\{\frac{1}{(s+1)^2}\right\} + a_2 \mathcal{L}^{-1}\left\{\frac{1}{(s+1)^3}\right\}.$$

$$= a_0 e^{-at} + a_1 e^{-at}(t) + a_2 e^{-at}\left(\frac{t^2}{2}\right)$$

Ans

Q44:- $\dfrac{a(s+K)+b\pi}{(s+K)^2+\pi^2}$

Sol:- We can write it as;

$$\frac{a(s+K)}{(s+K)^2+\pi^2}+\frac{b\,\pi}{(s+K)^2+\pi^2} \longrightarrow (1)$$

Since from table.

$$L\{\sin at\} = \frac{a}{s^2+a^2}$$

$$\mathcal{L}\{\cos at\} = \frac{s}{s^2+a^2}$$

$$\mathcal{L}\{F(s+K)\} = e^{-Kt}\{\mathcal{L}F(s)\}$$

Put $s+K = K$.

eq (1) =)

$$= a\,\frac{s}{s^2+\pi^2}+\frac{b\,\pi}{s^2+\pi^2}$$

Apply Inverse Laplace transformation

$$= a\,\mathcal{L}^{-1}\left\{\frac{s}{s^2+\pi^2}\right\}+b\,\mathcal{L}^{-1}\left\{\frac{\pi}{s^2+\pi^2}\right\}$$

$$= a\cos\pi t+b\sin\pi t \longrightarrow (2)$$

Replace s by $s+K$.

Just multiply e^{-Kt} with (2).

$$= e^{-Kt}(a\cos\pi t+b\sin\pi t)$$

Ans.

=

$\frac{k_1}{(s-a)}$

Qus:- $\frac{k_0}{s} + \frac{k_1}{(s-a)^2}$

Apply Inverse $\mathcal{L}$ trans.

$k_0 \mathcal{L}^{-1}\left(\frac{1}{s}\right) + k_1 \mathcal{L}^{-1}\left(\frac{1}{(s-a)^2}\right) \rightarrow ①.$

$\therefore \mathcal{L}^{-1}\left(\frac{1}{s}\right) = 1$

$\therefore \mathcal{L}^{-1}\left(\frac{1}{(s-a)^2}\right) = t e^{at}.$

eq ① $\Rightarrow$

$k_0(1) + k_1(t) e^{at}.$

$\boxed{= k_0 + k_1 t e^{at}}$ Ans.