Q4:- $y'' + 9y = 10e^{-t}$ $\quad y(0) = 0, \; y'(0) = 0$

Ex 6.2

Sol:- $\mathcal{L}\{y''\} + 9\,\mathcal{L}\{y\} = 10\,\mathcal{L}\{e^{-t}\}$.

$s^2\mathcal{L}\{$

$$s^2 y(s) - s\,y(0) - y'(0) + 9y(s) = 10\left(\frac{1}{s+1}\right)$$

$$s^2 y(s) + 9y(s) = 10\left(\frac{1}{s+1}\right)$$

$$y(s)\left[s^2+9\right] = 10\left(\frac{1}{s+1}\right)$$

$$y(s) = \frac{10}{(s+1)(s^2+9)}$$

$$\frac{10}{(s+1)(s^2+9)} = \frac{A}{s+1} + \frac{Bs+C}{s^2+9}.$$

$$10 = A(s^2+9) + (Bs+C)(s+1).$$

By Comparing co-efficients.

s^2: $A + B = 0 \longrightarrow (1)$.

s: $B + C = 0 \longrightarrow (2)$

s^0: $9A + C = 10 \longrightarrow (3)$

$A = -B$.
$B = -C$.
$\Rightarrow A = C$.

$9C + C = 10 \Rightarrow \boxed{C = 1}$

$\boxed{A = 1}$

$\boxed{B = -1}$

$$= \frac{1}{s+1} + \frac{-1s+1}{s^2+9}$$

$$= \frac{1}{s+1} - \frac{s}{s^2+9} + \frac{1}{s^2+9}$$

$$= \frac{1}{s+1} - \frac{s}{s^2+(3)^2} + \frac{1}{3}\frac{3}{(s^2+(3)^2)}$$

Apply Inverse Laplace

$$= \mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\} - \mathcal{L}^{-1}\left\{\frac{s}{s^2+(3)^2}\right\} + \frac{1}{3}\mathcal{L}^{-1}\left\{\frac{3}{s^2+(3)^2}\right\}$$

$$= e^{-1t} - \cos 3t + \frac{1}{3}\sin 3t$$

$$= e^{-t} - \cos 3t + \frac{1}{3}\sin 3t$$

Ans

Q3:- $y'' + y' - 6y = 0$, $y(0) = 1$, $y'(0) = 1$

Sol.- Apply Laplace transf

$$\mathcal{L}\{y''\} + \mathcal{L}\{y'\} - 6\mathcal{L}\{y\} = 0$$

$$s^2 y(s) - sy(0) - y'(0) + sy(s) - y(0) - 6y(s) = 0$$

$$s^2 y(s) - s - 1 + sy(s) - 1 - 6y(s) = 0$$

$$y(s)\left[s^2 - s - 6\right] = s + 2$$

$$y(s) = \frac{s+2}{s^2 - s - 6}$$

$$= \frac{s+2}{s^2 - 3s + 2s - 6} = \frac{s+2}{s(s-3) + 2(s-3)}$$

$$= \frac{\cancel{s+2}}{(s-3)\cancel{(s+2)}} = \frac{1}{s-3}$$

Apply Inverse Laplace transform.

$$\mathcal{L}^{-1}\left\{\frac{1}{s-3}\right\}$$

$$= e^{3t} \quad \text{Ans}$$

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Q2:- $y' + 2y = 0, \quad y(0) = 1.5$

Sol:- Apply Laplace transform

$$\mathcal{L}\{y'\} + 2\mathcal{L}\{y\} = 0$$

$$s\,y(s) - y(0) + 2y(s) = 0$$

$$s\,y(s) - 1.5 + 2(y(s)) = 0$$

$$y(s)(s+2) = 1.5$$

$$y(s) = \frac{1.5}{s+2}$$

Apply Inverse Lap tra

$$y(t) = \mathcal{L}^{-1}\left\{\frac{1.5}{s+2}\right\}$$

$$= 1.5\left\{\frac{1}{s+2}\right\}$$

$$= 1.5e^{-2t}$$

Ans

Q) S:-

$$y'' - \frac{1}{4}y = 0 \quad , y(0) = 12 \quad , y'(0) = 0$$

Sol:- Apply Laplace transfor.

$$\mathcal{L}\{y''\} - \frac{1}{4}\mathcal{L}\{y\} = 0$$

$$s^2 y(s) - s\,y(0) - y'(0) - \frac{1}{4}y(s) = 0$$

$$s^2 y(s) - 12s - \frac{1}{4}y(s) = 0.$$

$$y(s)\left(s^2 - \frac{1}{4}\right) = 12s.$$

$$y(s) = \frac{12s}{s^2 - 1/4}.$$

$$= 12 \frac{s}{s^2 - (1/2)^2}.$$

Apply Inverse Lap. Tr.

$$= 12\,\mathcal{L}^{-1}\left\{\frac{s}{s^2 - (1/2)^2}\right\}.$$

$$= 12 \cdot \cos\left(\frac{1}{2}\right)t \quad \text{Ans}$$

Q7:- $y'' + 7y' + 12y = +21e^{3t}$, $y(0) = 3.5$, $y'(0) = -10$

Sol:- Apply Laplace transformation.

$$\mathcal{L}\{y''\} + 7\,\mathcal{L}\{y'\} + 12\,\mathcal{L}\{y\} = 12\,\mathcal{L}\{e^{3t}\}.$$

$$s^2y(s) - sy(0) - y'(0) + 7(sy(s) - y(0)) + 12y(s) = 12\left(\frac{1}{s+3}\right)$$

$$s^2y(s) - 3.5s + 10 + 7(sy(s)) - 24.5 + 12y(s) = \frac{12}{s+3}.$$

$$y(s)[s^2 + 7s + 12] - 3.5s - 14.5 = \frac{12}{s+3}.$$

$$y(s)[s^2+7s+12] = \frac{12}{s+3} - 3.5s - 14.5.$$

$$= \frac{12 - 3.5s(s+3) - 14.5(s+3)}{(s+3)}$$

$$y(s)[s^2 + 3s + 4s + 12] = \frac{12 - 3.5s^2 - 10.5s - 14.5s - 43.5}{s+3}.$$

$$y(s)[(s+3)(s+4)] = \frac{-3.5s^2 - 24.5s - 31.5}{(s+3)}$$

$$y(s) = \frac{-3.5s^2 - 24.5s - 31.5}{(s+3)(s+3)(s+4)}.$$

$$\frac{-3.5s^2 - 24.5s - 31.5}{(s+3)^2(s+4)} = \frac{A}{s+3} + \frac{B}{(s+3)^2} + \frac{C}{s+4}.$$

$$\frac{-3.5s^2 - 24.5s - 31.5}{(s+3)^2(s+4)} = \frac{A(s+3)\overset{(s+4)}{} + B(s+4) + C(s+3)^2}{(s+3)^2(s+4)}$$

$$-3.5s^2 - 24.5s - 31.5 = A(s+3)(s+4) + B(s+4) + C(s+3)^2.$$

Put $s = -3$.

or ?

$$-3.5s^2 - 24.5s - 31.5 = A(s^2+7s+12) + B(s+4) + C(s^2+9+6s)$$

Comparing coefficients.

s^2: $A + C = -3.5. \Rightarrow \boxed{A = -3.5 - C} \rightarrow ①$

s: $7A + B + 6C = -24.5. \Rightarrow 28A + 4B - 24C = -98$

s^0: $12A + 4B + 9C = -31.5. \Rightarrow \underline{\pm 12A \pm 4B \pm 9C = \mp 31.5}$

$$16A + 33C = -66.5.$$

from ① $\Rightarrow$ $16(-3.5 - C) + 33C = -66.5.$

$$-56 - 16C + 33C = -66.5.$$

$$17C = 10.5$$

$$\boxed{C = 0.62.}$$

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$A = -3.5 - 0.62$

$\boxed{A = -4.12}$

$7(-4.12) + B + 6(0.62) = -24.5$

$B = -24.5 + 28.84 - 3.72$

$\boxed{B = 0.62}$

$$= \frac{-4.12}{s+3} + \frac{0.62}{(s+3)^2} + \frac{0.62}{s+4}$$

Apply Inverse Lapl. Trans

$$= -4.12\, L^{-1}\left(\frac{1}{s+3}\right) + 0.62\, L^{-1}\left(\frac{1}{(s+3)^2}\right) + 0.62\, L^{-1}\left\{\frac{1}{s+4}\right\}$$

$$= -4.12\, e^{-3t} + 0.62\, e^{-3t}\frac{t}{1} + 0.62\, e^{-4t}$$

$$= -4.12 e^{-3t} + 0.62 e^{-3t} t + 0.62 e^{-4t}$$

Ans

$L\left(\frac{1}{s}\right) = t^0$

$L\left(\frac{1}{s^2}\right) = \frac{t^2}{2!}$

$L^{-1}\frac{1}{(s+a)^n} = \frac{e^{-at}\, t^{n-1}}{(n-1)!}$

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Q5:- $y'' - 4y' + 4y = 0 \quad y(0) = 8.1, \quad y'(0) = 3.9.$

Sol:- Apply Lap. Transf-

$\mathcal{L}\{y''\} - 4\mathcal{L}\{y'\} + 4\mathcal{L}\{y\} = 0.$

$s^2 y(s) - y(0) - y'(0) - 4(sy(s) - y(0)) + 4y(s)) = 0.$

$s^2 y(s) - 8.1 - 3.9 - 4(sy(s) - 8.1) + 4y(s) = 0.$

$s^2 y(s) - 12 - 4s y(s) + 32.4 + 4y(s) = 0.$

$y(s)(s^2 - 4s + 4) = 12 - 32.4$

$y(s)((s)^2 - 2(2)(s) + (2)^2) = -20.4.$

$$y(s) = \frac{-20.4}{(s-2)^2}$$

$$y(s) = \frac{-20.4}{(s-2)^2}$$

Apply Inverse Laplace transformation.

$$= -20.4\, \mathcal{L}^{-1}\left\{\frac{1}{(s-2)^2}\right\}.$$

$$= -20.4\, e^{2t} \cdot \frac{t}{1!}$$

$= -20.4\, t\, e^{2t}$ Ans

Q10: $y'' + 0.04y = 0.02t^2, \; y(0) = -25, \; y'(0) = 0$

Sol: Apply Lap trans.

$$\mathcal{L}\{y''\} + 0.04\,\mathcal{L}\{y\} = 0.02\,\mathcal{L}\{t^2\}$$

$$s^2 y(s) - s y(0) - y'(0) + 0.04\, y(s) = 0.02 \frac{(2)}{s^3}$$

$$(s^2 + 0.04)\,y(s) + 25s = \frac{0.04}{s^3}$$

$$(s^2 + 0.04)\,y(s) = \frac{0.04}{s^2} - 25s$$

$$= \frac{0.04 - 25s^3}{s^2}$$

$$y(s) = \frac{0.04 - 25s^3}{s^2(s^2 + 0.04)}$$

$$\frac{0.04 - 25s^3}{s^2(s^2+0.04)} = \frac{A}{s} + \frac{B}{s^2} + \frac{Cs + D}{s^2 + 0.04}$$

$$0.04 - 25s^3 = A(s)(s^2 + 0.04) + B(s^2 + 0.04) + (Cs + D)s^2$$

By Comparing Co-efficients,

s^3: $A + C = -25 \longrightarrow (1)$

s^2: $B + D = 0 \Rightarrow \boxed{B = -D}$

$s: \quad 0.04A = 0$

$$\boxed{A = 0}$$

$s^0: \quad 0.04B = 0.04.$

$$\boxed{B = 1}$$

$$\boxed{D = -1}$$

$eq\ ① \Rightarrow \quad A + C = -25.$

$$\boxed{C = -25}$$

$$= \frac{1}{s^2} + \frac{(-25)s - 1}{s^2 + 0.04}$$

$$= \frac{1}{s^2} - \frac{25s + 1}{s^2 + 0.04}.$$

$$= \frac{1}{s^2} - 25\frac{s}{s^2 + (0.2)^2} - \frac{1}{s^2 + (0.2)^2}.$$

$$= 1/s^2 - 25\frac{s}{s^2 + (0.2)^2} - \frac{1}{0.2}\frac{0.2}{s^2 + (0.2)^2}.$$

Apply Inverse Laplace

$$= L^{-1}\left(\frac{1}{s^2}\right) - 25L^{-1}\left(\frac{s}{s^2 + (0.2)^2}\right) - \frac{1}{0.2}L^{-1}\left(\frac{0.2}{s^2 + (0.2)^2}\right).$$

$$= t - 25\cos 0.2t - \frac{1}{0.2}\sin 0.2t$$

Ans

Q23 :- $\frac{2}{s^2+s/3} = \frac{2}{s(s+1/3)}$

$$\frac{A}{s} + \frac{B}{s+1/3} = \frac{2}{s(s+1/3)}$$

$$2 = A(s+1/3) + B(s)$$

$$\boxed{B = -6}$$

$$\boxed{A = 6}$$

$$= \frac{6}{s} - \frac{6}{s+1/3} \Rightarrow \boxed{6\cdot t - 6e^{-1/3 t}}$$

Ans

Q27 :- $\frac{s+8}{s^4+4s^2}$

Sol :- $\frac{s+8}{s^2(s^2+4)} = \frac{A}{s^2} + \frac{(Bs+C)}{s^2+4}$

~~s+8 = A(s^2+4) + [B(s)+C] [s^2]~~

$$\frac{s+8}{s^2(s^2+4)} = \frac{A}{s} + \frac{B}{s^2} + \frac{(Cs+D)}{s^2+4}$$

$$s+8 = A(s)(s^2+4) + B(s^2+4) + (Cs+D)(s^2)$$

By Comparing Coefficients.

s^3: $A + C = 0$.

$\boxed{A = -C}$

s^2: $B + D = 0$.

$\boxed{B = -D}$

s: $\boxed{A = 1}$

$\boxed{C = -1}$

s^0: $4B = 8$.

$\boxed{B = 2}$

$\boxed{D = -2}$

$$\frac{s+8}{s^2(s^2+4)} = \frac{1}{s} + \frac{2}{s^2} + \frac{(-1s-2)}{s^2+4}.$$

$$= \frac{1}{s} + \frac{2}{s^2} - \frac{s+2}{s^2+4}.$$

$$= L^{-1}\left\{\frac{1}{s}\right\} + 2L^{-1}\left\{\frac{1}{s^2}\right\} - L^{-1}\left\{\frac{s}{s^2+(2)^2}\right\} + L^{-1}\frac{2}{s^2+(2)^2}.$$

$$= t^0 + t^2 - \cos 2t + \sin 2t.$$

Ans

Q29:- $\dfrac{1}{s^3+as^2}$.

Sol:- $\dfrac{1}{s^2(s+a)} = \dfrac{A}{s} + \dfrac{B}{s^2} + \dfrac{C}{s+a}$

$1 = A(s)(s+a) + B(s+a) + C(s^2)$

s^2: $A + C = 0$

$\boxed{A = -C}$

where a is Constant.

s: $Aa + B = 0$

$\boxed{A = -\dfrac{B}{a}}$

s^0: $Ba = 1 \Rightarrow \boxed{B = 1/a}$

$\boxed{A = -1/a^2}$

$\boxed{C = 1/a^2}$

$= -\dfrac{1}{a^2} \mathcal{L}^{-1}\left(\dfrac{1}{s}\right) + \dfrac{1}{a} \mathcal{L}^{-1}\left\{\dfrac{1}{s^2}\right) + \dfrac{1}{a^2} \mathcal{L}^{-1}\left(\dfrac{1}{s+a}\right\}$

$= -\dfrac{1}{a^2} t^0 + \dfrac{1}{a} t + \dfrac{1}{a^2} e^{-at}$ Ans

$= 1/a\, t + (e^{-at} - 1) 1/a^2$ Ans

Q25:- $\frac{1}{s(s^2+\omega^2/4)}$

Sol:- $$\frac{1}{s(s^2+\omega^2/4)} = \frac{A}{s} + \frac{(Bs+C)}{s^2+\omega^2/4}$$

$$1 = A(s^2+\omega^2/4) + (Bs+C)(s)$$

Put $s=0$

$\boxed{A = 4/\omega^2}$

s^2: $A + B = 0$

$\boxed{A = -B}$

s: $\boxed{C = 0}$

s^0: $A\,\omega^2/4 = 1$.

$\boxed{A = 4/\omega^2}$, $\boxed{B = -4/\omega^2}$

$$1 = \frac{4}{\omega^2(s)} - \frac{4s}{\omega^2(s^2+\omega^2/4)}$$

$$= \frac{4}{\omega^2} t^0 - \frac{4}{\omega^2} L^{-1}\left(\frac{s}{s^2+(\omega/2)^2}\right)$$

$$= 4/\omega^2 - 4/\omega^2 \cos \omega/2\, t \Rightarrow \frac{4}{\omega^2}\left(1-\cos \omega/2\, t\right)$$

Ans