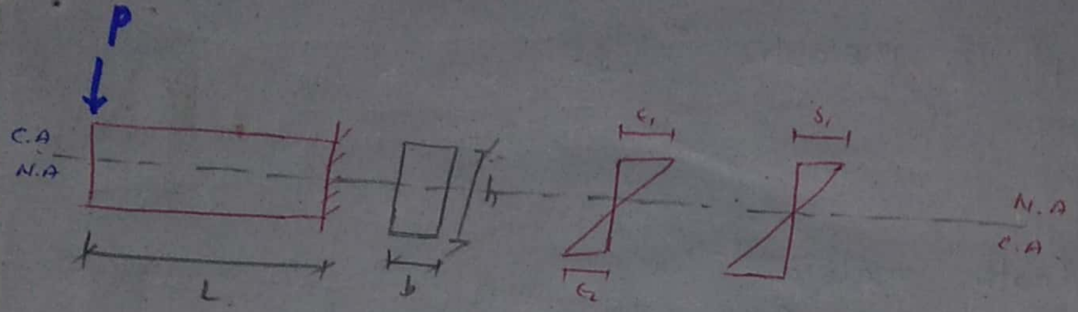
stresses (max)

Analysis of Inelastic Beam:-



consider a cantilever beam of length l, The load P is applied at end of beam as shown in fig. The load P is applied in such a way that cause symmetrical bending. In this case N.A & C.A are considing with each other.

- The strain will varry linearly. Zero at N.A & increase to extreme fibre
- So we multiply modulus of elasticity as matrial obey Hook's law to get corresponding stress profile as shown.
- The strain & stresses at any two fibres equidistant from N.A will be same.
- Stresses are also zero at N.A & increase linearly till extreme fibre.

Assumptions:-

(i) plane section remain plane

(ii) Matrial obeys Hook's law.

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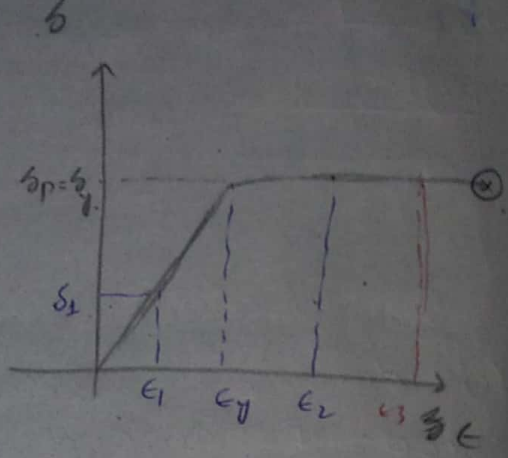
let consider fully plastic material in order to understand the concept of inelastic fully plastic

For plastic material the stress strain is linear till proportional limit as shown so after that its graph become linear & fracture after some time



Fully plastic material

To find $\sigma_1 = ?$

As we know that;

$$\sigma_f = \frac{My}{I}$$

To find max stresses which takes place at extreme fibre.

$$\sigma^{max} = \sigma_1$$

$$\sigma_1 = \frac{M\, y_{max}}{I} = \frac{Mc}{I} \qquad \because y_{max} = c.$$

$$\sigma_1 = \frac{M}{(I/c)}$$

$\because$ c & I are constant.

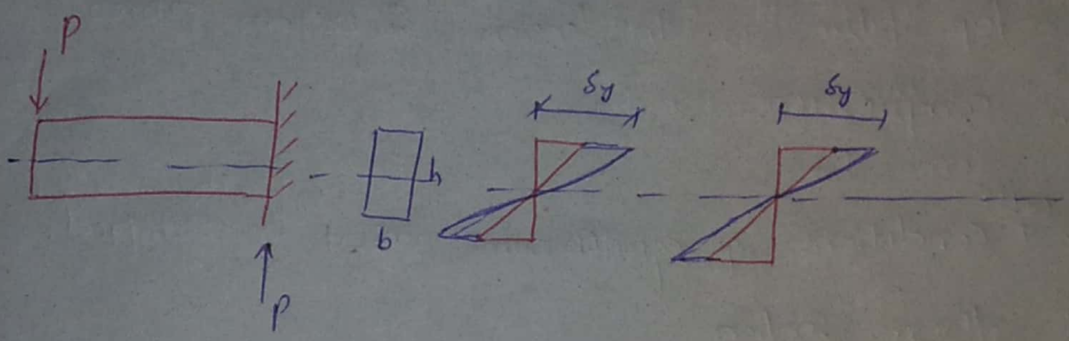
$$\boxed{\sigma_1 = \frac{M}{S}} \longrightarrow ①$$

$\because I/c = S =$ section modulus or elastic section modulus.

σ_1 is not max. stresses, it is those stresses (max) which is generated by load P.

Now let increase the load from P to P_y

This load will increase stresses to σ_y & also strain will increase. It still obey Hooke law.



The max. stress will be;

$$\sigma^{max} = \sigma_y = \frac{Mc}{I} = \frac{M_y}{I/c}$$

$\because M_y$ = the moment which correspond to yielding moment or yielding stresses.

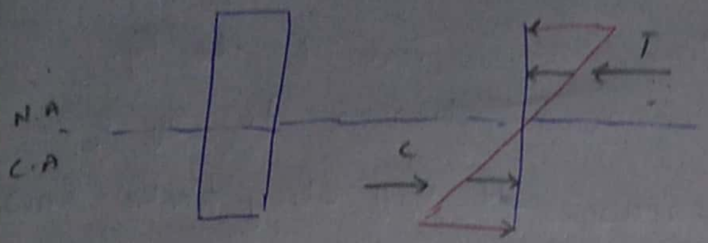
$$\boxed{\sigma_y = \frac{M_y}{S}}$$

To find the maximum capacity that the load can carry is;

$$\boxed{M_y = \sigma_y S}$$

Total capacity of beam.

Maximum stresses at time of Yielding:-



The top fibre is in tension & bottom fibres are in compression.

Two (call) conditions of equillibrium must be satisfied at these section.

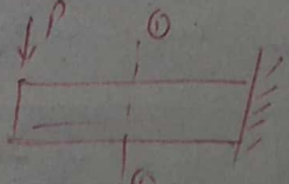
1) Sum of Horizontal force must be equal to zero for equillibrium. i.e

$\Sigma F_x = 0$.

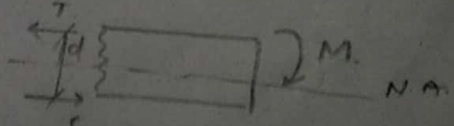
If T is Resultant tensile force & C is resultant compression force

$C - T = 0$

$\boxed{C = T}$



Consider section 1-1 & take right side of section.



$\Sigma M = 0$

External moment = Internal moment.

$$\boxed{M = C*d = T*d.}$$

As we know;

$$M_y = \sigma_y * S = \sigma_y * I/c.$$

But $I = \frac{bh^3}{12}$ & $c = h/2$.

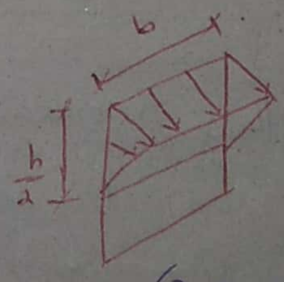
So;

$$M_y = \sigma_y * \frac{bh^3}{12} * \frac{2}{h}.$$

$$\boxed{M_y = \frac{1}{6} bh^2 \sigma_y}$$

For symmatric & rectangular section the stress distribution is linear. i.e top fibre will be in tension & bottom fibre will be in compression.

The resultant of that upper block is T. which is volume of that block.



(3-D view)

$$T = \frac{1}{2} \sigma_y * b * \frac{h}{2}.$$

$$\boxed{T = \sigma_y \frac{bh}{4}}$$

Similarly $\boxed{C = \sigma_y \frac{bh}{4}}$

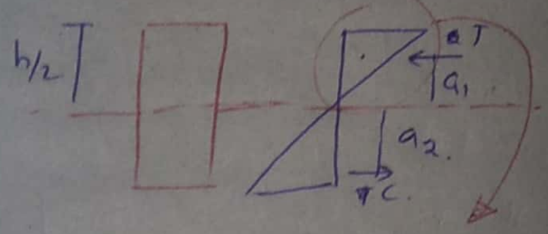
$\Sigma F_x = 0$

$\boxed{C = T}$ (Equillibrium hold).

(2) $\Sigma M = 0$

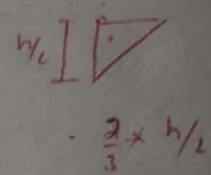
External moment = Sum of Internal moments.

$$M = \Sigma M_i$$



$$M = T * a_1 + C * a_2$$

$$M = \sigma_y \frac{bh}{4} * \frac{2}{3}\frac{h}{2} + \sigma_y \frac{bh}{4} * \frac{2}{3}\frac{h}{2}$$



$$M = 2 * \left[\sigma_y \frac{bh}{4} * \left(\frac{2}{3}\frac{h}{2}\right)\right]$$

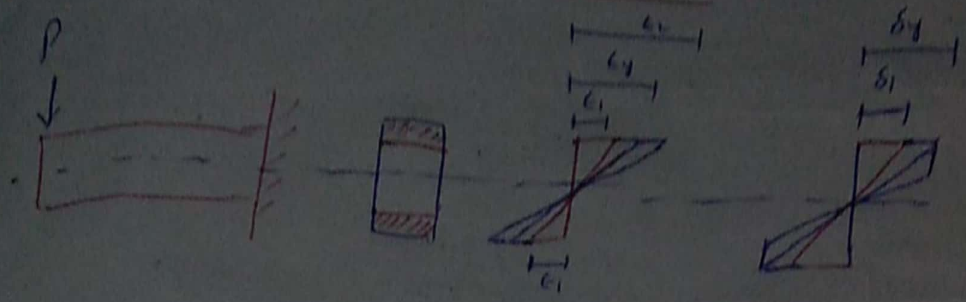
$$\boxed{M = \frac{1}{6}\sigma_y b h^2}$$

As this moment cause extreme fibre to yield so; $M = M_y$.

$$\boxed{M_y = \frac{1}{6}\sigma_y b h^2}$$

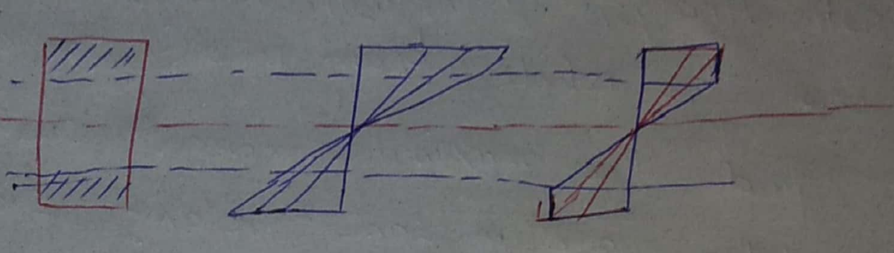
If we know the stress diagram for beam we can calculate the capacity of that beam.

Now if load is further increase



If load is further increase to P_2 the strain will also increase but the stresses will not increase for those fibres (extreme) whose stresses are equal to σ_y.

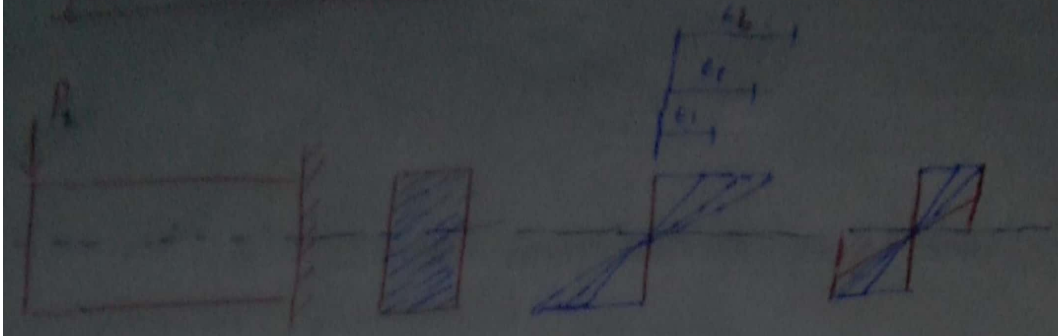
- Those fibres whose strain are larger than σ_y their stresses will remain same as extreme fibre & those whose strain is smaller than ϵ_y their stress will increases.



The shaded portion which cannot take more load when intermediate portion can take more load. bcz the stress & strain are still below the proportional limit.

- The failure depend upon the type of building is & on engineer that how much stresses he wanted allow

If load increase

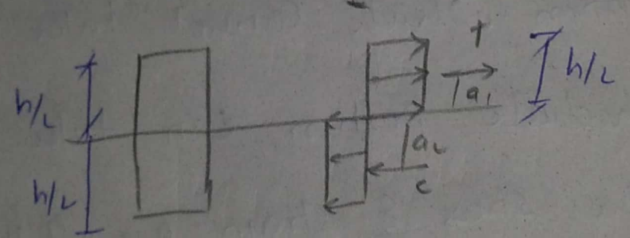


As the beam can carry more load so let some more load P_2 is applied. Now consider at this load P all stresses in cross section is σ_p i.e complete cross is yielding or we can say that whole cross section become fully plastic

Plastic Analysis & Inelastic Analysis:-

As the whole beam under yielding stress

To find the maximum moment such that all the fibres are in same yielding stresses



$$\Sigma F_x = 0 \cdot \quad T = C$$

Since $T = \sigma_y * b * \frac{h}{2}$

$$T = \sigma_y \frac{bh}{2}$$

As $T = C$

so;

$$\boxed{C = \sigma_y \frac{bh}{2}}$$

$\Sigma M = 0$

$M = \Sigma M_i$

As this ext. moment cause all fibres to yield so;

$$M_p = \Sigma M_i$$

$$M_p = T * a_1 + C * a_2$$

$$M_p = \left(\sigma_y \frac{bh}{2} * \frac{h}{4}\right) + \left(\sigma_y \frac{bh}{2} * \frac{h}{4}\right) \longrightarrow 1$$

$$= \cancel{2} \, \sigma_y \frac{bh}{\cancel{2}} * \frac{h}{4}$$

$$\boxed{M_p = \sigma \frac{bh^2}{4}}$$

$\because \frac{bh^2}{4}$ is constant which is equal to Z where Z is called plastic section modulus.

so;

$$\boxed{M_p = \sigma_y Z}$$

& 1 As $\frac{bh}{2}$ is area of cross section above N.A or which & T is acting & $h/4$ is distance from centroid to N.A so, it is first moment of area

So it means we can easily find elastic moment by multiplying & σ_y & first moment of area.

& if we required elastic modulus which will simply mean first moment of area.

→ For elastic analysis if cross is symmetrical & symmetrical bending takes place than C.A & N.A will conside.
But for plastic analysis if cross section symmetrical the N.A & C.A will consid.
• For unsymmetrical or monosymmetrical the N.A will not conside with C.A in plastic analysis.

Shape factor:-

The ratio of plastic ... ela... moment.

or The ratio of pl... ...ulu... modus is called ...pe factor.

to elastic section modus is called shape factor.

(How much load beam can carry)

It tells us reserve strength of beam beyond elastic limit

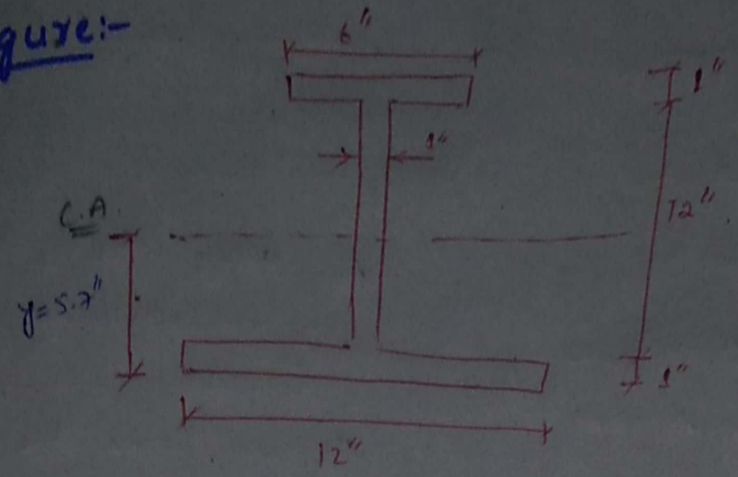
$$K = \frac{M_p}{M_y} = \frac{Z}{S} = \frac{\sigma_y bh^2}{\sigma_y 4} \times \frac{6}{bh^2}$$

$$K = \frac{3}{2} = 1.5$$

It mean the beam can carry 50% more load after yield point.

Pb# 14.08 The centroidal axis of section shown in fig is 5.7" above the bottom and the moment of inertia about centroidal axis is 855 in^4. Determine the ratio of limit moment to the yield moment for beam having this section.

Figure:-



Solution:-

~~First~~ First step is to find location of N.A which is already given that C.A is i.e 5.7" above bottom end. in order to satisfy this condition;

$T = C$

$\cancel{\sigma_y} A_T = \cancel{\sigma_y} A_C$

$\boxed{A_{\text{above N.A}} = A_{\text{below N.A}}}$

In order to satisfy equillibrium this condition must satisfy

As for elastic analysis, both ~~sym~~ N.A & C.A will be same even for mono & unsymmatrical.

As; $K = \dfrac{M_p}{M_y} \longrightarrow ①$

To find $M_y = ?$

As $M_y = F_y (S)$

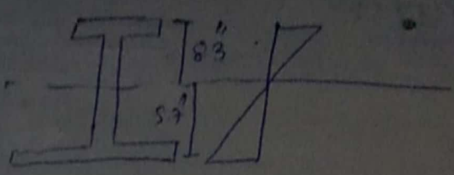
$= F_y (I/c)$

$= F_y \dfrac{855 \text{ k-in}}{8.3}$

$= F_y (103.01) \text{ k-in}$

$\hookrightarrow$ (1)

where c is distance from N A to extreme fibre. So first find the stress curve to determine which one is extreme fibre



To find $M_p = ?$

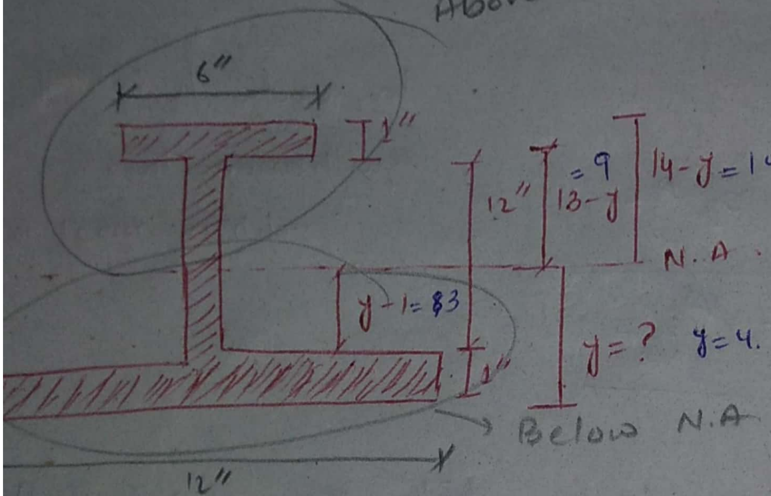
As for plastic analysis the location of N.A will change.

To find location of N.A consider

As obey Hook's law so it will be linear.

As we know the flexural stresses will increases as the distance from extreme fibre increase. So the stress in top fibre will be more & will yield first.

So consider 8.3"



$A_A = A_B.$

$(6*1) + (13-y)*1 = (y-1)*1 + 12*1.$

$6 + 13 - 13y = y - 1 + 12$

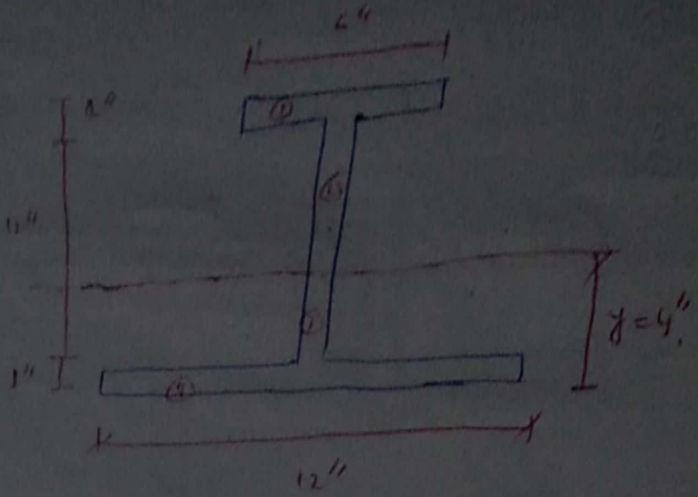
$2y = 8 \Rightarrow \boxed{y = 4"}$

So for plastic ~~moment~~ moment;

$M_p = \sigma_y Z$

As Z is first moment of area

So;



$$M_p = \sigma_y \left[6\times1 \times 9.5 + 9\times1\times4.5 + 3\times1\times1.5 + 12\times1\times3.5 \right].$$

$$M_p = \sigma_y \, [144] \text{ kip-in.}$$

$$M_p = 144\, \sigma_y \text{ kip-in}$$

As of (c) =>

$$K = \frac{M_p}{M_y}.$$

$$\therefore \frac{M_p - M_y}{M_p} \times 100.$$

$$= \frac{144\, \sigma_y \text{ kip in}}{\sigma_y\,(103.01) \text{ kip-in.}}$$

$$\boxed{K = 1.39}$$

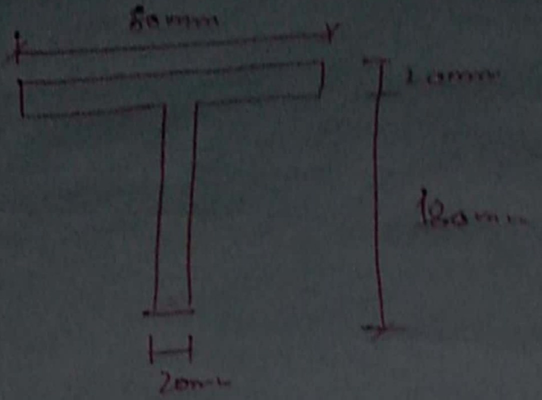
So it means this beam can carry 40% more load after yield point.

Pb# For the fig as shown,

(a) Find the shape factor.

(b) Find P at which yield stresses reach to 250 MPa

Figure:-



Solution:- As shape factor is;

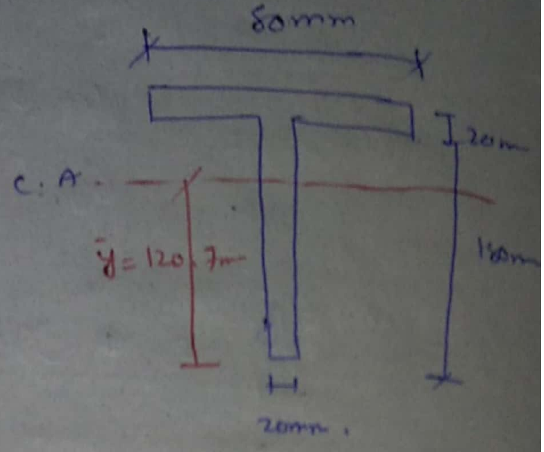
$$K = \frac{M_p}{M_y} \longrightarrow (1).$$

To find $M_y = ?$

as $M_y = \sigma_y \left(\frac{I}{c}\right) \longrightarrow (A)$.

To find $I = ?$

First find Centroid.



$$\bar{y} = \frac{80 \times 20 \times (190) + 180 \times 20 \times 90}{80 \times 20 + 180 \times 20}.$$

$$\boxed{\bar{y} = 120.7\,mm}$$

Now; $I_x = \frac{80 \times (20)^3}{12} + 80 \times 20 (190 - 120.7)^2$

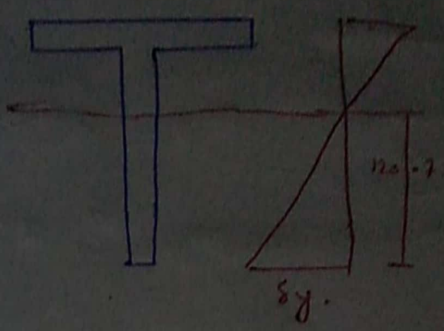
$+ 20(180)^3/12 + 20(180)(180 - 120.7)^2.$

$$\boxed{I_x = 20.8 \times 10^6\ mm^4}$$

A3

$$M_y = \sigma_y \frac{I}{c}$$

So select the distance to extreme fibre so draw stress profile about C.A



So, $c = 120.7\ mm$

$$\Rightarrow M_y = \sigma_y \frac{20.8 \times 10^6}{120.7}$$

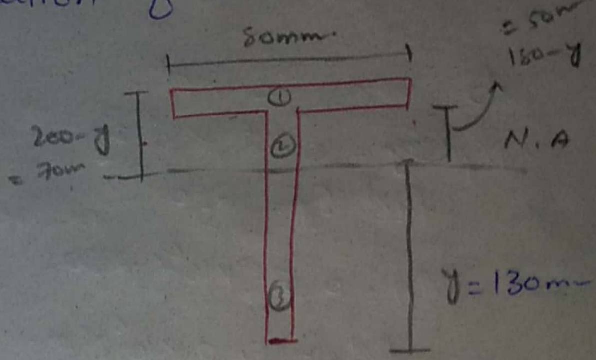
$$M_y = 0.17 \times 10^6\ \sigma_y \longrightarrow (2)$$

To find $M_p = ?$

We first find location of N.A.

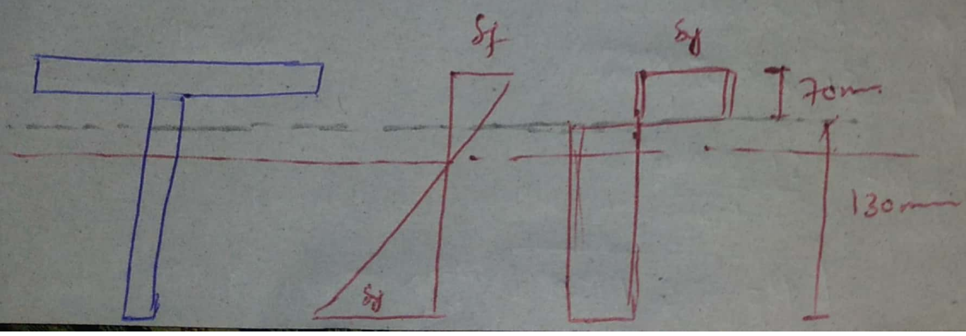
For equillibrium

$A_{above} = A_{below}$.



$$80 \times 20 + (150 - y) \times 20 = 20 \times y$$

$$\boxed{y = 130 mm}$$



A $M_p = \sigma_y Z$

$= \sigma_y [80 \times 20 \times 60 + 50 \times 20 \times 25 + 130 \times 20 \times 65]$

$M_p = 290000\ \sigma_y$

$M_p = \sigma_y\ 0.29 \times 10^6\ Nmm$

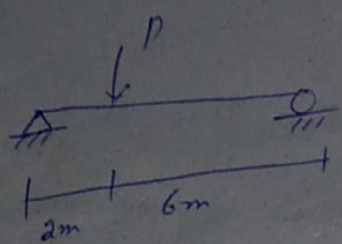
(b);

$$K = \frac{M_p}{M_p}$$

$$= \frac{\sigma_y (0.29 \times 10^6)\ Nmm}{\sigma_y (0.17 \times 10^6)\ Nm} \Rightarrow \boxed{1.7 = k}$$

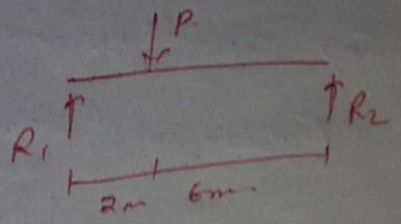
It carries 70% more load after Y.P.

(b)



First find Rxn

Σ moment



As we found.

$M_p\ \cancel{\sigma_y} = 0.29 \times 10^6\ \sigma_y$.

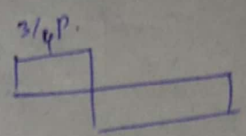
Put $\sigma_y = 280$

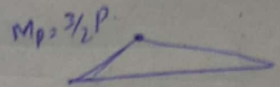
$M_p = 0.29 \times 10^6 \times 280$

$M_p = 81.2 \times 10^6\ Nm$.

$\Sigma R_1 = P \times 6$.

$\boxed{R_1 = \frac{3}{4} P}$, $\boxed{R_2 = \frac{1}{4} P}$





As The max. stress will cause by Max. M so M_{max}.

$M_{max} = 81.2 \times 10^6\ Nm$.

$3/2\ P = 81.2 \times 10^6\ N/m$

$$P = \frac{81.2 \times 10^6}{1.5}$$

$\boxed{P = 54.1\ KN}$ Ans

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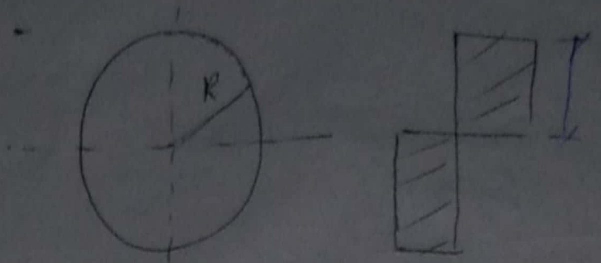
Inelastic Beam Problems

Pb#1404:- Verify the ratios M_L/M_{yp} specified in table 14-1 for solid circle & thin walled tube?

Ans:- For Circle



Solution:-

As;

$$M_y = \sigma_y S.$$

$\therefore D/_2 = c = R.$

$$M_y = \sigma_y (I/c).$$

$$= \sigma_y \frac{\left(\frac{\pi R^4}{4}\right)}{R}.$$

$$M_y = \sigma_y \frac{\pi R^3}{4} \longrightarrow (1).$$

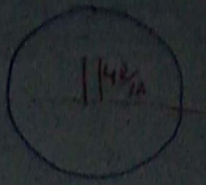
Plastic moment:-

$$M_p = \sigma_y Z.$$

$$= \sigma_y \left(\frac{1}{2} A\right)(\text{centroidal distance}).$$

$$= \sigma_y \left(\frac{1}{2} \pi R^2\right)(y_1 + y_2).$$

$$= \sigma_y \frac{\pi R^2}{2}\left(\frac{4R}{3\pi} + \frac{4R}{3\pi}\right)$$



$$M_p = \frac{4}{3}\sigma_y \pi R^3$$

So k factor is;

$$K = \frac{M_p}{M_y}$$

$$K = \frac{\frac{4}{3}\cancel{\sigma_y}\,\cancel{\pi}\,\cancel{R^3}}{\cancel{\sigma_y}\,\frac{\pi \cancel{R^3}}{4}}$$

$$= \frac{4 \times 4}{3 \times 3.14} \Rightarrow 1.69$$

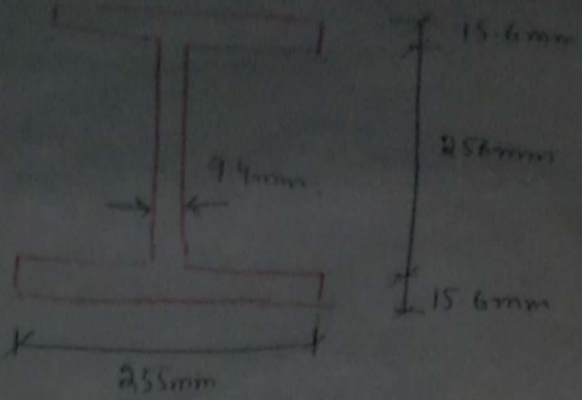
$$\boxed{K \simeq 1.7}$$

For thin walled tube:-

Pbm 1405

Compute the ratio of limit moment to yield moment for W250×80 beam.

Figure:



From table properties of W250×80 are:

$A = 10{,}200\ mm^2$.

Depth = 256mm.

Flange wedth = 255mm

F. Thickness = 15.6mm.

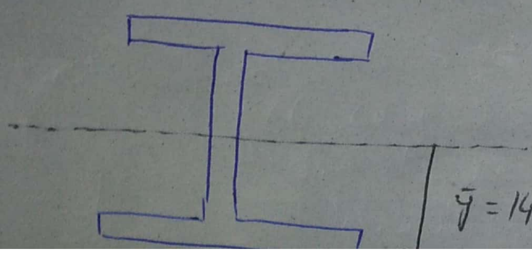
W. Thickness = 9.4mm.

$I_x = 126 \times 10^6\ mm^4$. $I_y = 43.1 \times 10^6\ mm^4$.

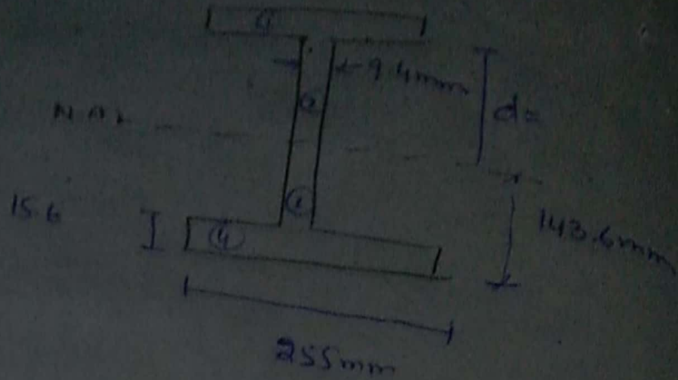
Solution:- As

$$K = \frac{M_p}{M_y} \longrightarrow (A).$$

Since $M_p = \sigma_y Z$ - where Z is first moment of area.



$\bar{y} = 143.6mm$,



$$M_{py} = \sigma_y\left((255 \times 15.6 \times 135.8)\times 2 + (64 \times 9.4 \times \overset{126}{\cancel{32}}) \times 2\right)$$

$$= \left(1080424.8 + \overset{154009.6}{\cancel{38\ldots}}\right)\sigma_y$$

$$M_{yp} = \overset{1234434.4}{\cancel{1118927.2}}\ \sigma_y \longrightarrow (1)$$

$$M_y = \sigma_y S$$

$$= \sigma_y\, I/c$$

$$= \sigma_y\left(\frac{43.1\times10^6}{143.6}\right)$$

$$M_y = 300139.27\ \sigma_y$$

{(4) $\Rightarrow$

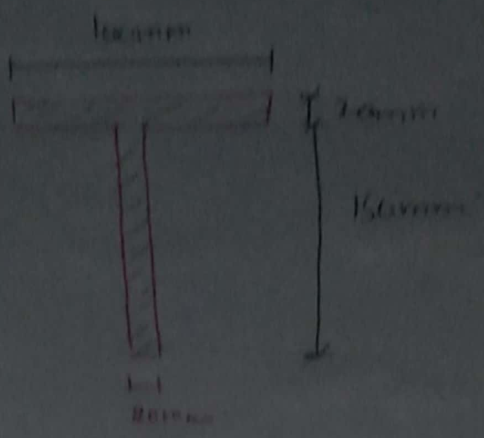
$$K = \frac{111892 7.2\ \sigma_y}{300139.27\ \sigma_y}$$

Confused in selection of I.

Pb# 1408

Find shape factor for T-section as shown.

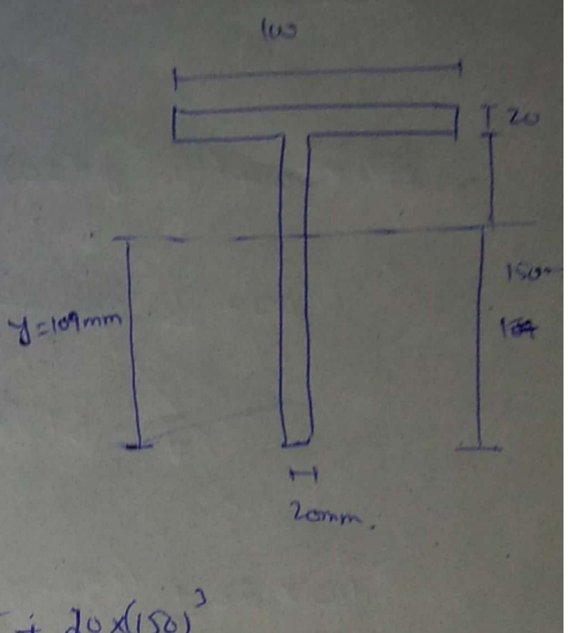
Figure:



Solution:- First find Centroid.

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2} = \frac{100 \times 20 \times 160 + 150 \times 20 \times 75}{100 \times 20 + 150 \times 20}$$

$$\boxed{\bar{y} = 109mm}$$



To find $I = ?$

$$I = \Sigma \left(\frac{bh^3}{12} + Ad^2 \right)$$

$$= \frac{100 \times 20^3}{12} + 100 \times 20 \times (51)^2$$

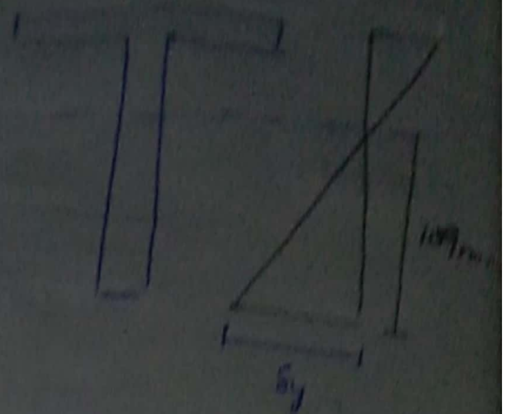
$$+ 150 \times 20 \times (109 - 75)^2 + \frac{20 \times (150)^3}{12}$$

$$I = 66666.67 + 5202000 + \frac{5625000}{\cancel{12}} + 3468000$$

$$\boxed{I = 14.36 \times 10^6 \, mm^4}$$

$M_y = \sigma_y \, S$

$= \sigma_y \, I/c$

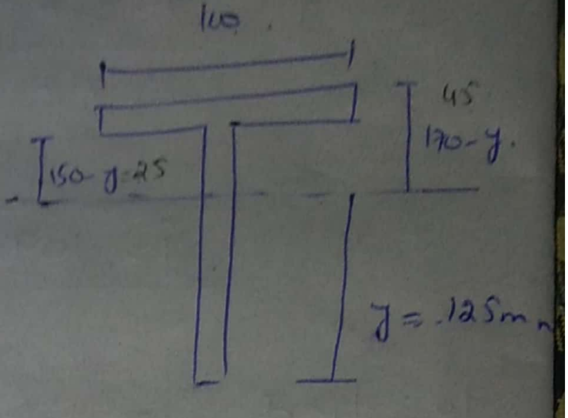


$= \sigma_y \left(\dfrac{14.36 \times 10^6}{109} \right)$

$= \sigma_y (0.1317 \times 10^6)$

To find $M_p = ?$

As monosymmetrical
so first find location of N.A.



$A_{above} = A_{below}$

$\cancel{180 \times 20 + (150 - y)(20) = 109y}$

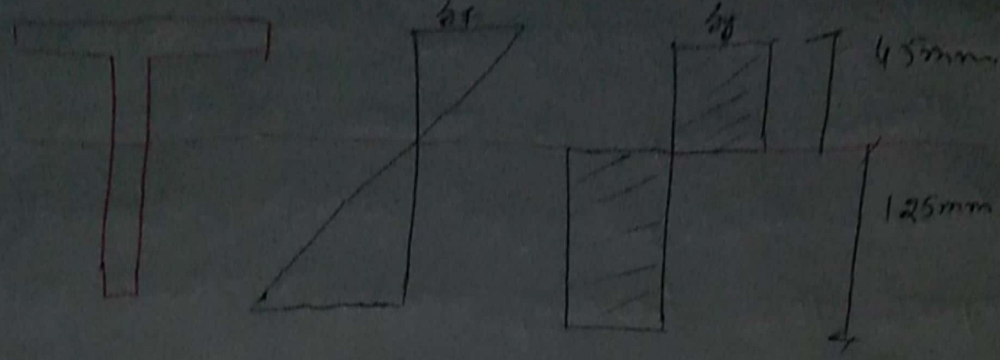
$\cancel{3600 + 3000 = 109y + 20y}$

$\cancel{y = 51.16}$

$100 \times 20 + (150 - y)20 = y \times \cancel{129}(20)$

$2000 + 3000 = \cancel{129}\,40\, y$

$\boxed{y = 125 \text{ mm}}$



$$M_p = \sigma_y Z$$

$$= \sigma_y(100\times 20\times 35 + 25\times 20\times 12.5 + 12.5\times 20\times 62.5)$$

$$M_p = \sigma_y(232500)$$

As,

$$K = \frac{M_p}{M_y}$$

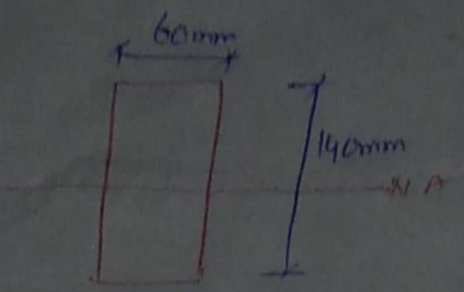
$$= \frac{232500\,\cancel{\sigma_y}}{0.1317\times 10^6\,\cancel{\sigma_y}}$$

$$\boxed{K = 1.76}$$

M = 134746.5

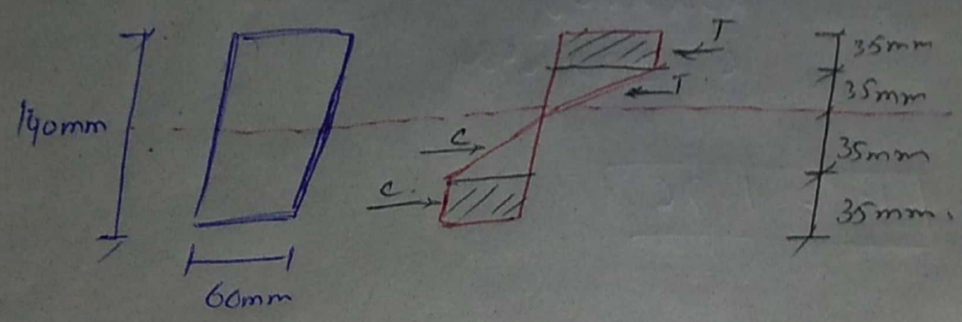
Pb # 1409 A rectangular beam 60mm wide & 140mm deep is made of elastic perfectly plastic material for which $\sigma_{yp} = 240MPa$. Compute the bending moment that will cause one half of the section to be in plastic range.

Figure:



Solution:-

Since M (bending moment) can be find as:



As;

$\Sigma F_x = 0$.

$\boxed{T = C.}$

$\Sigma M = 0$.

$M = T \times (35 + 35/2) + T \times \frac{2}{3} \times 35/2 + C \times (35 + 35/2) + C \times \frac{2}{3} \times \frac{35}{2}$

As & $T = \sigma_y \times A$

$$M = \left(\sigma_y \times 60 \times 35 (52.5) + \frac{1}{2}\sigma_y \times 60 \times 35 \times \frac{2}{3} \times 35\right) \times 2$$

$$M = 2\sigma_y (134750)$$

$$M = 2(240)(134750) = 2(240 \times 10^6)(134750 \times 10^{-9})$$

$$M = 64.6 \times 10^3 \; Nm$$

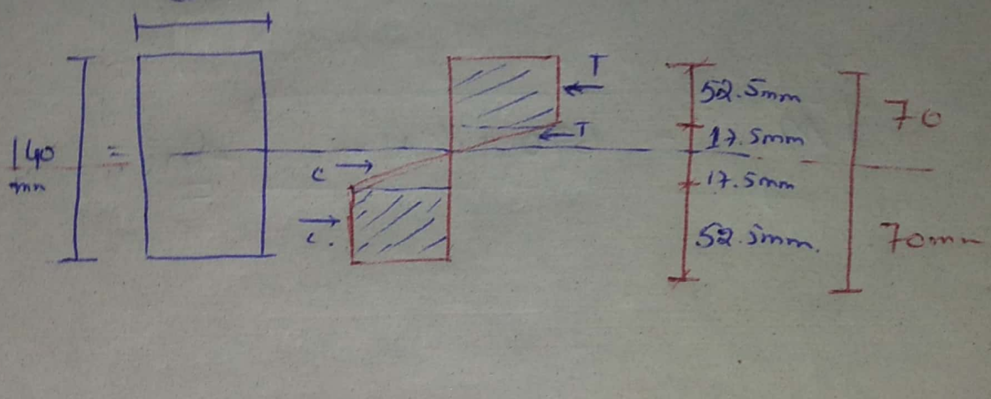
$$\boxed{M = 64.6 \; kNm} \; \underline{\underline{Ans}}$$

Pb#1410:-

In Prob 1409, determine the bending moment that will cause the middle three-fourth of section to be in plastic range.

$\sigma_{yp} = 240 MPa$

Solution:-



$\Sigma M = 0.$

$M = T\times(17.5+\frac{52.5}{2}) + T\times\frac{2}{3}(17.5) + C\times(17.5+\frac{52.5}{2}) + C\times\frac{2}{3}(17.5)$

As $\Sigma F_x = 0.$

$T = C$

so, $M = 2\left(T\times(17.5+\frac{52.5}{2}) + T\times{}^{2}/_{3}(17.5)\right)$

$M = 2\left(\sigma_y A\,(43.75) + \sigma_y A\,(11.67)\right)$

$= 2\sigma_y\left(60\times 52.5\times(43.75) + \frac{1}{2}(17.5)(60)(11.67)\right)$

$= 2\sigma_y(137812.5 + 6126.75)$

$= 2\sigma_y(143939.25)\,mm^3.$

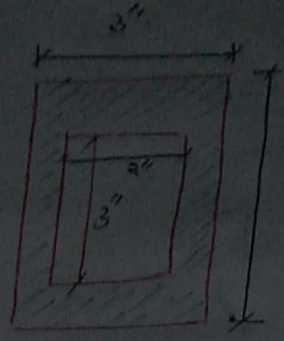
$= 2\left(\frac{240\times10^{6}}{m^2}\right)(143939.25\times10^{-9})\,m^3.$

$= 2(34545.42)$

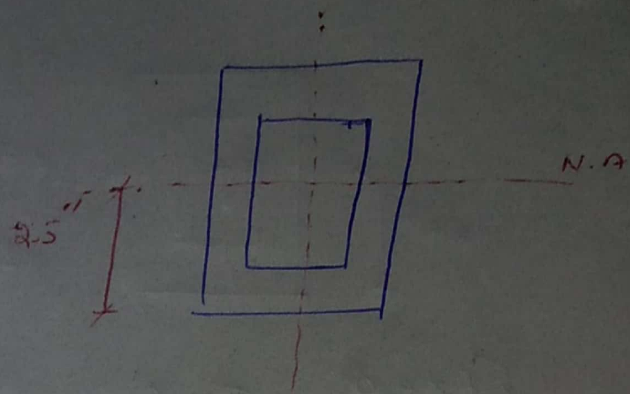
$\boxed{M = 690.1\,kNm}$ Ans

Page 411 If $\sigma_{yp} = 40\,ksi$, compute the limit moment for section shown in fig.

Figure:



Solution:- As symmatric about both axis so C.A will coincide with N.A

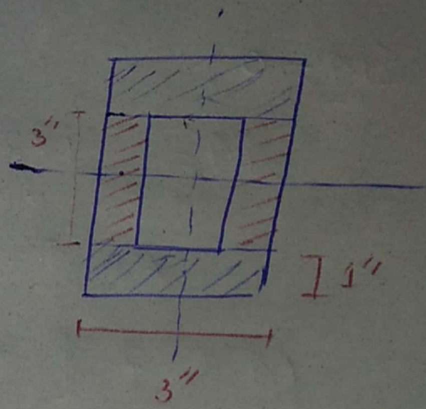


so;

$L.M = \sigma_y Z$

$M_L = \sigma_y Z$

$M_L = \sigma_y(3 \times 1 \times 2 \times 2 + 3 \times 1 \times 0)$



$M_L = \sigma_y(12)$

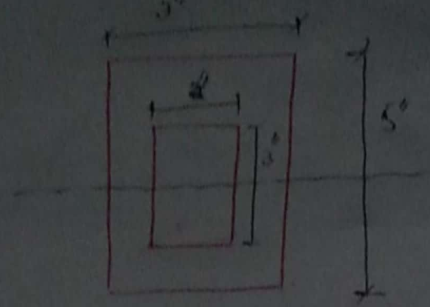
$= 40 \times 10^3 \times 12$

$M_L = 480 \times 10^3\ lb\ in.$

$$\boxed{M_L = 480\ Kin}$$

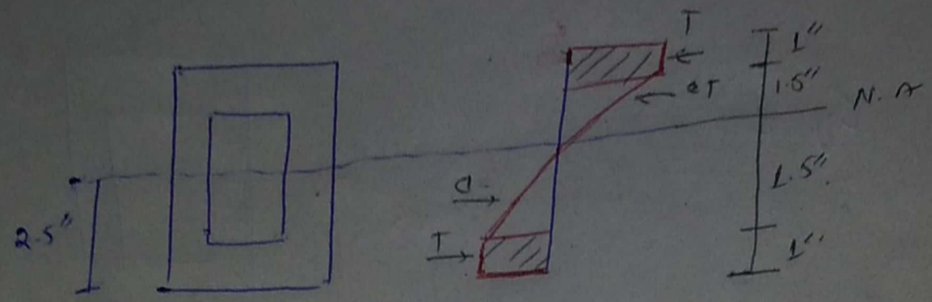
Pb# 1412 If $\sigma_{yp} = 40$ ksi, Compute the bending moment that will cause elastic region to extend 1.5" from the N.A of Section shown in fig P-1412.

Fig:



$\sigma_y = 40 \times 10^3$ psi

Sol:- Consider the fig.



$\Sigma F_x = 0$. $T = C$.

$\Sigma M_{N.A} = 0$.

$M = T \times 2 + T \times \frac{2}{3} \times 1.5 + C \times 2 + C \times \frac{2}{3} \times 1.5$

As $T = C$.

$M = 2 \left[T \times 2 + T \times \frac{2}{3} \times 1.5 \right]$. As $T = \sigma_y A$.

$M = 2 \left[\sigma_y \times A \times 2 + \cancel{\sigma_y A \times \frac{2}{3} \times 1.5}^{\,0} \right]$.

$= 2 \sigma_y [2 \times 3 \times 1]$.

$= 12 \sigma_y \Rightarrow 12 \times 40 \times 10^3$.

$\boxed{M = 480 \text{ Kip in}}$