Internal force System.

→ <u>Normal force</u>:-

The force component that is perpendicular to cross section, tending to elongate or shorten the bar is called normal force.

(*) <u>Shear force</u>:-

The force component laying in plane of cross section, tending to slide the part of bar laying on one another is called shear force.

→ N
Normal
force

(*) <u>Twisting moment</u>:-

The component of resultant moment that tend to twist the bar is called twisting moment.

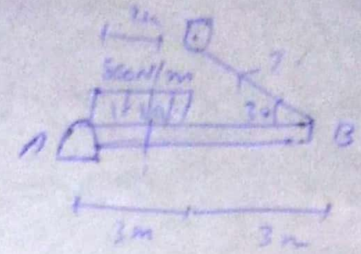
(*) <u>Bending moment</u>:-

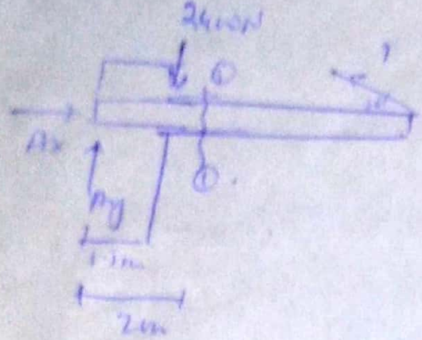
The component of resultant moment that tend to bend the bar is called bending moment.

→ The component of force laying in the plane of cross section, tending to slide (shear) the parts of bar laying on either side of cross section relative to one another is called shear force

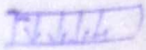
II 2

First Draw FBD of Fig.





$800 \times 2 = 1600N$



$\sum F_x = 0$

$A_x - T\cos 30° = 0 \longrightarrow ①$

$\sum F_y = 0$

$A_y - 2400 + T\sin 30° = 0 \longrightarrow ②$

$\sum M_A = 0$

$-2400(1.5) + T\sin(30°)(6) = 0$

$T = \frac{3600}{3}$

$T = 1200N$

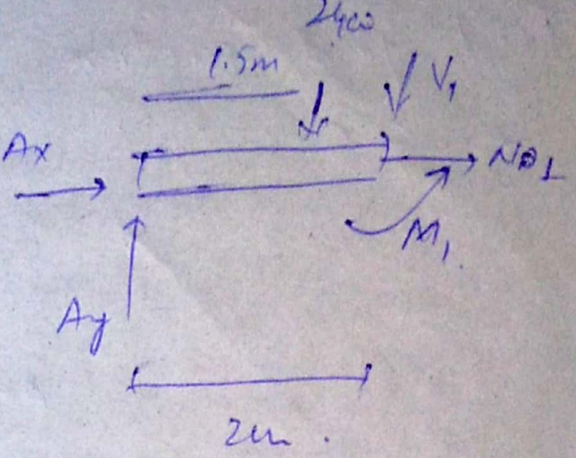
eq ① ⟶ $A_x = 1039.2N$

eq ② ⟶ $A_y - 2400 + 1200(0.5) = 0$

$A_y = 1800N$

Now take right of section 1

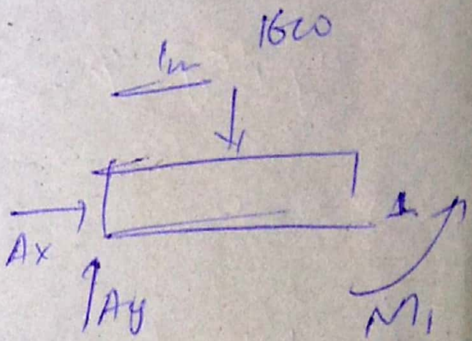
Draw FBD of section 1-1 to right.



$\Sigma F_x = 0.$

$A_x - N_D = 0.$

$N_D = -1039N$ → Assume direction wrong



$\Sigma F_y = 0.$

$\overset{1800}{A_y} - V_1 - \overset{1600}{\cancel{2400}} = 0$

$\boxed{V_1 = +200N}$

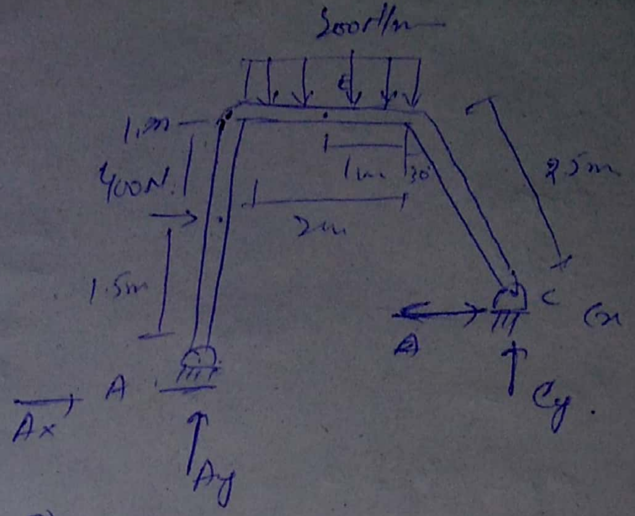
$\overset{+}{\circlearrowleft} \Sigma M_1 = 0$

$-1800 \times 2 + 1600 + M_1 = 0.$

$M_1 = 2000Nm$

Problem #11.2

First find reaction at A & C.

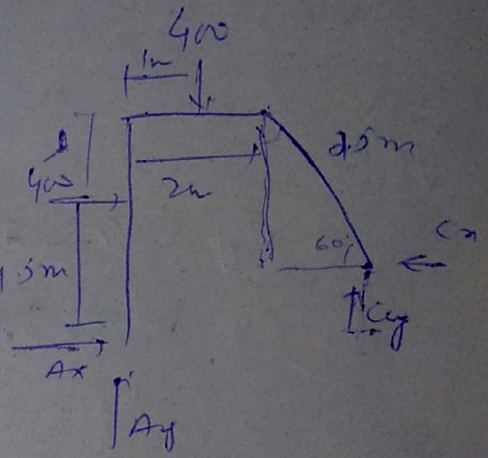


$\Sigma F_x = 0.$

$A_x - C_x + 400 = 0 \rightarrow (1).$

$\Sigma F_y = 0.$

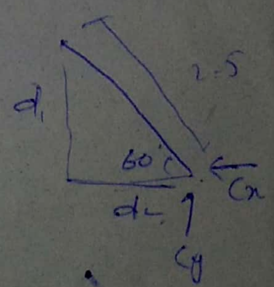
$A_y + C_y - 400 = 0 \rightarrow (2)$



We take B as moment centre as B act as internal Pin of the whole body is in equilibrium than Pin also in eq.

$\Sigma M_B = 0$

$C_y(1.25) - C_x(2.165) = 0.$



$C_y = 1.732\, C_x \rightarrow (3).$

$\cos 60 = \frac{d_2}{2.5}.$

$d_2 = 1.25.$

$\sin 60 = \frac{d_1}{2.5}$

$d_1 = 2.165.$

$\Sigma M_A = 0$

$-400(1) - 400(1.5) + C_x(2.5 - 2.165) + C_y(2 + 1.25) = 0.$

$C_x(0.335) + (3.25)C_y = 1000$

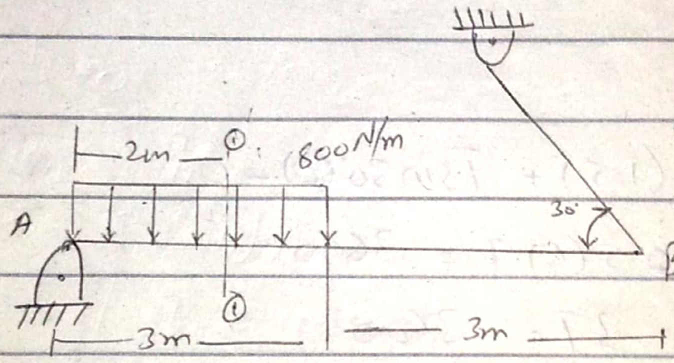
$0.335 C_x + 3.25(1.732)C_x = 1000$

$5.964\, C_x = 1000$

Module # ~~6~~ 11.

Problem 11.1

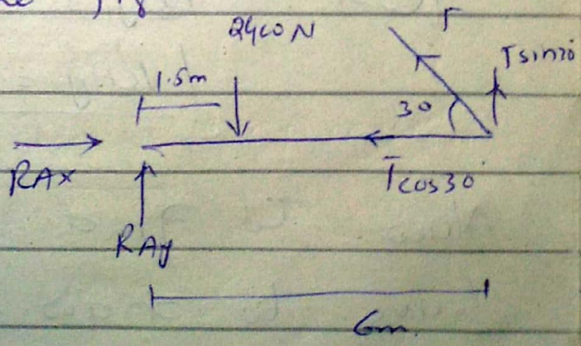
Determine the normal force, shear force and bending moment acting on cross section ①-① as shown in fig.



solution:-

We must find the ~~tota~~ reactions first in order to find the Internal forces.

So Draw FBD of the Fig.



Apply conditions of Equilibrium

$\Sigma F_x = 0.$

$R_{Ax} - T\cos 30^\circ = 0.$

$R_{Ax} - 0.866\,T = 0 \rightarrow (1).$

$\Sigma F_y = 0.$

$R_{Ay} + T\sin 30^\circ - 2400 = 0.$

$R_{Ay} + 0.5T = 2400 \rightarrow (2).$

$\overset{+}{\curvearrowleft} \Sigma M_A = 0$

$-2400 \times (1.5) + T\sin 30^\circ (6) = 0.$

$0.5(6)\,T = 3600.$

$3T = 3600.$

$\boxed{T = 1200\,N}$ (~~Tension~~)

eq (1) ⇒ $R_{Ax} - 0.866\,T = 0$

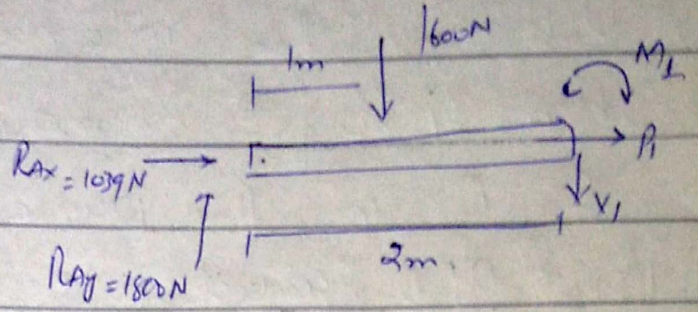
$R_{Ax} - 0.866(1200) = 0.$

$\boxed{R_{Ax} = 1039\,N}$ (~~[illegible]~~)

eq (2) ⇒ $R_{Ay} + 0.5(1200) = 2400.$

$\boxed{R_{Ay} = 1800\,N}$ (~~[illegible]~~

Now to find internal forces we have to draw FBD of body to left of section ①-①



Now Apply Conditions of equilibrium.

$\Sigma F_x = 0$.

$R_{Ax} + P_1 = 0$.

$P_1 + 1039\,N = 0$

$\boxed{P_1 = -1039\,N}$

-ve sign shows our assume direction is wrong.

$\Sigma F_y = 0$.

$R_{Ay} + (-1600) - V_1 = 0$.

$1800 - 1600 - V_1 = 0$.

$+200 - V_1 = 0$.

$-V_1 = -200$.

$\boxed{V_1 = 200\,N}$

$\Sigma M_c = 0$

$-1800 \times 2 + 1600 \times 1 - M_1 = 0$

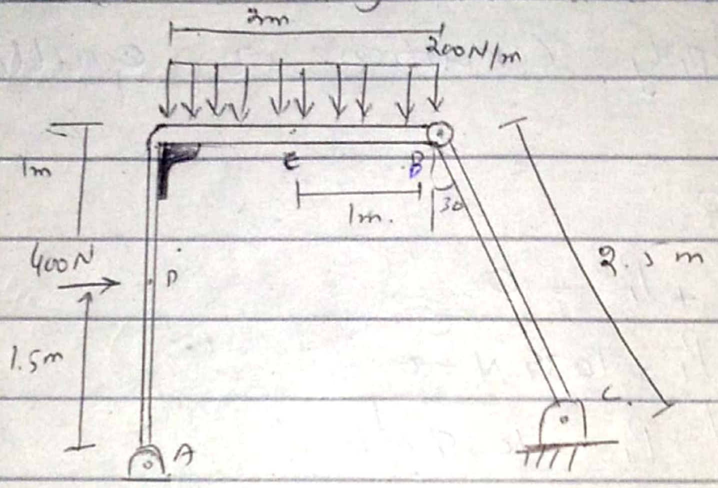
$-3600 + 1600 - M_1 = 0$ — -ve sign shows our assum direction is wrong.

$-M_1 - 2000 = 0 \Rightarrow \boxed{M_1 = -2000\,N.m}$

Problem 11.2

Determine the internal normal forces, Shear force & moment at point D & E in the frame.

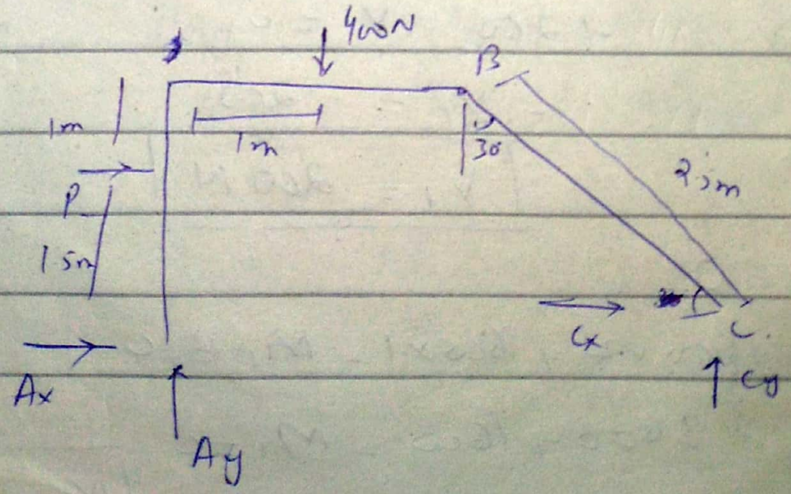
Point D is located just above 400 N force



Sol:- First step is to find the unknown reactions at A & C.

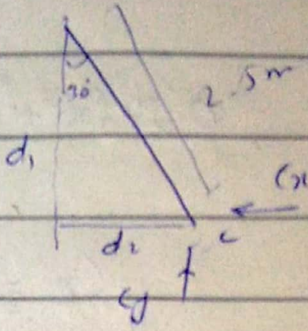
For that we have 4 unknown

So take B point as moment centre & draw FBD & fig.



$\circlearrowleft^{+} \; \Sigma M_B = 0$

We take B as moment Centre because B act as Internal pin where it has no moment there. Because if System is in equillibrium then Pin B will also in equillibrium.



from this eqn we will find reaction forces at C.

$$\cos 30° = \frac{d_1}{2.5}$$

$$-C_x \times 2.16 + C_y \times (1.25) = 0.$$

$$d_1 = 2.16$$

$$(1.25)\, C_y = (2.16)\, C_x$$

$$\sin 30° = \frac{d_2}{2.5}$$

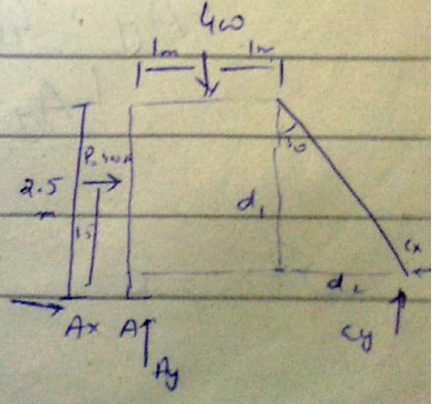
$$C_y = 1.73\, C_x \longrightarrow ①$$

$$d_2 = 1.25$$

Now take $\circlearrowleft^{+} \; \Sigma M_A = 0$

$$C_x(2.5 - 2.16) + C_y(2 + 1.25)$$

$$-400 \times 1 - 400 \times 1.5 = 0$$



$0.34 C_x + 1.73 C_x (3.25) = 1000$

$0.34 C_x + 5.6225 C_x = 1000$

∵ $C_y = 1.73 C_x$

∴ $400 \times 1.5 = 600$

$400 \times 1 = 400$

$C_x = \frac{1000}{5.96}$

$\boxed{C_x = 167.7 N}$

eq ① ⇒ $C_y = 1.73 C_x$

$= 1.73(167.7)$

$\boxed{C_y = 290.1 N}$

Now $\sum F_y = 0$

$A_y - 400 + C_y = 0$

$A_y - 400 + 290.1 = 0$

$\boxed{A_y = 109.8 N}$

$\sum F_x = 0$

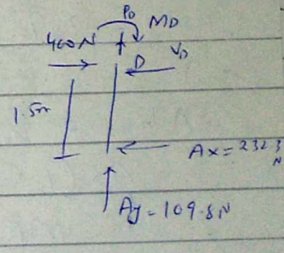
$A_x - C_x + 400 = 0$

$A_x - (167.7) + 400 = 0$

$\boxed{A_x = -232.3 N}$

It means our assume direction is wrong.

Now draw section ①-① and take down portion to find shear force, normal force and moment as shown.



Apply conditions of equilibrium.

$\sum F_x = 0$

$+400 - V_D - A_x = 0$

$-V_D + 400 - 232.3 = 0$

$-V_D + 167.7 = 0$

⇒ $\boxed{V_D = 167.7 N}$

$\sum F_y = 0.$

$A_y + P_D = 0.$

$P_D = -A_y \Rightarrow \boxed{P_D = -109.8N}$

-ve sign = Wrong assume direction

$\curvearrowleft +$

~~$\sum M_A = 0$~~

~~$-V_D \times (1.5) - 400 \times (1.5) - M_D \times (1.5) = 0$~~

~~$-167.7(1.5) - 600 - 1.5M_D = 0$~~

~~$-251.5 - 600 - 1.5M_D = 0.$~~

$\curvearrowleft +$

$\sum M_D = 0$

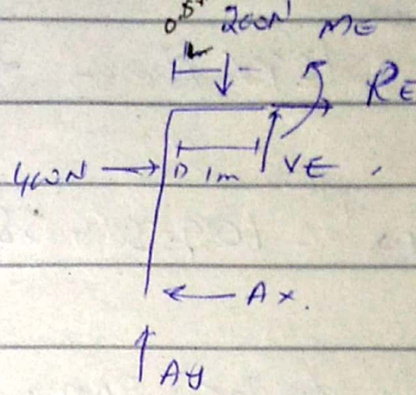
$-A \times (1.5) - M_D = 0.$

$M_D = -232.3(1.5)$

$\boxed{M_D = -348.5N}$

-ve sign show our assume direction wrong.

Now take section 2-2 to find the internal forces at E. So I select the left section. Draw the FBD.



Apply conditions of equilibrium.

$\Sigma F_x = 0$

$-A_x + R_E + 400 = 0$

$R_E = -400 + A_x$

$= -400 + (232.31)$

$\boxed{R_E = -167.7\ N}$ -ve sign show our assume direction wrong.

$\Sigma F_y = 0$

$A_y + V_E - 200 = 0$

$V_E = 200 - A_y$

$= 200 - 109.8$

$\boxed{V_E = 90.2N}$

$\curvearrowleft^{+} \Sigma M_E = 0$

$$-A_y(1) - A_x(2.5) - M_E = 0. \quad +400 \times (1) + 200(0.5)$$

$$400^{+100} - 109.8\,Nm - 222.3(2.5) - M_E = 0.$$

$$400^{+100} - 109.8 - 580.7 - M_E = 0.$$

$$500 - 690.55 - M_E = 0$$

$$-190.55 - M_E = 0.$$

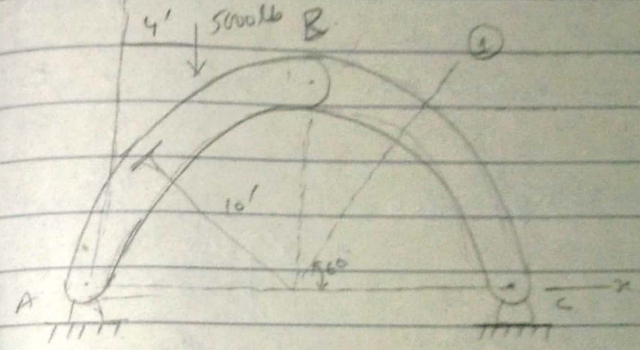
$$-M_E = 190.55\ Nm,$$

$$\boxed{M_E = -190.55\,Nm}$$

-ve sign show an assume direction is wrong.

Problem 11.3

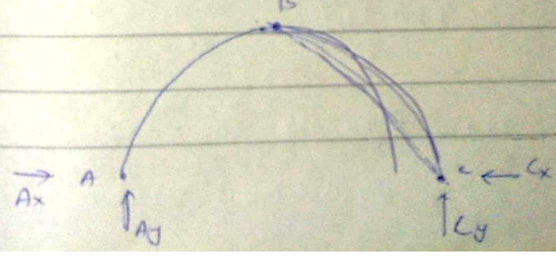
A pin connected circular arch supports a 500 lb vertical load as shown in fig. Neglecting the weight of members, determine the normal force, shear force & bending moment that acts on cross section At 1.



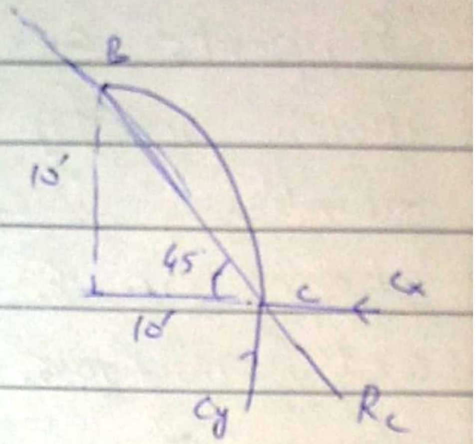
Solution:-

In order to find internal forces first we have to find the reactions.

So draw FBD of system



If we consider member BC as Two force member Then;



Now Take
$\circlearrowleft^{+} \Sigma M_A = 0$

$R_{Cy} \times 20 - 5000(4) = 0.$

$R_{Cy}\sin 45^\circ (20) - 20000 = 0.$

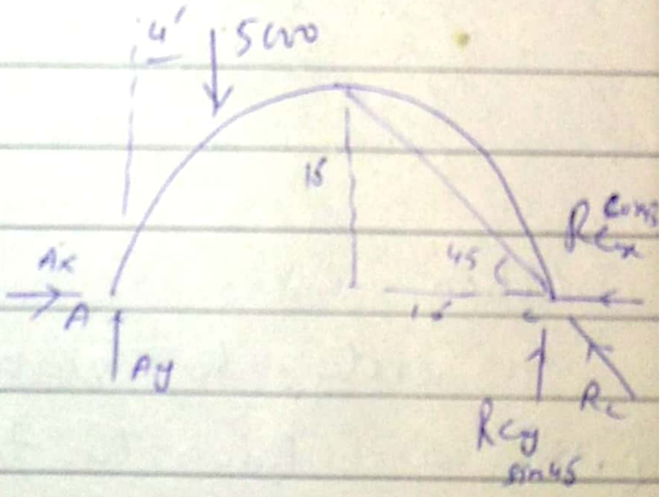
$\tan\theta = \frac{10}{10} = \quad \theta = \tan^{-1}(1)$

$\theta = 45^\circ$

$0.707(20) R_{Cy} = 20000$

$14.14 R_{Cy} = 20{,}000$

$\boxed{R_{Cy} = 1414.2 \text{ lb}}$



Now; $\Sigma F_x = 0.$

$A_x - R_{Cx} \cos 45^\circ = 0.$

$A_x - 1414.2(0.7071) = 0.$

$\boxed{A_x = 1000 \text{ lb}}$

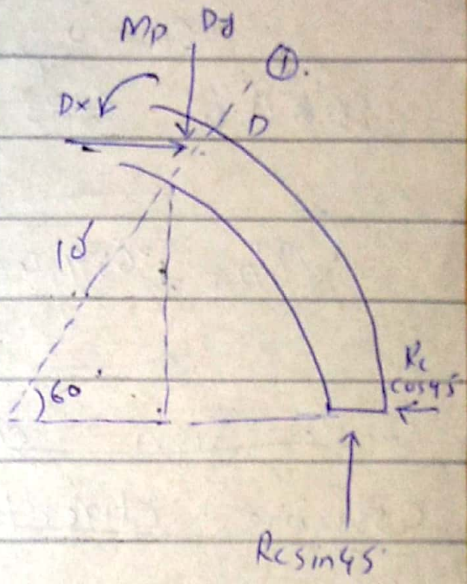
$\Sigma F_y = 0. \quad A_y + R_{Cy} \sin 45^\circ - 5000 = 0.$

$A_y + 1414.3(0.7071) - 5000 = 0$

$A_y + 1000 - 5000 = 0$

$\boxed{A_y = 4000\ lb}$

Now take The section ①.



Apply Conditions of equilibrium

$\sum F_x = 0$

$D_x - R_c \cos 45° = 0$

$D_x - 1414.3(0.7071) = 0$

$\boxed{D_x = 1000\ lb}$

$\sum F_y = 0$

$-D_y + R_c \sin 45° = 0$

$+D_y = +1414.3(0.7071)$

$\boxed{D_y = +1000\ lb}$

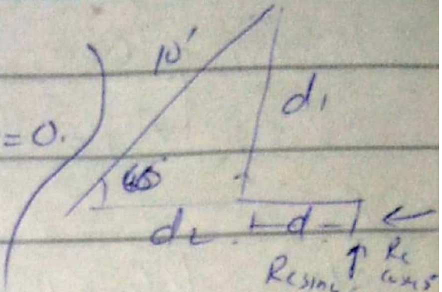
~~-ve sign show our assume direction is wrong.~~

$\overset{+}{D}\ \sum M_D = 0$

2)

$\Sigma M_D = 0$

$R_c \cos 45 \times 8.66 - R_c \sin 45 \times 5 - M_D = 0.$



$1414.3(0.7071) \times 8.66 - 1414.3(0.7071)5.$

$- M_D = 0$

$8658.6 - \cancel{6123.9}\ 4999.2 - M_D = 0$

$M_D = 3659.4\ Nm$

~~-ve sign show our assume direction is wrong.~~

$\cos 60 = \frac{d_2}{10}$

$d_2 = 5$

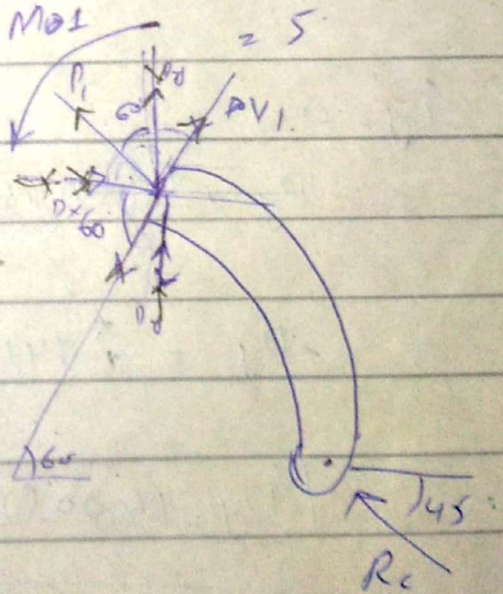
$\sin 60 = \frac{d_1}{10}$

$d_1 = \cancel{51.9}\ 8.66$

$d = 10 - 5$

$= 5$

$P_1 = Dx \sin 60 + Dy \cos 60$



$= 1000 \sin 60 + 1000 \cos 60$

$= 866.6 + 500$

$\boxed{P_1 = 1366\ lb}$

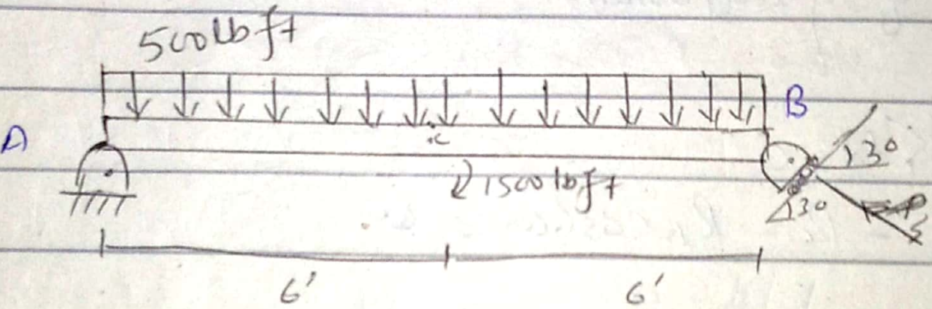
$V_1 = D$

$P_1 = Dy \cos 60 + Dx \sin 60$

$V_1 = Dy \sin 60 - Dx \cos 60$

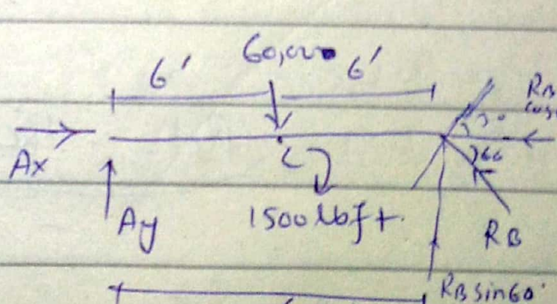
Excercise II

(a) Determine the Internal normal forces, shear force and moment at point C in the Simply supported beam. Point C is located just to right of the 1500 lbft couple moment.



Sol:- First draw FBD of fig to find Unknown reaction at B & A

Apply Conditions of equillibrium..



$\circlearrowright^{+} \Sigma M_A = 0$

$6000 \times 6 + 1500 + (-R_B \sin 60°)(12) = 0.$

$12(-0.866) R_B + 37500 = 0$

$12(-0.866) R_B = -37500$

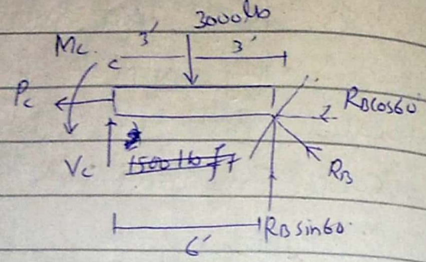
$10.392\, R_B = 37500$

$R_B = 3500/10.392$

36 08.54

$\boxed{R_B = \cancel{43302.5}\ lb}$

Now take the bar right from C.



Apply conditions of equilibrium.

$\sum F_x = 0$

$-P_c - R_B\cos60 = 0$

$-P_c - \overset{3608.54}{\cancel{43302.5}}(0.5) = 0$

$-P_c - 1804 = 0$

$P_c = -1804\ lb$,

-ve sign show our assume direction wrong.

$\sum F_y = 0 \quad V_c + R_B\sin60 - 3000 = 0$

$V_c + 3124.99 - 3000 = 0$

$\boxed{V_c = 124.99\ lb}$

$\sum M_c = 0$

$M_c - 3000\times3 + R_B\sin60\times6 = 0$

$M_c - \overset{9000}{\cancel{9000}} + 18749.97 = 0$

~~$M_c + 8.249.9 = 0$~~

~~$M_c = -8\ 2499$~~

$M_c - 9000 + 18749.97 = 0$

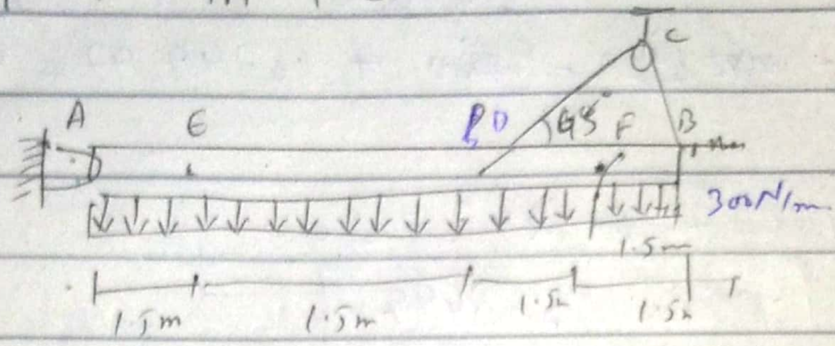
$M_c + 9749.9 = 0$

$M_c = -9749.9\ lbft$

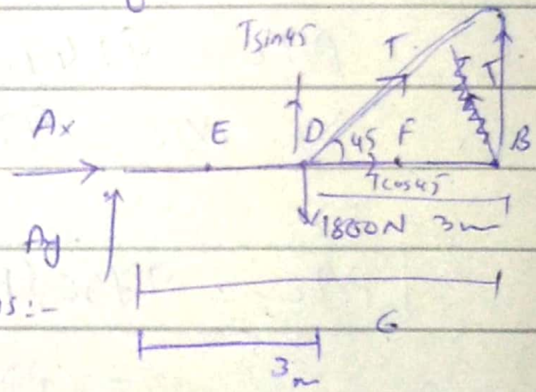
$\boxed{M_c = -9750\ lbft}$

-ve sign show our assume direction wrong it is clockwise.

(b) Determine the internal normal force, shear force, & moment at point E & F in the beam.



Sol:- Draw FBD of fig to find the unknown rxn forces.



To find T:-

Apply equillibrium Conditions:-

$\curvearrowleft + \sum M_A = 0.$

$-1800 \times 3 + T\sin 45^\circ \times 3 + T(6) = 0.$

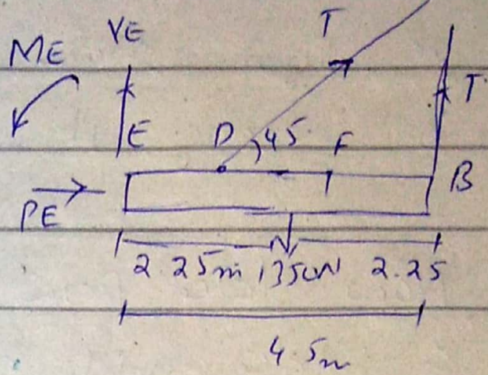
$8.1213T = 5400$

300×6
=1800 N

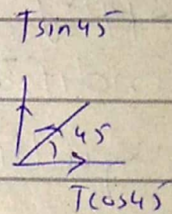
$\boxed{T = 6165\cancel{1}6\,N}$

Now Take section 1 on E & take right of the beam to find unknown at E.

Draw FBD



to find the internal forces,
Apply conditions of equilibrium



$\Sigma F_x = 0.$

$P_E + T\cos 45 = 0.$

$P_E = -665(0.7071)$

$\boxed{P_E = -470\ N}$

-ve sign → wrong assumption

$\Sigma F_y = 0.$

$V_E - 1350 + T + T\sin 45^\circ = 0.$

$V_E - 1350 + 665 + 665(0.7071) = 0.$

$V_E - 214.8 = 0.$

$\boxed{V_E = 214.8 N}$

$\Sigma M_E = 0.\ M_E - 1350 \times 2.25 + 665(0.7071)(1.5) + 665 \times 4.5 = 0$

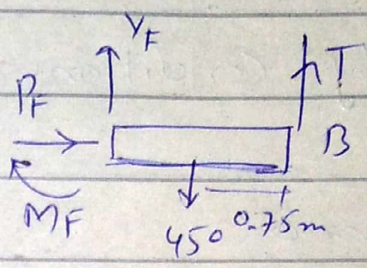
$M_E - 3037.5 + 705.33 + 2992.5$

$M_E + 660.3 = 0$

$\boxed{M_E = -660.3\ Nm}$

-ve sign show our assume direction is wrong it is Clockwise.

Now take point F & draw section & take right of F.



Apply Conditions of equilibrium.

$\Sigma F_x = 0$.

$P_F \cancel{+} 0 = 0$.

$P_F = 0$.

$\Sigma F_y = 0$. $V_F + T = 0$ -450

$V_F = -665 + 450$.

$V_F = -215N$ wrong assumption.

$\circlearrowright^{+} \Sigma M_F = 0$.

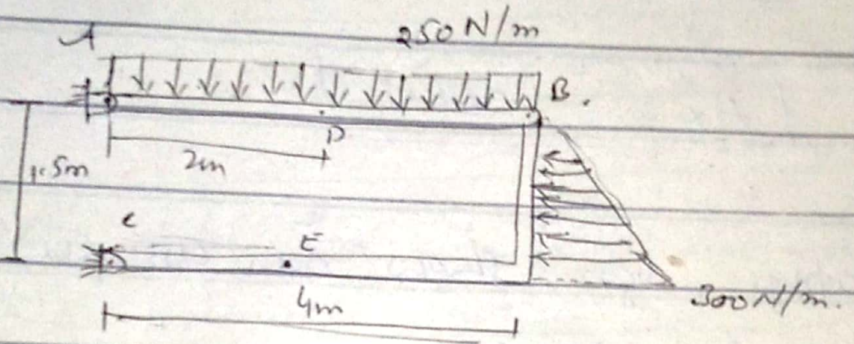
$M_F + 450 \times 0.75 - 665 \times 1.5 = 0$.

$M_F + 337.5 - 997.5 = 0$

$M_F - 660 = 0$

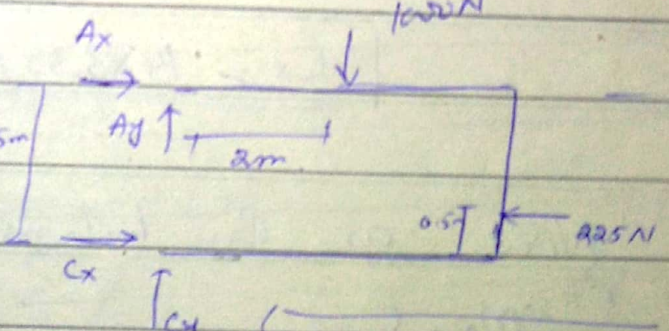
$\boxed{M_F = 660 \text{ Nm}}$

(c) Determine the internal normal force, shear force and moment at point D and E of two member frame?



Sol:- First we will find the unknown reactions at A and C.

So draw FBD of system.



Now apply condition of equilibrium to find unknown reactions.

For UDL = $(\omega \times L)$
For UVL = $(\omega \times \frac{L}{2})$
UDL act on $(\frac{L}{2})$
UVL act on "$L/3$"

$+\circlearrowleft \sum M_C = 0.$

$-Ax(1.5) - 1000 \times 2 + 225 \times 0.5 = 0.$

$-1.5(Ax) - 2000 + 112.5 = 0.$

Scanned with CamScanner

$-1.5 A_x + 1887.5 = 0.$

$\boxed{A_x = -1258.33 N}$

$\boxed{A_x = -1.26 kN}$

Negative sign show our assume direction is wrong.

Now; $\Sigma F_x = 0.$

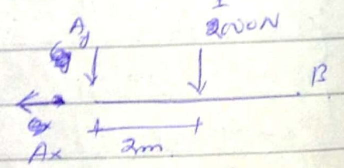
$A_x + C_x - 225 = 0.$

$C_x - 225 - 1258.33 = 0.$

$\boxed{C_x = 1483.33 N}$

Now as On Internal Pin B.

$\Sigma M_B = 0$



$-A_y \times 4 - 2000 \times 2 = 0.$

$4A_y = -2000.$

$\boxed{A_y = -500 N}$

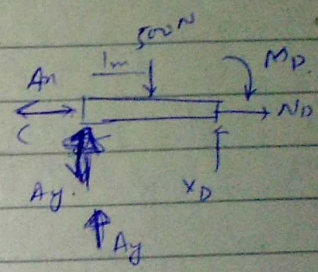
Now $\Sigma F_y = 0$ for all system/fig.

$500 - 1000 + C_y = 0$

$-500 + C_y = 0 \Rightarrow \boxed{C_y = 500 N.}$

Now take the D point to find the internal forces at D.

Apply conditions of equilibrium.



$\Sigma F_x = 0.$

$-A_x + N_D = 0.$

$N_D = A_x$

$N_D = 1258.33$

$\boxed{N_D = 1.26 kN}$

$\Sigma F_y = 0 \quad A_y + V_D - 500 = 0.$

$500 + V_D - 500 = 0$

$V_D = 0$

$\circlearrowleft +$

$\Sigma M_D = 0$

$500 \times 1 - M_D - A_y \times 2 = 0.$

$500 - M_D - 500 \times 2 = 0$

$500 - M_D - 1000 = 0$

$-M_D - 500 = 0.$

$-M_D = 500.$

$\boxed{M_D = -500\ Nm}.$

It means our assume direction is wrong it is anticlockwise.

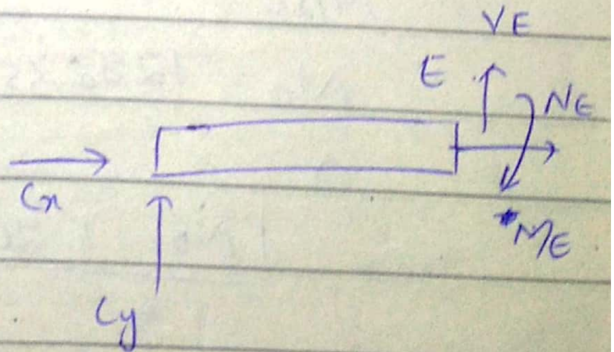
Now take point E:-

$\Sigma F_x = 0.$

$C_x + N_E = 0$

$C_x = -N_E.$

$N_E = -1488.33.$



$\boxed{N_E = -1.48\,kN}$

Negative sign show our assume direction is wrong.

$\Sigma F_y = 0.$

$C_y + V_E = 0$

$\boxed{V_E = -500\,N}$

Negative sign shows our assume direction is wrong.

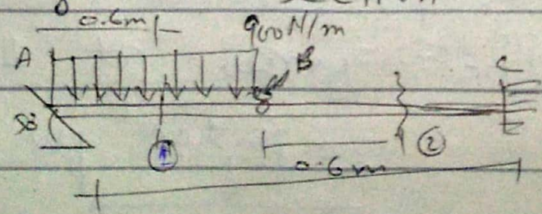
$\Sigma M_E = 0.$

$-500 \times 2 - M_E = 0.$

$M_E = -1000\,Nm.$

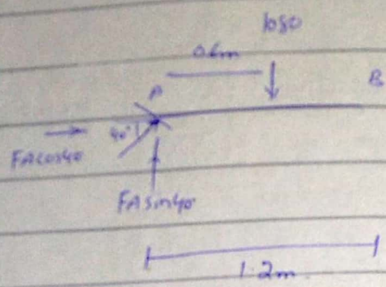
It means our assume direction is wrong it is anticlockwise.

Q_2:- The two bars, pinned together at B, are supported by frictionless surface at A & a built in support at C. Determine the Int. force system acting on section 1 & 2.



Sol_2:- First we find the reaction at either A or C.

Draw FBD from A to B of bar.



As B is Internal so moment about any Int pin is zero.

So, $\circlearrowleft^{+} \sum M_B = 0$

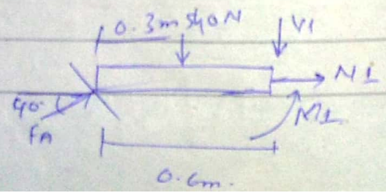
$- F_A \sin 40 \times 1.2 + 1080 \times 0.6 = 0.$

$F_A (0.7713) = 648$

$\boxed{F_A = 840.14\ N.}$

Now take the point/section 1 to find Internal forces at 1.

Draw FBD of section 1 to left as shown in fig.



Apply conditions of equilibrium

$\sum F_x = 0.$

$F_A \cos 40° + N_1 = 0$

$840.14 (0.7660) + N_1 = 0 \quad \because F_A = 840.14N.$

$\boxed{N_1 = -643.5\ N.}$

-ve sign shows assume direction is wrong.

$\sum F_y = 0$

$F_A \sin 40 - 540 - V_1 = 0$

$540 - 540 - V_1 = 0.$

$\therefore \boxed{V_1 = 0}$

$\circlearrowright^{+} \sum M_1 = 0$

$-M_1 - 540 \times 0.3 + F_A \sin 40 \times 0.6 = 0.$

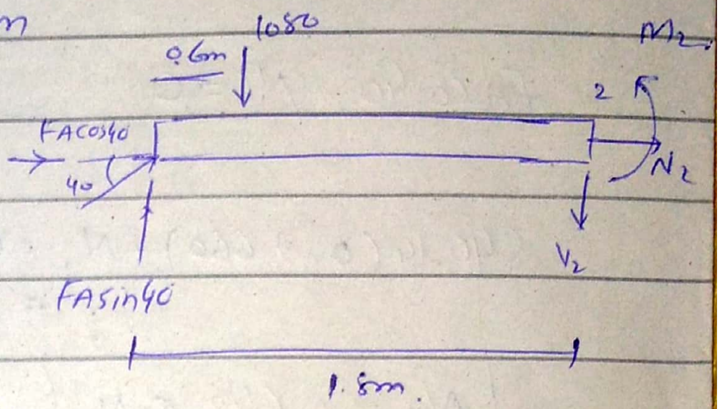
$-M_1 - 162 + 324 = 0$

$-M_1 = -162$

$\boxed{M_1 = 162\ N\,m}$ Ans

Now take section 2 & draw its FBD as shown.

Apply condition of equillibrium.



$\Sigma F_x = 0.$

$FA\cos 40 + N_2 = 0.$

$N_2 = -840.14\cos 40$

$\boxed{N_2 = -643.5\,N}$ ⇒ −ve sign assume direction wrong.

$\Sigma F_y = 0.$

$FA \sin 40 - 1080 - V_2 = 0.$

$540 - 1080 - V_2 = 0.$

$-V_2 - 539.9 = 0.$

$\boxed{V_2 = -539.9\,N}$ Assume direction is wrong.

$\overset{+}{\curvearrowleft}\ \Sigma M_2 = 0. \Rightarrow M_2 + 1080 \times (1.2) - FA\sin 40 \times 1.8 = 0$

$M_2 + 1296 - 972 = 0$

$\boxed{M_2 = -323.9\,N.}$

−ve signs shows our assume direction is wrong.