

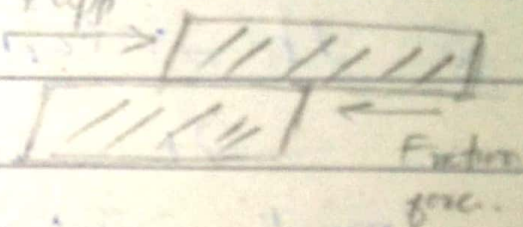
How to find value of μ_s ?
→ Applied force when the body slide. So divide it on normal force give μ_s .

Module # 12

DRY FRICTION.

Consider two bodies which are placed on each other.

If no lateral force F_{app} is applied on it then there is no friction in between those bodies.



But if applied by a force F on one body then body on which force is applied will tend to slide.

But there are maximum friction between two bodies that is equal to $\mu_s N$.

Here μ_s is coefficient of static friction. It is experimental value calculated after several experiments while N is normal reaction force.

Mathematically:-

Mathematically friction is;

$$F = \mu_s N \rightarrow (1)$$

Friction depend upon:

(a) $\mu_s \rightarrow$ Coefficient of static friction:

It means if we have two smooth surface then their μ_s value is small. While if we have two rough surface then their μ_s value will be more and the friction will be more and the body will be difficult to slide. $\rightarrow$ More friction.

(b) Normal force:-

Similarly if we increase load on either surface. The normal reaction will also increase. That will increase the friction b/w them so it will need more force to slide.

Between two interacting bodies the maximum friction force is $\mu_s N$.

→ When the body start sliding then $\mu_s \rightarrow$ change $\rightarrow \mu_k$.

$\mu_k \rightarrow$ coefficient of kinetic friction.

There are three condition:-

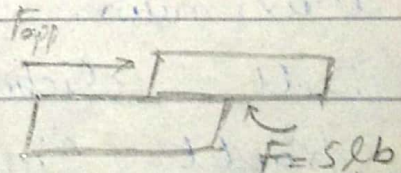
(a) Consider two surface having ~~5 lb~~ μ_s or max. friction is 5 lb.

Now if we applied a force of 1 lb.

So the friction force will also 1 lb. So

In this stage the body is at state of rest.

So "If the applied force is less than max. friction force the body will be in rest".



(b) Now if we increase the applied force from 2 to 4 lb, the friction force

increase from 2 to 4 lb. But
When applied force equal to 5 lb
This stage of body is called
impending sliding.

In this case the body is
in verge of sliding.

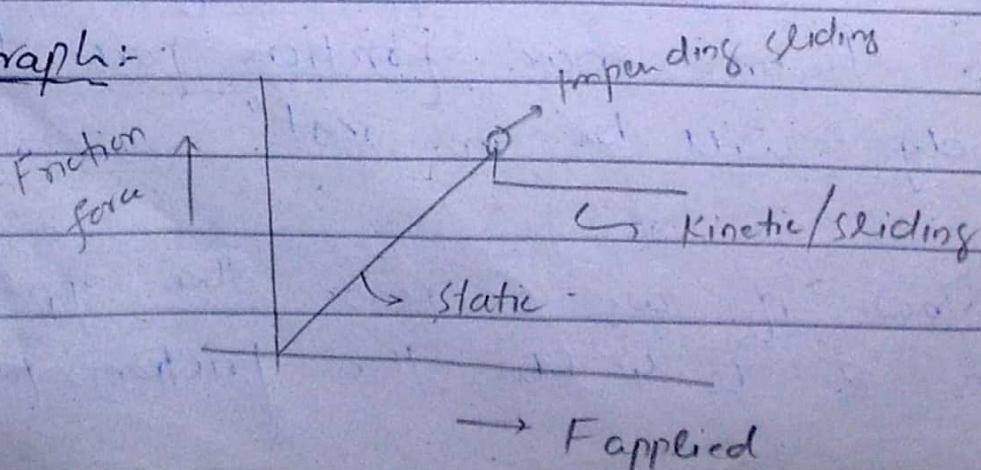
So this happens when

$$F_{app} = F_{max} = \mu_s N.$$

(c) When applied force is greater than
maximum friction then the body
will sliding. The graph will then
straight line.

This applied force will not then
overcome friction force but will used
in acceleration & motion etc.

Graph:-



Importance:-

- (*) Friction enable us to walk on ground without it we will slip.
- (*) Hold nail & screw in place.
- (*) Transmit power by mean of clutches & belts.

Disadvantage:-

- ↔ Reduce efficiency in transmission of power by converting mechanical energy into heat.
- ↳ Causes wear in machinery.

Types of Friction:-

(a) Dry friction:-

Friction between two unlubricated solid surfaces.

Also referred as Coulumb friction.

Small lubricants or object moving with slow speed on each other the friction between them can be consider as dry friction.

(b) fluid friction:-

The friction between moving surfaces separated by layer of fluid.

Example:-

The friction in a lubricated journal bearing is classified as fluid friction. As the two halves of bear are not in direct contact but are separated by thin layer of lubricant liquid.

Columb's Theory of Dry friction:-

Columb's Theory of dry friction is important theory that gives satisfactory result in many practical problems.

Columb's 'Theory of dry friction'

can be define in three cases.

Case #01 Static Condition:-

Columb's law states that;

When two bodies are in contact with each other and lateral force is applied, but due to lateral force the body not in motion, then the applied force and friction force may satisfy the following relationship.

$$F \leq F_{\max} = \mu_s N.$$

F = Applied force

$F_{\max}$ = Max friction force b/w surfaces.

μ_s = Co-efficient of static friction.

Condition #02

Now consider if the applied force is equal to max. friction in between two surface then the body will be in verge of motion. This state of

body is known as impendence state.

Because at this stage a little change in force (applied) will cause sliding (motion) & μ_s will convert to μ_k .

So in this case we can write.

$$F = F_{\max} = \mu_s N$$

& The direction in this ^{case} of friction will be opposite to direction of sliding.

Case #03:-

But at impending stage if we further increase our applied load (force) then the body will slide and will cause movement.

So in this case.

$$F > F_{\max} = \mu_s N \quad \mu_k N$$

$$\text{or } F = F_k = \mu_k N$$

Where μ_k coefficient of kinetic friction

Lamitation of Columb Theory:-

(1) Columb did not ^{tell} about effect of area on friction in his theory. Because in our daily life if area is more, the friction will also more because surface of contact increase.

(2) → Columb's theory is only applicable to surface that are dry or that contain small amount of lubricant. Also this theory valid for small motion only. At high speed the theory failed.

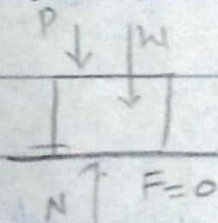
(3) → There is theoretical (Mathematical) proof (explanation) about friction. We just rely on experimental value of constant i.e. μ . But these experimental values may be vary because of environment conditions etc. So not constant always. We used appropriate (approximate)

Value of these constant.

Summary:-

Case #01

When no lateral force is applied.

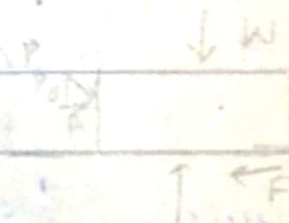


$$N = P + W$$

(No friction)

Case #02

When lateral force is applied but $< F_{max}$.



$$F = F_f$$

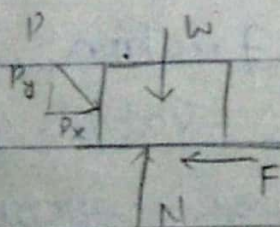
$$N = W + P_y$$

$$F < F_{max} = \mu_s N$$

(No motion)

Case #03

(Impending sliding)



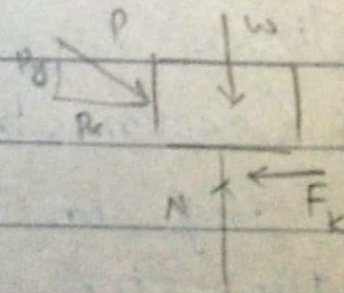
$$P_x = F$$

$$F = F_{max} = \mu_s N$$

$$N = P_y + W$$

$$(P_x = F_m)$$

Case #04



$$F_k < P_x$$

$$F_k = \mu_k N$$

$$N = P_y + W$$

Motion ($F \geq F_{max}$)

Angle of friction:-

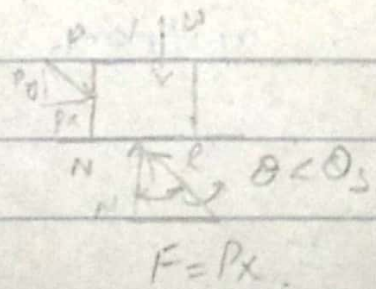
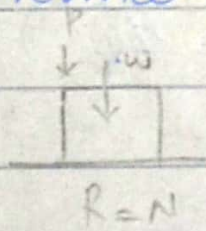
Sometime we can write the normal force and frictional force as a resultant i.e.

$$R = \sqrt{F^2 + N^2}$$

In case of no lateral force applied

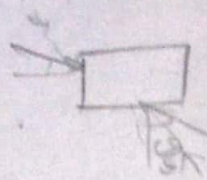
$$R = \sqrt{N^2} \Rightarrow R = N$$

So normal acts as resultant.



In second state when applied force become equal to max. friction force then the body is in verge of motion called impending state.

In this case the ^{frictional} applied force and normal can be written in term of resultant. The angle which R makes with normal is called angle of static friction.



→ In this case: ~~not~~ generally the angle b/w resultant & normal is ϕ .

In ^{no motion} impending state it should be noted that

$\phi \leq \phi_s$. (This is in case of no motion).

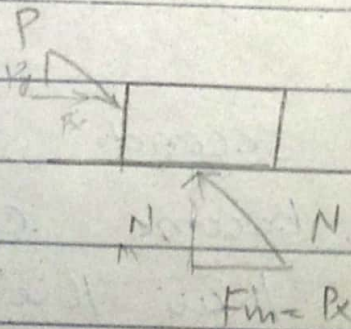
While in case of impending sliding $\phi = \phi_s$.

In case of motion $\phi = \phi_k$.

From fig.

$$\tan \phi_s = \frac{F_{ms}}{N}$$

$$= \frac{\mu_s N}{N}$$



$$\boxed{\tan \phi_s = \mu_s}$$

In case of motion.

$$\tan \phi_k = \frac{F_{mk}}{N} = \frac{\mu_k N}{N}$$

$$\boxed{\tan \phi_k = \mu_k}$$

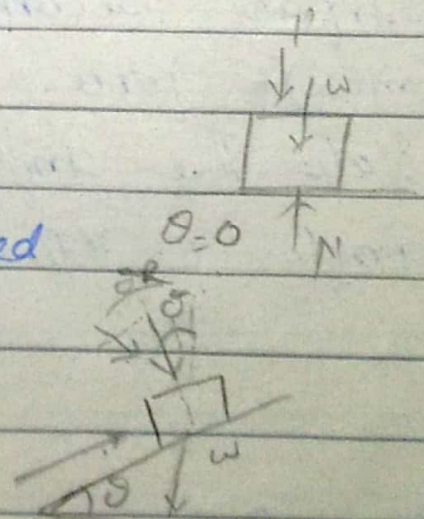
Angle of Repose:-

The angle of resultant with normal in case of the surface is inclined and impending state, the angle is known as angle of repose or

The angle of inclined surface at which the body is in verge of sliding in downward direction called angle of repose.

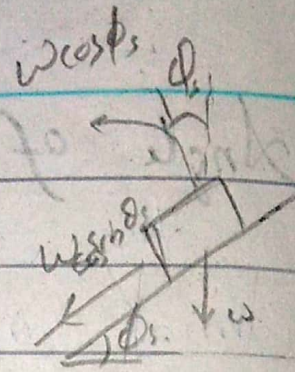
Explanation:-

In first case There is no angle of repose as the body is not inclined also lateral force not applied.



→ Now if we increase the inclination so in that case the weight has two components one is acting

downward while other one is along inclined surface. This component now act as literal load. If we increase the inclination so the sin component also increase which means the literal force is increasing. So at first the literal force is less than max. friction b/w those surface but after some inclination the that literal component of weight along inclined surface become equal to friction (max) force. So now in impending state the angle is now called angle of repose.



If we increase the angle of repose more than the body will slide

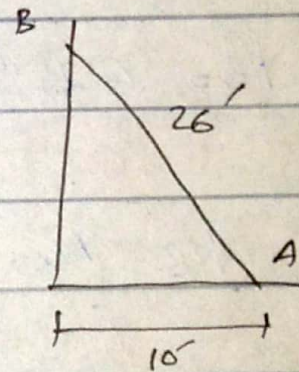
Problem 12.1..

The uniform thin pole has a weight of 30 lb and length of 26' foot.

96 it is placed against the smooth wall and on rough floor in the position.

Will it remain in this position when it is released? The coefficient of static friction is $\mu_s = 0.3$.

Sol:- First step is to draw FBD of system (thin pole).



Step #02

Apply conditions of equilibrium.

$$\sum F_x = 0.$$

$$R_B - A_x = 0.$$

$$\boxed{R_B = A_x} \rightarrow \textcircled{1}.$$

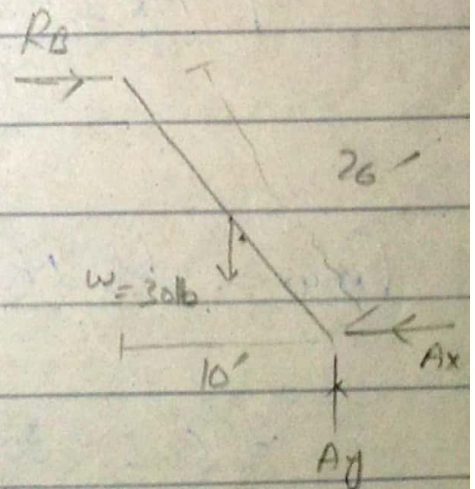
$$\sum F_y = 0.$$

$$A_y - 30 = 0.$$

$$A_y = 30 \text{ lb.}$$

$$\therefore N = A_y.$$

$$\boxed{N = 30 \text{ lb.}}$$



$$\sum M_A = 0$$

$$-R_B \times 24 + 30 \times 5 = 0$$

$$R_B (24) = 150$$

$$R_B = 6.25 \text{ lb.}$$

$$\cos \theta = \frac{10}{26}$$

$$\theta = 67.4^\circ$$

$$R_B = A_x$$

$$\cos(67.4) = \frac{b}{13}$$

$$A_x = 6.25 \text{ lb.}$$

$$b = 5'$$

$$F_{\max} = \mu_s N$$

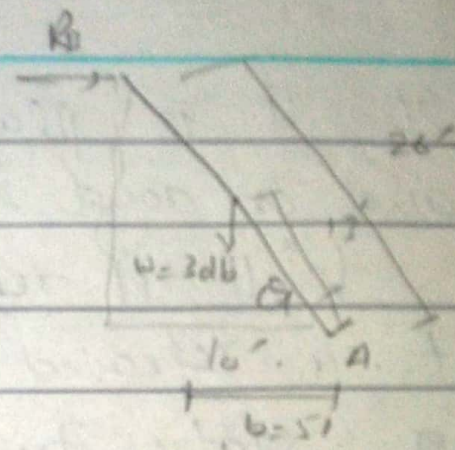
$$= (0.3)(30)$$

$$F_{\max} = 9 \text{ lb.}$$

$$\sin(67.4) = \frac{P}{26}$$

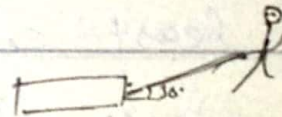
$$P = 24'$$

As $F_{\max} > A_x$ (lateral force) so the body will be at rest.



Problem 12.2

The coefficient of static friction between the 150 kg crate and ground is $\mu_s = 0.3$ while coefficient of friction b/w 80 kg man's shoes and the ground is $\mu_s = 0.4$. Determine if man can move the crate?



Solution:-

First we have to draw FBD of crate.

Step # 02

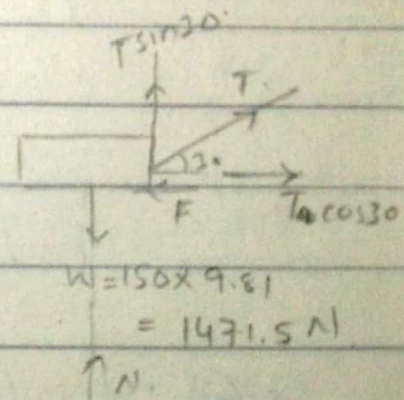
Apply conditions of equilibrium

$$\sum F_x = 0$$

$$-F + T \cos 30^\circ = 0$$

$$T \cos 30^\circ = F$$

$$T = \frac{F}{0.866} \quad \text{--- (1)}$$



$$\sum F_y = 0.$$

$$T \sin 30^\circ - 1471.5 + N = 0.$$

$$0.5T - 1471.5 + N = 0. \rightarrow (2)$$

The man will move the crate when the lateral force i.e. $T \cos 30^\circ$ is at least equal to maximum friction force i.e.

$$T \cos 30^\circ = F_{\max} = \mu_s N.$$

$$T = \frac{(0.3) N}{0.866}$$

$$T = 0.3462 N \rightarrow (3)$$

eq (2) $\Rightarrow$

$$0.5 (0.3462 N) - 1471.5 + N = 0$$

$$0.1732 N + N = 1471.5$$

$$1.1732 N = 1471.5$$

$$N = 1254.25 N.$$

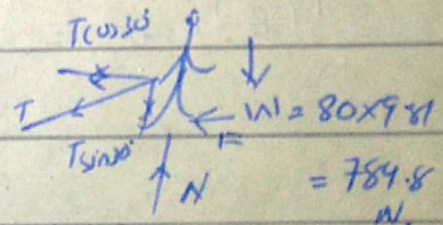
$$\text{eq (3)} \Rightarrow T = 0.3462(1254.25)$$

$$\boxed{T = \cancel{432} \cdot 434.2 \text{ N.}}$$

In order to move the crate at least 434.2 N are required.

Now to check when the man apply 434.2 N force either he slip or not. So draw FBD of man now.

Apply condition of equilibrium to find the normal force. $\sum F_y = 0$.



$$-T \sin 30 + N - W = 0$$

$$N - 434.2(0.5) - 784.8 = 0$$

$$\boxed{N = 1002 \text{ N.}}$$

$\sum F_x = 0$ To find the friction force

between man's shoes & ground.

$$-F_B - T \cos 30^\circ = 0$$

$$-F_B - 434.2(0.866) = 0$$

$$\boxed{F_B = -376 \text{ N}} \quad \therefore \text{Assume direction is wrong.}$$

Force that required to slip on floor.
i.e. Max friction.

$$(F_B)_{\max} = \mu_s N$$

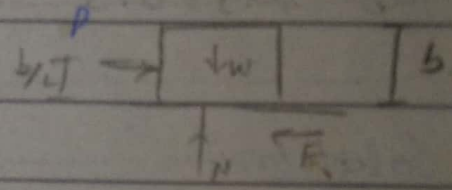
$$= (0.4)(1002)$$

$$\boxed{F_{\max} = 400.8 \text{ lb.}}$$

Since the force required to cause slip on floor is $>$ Friction force b/w man's shoes & floor. So the man can slide the crate.

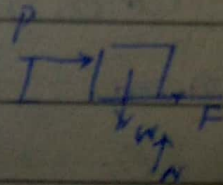
Impending Tipping:-

When we literally force applied on a body then there are two conditions. One as discuss earlier i.e. the body will slide. Now if the force is applied at distance greater than $b/2$ then the body will overturn as shown in fig.



Consider the block, P is literal force applied. If P is applied at $\leq b/2$ then the body (block will slide). But if applied at upper then the block will overturn. This is also called impending tipping.

P & F create
clock wise moment. In



order to balance the normal will be at right of W , at how many distance we will calculate.

Problem 12.3

The man in fig is trying to move a packing crate across the floor by applying a horizontal force P . The centre of gravity of 250N crate at geometric Centre. Does crate move if $P=60\text{N}$?

The $\mu_s = 0.3$ b/w crate & floor.

Solution:-

First we will draw FBD of crate to find Unknown.

i.e. $F_f = ?$

$N_1 = ?$

$\sum x = ?$ (where Normal act).

Step #1

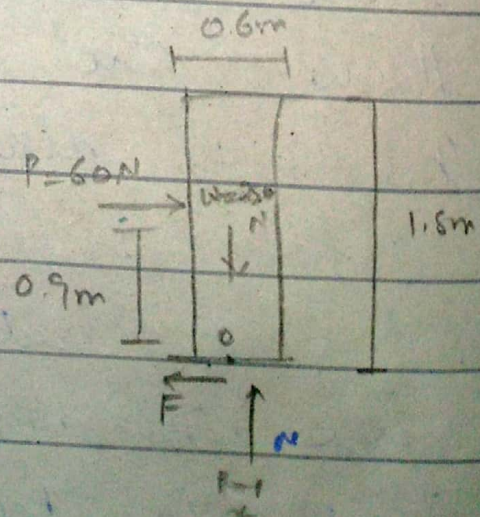
Draw FBD of crate.

Apply conditions of equilibrium.

$$\sum F_x = 0.$$

$$P - F_f = 0$$

$$F_f = 60\text{N.}$$



$$\sum F_y = 0$$

$$N_2 - 250 = 0.$$

$$\boxed{N_2 = 250 \text{ N}}$$

$$\sum M_o = 0$$

$$N \times x - 60 \times (0.9) = 0.$$

$$N \times x = 54.$$

$$x = \frac{54}{250}$$

$$\boxed{x = 0.216 \text{ m}}$$

Checking for Overturning:-

↳ The smallest largest possible value for x to be in equilibrium is 0.3 m (half of width of block) which is larger than 0.216 m . So we can say, the crate will not overturn.

Checking for sliding:-

The max. frictional force is;

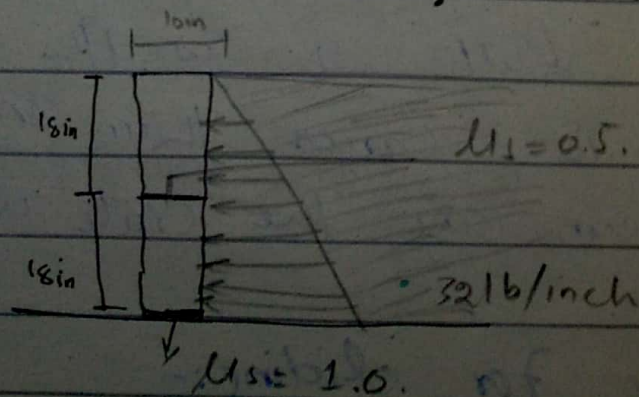
$$\begin{aligned}
 F_{\max} &= \mu_s N. \\
 &= (0.3)(250) \\
 &= 75 \text{ N.}
 \end{aligned}$$

As $F_{\max}$ is larger than lateral force applied $P = 60 \text{ N}$. So we conclude the block will not slide.

Problem # 12.4

Two concrete block weighing 320 lb each form part of retaining wall of swimming pool.

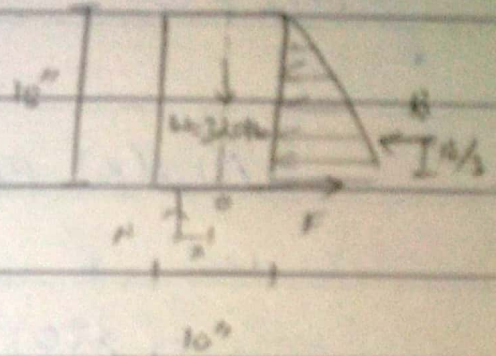
Will the block be in equilibrium when the pool is filled & water exerts the line loading shown.



Sol:- First we will draw FBD.

of above block.

Apply conditions of equilibrium to find unknown.



We have $P = 32 \text{ lb/inch}$.

But at mid we get $\frac{P}{2} = 16 \text{ lb/in}$

$$\text{So; } R_1 = \frac{1}{2} \times 16 \times 18.$$

$$= 144 \text{ lb}$$

$$\sum F_x = 0.$$

$$F_1 - 144 = 0.$$

$$\boxed{F_1 = 144 \text{ lb}}$$

$$\sum F_y = 0.$$

$$N_1 - 320 = 0.$$

$$\boxed{N_1 = 320 \text{ lb}}$$

$$\sum M_o = 0$$

$$-N_1 x(x) + R_1 x(18/3) = 0$$

$$-(320)x + 144(18/3) = 0$$

$$x = \frac{864}{320}$$

$$x = \frac{2.7}{2.0} \text{ inch.}$$

Criteria for sliding of upper block.

Max friction is,

$$F_{\max} = \mu_s N$$

$$= (0.5)(320)$$

$$= 160 \text{ lb.}$$

Since the max. friction (160 lb) is greater than lateral force i.e. 144 lb. So the block will not slide & will be at rest.

Criteria for overturning:-

As the normal act $x = 2.7$ which is less than half of the width of upper block i.e. 5 inch. So the block will not overturn.

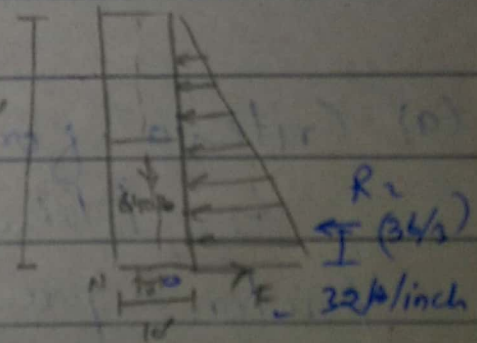
Now Consider Complete blocks.

Step #1

Draw FBD of the block.

Fig:

Apply condition of equilibrium.



$$R_2 = \frac{32 \times 36}{2} = 576 \text{ lb}$$

$$\sum F_x = 0$$

$$F - R_2 = 0$$

$$\boxed{F = 576 \text{ lb.}}$$

$$\sum F_y = 0$$

$$N - 640 = 0$$

$$\boxed{N = 640 \text{ lb}}$$

Find $x = ?$: $\sum M_o = 0$.

$$N \times (x) - R_2 \times (36/3) = 0.$$

$$640 \times (x) - 576(12) = 0.$$

$$640(x) = 6912$$

$$x = 10.8 \text{ inch}$$

(a) Criteria for sliding:-

As $F_{\text{literal}} = R_2 = 576$ while max friction force is; $F_{\text{max}} = \mu_s N$

$$= 1(640)$$
$$= 640 \text{ lb.}$$

As $F_{\text{max}} > R_2$ (literal force)
So the block will not slide
as a result of pressure of water
& remain at rest.

Criteria for Overturning:-

As $x_a = 10.8 > 5$ (half of the width of block). So this is not essential for equilibrium. Therefore the block will overturn.

EXERCISE #12

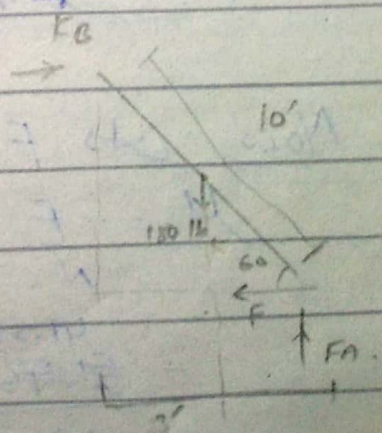
The 180 lb man climbs up the ladder and stop at position as shown. Determine the coefficient of static friction b/w friction pad A & ground if $\theta = 60^\circ$ & wall at B is smooth.

Sol:- First step is to draw FBD of ladder to find unknown reactions.

Apply conditions of equilibrium.

$$\sum F_x = 0.$$

$$F_B - F = 0 \rightarrow \text{①}$$



$$\sum F_y = 0 \quad \therefore \text{reaction at } A = F_A = A_y$$

$$A_y - 180 \text{ lb} = 0$$

$$A_y = 180 \text{ lb} \rightarrow \textcircled{2}$$



$$\sum M_B = 0$$

$$180 \times \left(\frac{3}{5}\right)$$

$$A_y(5) - F(8.66) = 0$$

$$\cos 60^\circ = \frac{b}{10}$$

-540

$$-540 + 180(5) - 8.66F = 0$$

$$b = 5'$$

$$8.66F = 900 - \frac{540}{1}$$

$$\sin 60^\circ = \frac{p}{10}$$

$$8.66F = 450$$

$$p = 8.66'$$

$$F = \frac{41.57}{1.576} \text{ N or lb}$$

$$F_{\max} = \mu_s N$$

$$\text{Now } F = \mu_s N$$

$$\mu_s = \frac{F}{N} \rightarrow \textcircled{1}$$

$$\cos 60^\circ = \frac{b}{5}$$

$$= \frac{41.57}{180}$$

$$b = 2.5'$$

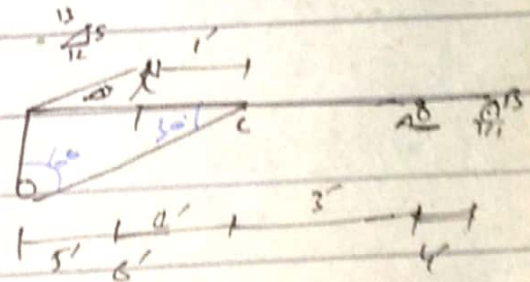
$$\therefore N = A_y$$

$$\mu_s = 0.2309 \quad \text{Ans.}$$

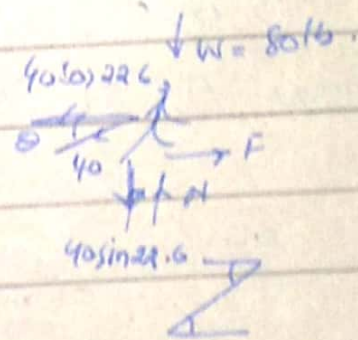
Q2:- Ex: 12

The 80 lb boy stand on beam & pull with a force of 40 lb, determine the frictional force b/w his shoes & beam & reactions at A & B. The beam is of uniform & weight 100 lb. Neglect size of pulley & thickness of beam?

Sol:- First draw FBD of man to find the frictional force b/w his shoes & beam.



Apply conditions of equilibrium.



$$\sum F_x = 0$$

$$F - 40 \cos 22.62 = 0$$

$$F = 40 \cos 22.62$$

$$F = 36.92$$

$$\tan \theta = \frac{5}{12}$$

$$\theta = 22.62^\circ$$

Now $\sum F_y = 0$.

$$N - 80 - 40 \sin 22.62 = 0$$

$$\boxed{N = 95.38 \text{ lb}}$$

$$F_{\max} = \mu_s N$$

$$= 0.4(95.38)$$

$$F_{\max} = 38.15$$

$$\text{As } F_{\max} > F_D$$

so he will not slip

Now Draw FBD of whole beam to find unknown reactions at A & B.

$\sum M_B = 0$

$$-40 \sin 22.6(13) + 40(13) + 95.38(8) + 100(6.5)$$

$$40 \sin 30(7) - A_y(4) = 0$$

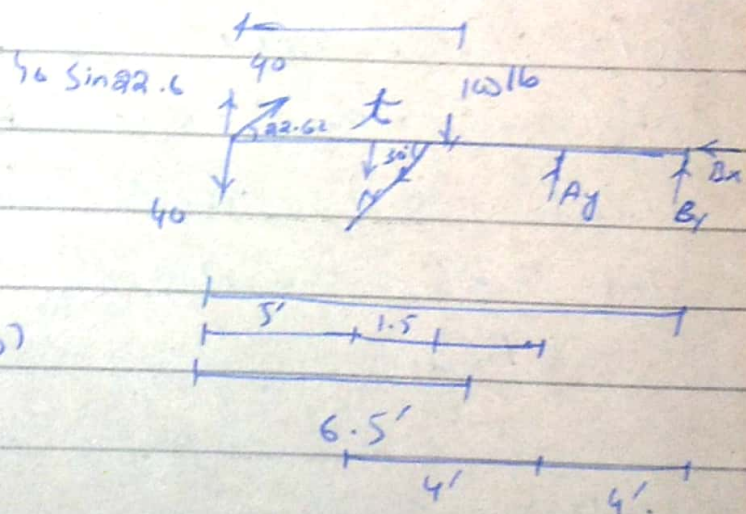
$$100 - 199.83 + 520 + 763 + 650 - 4A_y = 0$$

$$-4A_y = -1733.17 - 140$$

$$\boxed{A_y = 488.3 \text{ lb}}$$

$\sum M_A = 0$

$$-40 \sin 22.62(9) + 40(9) + 40 \sin 30(3) + 100(2.5) + B_y(4) = 0$$

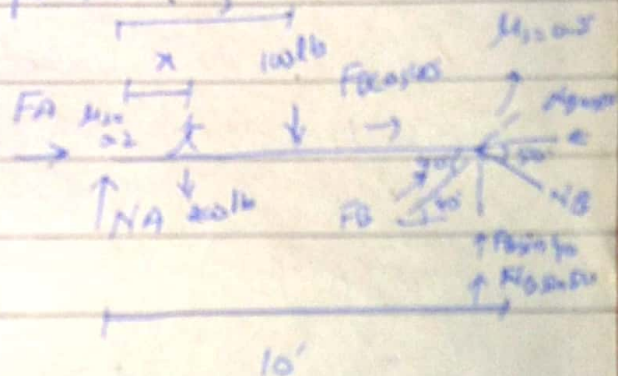


Ex₂
Q3:- (a).

The uniform 100 lb plank in fig is resting on friction surface A & B. Co-efficient of static friction is shown in fig. If 200 lb man start walking from A towards B. Determine the distance when the plank will start slide.

Sol:- Draw FBD of plank.

Apply conditions of equilibrium.



$$\sum F_x = 0.$$

$$F_A + F_B \cos 40^\circ - N_B \cos 50^\circ = 0. \rightarrow (1)$$

$$\sum F_y = 0$$

$$-200 + N_A - 100 + F_B \sin 40^\circ + N_B \sin 50^\circ = 0.$$

$$N_A + F_B \sin 40^\circ + N_B \sin 50^\circ = 300 \rightarrow (2)$$

$$\sum M_A = 0$$

$$-200(x) - 100(5) + F_B \sin 40^\circ(10) + N_B \sin 50^\circ(10)$$

$$\Rightarrow -200x - 500 + 6.42 F_B + 7.66 N_B = 0.$$

$\hookrightarrow (3)$

$$\text{As } F_A = \mu_s N_A$$

$$F_A = 0.2 N_A \rightarrow (4)$$

$$F_B = \mu_s N_B$$

$$F_B = 0.5 N_B \rightarrow (5)$$

Putting eq (4) & (5) in eq (1)

$$0.2 N_A + 0.5 N_B \cos 40^\circ - N_B \cos 50^\circ = 0$$

$$0.2 N_A + 0.383 N_B - 0.6427 N_B = 0$$

$$0.2 N_A - 0.26 N_B = 0 \rightarrow (6)$$

Putting eq (5) in eq (2)

$$N_A + 0.5 N_B \sin 40^\circ + N_B \sin 50^\circ = 300$$

$$N_A + 0.3214 N_B + 0.766 N_B = 300$$

$$N_A + 1.087 N_B = 300 \rightarrow (7)$$

$$0.2 N_A + 0.217 N_B = 60 \rightarrow (8)$$

Solving eq (6) & (8)

$$0.2 N_A - 0.26 N_B = 0$$

$$0.2 N_A + 0.217 N_B = 60$$

$$-0.477 N_B = -60$$

$$N_B = 125.7 \text{ lb}$$

Multiply

0.2 to eq (7)

eq ③ $\Rightarrow$

$$-200x - 500 + 6.43 F_B + 7.6 N_B = 0.$$

$$-200x - 500 + 6.43 (0.5) N_B + 7.6 N_B = 0.$$

$$-200x - 500 + 3.21 N_B + 7.6 N_B = 0.$$

$$-200x - 500 + 10.8 N_B = 0.$$

By Putting value of N_B .

$$-200x - 500 + 10.8 (125.7) = 0.$$

$$-200x - 500 + 1359.8 = 0.$$

$$-200x + 859.8 = 0.$$

$$-200x = -859.8$$

$$\boxed{x = 4.30'} \text{ Ans}$$

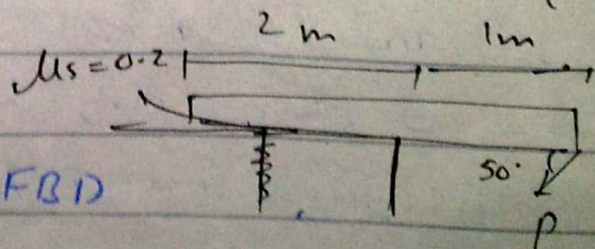
12.4 :-

Determine the largest force P for which the 18 kg uniform bar in equilibrium.

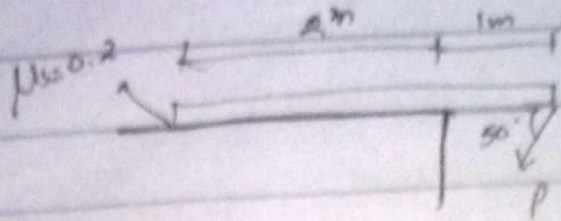
Sol:- To find

P first draw FBD

of Uniform bar.

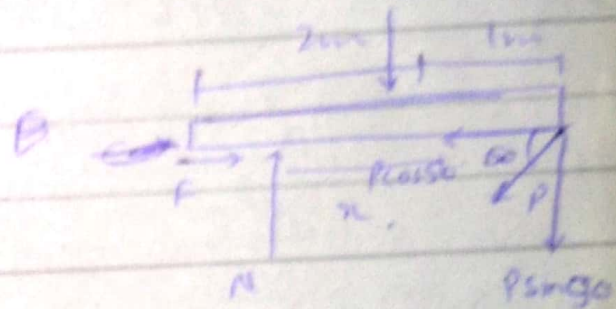


(12.4) Determine the ^{largest} force P for which the 18 kg uniform bar remains in equilibrium



Sol: In order to find the largest force due to which the bar will be in equilibrium. So first draw FBD of whole fig (bar)

Apply conditions of equilibrium



$$\sum F_x = 0$$

$$F - P \cos 50 = 0 \rightarrow (1)$$

$$\text{or } F = P \cos 50 \rightarrow (1)$$

$$\sum F_y = 0$$

$$N - 176.5 - P \sin 50 = 0$$

$$N = P \sin 50 + 176.5 \rightarrow (2)$$

Checking for sliding,

$$F = \mu N \rightarrow (3)$$

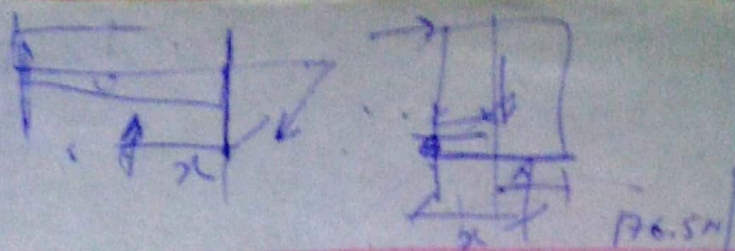
$$F = 0.2(P \sin 50 + 176.5) \rightarrow (3)$$

Comparing (1) & (3)

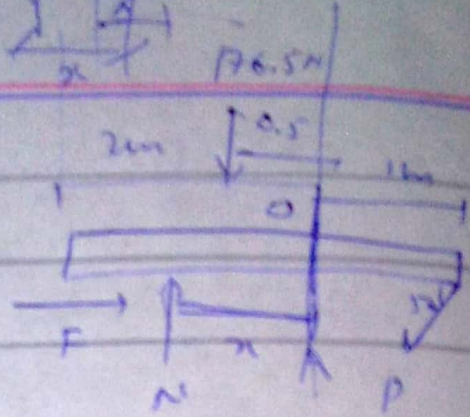
$$P \cos 50 = 0.2(P \sin 50 + 176.5)$$

$$P = 72.1 \text{ N}$$

The max force need to slide a body



Now for Overturning:-



$$\sum M_0 = 0$$

$$-P \sin 50^\circ (1) + N \times (2) + 176.5(0.5) = 0.$$

$$-P(0.766) + 88.25 = 0.$$

$$-P(0.766) = -88.25.$$

$$\boxed{P = 115.2 \text{ N}}$$

As for overturning the minimum force required is 115.2N. i.e. for overturn P should be at least 115.2N. But when P is 115.2N then sliding will also occur. So for a body to be in equilibrium the force P should not be exceed from 72.1N.

Ex:- 12.5

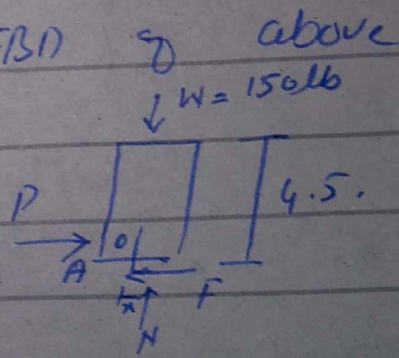
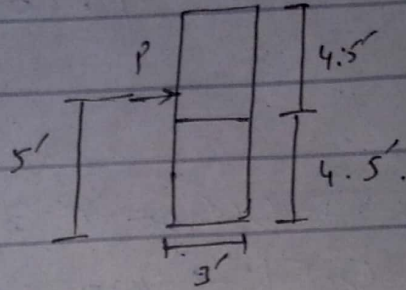
Ex: - 14.1
96 each box weight 150lb, determine the least horizontal force that must exert on the top box in order to cause motion. The coefficient of static friction b/w the boxes is $\mu_s = 0.5$ & coefficient of static friction b/w box & floor is $\mu_s' = 0.2$.

Solution:-

To find the least horizontal force P that must

exist in order to cause motion.

First we will draw FBD of above box.



Apply conditions of equilibrium.

$$\sum F_x = 0.$$

$$P_F = 0.$$

$$p = \dot{F} \rightarrow (1).$$

$$\sum F_y = 0$$

$$N - K! = 0.$$

$$N = 150 \text{ lb}$$

to for force (least) i.e. applied

to cause motion should be equal to $F = \mu_s N$.

$$P = F = (0.5)(150).$$

$$P \neq F = 75 \text{ N.}$$

Now checking for overturning.

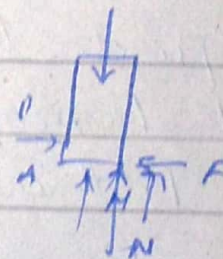
$$\sum M_A = 0$$

$$-P(0.5) - 150(1.5) + N(3) = 0.$$

$$0.5P = 225 - 450$$

$$0.5P = 11.450 - 225$$

$$P = 450 \text{ lb.}$$



$$\therefore P = 450.$$

Now for both blocks.

$$\sum F_x = 0.$$

$$P - F = 0$$

$$P = F \rightarrow (2)$$

$$\sum F_y = 0 \quad N - W = 0.$$

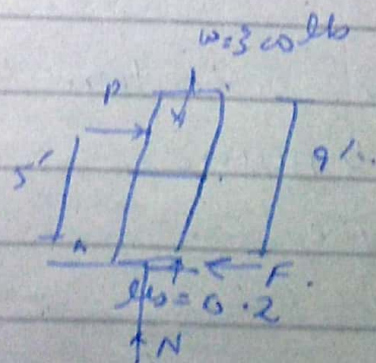
$$N = 300 \text{ lb}$$

For sliding:-

$$F = \mu_s N$$

$$F = 0.2(300)$$

$$P = F = 60 \text{ lb} \quad \therefore P = F$$



For overturning:-

$$\sum M_A = -P(5) + N(3) - W(1.5) = 0.$$

$$\Rightarrow -5P + 300(3) - 300(1.5) = 0$$

$$5P = 450$$

$$P = 90 \text{ lb}$$

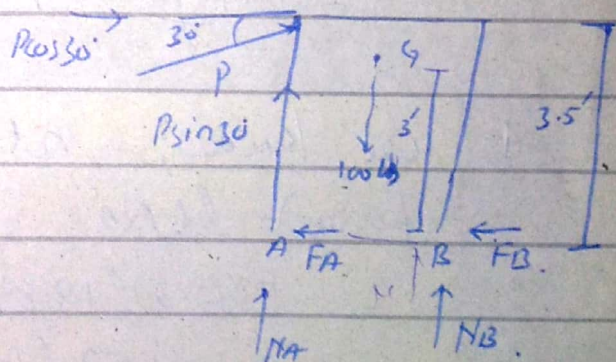
Ex:- 12.6

If the centre of gravity of stacked table is at G, and the stack weighs 100 lb, determine the smallest force P and boy must push the stack in order to cause movement. The coefficient of static friction at A & B is 0.3. The tables are locked together.

Solution:-

To find the least force P that cause movement in stack. We have to draw FBD of fig.

Apply conditions of equilibrium.



$$\sum F_x = 0.$$

$$P \cos 30^\circ - F_A - F_B = 0.$$

$$P \cos 30^\circ = F_A + F_B.$$

$$0.866 P = F_A + F_B \rightarrow (1).$$

$$\text{Eq (1)} \Rightarrow 0.866 P = 0.3 N_A + 0.3 N_B \rightarrow (2) \therefore F = \mu_s N.$$

$$\sum F_y = 0$$

$$N_A + N_B - 100 + P \sin 30^\circ = 0. \rightarrow (3).$$

Multiply 0.3 to eq (3).

$$0.3 N_A + 0.3 N_B - 30 + 0.15 P = 0 \rightarrow (4)$$

Subtract eq (2) from (4)

$$0.3 N_A + 0.3 N_B - 30 + 0.15 P = 0$$

$$\begin{array}{r} + 0.3 N_A + 0.3 N_B \\ - 0.866 P = \end{array}$$

$$-30 + 1.016 P = 0$$

$$P = \frac{30}{1.016}$$

$$\Rightarrow \boxed{29.52 \text{ lb}}$$

Now checking for sliding:-

As checking for overturning point.

$$\sum M_A = 0$$

At

$$= 21.72 \text{ lb}$$

$$F_{\max} = (F_{\max})_1 + (F_{\max})_2$$

$$= 3.852 + 21.72$$

$$\boxed{F_{\max} = 25.6 \text{ lb}}$$

$$\sum M_A = 0$$

$$-P \cos 30(3.5) - 100(2) + 4(N_B) = 0$$

$$-89.47 - 200 + 4N_B = 0$$

$$4N_B = 289.47$$

$$N_B = 72.4 \text{ lb}$$

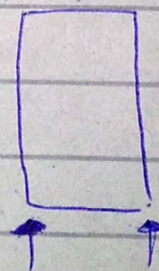
$$\sum F_y = 0$$

$$P \sin 30 - 100 + N_A + N_B = 0$$

$$29.5(0.5) - 100 + N_A + 72.4 = 0$$

$$N_A = 12.84 = 0$$

$$N_A = 12.84 \text{ lb}$$



$$As F_{\max} < P$$

So body

will slide