Module # 06

Equillibrium.

Equillibrium:-

Equillibrium can be defined as;

"If the resultant of applied and reactive forces on a body is zero than the body is said to be in equillibrium."

OR.

"If number of forces are applied on a body and the body does not change its state of rest or uniform motion. The body is said to be in equillibrium."

OR.

"When the body is in equillibrium the resultant of all forces acting on it is zero."

Explaination:-

For a body to be in equillibrium the following three conditions should be necessary.

→ If same number of forces are

applied on x-direction (in one direction) and same number of forces are resist (in opposite direction) in x-direction then body will not move in x-direction. The body will said to be in equilibrium along x-direction, Mathematically;

$$\Sigma F_x = R_x.$$

Similarly if number of forces are applied on y-direction (in one direction) and same number of forces are resist (in opposite direction) then body will be said to be in equilibrium along y-direction

Mathematically;

$$R_y = \Sigma F_y.$$

Similarly if number of moment (clockwise) and same number of moment also resist in opposite direction, then body will not move in any direction and there will be equilibrium. Mathematically;

$$MR = \Sigma M$$

These three conditions are required for a body to be in complete equillibrium.

The mathematical equation's can also be written as;

$$R_x = \Sigma F_x = 0$$

$$R_y = \Sigma F_y = 0.$$

$$M_1 = \Sigma M = 0.$$

"Summation of moment due to applied and reactive force should be zero, for a body to be in equillibrium."

→ If sum of forces at any direction is not zero, then will be translation at/along that direction and body will not be in equillibrium.

In order to understand and to represent the body in equillibrium we have to first understand free body diagram (FBD).

FREE BODY DIAGRAM:-

It is defined as;

"Separating a body from all of its intersection points, then showing the applied forces and reactive forces i.e reactions is called free body diagram."

OR

"Sketching or drawing a body along all its forces (both applied and reactive) that act on that body is called free body diagram."

∴ Separated from all of its intersection.

Explaination:-

It is called free body diagram because when we separated or remove the body from its intersection points that can be replaced by reactive forces.

Free body diagram is the most important single step in solution of problems in mechanics."

For example, if we have a chair that is placed on a floor. Its weight acting downward. Since the weight

is acting downword. so it is necessary that chair is translated downword but is prevented by ground i.e reactive force. So the direction of reactive force in this case along y-direction as translation or movement along y-axis (downword) is restricted, upword

Types of Support:-

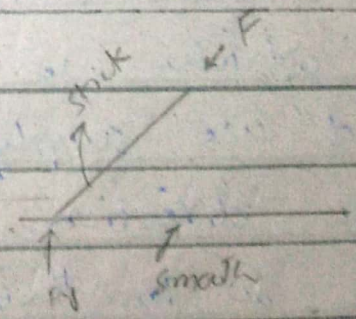
Types of support depend upon the surface of ground or on which the object is kept.

(a) Roller Support:-

It only restrict only vertical translation.

For example:-

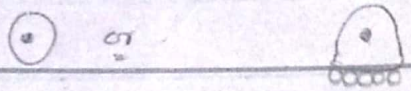
If we have smooth surface and we apply force on stick that touch that surface as shown in fig. So this will restrict the translation of stick only along y-direction i.e create or provide



reactive force along y-direction while the stick is free to move along x-direction. This is example of Roller support.

Symbol:-

The roller support can be shown as;

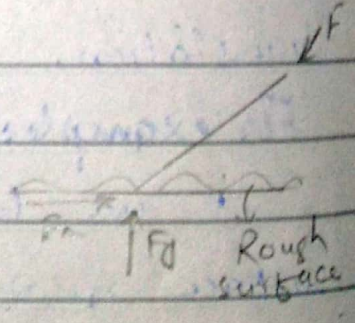


(b) Pin or Hinge Support:-

That support that provide reactive force along x and y-direction.

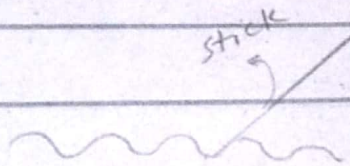
For Example:-

Consider the rough surface in which we have stick and we supply force in given direction as shown in fig. So in this the surface will provide both forces i.e along the x and y-direction but this will reactive forces. Now if force is



is applied as shown in figure then absolutly it provides forces (reactive) along x and y-direction but at same time moment will also produce. i.e it cannot restrict moment.

Figure:-



Symbol:-

© Fixed Support:-

It gives horizontal, vertical and also moment reactions.

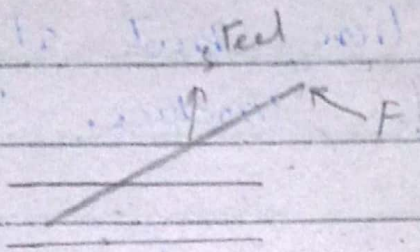
It means it restricts the horizontal translation, vertical translation and also restrict moment.

For Example:-

For example consider a steel flat is depth upto some foot into the beam then now if we apply a force at any direction then it will show resistance means provide resistive forces

and resistive moment.

Figure:-



Symbol:-

1

Rules or Steps To Draw: FBD:-

Rule Number # 01.

1. Isolate or separate the body from all the intersection points.

Rule # 02

② Show the all applied forces.

Rule # 03

③ Replacement of support by support reactions or reactive forces.

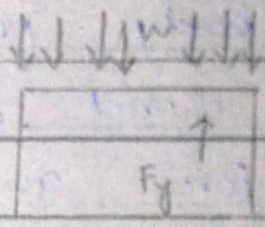
Rule # 04

④ Write dimension or angle or any information given with that diagram.

How to find Reactive force :-

The reactive force will be in that direction in which the translation is prevented. If rotation is prevented, then couple moment is exerted on the body.

Consider the diagram, here the translation is restricted only along vertical direction.



So reactive force is along y direction.

Significance of FBD :-

By drawing FBD we can write the equilibrium equation.

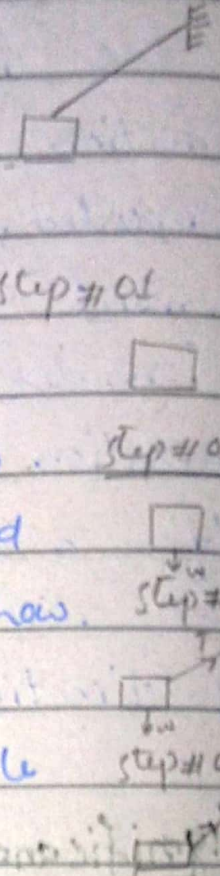
→ We can find the reactive forces i.e. reactions that acts on the body also we can find its direction.

→ It gives information about the forces acting on the body and any significant geometry.

Types of Connection:-

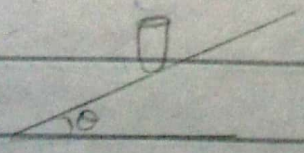
(a) If a body is support by cable:-

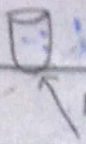
Consider the diagram in which a body is support by cable. In order to show its FBD. First step is to separate that body from all intersection points. Second step is to show applied force and third step is to show the reactive force. Here the reactive force is shown by Tensile force i.e in direction of cable. Last step is to show geometry, dimension or any other information about that body.



(b) Frictionless Surface:-

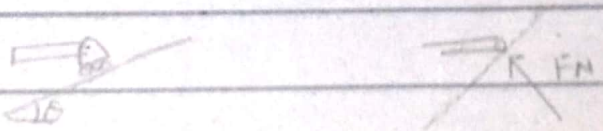
If we have an inclined frictionless surface then in this case the reaction will be only one and that will vertical perpendicular to that surface.





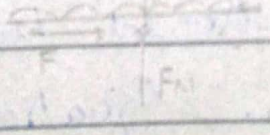
Roller Support:-

Roller support also give only one reaction i.e perpendicular, or ↑ vertical to surface directions as shown.



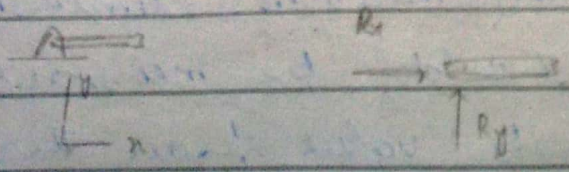
Surface with Friction:-

Give two types of reactions i.e in horizontal and vertical direction.



Pin Support:-

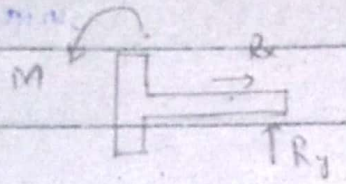
It gives two types of reactions forces that is in horizontal and vertical direction



Literall mean horizontal direction movement.

Built in or fixed Support:-

It restrict movement in both horizontal and vertical direction as well as do not allow any moment at any direction. So that the body to be in equilibrium.



Tips to Draw Correct : FBD:-

The following tips should be kept in mind in order to draw correct FBD.

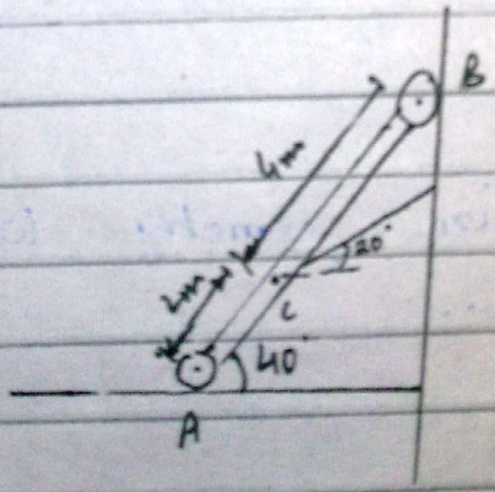
1. The diagram that we draw that is free body diagram should be neat because the equilibrium equations are derived from this diagram. That make us enable to calculate reactive forces.

2. All the given informations, dimensions and ~~dimension~~ angles etc should be mensioned and if not known its value than that should be shown by symbols.

(3) The support reaction should be taken on the basis of nature of support. For e.g for roller support we have only one support reaction i.e in vertical direction.

4. Only show that forces that are external to the body (It includes weight and reactive forces). Internal forces are equal and in opposite pair so they will not appear in free body diagram.
↳ It means where pair of forces occurs that act one body on other & vice versa that should be neglected.

Problem # 6.1. Draw the FBD of following figure.



Sol:- Step #1

Separate the body from all of its intersecting points.

Step #2

Show the applied force i.e weight.

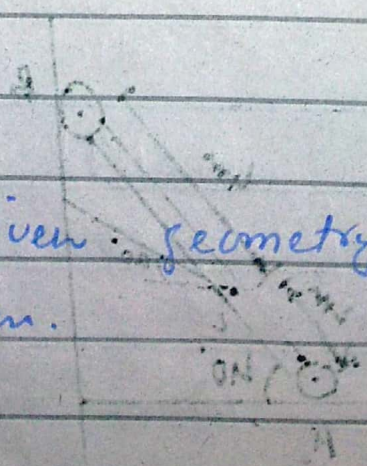
Since $w = mg \Rightarrow 50 \times 9.81$

$= 491 N$.

That will act on the middle i.e 3m

Step #03.

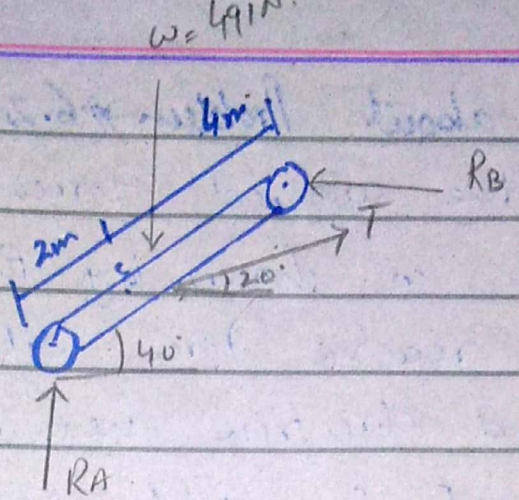
At A & B there are normal support that will provide only one reaction i.e perpendicular to the plane, as shown. Since we also separate it from cable so. This reactive force is shown by tensile force as shown.



Step #04

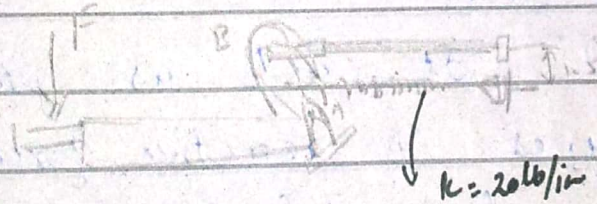
Show the given geometry with free body diagram.

FBD:-

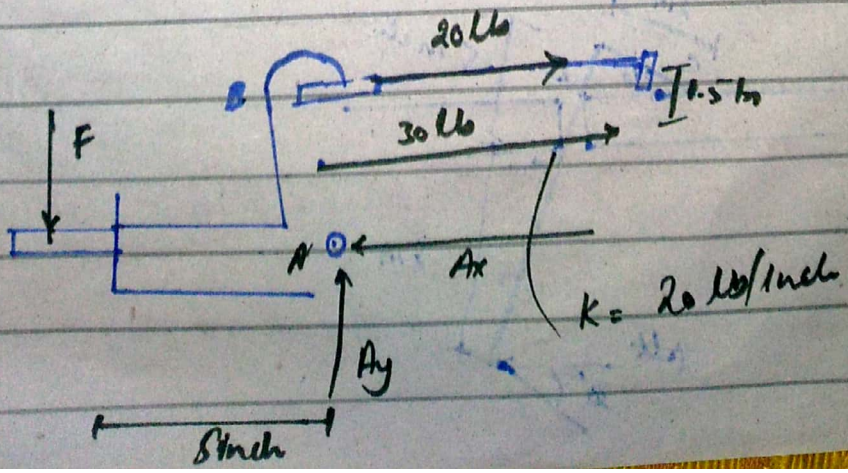


Problem #6.2

Draw FBD diagram.



FBD:-



Explaination about Problem #6.2

↳ Show all the applied forces since here is only one so shown by F.

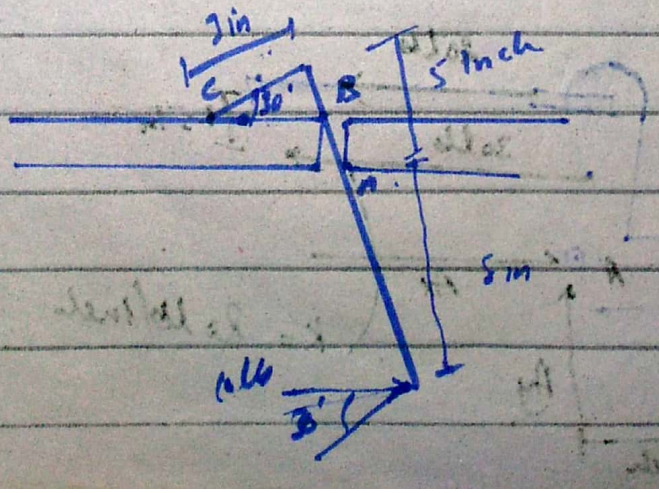
Now 20lb reactive force will create in right word direction here point B act as roller support in which reactive force produce in perpendicular to that plane.

→ In spring as elongation of 1 inch required produce and a force of 20lb is applied so the reactive force will be 30lb in right word direction.

At point A it acts as pin support so two types of reactive force will be created in horizontal & vertical direction as shown in figure.

↳ At last show all of its dimension & angles.

Problem 6.3 Draw FBD.



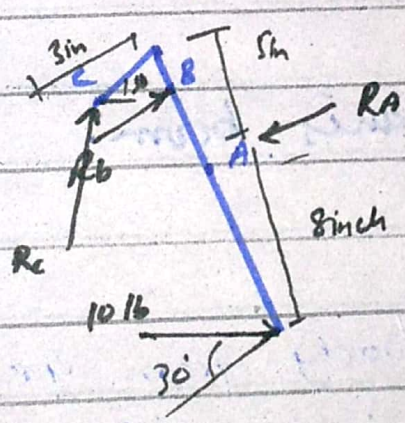
FBD Diagram:-

↳ Separate the body (bar) from all of its intersecting points.

↳ Show the applied force.

↳ Show the reactive forces since they are smooth only one reaction will produce that will normal to plane or surface

↳ Show all dimensions and angles.



Problem 6.4:- Draw FBD.

↳ Separate the body

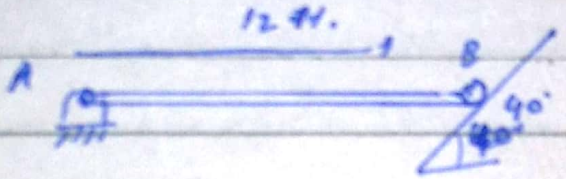
↳ Show the applied weight at centre of dumpster.

↳ Now at A as pin support so two types of reactive forces & at B only one reactive force as shown in fig.

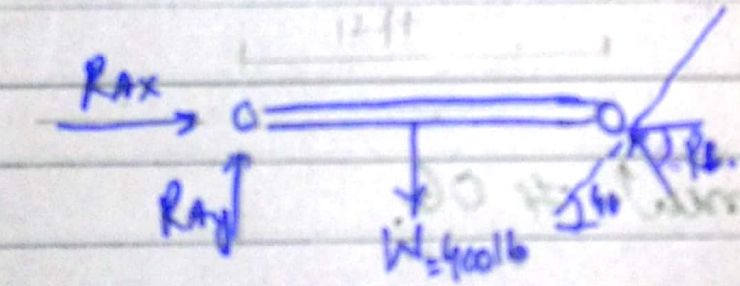
Exercise 6.

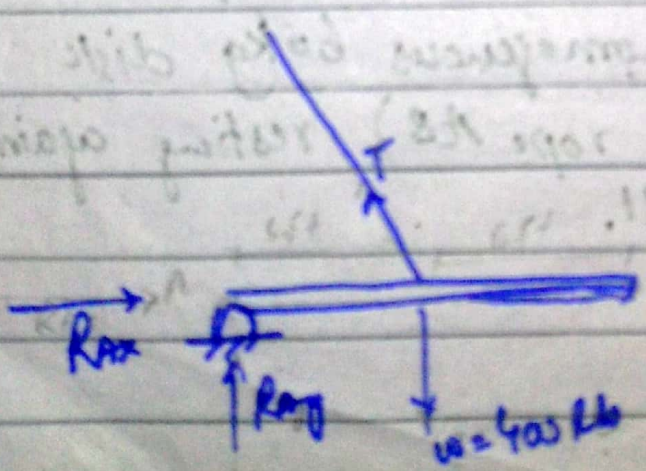
Q1:- Draw FBD.

Sol:- (a)

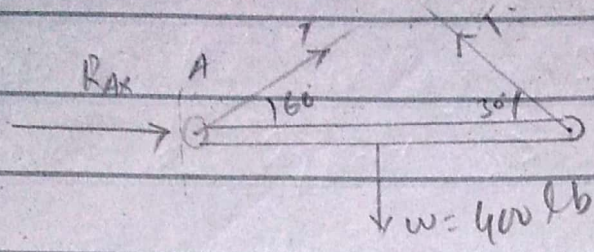


FBD:-

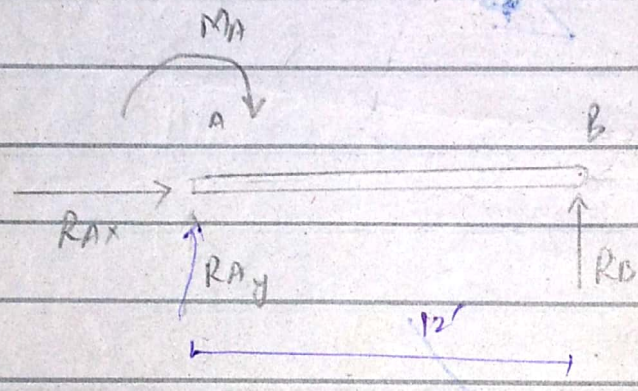




(c)



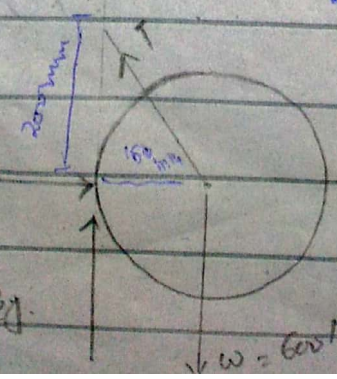
(d)



Home Assignment # 06.

2. Draw FBD of homogenous 60kg disk (supported by one rope AB) resting against rough vertical wall.

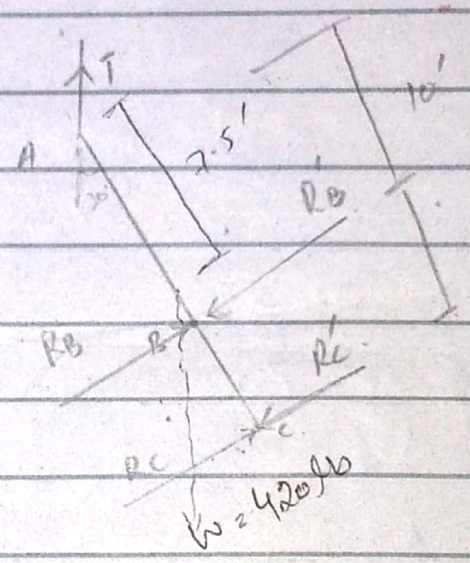
FBD :-



$60 \times 9.81 \Rightarrow 600$ N.

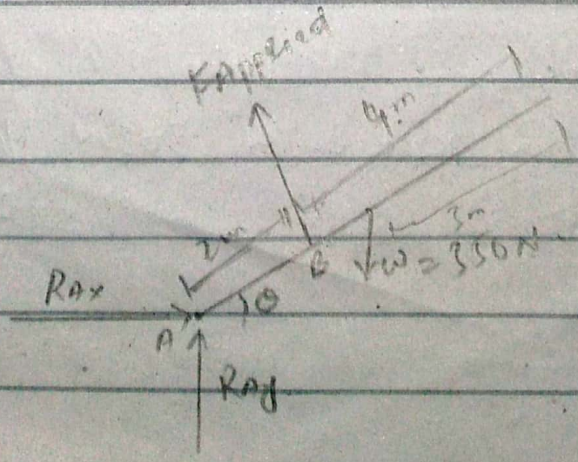
Q3:- Draw FBD of the 420 lb homogenous log which is supported by a rope at A and loose fitting rollers at B and C as it is being fed into a saw mill.

Sol:- **FBD:-**



Q4:- A man is holding up the 35 kg ladder ABC by pushing perpendicular to the ladder. Draw FBD of ladder?

Sol:- **FBD**

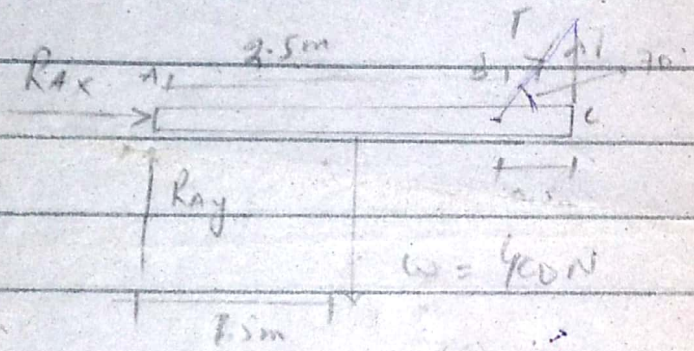


$W = 35 \times 10$

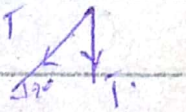
$= 350\ N$

Q5:- Uniform plank weight of 400N supported by Pin at A & Cable that runs around the pulley D. Draw FBD of plank & support D.

Sol:- FBD:-

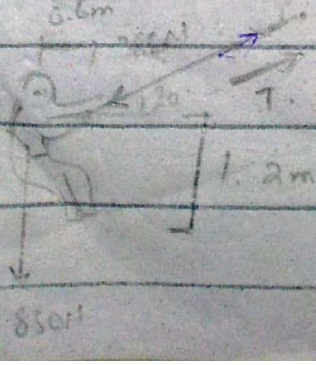


Support D:-



Home assignment #06

The centre of gravity of 850N man is at G. Draw his FBD if he pulls the rope with 348N force



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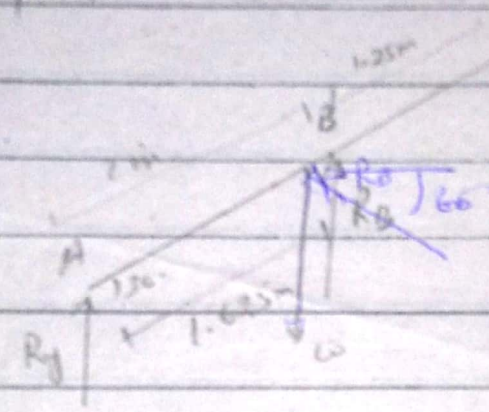
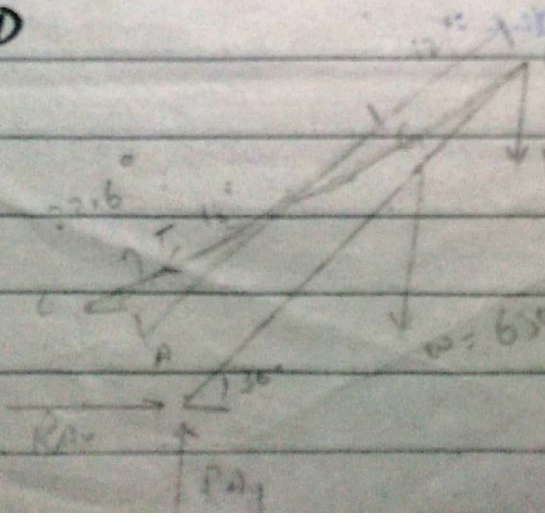
Q7:- The homogeneous 40 kg bar ABC is held in position by a horizontal rope A attached to end C. Draw FBD of bar, neglecting friction.



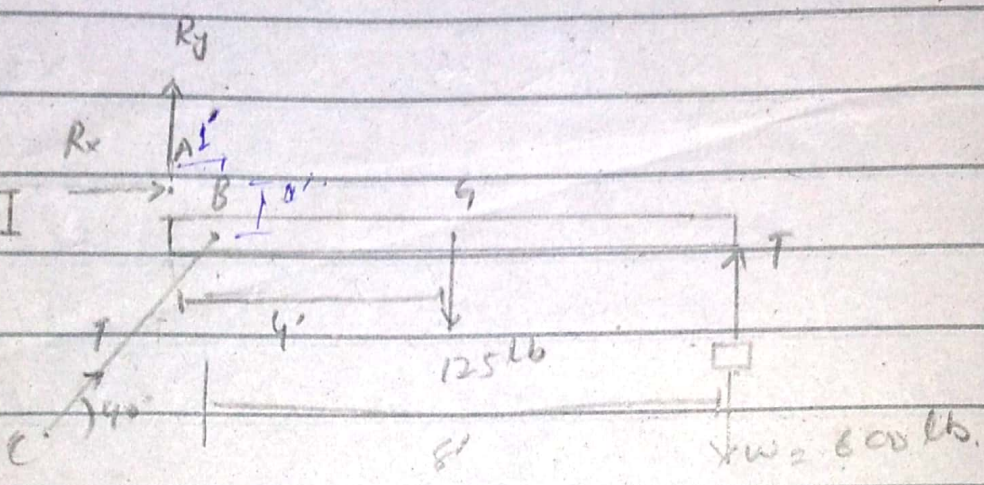
Q8 Draw the FBD of Crane boom AB having 650 lb weighting acting at centre of gravity G. The boom is supported by Pin at A & Cable BC with a load of 1250 lb suspended from Cable attached at B

Sol:- FBD



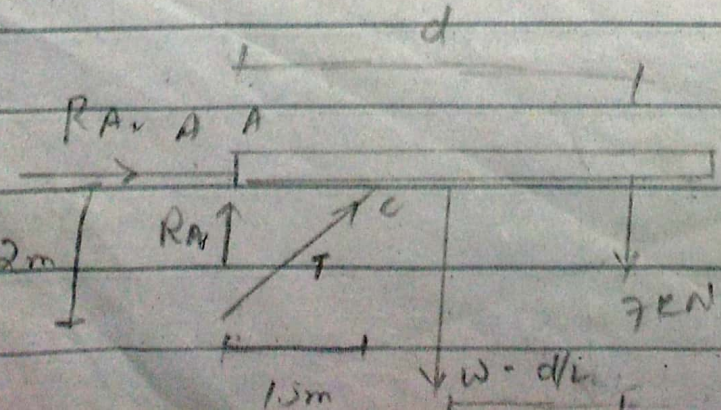
Q9:- The articulated crane boom has a weight of 125 lb and centre of gravity at G. Draw FBD of boom if it supports a load of 600 lb.

Sol:- FBD:-

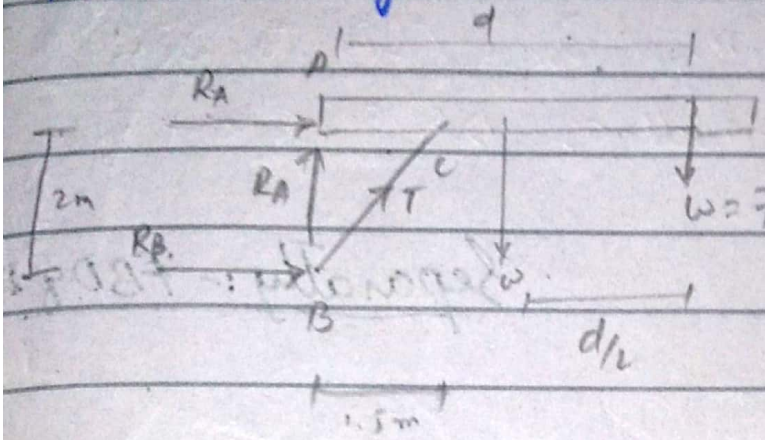


Q10:- The mass of 700 kg is suspended from a trolley which moves along the crane rail. Draw FBD of crane as well as of the whole system.

Sol:- FBD for Crane :-

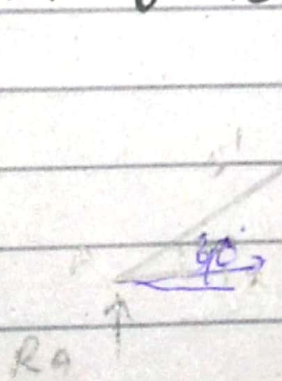


For whole System:-



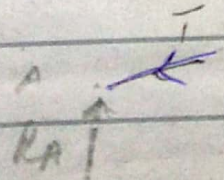
Q11:- The uniform rod AB has a weight of 15 lb. Draw FBD of rod, support at A, B & C

Sol:- FBD of Rod:-

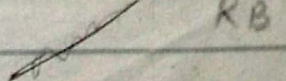


- Nut & bolt
- At A
roller Suppor

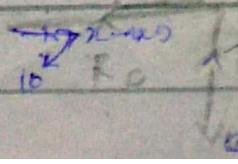
↳ Support At A



Support at B.

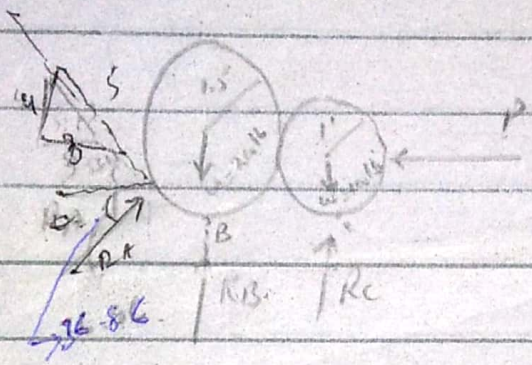


Support at C

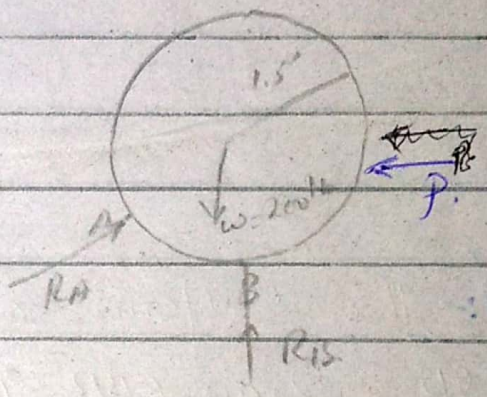


Q12:- The smooth disk D & E have a weight of 200 lb & 100 lb resp. Draw FBD of both disks together as well as D & E separately.

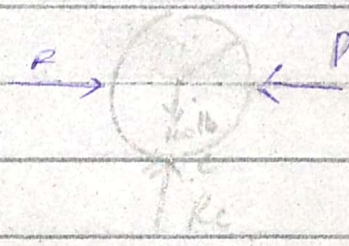
Sol:- Together FBD:-



Separately:- FBD of D

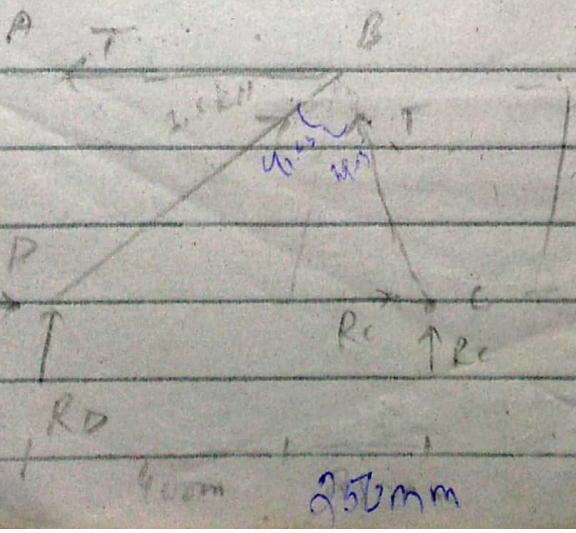


FBD of E:-

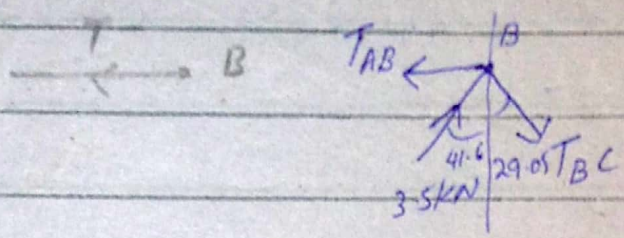


Q13:- The device shown is used to straighten the frame of wrecked autos. Draw FBDs of device, B, C & D.

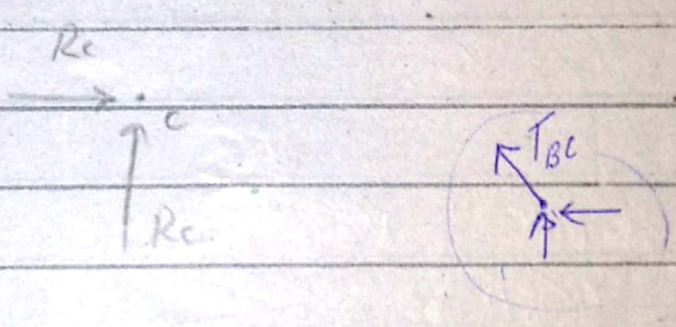
FBD of device:-



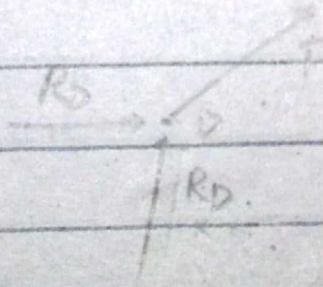
FBD at B:-



FBD at C.

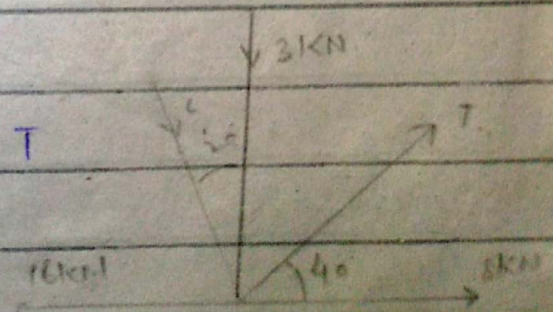


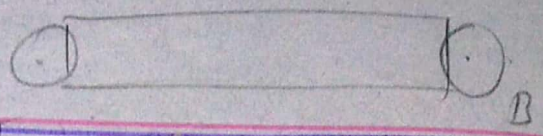
FBD at D:-



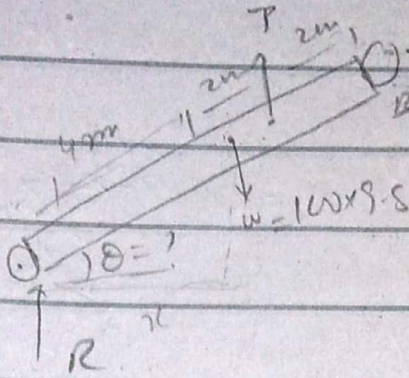
Sample problem 3/1 :-

Determine magnitude & T & C.





Problem # 3/3



$\cos\theta = \frac{2}{6}$

$x = \cos\theta\,(6)$

T - 981 N

Sol: From def ⁹ equilibrium

$\Sigma M_A = 0.$

$W(4\cos\theta) - P(6\cos\theta) = 0$ → ①

$4(981)\cos\theta = 6P\cos\theta$

$6P = \Rightarrow \boxed{P = 654\,N}$

For R = ? $\Sigma R_y = 0$

$R - 981\,N + (654\,N) = 0$

$\boxed{R = 327\,N}$

θ = ?

$981(4)\cos\theta - 654(6)\cos\theta = 0.$

$3924\cos\theta$

$\sin\theta = \frac{Per.P}{Hyp} = \frac{3}{8} \Rightarrow$

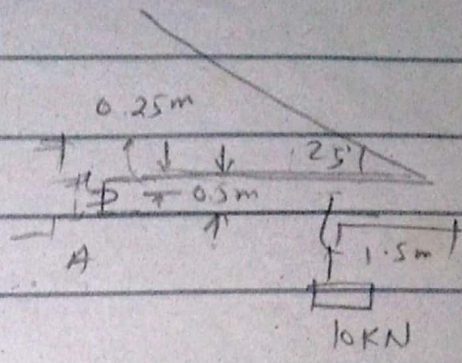
$\boxed{\theta = 22^\circ}$ Ans.

Sample Problem 3/4 :-

Sol:-

$\Sigma F_x = 0$.

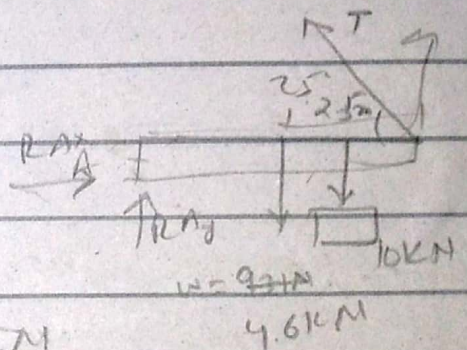
$\Sigma M_A = 0$.



$T\cos 25 \times 0.5 + T\sin 25 \times 5 = 0$

$-4.6 \times (2.5) - 10 \times (3.5)$

$= -11.5 - 35$



$T(0.4531) + 2.113T = 11.5$ KN

$T =$

$T\cos 25 \times 0.5 + T\sin 25 \times 5 - 10(5 - 1.5 - 0.12) - 4.6(2.5 - 0.12) = 0$.

$\Rightarrow 0.4531T + 2.113T = 33.88 + 10.948$

$= 44.748$.

$\therefore T = 19.6$ KN

To find A_x = ?

$\Sigma F_x = 0$

$R_{Ax} - 19.6(\cos 25°) = 0$.

$R_{Ax} = 17.76$ KN

To find A_y = ? $\Sigma F_y = 0$.

$R_{Ay} = 4.6$ KN $- 10$ KN $+ 19.6(\sin 25°) = 0$.

$R_{Ay} = 6.31$ KN