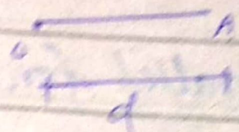
Module #07.

As in equilibrium, summation of all forces along x-direction & y-direction are zero i.e $\Sigma F_x = 0$ & $\Sigma F_y = 0$. It means resultant is zero. Resultant is both applied and reactive force. The body is said to be in equilibrium.

Since resultant is zero

∴ moment about any point

Let O is,



$M_o = R \times d$

$= 0 \times d$

$\boxed{M_o = 0}$

Thus $\boxed{\Sigma M = 0}$

So for a body to be in equilibrium the above three conditions should be satisfied.

Points that should Noted:-

→ **Collinear force System:-**

As in collinear force there is only one forces acting (acts either in horizontal or vertical di) may be acting horizontally or vertically

So in that system the unknown

reaction can be find either from $\Sigma F_x = 0$ or $\Sigma F_y = 0$.

& In concurrent force system ΣM_o about any can be used as check or for verification.

2. Concurrent force System:-

In concurrent force system we are required two conditions i.e to find the unknown reaction.

Condition # 01 And Condition # 02

$\Sigma F_x = 0$ $\Sigma F_y = 0$.

. Parallel force System:-

In Parallel force system we need to find/required the two conditions. One is $\Sigma M = 0$ and other is either $\Sigma F_x = 0$ or $\Sigma F_y = 0$ to find the unknown reactions.

Non Parallel & non current force System:-

For such system the unknown reactions forces can be determined from 3 conditions.

a) $\Sigma F_x = 0$ (b) $\Sigma F_y = 0$ & (c) $\Sigma M = 0$.

raphical methods for equillibrium Analysis:-

) Triangle law:-

If we have concurrent forces then

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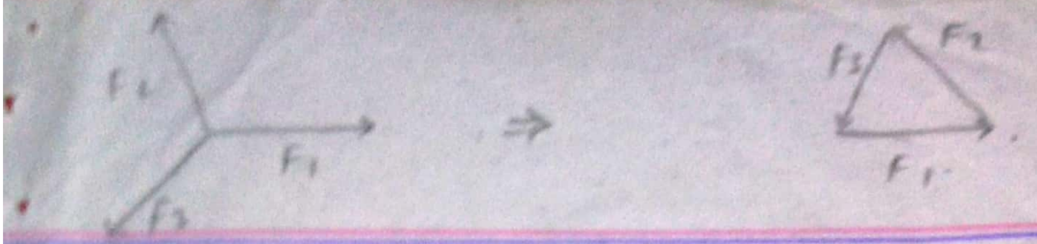


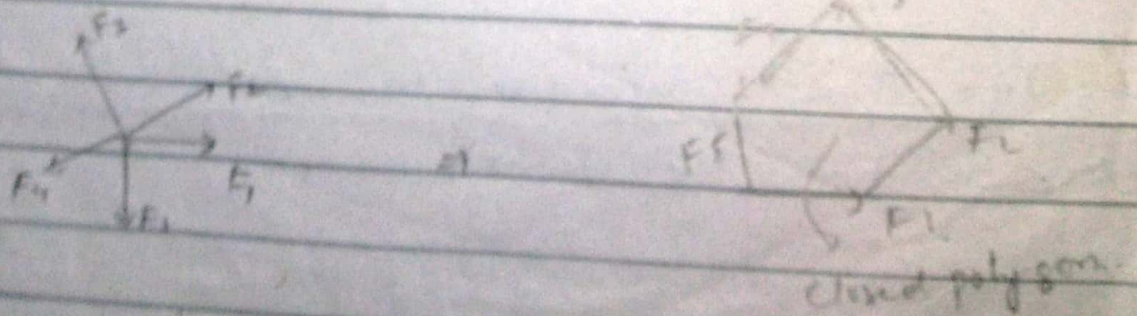
and if they are in equilibrium we can simply represent in form of triangle (closed) if the system is in equilibrium.

→ If a body is in equilibrium as a result of three forces those forces must be concurrent & triangle is made from them

→ But If a body is in equilibrium because of ~~the~~ more than three forces they may or may not be concurrent.

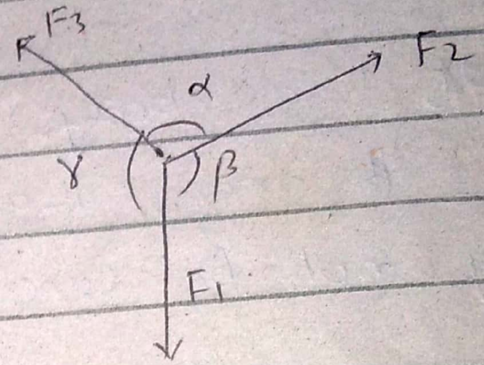
(b) Polygon law:-

If more than three forces acting on a body and the body is in equilibrium then by head to tail rule they form a closed polygon.



LAMI's Theoram:-

If a system of three forces ~~are~~ is in equilibrium then each force of the System is proportional to sin of angle between the other forces of system (Constant of proportionality is same for all forces).



Mathematically:-

$$\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\gamma} = \frac{F_3}{\sin\beta}$$

Important to note:-

In Lami's Theoram the direction of all three concurrent forces should be either towards or away from the point of cuncurrence.

→ By this Theoram we can also find the reactions (Unknown).

→ Also angles (α, β & γ) can be find

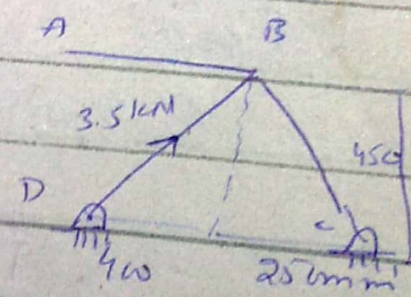
Equillibrium Conditions for ~~Concurrent~~ Coplanar.
Force System:
Since ~~Concurrent~~ Coplanar force system occur ~~in string~~ allong either x axis or y-axis so to find unknown reactions we required ~~three~~ 2 conditions of equillibrium

$\Sigma Fx = 0$, $\Sigma Fy = 0$.

$\Sigma M = 0$ is used to check our results. For moment the centre should not be the point of Intersection of forces but may be other point.

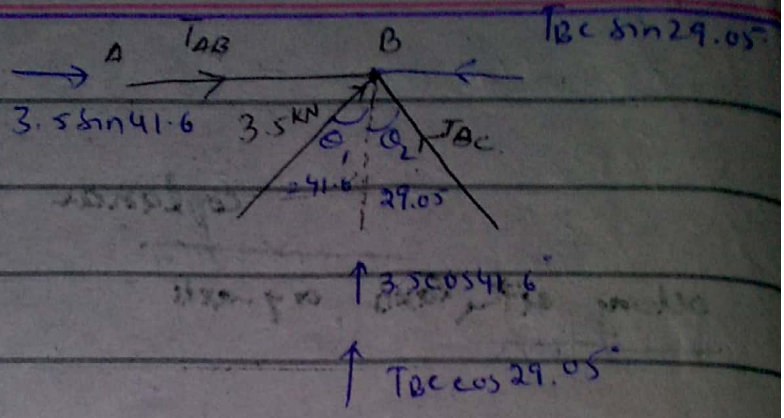
Problem # 7.1

Given data
Force from D to B = 3.5 KN.
Required data
$T_{AB} = ?$
$T_{BC} = ?$



Solution:- If we draw FBD of given fig at D we ~~get the required~~ get all the required. So therefore we draw FBD of point B.

$\tan\theta_1 = \frac{400}{450}$



$\theta_1 = 41.6^\circ$

$\tan\theta_2 = \frac{250}{450}$

$\theta_2 = 29.05^\circ$

By applying Condition of equillibrium as forces are concurrent. So,

$\Sigma F_x = 0 \quad \& \quad \Sigma F_y = 0$

$3.5 \sin 41.6 + T_{AB} - T_{BC} \sin 29.05 = 0 \rightarrow (1)$

$3.5 \cos 41.6 = 2.6$
$3.5 \sin 41.6 = 2.32$

$\Sigma F_y = 0$

$3.5 \cos \cancel{41.6}^\circ + T_{BC} \cos 29.05 = 0$ 2.3237

$T_{BC} \cos 29.05 = -2.3237$

$T_{BC} = \frac{-2.3237}{0.8741}$

$\boxed{T_{BC} = -2.99 \text{ KN}}$

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-ve sign shows our assume direction is wrong. It is away from point B.

Now Eq ① ⇒

$$2.32 + T_{AB} - T_{BC} \sin 29.05 = 0.$$

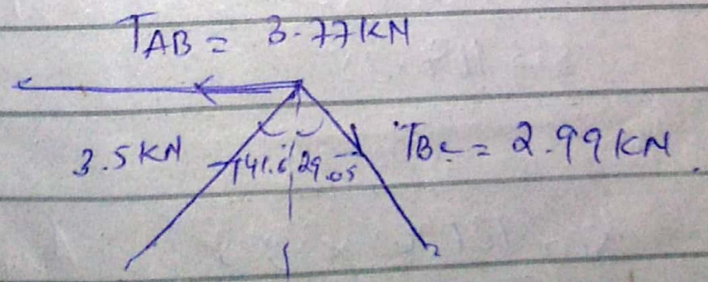
$$2.32 + T_{AB} - (-2.99)(0.4855) = 0$$

$$T_{AB} + 3.77 = 0$$

$$\boxed{T_{AB} = -3.77 \text{ KN}}$$

Here it again show our assume direction is wrong and it is away from point B.

So the correct FBD will be;



→ Away from joint → tension
→ Toward joint → Compression.

Verify the result:-

$+\circlearrowleft \sum M_D = 0.$

$= +3.77 \times 450 - 2.99 \times \cos 29.05 \times 400 - 2.96 \sin 29.05 \times 4$

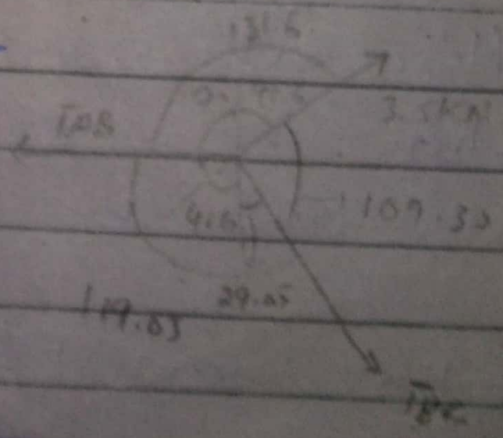
$= 1696.5 - 1045.538 - 651.15$

$= 1696.5 - 1696.5$

$\boxed{\sum M_D = 0}$ Proved

Now 2nd method:-

Lami's Theoram:-



As we know that;

$$\frac{T_{AB}}{\sin 109.35} = \frac{3.5}{\sin 119.35}$$

$$\boxed{T_{AB} = \frac{\sin 109.35 \times 3.5}{\sin 119.35}}$$

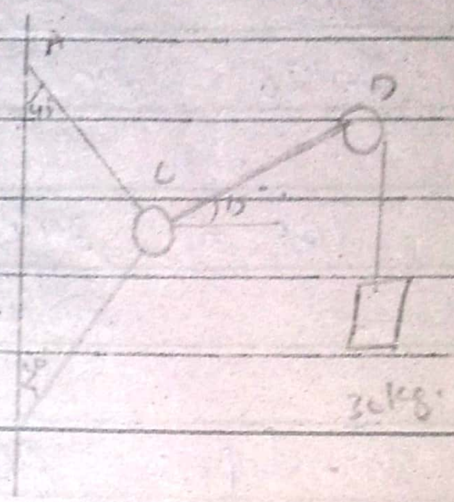
$$\boxed{T_{BC} = \frac{\sin 131.6 \times 3.5}{\sin 119.35}}$$ Ans

Excercise 7.1

Q1:-

Sol:- Given data ;

Mass of Cylinder = 30 kg.



Required data ;

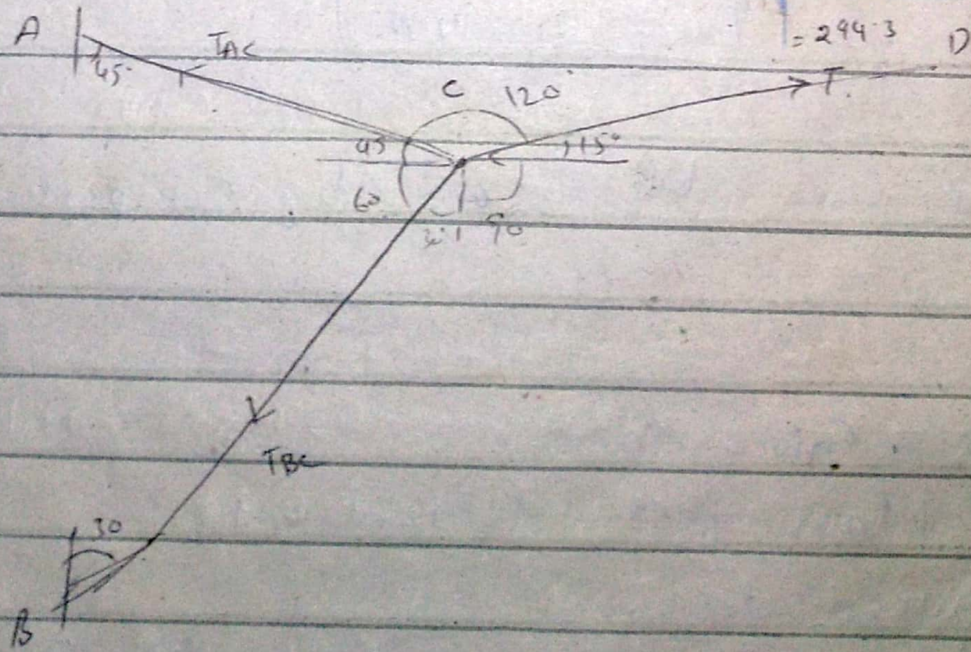
Tension In Cable AC = T_{AC} = ?

Tension In Cable BC = T_{BC} = ?

Solution:-

We will draw FBD at C to get the required unknown.

FBD at C:-



By Lemi Theoram

$$\frac{T_{BC}}{\sin 120°} = \frac{T_{AC}}{\sin 135} = \frac{294.3}{\sin 105°}$$

$$T_{BC} = \frac{294.3}{\sin 105°} \times \sin 120°.$$

$$\boxed{T_{BC} = 263.8\ N.}$$

$$T_{AC} = \frac{294.3}{\sin 105°} \times \sin 135°.$$

$$= \frac{294.3 \times 0.7071}{\sin 105°}$$

$$\boxed{T_{AC} = 215.4 N}$$

It can be done by Component method.

Q2:-

Ans:- Given data;

Mass of chandelier = 50kg

Required data;

$T_{CD} = ?$, $T_{BD} = ?$, $T_{AB} = ?$, $T_{BC} = ?$

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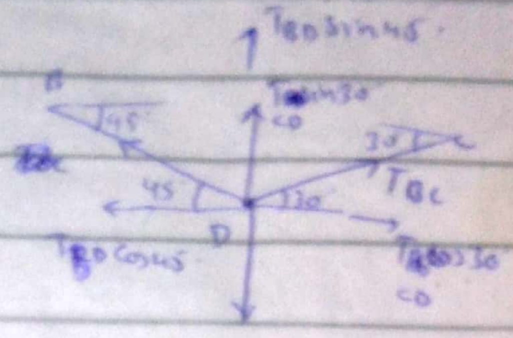
Bcz if we draw FBD at other joint we have more unknown than no. of eqns

Solution:- First we will draw FBD at D.



$\Sigma F_x = 0$

$-T_{BD}\cos 45^\circ + T_{CD}\cos 30^\circ = 0$

$W = 50 \times 9.81$

$= 490.5\,N$

$T_{CD}\cos 30^\circ = T_{BD}\cos 45^\circ \longrightarrow (1)$

$\Sigma F_y = 0$

$T_{BD}\sin 45^\circ + T_{CD}\sin 30^\circ - 490.5 = 0$

$T_{BD}\sin 45^\circ + T_{CD}\sin 30^\circ = 490.5 \longrightarrow (2)$

eq (1) $\rightarrow$ $0.866\,T_{CD} - 0.7071\,T_{BD} = 0 \rightarrow (4)$

eq (2) $\rightarrow$ $0.7071\,T_{BD} + 0.5\,T_{CD} - 490 = 0$

$0.5\,T_{CD} + 0.7071\,T_{BD} - 490 = 0 \longrightarrow (3)$

Add eq (3) & (4)

$0.866\,T_{CD} - 0.7071\,T_{BD} = 0$

$+0.5\,T_{CD} + 0.7071\,T_{BD} - 490 = 0$

$1.366\,T_{CD} = 490$

$\boxed{T_{CD} = 358.71}$

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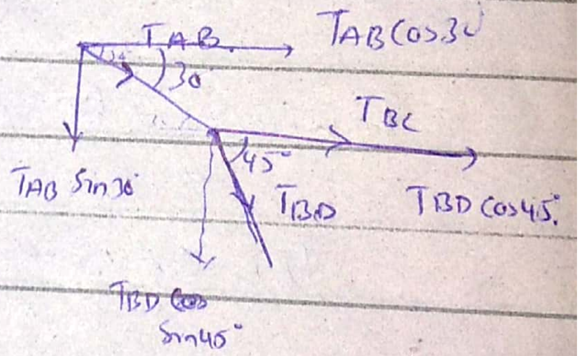
Put in eq (4).

$0.866(358.71) - 0.7071\, T_{BD} = 0.$

$(0.7071)\, T_{BD} = 316.64$

$$\boxed{T_{BD} = 439.3\,N}$$

Now Draw FBD at B.



$\Sigma F_x = 0.$

$T_{BD}\cos 45° + T_{AB}\cos 30° + T_{BC} = 0.$

$439.3(0.7071) + 0.866(T_{AB}) + T_{BC} = 0.$

$0.866\, T_{AB} + T_{BC} + 310.629 = 0. \rightarrow (1)$

$\Sigma F_y = 0.$

$-T_{AB}\sin 30° - T_{BD}\sin 45° = 0.$

$-T_{AB}(0.5) = 439.3(0.7071)$

$= 310.62$

$$\boxed{T_{AB} = -621.25\,N}$$

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Apply condition of equillibrium

$\Sigma F_x = 0$

$-200 + 400 \cos\theta = 0.$

$400 \cos\theta = 200.$

$\Rightarrow \cos\theta = \frac{1}{2}$

$\boxed{\theta = 60^\circ}$

$\Sigma F_y = 0.$

$400 \sin 60^\circ - 800 + N = 0.$

$N - 453.589 = 0.$

$\boxed{N = 453.58 lb}$

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The negative show our assume direction is wrong.

Put in eq ① to get T_{BC}

$$0.866(-621.25) + T_{BC} + 310.629 = 0$$

$$\boxed{T_{BC} = 227.38\,N} \quad \text{Ans.}$$

Qs:-

Ans:- Given data

Total weight = 800 lb

weight of 1st box = 200 lb

weight of 2nd box = 400 lb

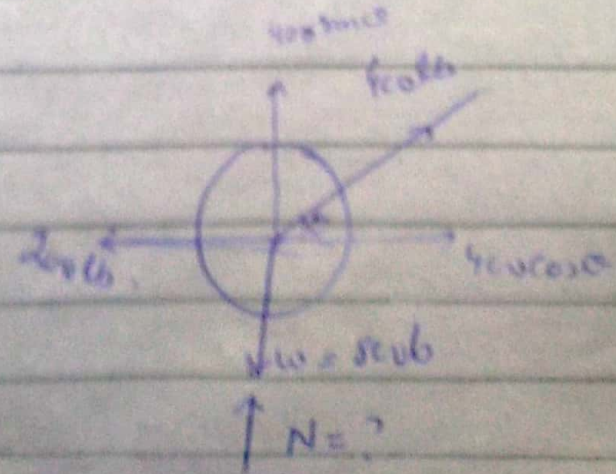
Required data:-

Angle θ = ?

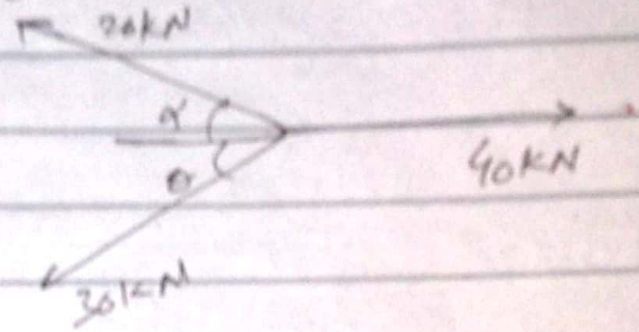
Normal reaction force = N = ?

Solution:-

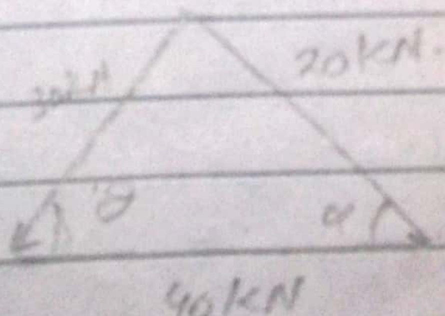
FBD:-



Q4:- Determine the value of α & θ so that forces shown in fig will be in equillibrium.



Solution:-
to find the values of α & θ so that the forces shown in fig are in equillibrium.
We used triangle law;
So according to triangle law, the forces (Three) that are in equillibrium can be joined by head to tail rule.



Now using law of Cosine

$$\cos\theta = \frac{(30)^2 + (40)^2 - (20)^2}{2(30)(40)}$$

3. Truss joint with 2 non collinear

$$= \frac{900+1600-400}{2400}$$

$$\cos\theta = \frac{2100}{2400}$$

$$\cos\theta = 0.875$$

$$\theta = \cos^{-1}(0.875) \Rightarrow \boxed{\theta = 28.95^\circ}$$

Now α:-

$$\cos\alpha = \frac{(40)^2+(20)^2-(30)^2}{2(40)(20)}$$

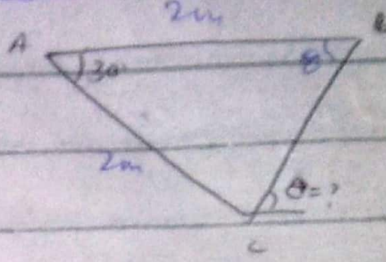
$$= \frac{1600+400-900}{1600} \Rightarrow \frac{1100}{1600}$$

$$\cos\alpha = \cancel{0.5625}\ 0.6875$$

$$\alpha = \cos^{-1}(0.6875)$$

$$\alpha = 46.567^\circ$$

Q6.- Given data;



$\theta = 30°$

mass = $m = 200kg$.

Required data;

force, $P = ?$

Sol.- In order to solve it first we have to find the angle which P makes with x-axis.

Using law of sine,

$$\frac{\sin 30°}{BC} = \frac{\sin\theta}{2} \rightarrow ①$$

For side "BC" using law of Cosine;

$$(BC)^2 = (2)^2 + (2)^2 - 2(2)(2)\cos 30°$$

$$= 8 - 8(0.866) = 1.071$$ Taking square root on b/s.

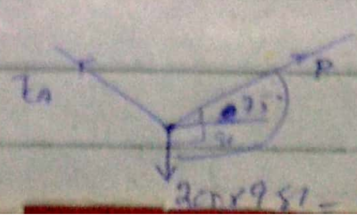
$$\boxed{BC = 1.035\,m}$$

eq ① ⇒ $\dfrac{\sin 30°}{1.035} = \dfrac{\sin\theta}{2} \Rightarrow \sin\theta = \dfrac{\sin 30° \times 2}{1.035}$

$\sin\theta = 0.966 \rightarrow \boxed{\theta = 75°}$

Now Draw FBD at C, to Calculate P,

→



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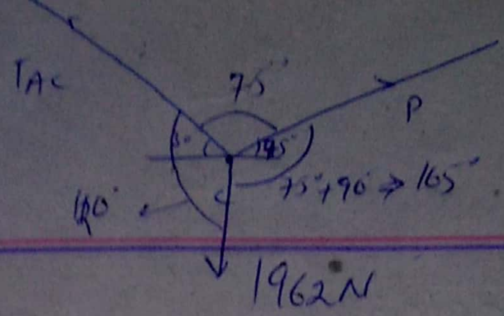


Using Lami's theoram;

$$\frac{\sin 120^\circ}{P} = \frac{\sin 75}{1962}$$

$$1/P = \frac{\sin 75^\circ}{\sin 120^\circ \times 1962}$$

$$P = \frac{\sin 120^\circ \times 1962}{\sin 75^\circ}$$

$$\boxed{P = 1759\ N}$$

Equillibrium of Coplanar Parallel Force System:-
In this case the forces are either parallel to vertical or horizontal. So the reactions are find from $\Sigma F_x = 0$ or $\Sigma F_y = 0$ & $\Sigma M = 0$. because the forces is not parallel to other plane & we neglect that components.

for Find reaction of forces take $\Sigma M = 0$ at that point due to which moment due to one of reaction become zero Then from that find one force & put it in other eqn to find other unknown force

Problem # 7.2

Ans:- Given that;

weight of beam = 100 lb/ft

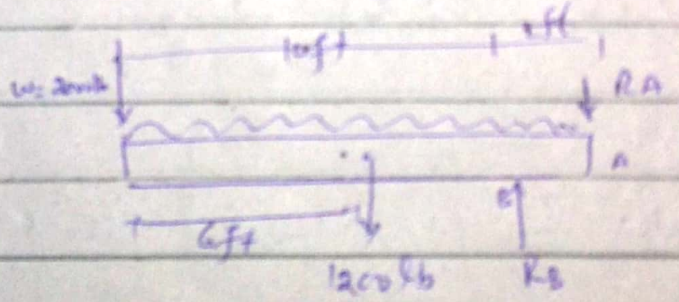
Total length of beam = 12 ft.

weight / force acting = 2000 lb.

Required:-

Reaction at A & B.

Solution:- Draw FBD of beam.



$\frac{100\ lb}{ft} \times 12\ ft$

$= 1200\ lb.$

$\Sigma F_y = 0.$

$-2000 - 1200 + R_B - R_A = 0.$

$R_B - R_A = 3200 \longrightarrow (1)$

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$$\sum M_B = 0.$$

$$2000 \times 10 + 1200 \times 4 - R_A \times 2 = 0$$

$$2 \times R_A = 20000 + 4800$$

$$2 \times R_A = 24800.$$

$$\boxed{R_A = 12400 \text{ lb}}$$

$$\text{eq } ① \Rightarrow . \quad R_B - 12400 = 3200$$

$$R_B = 3200 + 12400.$$

$$\boxed{R_B = 15600 \text{ lb}}$$

Verification:-

Let take B as moment Centre;

$$\sum M_B = 2000 \times 10 + 1200 \times 4 + R_B \times 0 - 12400 \times 2.$$

$$\boxed{\sum M_B = 0}$$

The result is verified.

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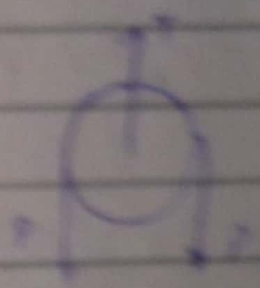
Problem # 7-3

Given Data,

Weight of block = 900 lb

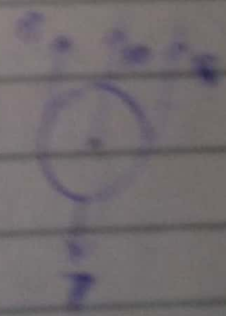
We have to find $P = ?$

Sol: FBD at A.



$2P = T$

FBD at C.

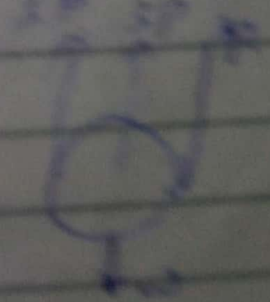


$\Sigma F_y = 0$

$-T + 3P = 0$

$\boxed{T = 3P}$

FBD at D.



$\Sigma F_y = 0$

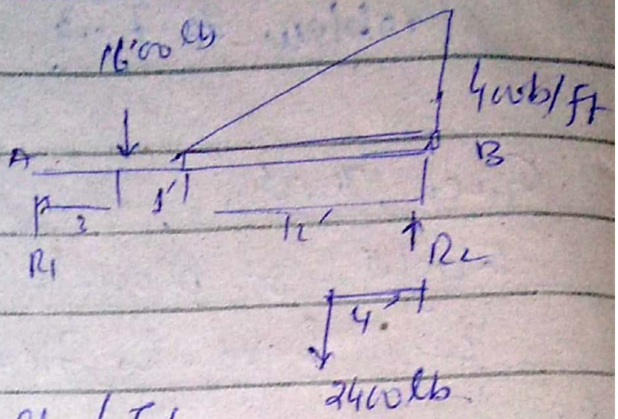
$-W + 3(3P) = 0$

$-W + 9P = 0$

$9P = W \Rightarrow P = \frac{900}{9}$

$\boxed{P = 100 \text{ lb}}$

Excercise 7.2



Q1:-

Given that;

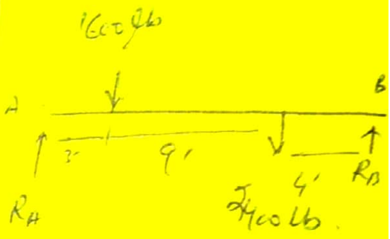
load = 1600 lb.

weight per foot = 400 lb / ft

Weight. $= \frac{400 \times 12}{2} \Rightarrow 2400\,lb.$

Apply conditions of equillibrium.

Draw FBD of Fig.



As from big side the rxn of distributed (variable) load occur at $L/3 \Rightarrow \frac{12}{3} = 4'$

$\Sigma F_y = 0.$

$R_1 + R_2 - 1600 - 2400 = 0.$

$R_1 + R_2 = 4000 \longrightarrow (1)$

$\circlearrowleft +$

$\Sigma M_B = 0.$

$-R_1 \times 16 + 1600 \times 13 + 2400 \times 4 = 0$

$-16 R_1 + 20800 + 9600 = 0$

$\boxed{R_1 = 1900\,lb}$

eq(1) ⇒

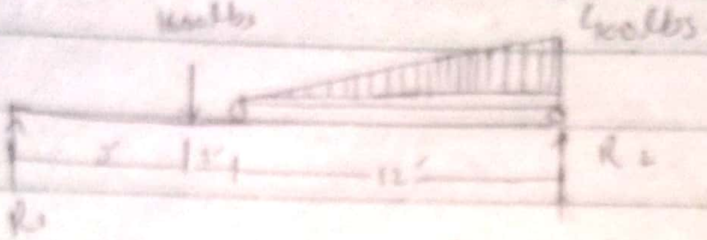
$R_2 = 4000 - 1900$

$\boxed{R_2 = 2100\,lb}$ Ans

{Second Method}

Ex - 7.2

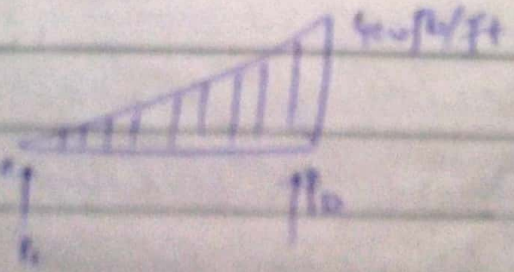
Q1. Determine the reactions R_1 & R_2 of the beam loaded with a concentrated load of 1600lb and a load varying from zero to an intensity of 400 per ft.



Solution:-

If we draw the FBD of lower beam we have four unknown so we cannot find four unknown from three eqns i.e $\Sigma F_x = 0$, $\Sigma F_y = 0$ & $\Sigma M = 0$

So therefore we will draw FBD of upper beam as shown in fig below.



Apply condition of equilibrium as it is case of parallel force system,

$\Sigma F_y = 0$

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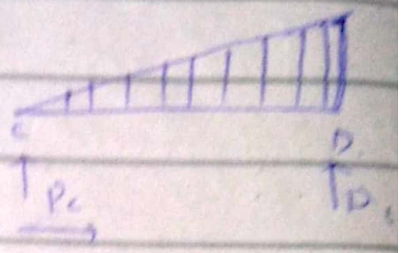
$$P_c + P_d - \frac{400 \times 12}{2} = 0$$

$$P_c + P_d = 2400 \longrightarrow ①$$

$$\sum M_c = 0$$



$$-\frac{400 \times 12}{2} \times 8 + P_D \times 12 = 0$$

$2L/3$

$$12 P_D = 400 \times 12 \times 4$$

$= \frac{2(12)}{3}$

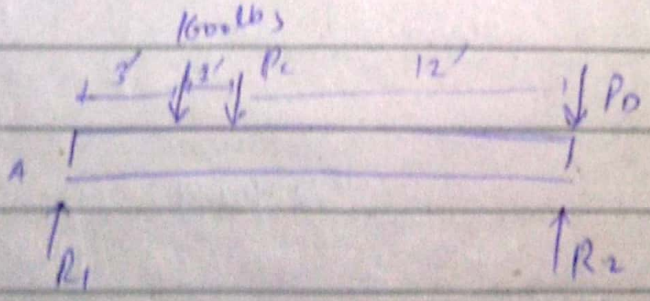
$= 8$

$$\boxed{P_D = 1600 \text{ lbs}}$$

eq ① ⇒ $P_c = 2400 - 1600$

$$\boxed{P_c = 800 \text{ lbs}}$$

Now draw FBD of lower beam;



Apply condition of equillibrium;

$$R_1 + R_2 - 1600 - P_c - P_d = 0$$

$$R_1 + R_2 - 1600 - 800 - 1600 = 0$$

$$R_1 + R_2 = 4000 \longrightarrow ③$$

$\Sigma M = 0$

$-1600 \times 3 - 300(4) - [illegible](16) + R_2(16) = 0$

$-4800 - 1200 - 27600 + 16R_2 = 0$

$16R_2 = 33600$

$\boxed{R_2 = 2100 \text{ lbs}}$

Now, eq (i)

$R_1 + R_2 = 4000$

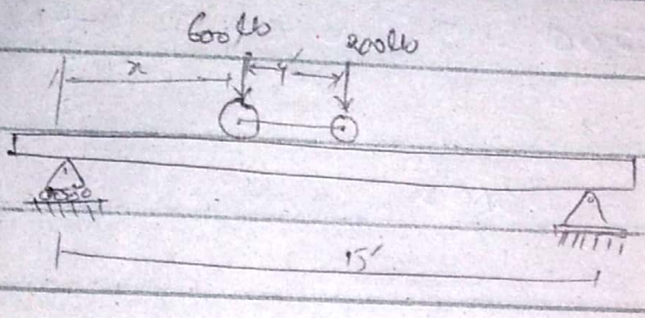
$R_1 = 4000 - R_2$

$R_1 = 4000 - 2100$

$\boxed{R_1 = 1900 \text{ lbs}}$

Ans

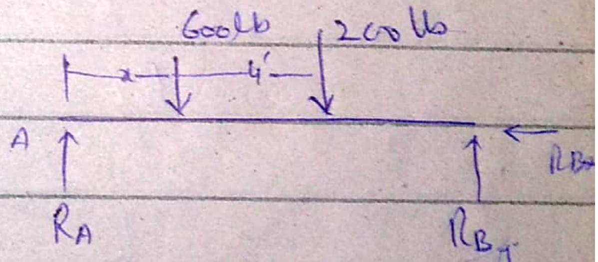
Q2:- The wheel loads on a jeep are given in fig. Determine the distance x so that reaction of beam at A is twice as great as reaction at B.



Solution:- Given that,

~~$R_B = 2R_A$~~ $R_A = 2R_B$.

Draw FBD of the fig.



$\Sigma F_y = 0$.

$R_A + R_{B_y} - 600 - 200 = 0$.

$R_A + R_{B_y} = 800 \longrightarrow ①$.

Since $R_A = 2R_B$.

eq① ⇒ $2R_B + R_B = 800$

$3R_B = 800$.

$R_B = \frac{800}{3}$ ⇒ $\boxed{266.67 \text{ lbs}}$

$\sum M_B = 0$

$-600 \times x - 200(x+4) + 266.67(15) = 0$

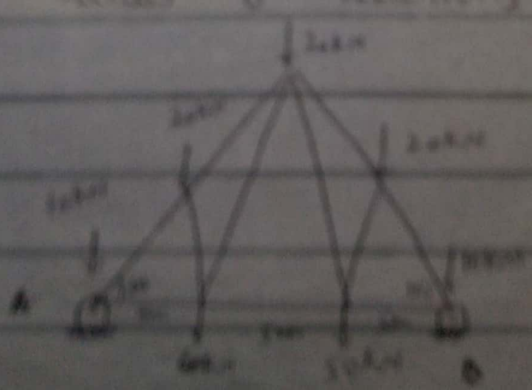
$-600x - 200x - 800 + \cancel{4000} = 0$

$-800x + 3200 = 0$

$-800x = -3200$

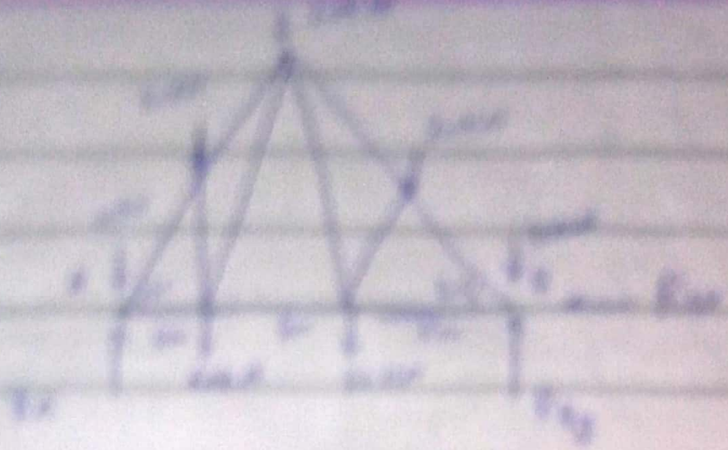
$\boxed{x = 4\,m}$ Ans

Q2:- The roof truss in figure is supported by a roller at and a hinge at B. Find the values of reactions



Solution:-

Draw the FBD of figure.



Apply conditions of equilibrium

$\Sigma F_x = 0$

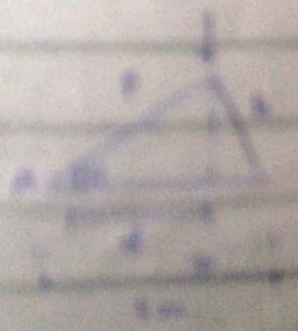
$\boxed{R_{Ax} = 0}$

$\Sigma F_y = 0$

$R_A + R_{By}$ [illegible] $= 0$

$R_A + R_{By} - 190 = 0$

$R_A + R_{By} = 190 \rightarrow (1)$



$\Sigma M_A = 0$

[illegible]

[illegible]

[illegible]

$-437.3 - 360 - 150 - 560 - 256.6 - 150 + 15R_{By} = 0$

$15R_{By} - 1399.9 = 0$

$15R_{By} = 1399.9$

$R_{By} = 93.3\,KN$

eq ② ⇒ $R_A + R_{By} = 190$

$R_A = 190 - R_{By}$

$R_A = 190 - 93.3$

$\boxed{R_A = 96.7\,KN}$ Ans

Equillibrium of non Parallel, non concurrent force System:-

Here the forces are neither Parallel nor Concurrent So we use both the force condition and moment condition of equilibrium i.e.

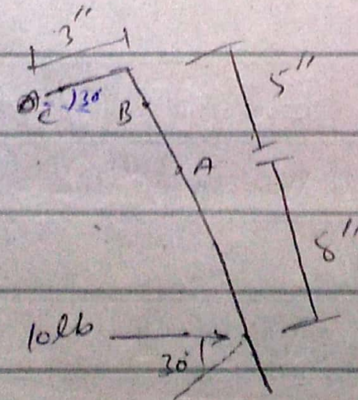
$$\Sigma F_x = 0, \quad \Sigma F_y = 0 \quad \& \quad \Sigma M = 0$$

to find our unknown reactions.

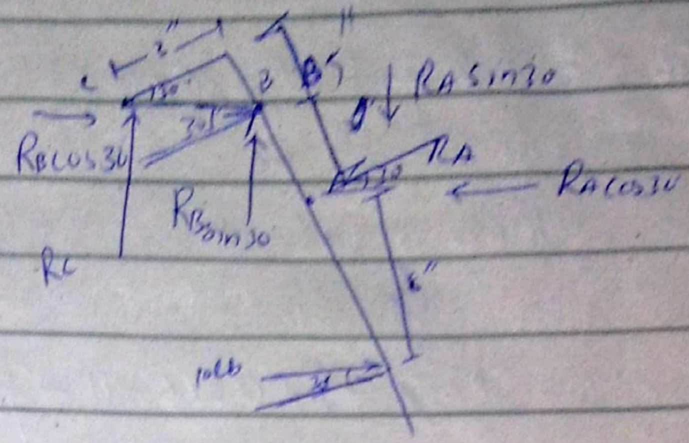
Other approach is used two moment equillibrium equations and one force eqn.

Problem # 7.4

Determine the reactions at the contact points of the rod due to 10lb force. Assume that bar has negligible thickness and smooth point of contact at A, B & C.



Solution:- Draw the FBD of the System.



Apply Conditions of equillibrium.

$\Sigma F_x = 0$.

$R_B \cos 30^\circ - R_A \cos 30^\circ + 10 = 0$

$0.866 R_B - 0.866 R_A + 10 = 0$

$R_B - R_A + \frac{10}{0.866} = 0$.

$R_B - R_A + 11.55 = 0$

$R_B = R_A - 11.55 \rightarrow (1)$.

$\Sigma F_y = 0$.

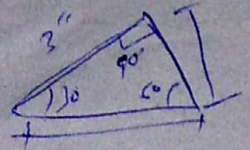
$R_B \sin 30^\circ - R_A \sin 30^\circ + R_C = 0$.

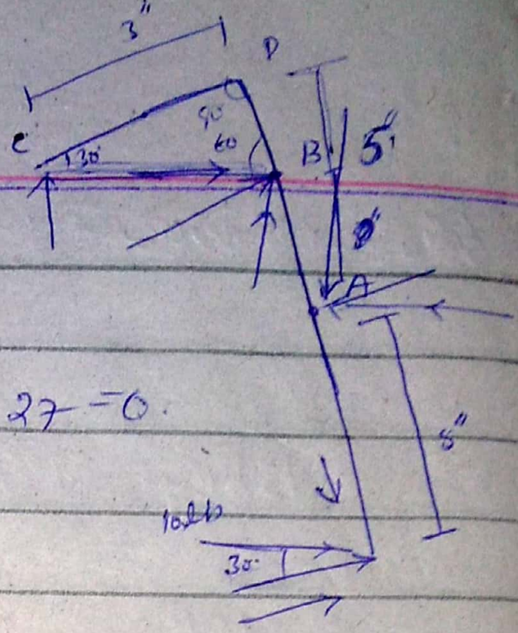
$0.5 R_B - 0.5 R_A + R_C = 0$.

$0.5(R_A - 11.55) - 0.5 R_A + R_C = 0$ Put eq (1) in eq.

$0.5 \cancel{R_A} - 5.775 - \cancel{0.5 R_A} + R_C = 0$

$\boxed{R_C = 5.775 \text{ lbs}}$





Now $\Sigma M_B = 0$:

$-R_C \times 3.46 - R_A \times 3.27 + 10\cos 30° \times 11.27 = 0$

$R_C = 5.775$

$-5.775 \times 3.46 - R_A(3.27) + 8.66 \times 11.27 = 0$

$-19.9815 - 3.27 R_A + 97.59 = 0$

$-3.27 R_A + 77.6167 = 0$

$3.27 (R_A) = 77.6167$

$R_A = \frac{77.6167}{3.27}$

$\boxed{R_A = 23.73 \text{ lbs}}$

Eq(1) → $R_B = R_A - 11.55$

$R_B = 23.73 - 11.55$

$\boxed{R_B = 12.18 \text{ lbs}}$

$\frac{\sin 30°}{BD} = \frac{\sin 60}{3}$

$BD = \frac{\sin 30 \times 3}{\sin 60}$

$BD = 1.732"$

$\frac{\sin 60}{3} = \frac{\sin 90}{BC}$

$BC = \frac{3}{\sin 60}$

$= 3/0.866$

$\boxed{BC = 3.46"}$

$AD = 5"$

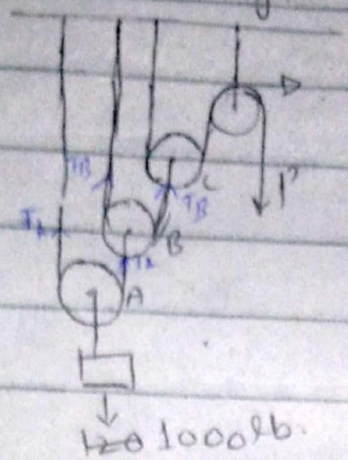
$AD = AB + BD$

$AB = AD - BD$

$= 5 - 1.732$

$= 3.27"$

Q5:- Determine the force required to hold the 1000 lb weight in equilibrium.



Sol:-

FBD at A:-

$\Sigma F_y = 0. \Rightarrow T_1 + T_1 - 1000 = 0 \Rightarrow 2T_A = 1000.$

$T_A = 500\ lb.$

FBD at B:-

$\Sigma F_y = 0.$

$T_B + T_B - T_A = 0 \Rightarrow 2T_B = 500\ lb$

$\boxed{T_B = 250\ lb}$

FBD at C:- $\Sigma F_y = 0.$

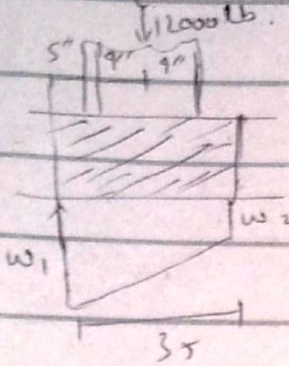
$T_c + T_c - T_B = \cancel{250}0.$

$2T_c = 250.$

$T_c = 125$

So P will be 125 lb. Ans.

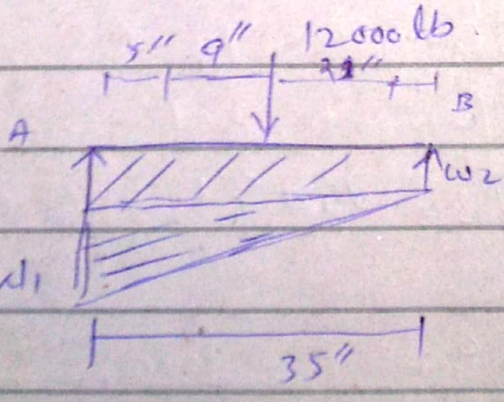
Q4:- The pad foot is used to support load of 12,000 lb. Determine the intensities W_1 & W_2 of distributed loading acting on the base of footing for equillibrium.



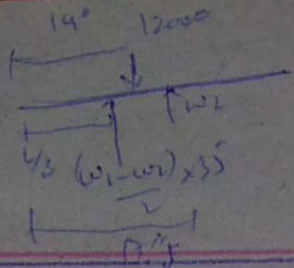
Sol:- The Intensities $W_2 \times 35$ will act at middle that is "35/2" as uniform distributed load. while $\left(\frac{W_1 - W_2}{2}\right) \times 35$ will act at

L/3 from left to right.

So the FBD of system is,



Apply Conditions of equillibrium.



$\Sigma F_y = 0$.

$$35w_2 + \frac{(w_1 - w_2)35}{2} - 12000 = 0$$

$$2\left(35w_2 + \frac{(w_1 - w_2)35}{2} - 12000\right) = 0 \times 2$$ ∴ Multiply 2 on b/s

$$70W_2 + 35w_1 - 35W_2 - 24000 = 0.$$

$$35W_1 + 35W_2 = 24000 \rightarrow (1).$$

↺+

$\Sigma M_{A} = 0$.

As $\frac{(W_1-W_2)\times 35}{2}$ will act 1/3 from right which $35/3 = 11.66$. So we take moment about that Point zero When $\frac{(W_1-W_2)35}{2}$ act.

$$-12000 \times (14 - (11.66)) + 35 \times w_2 \times (17.5 - 11.66) = 0.$$ In order to vanish $\frac{(W_1-W_2)\times 35}{2}$

$$\Rightarrow -12000 \times (14 - 11.66) + 35W_2 \times (17.5 - 11.66) = 0.$$

$$-12000(2.34) + 35W_2 \times (5.84) = 0.$$ a point where $\frac{(W_1-W_2)\times 35}{2}$ act.

$$-28080 + 204.4W_2 = 0$$

$$204.4W_2 = 28080$$

$$\boxed{W_2 = 137.37 \text{ lb}}$$

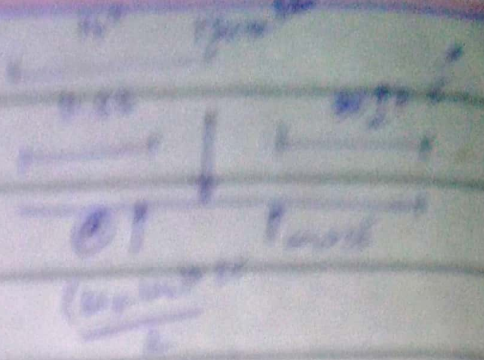
eq (1) $\Rightarrow$ $35W_1 + 35(137.37) = 24000.$

$$35W_1 = 24000 - 4808.219$$

$$35W_1 = 19191.78$$

$$\boxed{W = 548.3 \text{ lb}}$$ Ans

Checking



$\sum M_B = 0$

$= -12000 \times (14 - 11.66) + 40 \times (55) \times (17.5 (11.66)) = 0$

$= -12000(2.4) + 137.37(55)(5.9)$

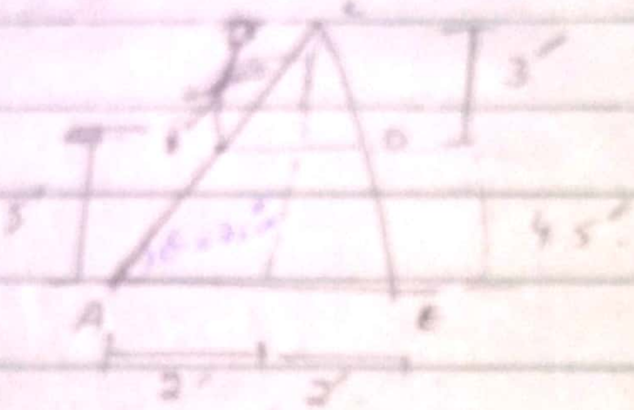
$= -2808 + 4507950(5.9)$

$= 2808 = 2808$

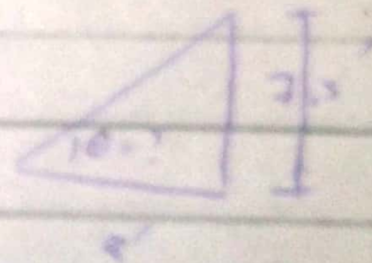
$\sum M_A = 0$

Verified

Compute the tension in the cable BD when the 165lb man stands 3ft off the ground, as shown. The weight of step ladder and friction may be neglected



Solution. If we seperate the whole system we will just get the reactions but we are required tension in cable BD so if we will draw FBD of any beam or side so have 4 unknown which cannot be possible so we will draw more FBD in order to solve them & find the tension in cable

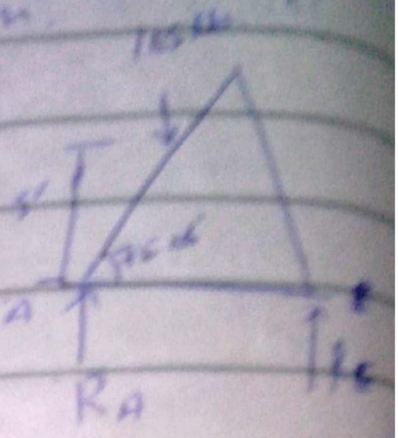


$\tan\theta = \frac{7.4}{2}$

$\theta = 75.06$

First draw the FBD of the whole system (Reactions two) → (Parallel force system)

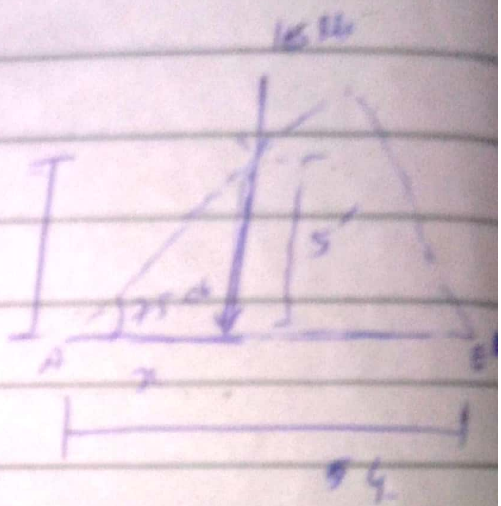
Apply Conditions of equilibrium



$\Sigma F_y = 0$

$R_A + R_E - 165 = 0$

$R_A + R_E = 165 \longrightarrow (1)$



$\tan 75.06 = \frac{5}{x}$

$x = \frac{5}{\tan 75.06}$

$x = 1.33 ft$

Now $\Sigma M_A = 0$

$-165 \times 1.33 + R_E \times 4 = 0$

$4R_E = 220.132$

$\boxed{R_E = 55 lb}$

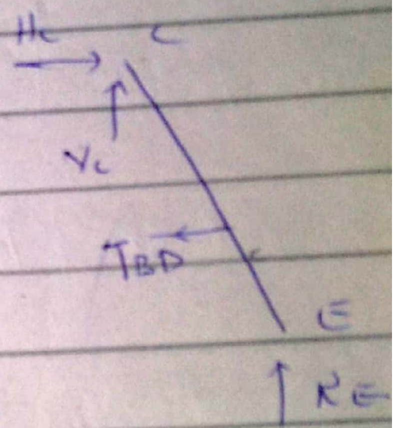
eq ① ⇒

$$R_A + R_E = 165$$

$$R_A = 165 - 55$$

$$\boxed{R_A = 109.9 \approx 110\, lb}$$

Now Draw FBD of rod / CE :-



As condition of non-parallel & non concurrent ..

So; $\Sigma F_x = 0$

$$H_c - T_{BD} = 0$$

$$H_c = T_{BD} \rightarrow ②$$

$\Sigma F_y = 0$ $\quad Y_c + R_E = 0$

$$Y_c + 55 = 0$$

$$\boxed{Y_c = -55\, lb}$$

It means our assume direct is wrong it is downword.

$+\circlearrowleft \Sigma M_c = 0$

$-T_{BD} \times 3 + R_E \times 2 = 0.$

$-3T_{BD} + 2(55) = 0.$

$-3T_{BD} = -110.$

$T_{BD} = \frac{+110}{3}$

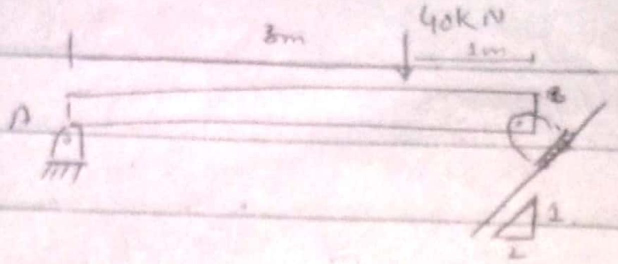
$\boxed{T_{BD} = 36.66\ lb}$

or $\boxed{T_{BD} = 36.67\ lb}$ Ans

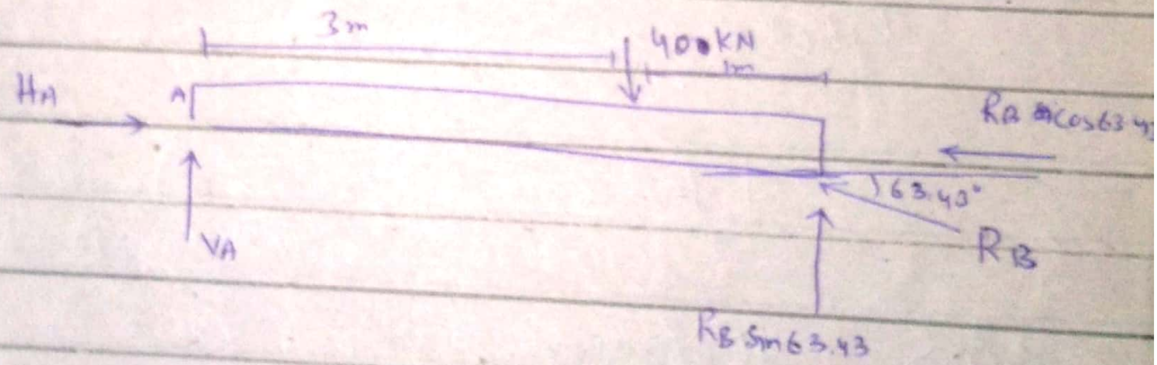
So the tension in cable is 36.67 lb.

Ex - 7.3

Q1. The beam shown in fig is supported by hinge at A and a roller on 1 to 2 slope at B. Determine resultant rxn. at A & B.

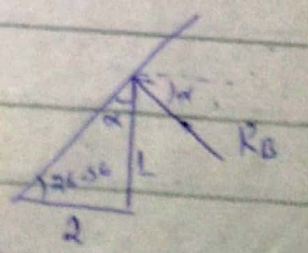


Solution:- Draw the FBD of the beam.



Now Apply Conditions of equilibrium;

As they are perpendicular to each other so their angle will also same



$\tan\theta = 1/2$

$\theta = 26.56$

$\alpha = 90 - 26.56 + 180$

$\alpha = 63.43°$

$\Sigma F_x = 0$

$H_A - R_B \cos 63.43° = 0$

$H_A = R_B \cos 63.43°$

$\boxed{H_A = 0.4472 R_B} \rightarrow (1)$

$\Sigma F_y = 0$

$$Y_A - 40 + R_B \sin 63.43^\circ = 0.$$

$$Y_A - 40 + 0.8943 R_B = 0 \longrightarrow ②.$$

$\Sigma M_A = 0.$

$$-40 \times 3 + (R_B \sin 63.43) \times 4 = 0.$$

$$-120 + (0.8943\ R_B)4 = 0.$$

$$(R_B)\ 3.5775 = 120,$$

$$\boxed{R_B = 33.54\ lb}$$

eq ① ⇒ $R_B(0.4472) = H_A$

$$H_A = 0.4472(33.54).$$

$$\boxed{H_A = 15\ lb}$$

eq ② ⇒ $V_A - 40\ \ 0.8943(33.54) = 0$

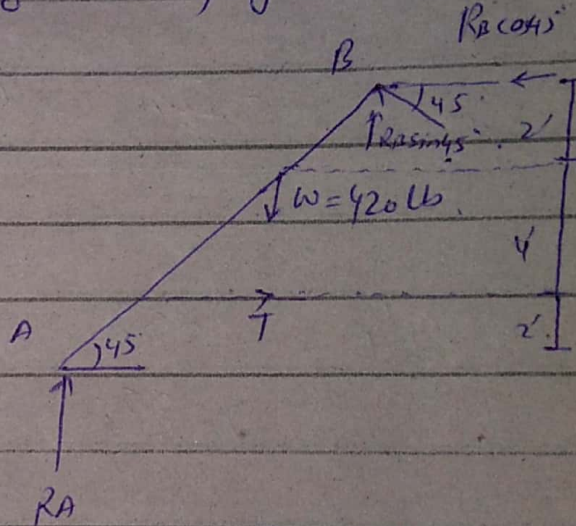
$$V_A - 40 + 29.99 = 0$$

$$Y_A - 10 = 0$$

$\boxed{V_A = 10\ lb}$ Ans.

Q2:- The uniform rod in fig weight 420 lb act at centre of gravity. Determine the tension in cable & reactions at smooth surface A & B?

Solution:- Draw the FBD of the fig/rod.



Apply conditions of equilibrium.

$\sum F_x = 0.$

$T - R_B \cos 45° = 0.$

$-R_B(0.7071) + T = 0.$

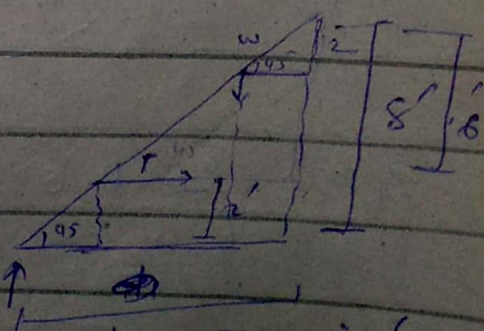
$T = R_B(0.7071) \rightarrow ①$

$\sum F_y = 0.$

$R_A + R_B \sin 45° - 420 = 0.$

$R_A + 0.7071 R_B = 420 \rightarrow ②$

$\sum M_B = 0.$



$R_A \times 8 + 420 \times 2 + T \times 6 = 0.$

$-8R_A + 840 + 6T = 0.$

$-8R_A + 6(R_B(0.7071)) + 840 = 0.$

$-8R_A + 4.2426 R_B + 840 = 0 \rightarrow ③$

$\tan 45° = \frac{6}{d_1}$

$d_1 = 6'$

$\tan 45° = \frac{8}{d_2}$

$d_2 = ...$

Scanned with CamScanner

Multipy 8 with eq ②.

eq ② ⇒ $8(R_A + 0.7071 R_B - 420) = 0$

$8R_A + 5.6568 R_B - 3360 = 0 \rightarrow ④$

Add eq ③ & ④.

$\cancel{8R_A} + 5.6568 R_B - 3360 = 0.$

$-\cancel{8R_A} + 4.2426 R_B + 840 = 0$

$9.899 R_B - 2520 = 0$

$R_B = \dfrac{2520}{9.899} \Rightarrow \boxed{R_B = 254.6 \, lb}$

eq ① ⇒

$T = 0.7071 R_B.$

$= 0.7071 (254.6)$

$T = 18 \, lb$

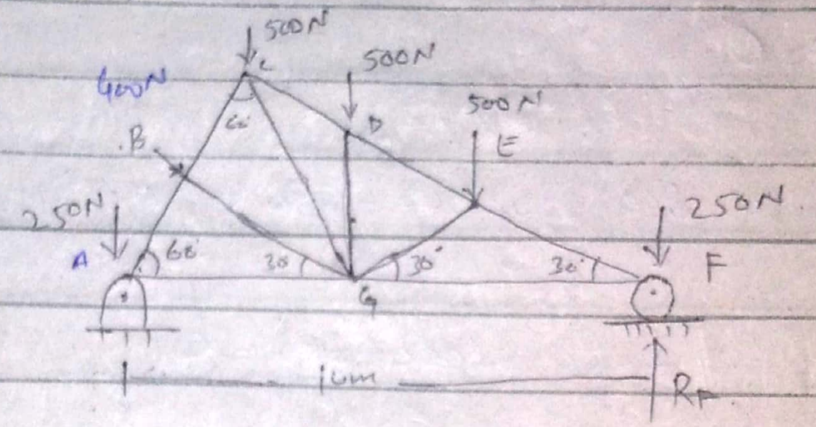
eq ② ⇒ $R_A + 0.7071(254.6) = 420.$

$R_A = 420 - 180.0$

$R_A = 240 \, lb$ Ans

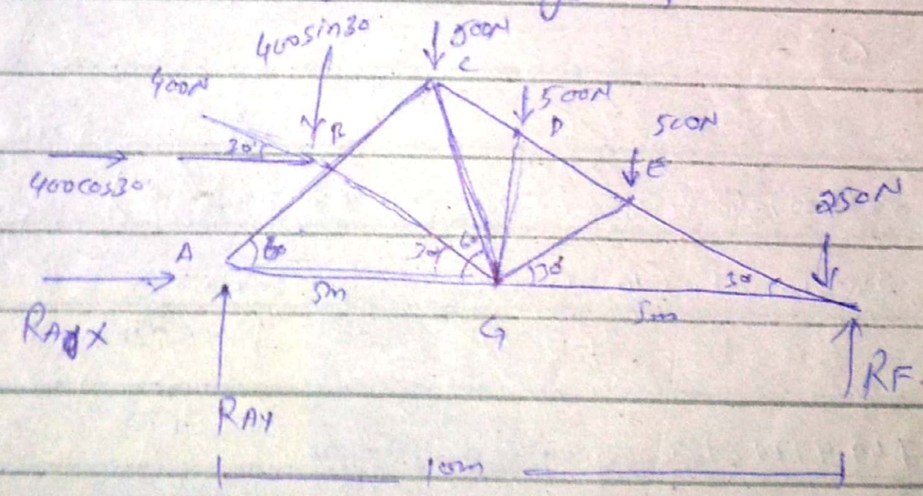
Exercise # 7.3

Q3:- Determine the ext reactions at A & F for the roof truss loaded as shown. The vertical loads represent the effect of supported roofing materials while 400N force represents a wind load.



Solution:-

Draw FBD of the system;



Apply Conditions of equilibrium.

$\sum F_x = 0$

$R_A^x + 400\cos 30° = 0$

$R_A^x + 400(0.866) = 0$

How we assume all things right

$\boxed{R_{Ay} = -346.4\,lb} \rightarrow ①$

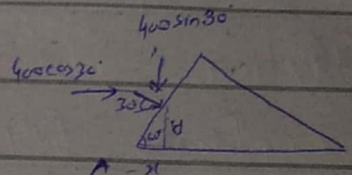
The negative sign show our assume direction is wrong. It is downword.

$\Sigma F_y = 0.$

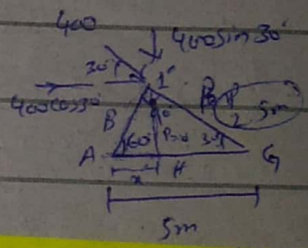
$-250 - 500 - 500 - 500 - 250 + R_{Ay} - 400\sin 30^\circ + R_F = 0.$

$R_{Ay} + R_F - 2000 - 200 = 0.$

$R_{Ay} + R_F = 2200 \rightarrow ②$



$\Sigma M_A = 0$



$-400\sin 30^\circ \times (1.25) - 400\cos 30^\circ \times (2.165) - 500(2.5) - 500(5) - 500(7.5)$

$- 250 \times 10 + R_F \times 10 = 0.$

$-250 - 749.9779 - 1250 - 2500 - 3750 - 2500 + 10R_F = 0.$

$10R_F - 10999.779 = 0.$

$10R_F = 10999.779.$

$R_F = 1099.97.$

$\boxed{R_F = 1100\,lb}$

$\cos 60^\circ = \frac{Base}{5}$

$Base = 5(0.5)$

$\boxed{Base = 2.5\,m}$

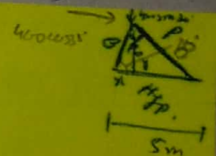
$\cos 60^\circ = \frac{x}{Base} \Rightarrow \frac{x}{2.5}.$

$x = 0.5(2.5) \Rightarrow \boxed{x = 1.25\,m}$

$\sin 60^\circ = y/Base \Rightarrow y/2.5.$

$y = 0.866(2.5)$

$\boxed{y = 2.165\,m}$



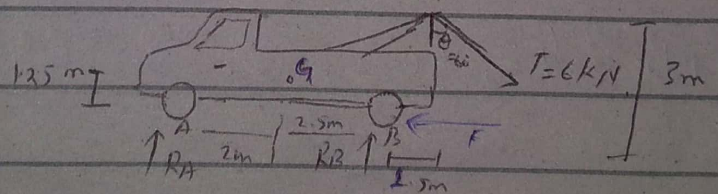
eq (2) ⇒ $R_{Ay} + R_F = 2200$.

$R_{Ay} + 1100 = 2200$.

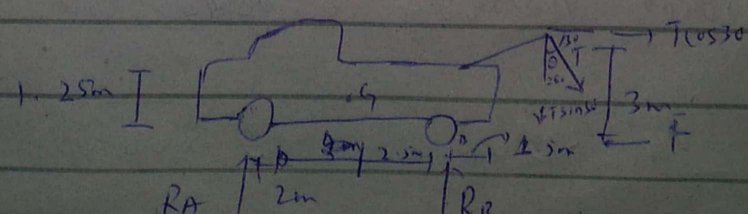
$R_{Ay} = 2200 - 1100$.

$\boxed{R_{Ay} = 1100 lb}$ Ans

Q4:- The winch cable on a tow truck is subjected to a force of T= 6KN when the cable is directed at θ = 60°. Determine the magnitude of total brake friction force F for the rear set of the wheels B and the total normal forces at A & B wheel. The total mass of truck is 4Mg act at Centre at G?



Solution:- Draw the FBD of truck.



Apply condition of equillibrium.

$\Sigma F_x = 0$

$-F + 7\cos 30^\circ = 0$

$F = 6(\cos 30^\circ) \Rightarrow F = \cancel{ces}\ 6(0.866)$

$F = 5.19\,kN$

or $\boxed{F = 5.2\,kN}$ This is friction force $\hookrightarrow$ ①

$\Sigma F_y = 0$

$R_A + R_B - 39240 - 6\sin 30^\circ = 0$

$m = 4Mg$

$R_A + R_B - 39240 - 3\,KN = 0$

$\omega = 4 \times 10^6 \cdot 10^{-3} kg \times 9.81$

$R_A + R_B - 39240 - 3000 = 0$

$\omega = 39240\,N$

$R_A + R_B = 42,240 \rightarrow$ ②

$\stackrel{+}{\curvearrowleft} \Sigma M_A = 0$

$R_B(4.5) - 6\cos 30^\circ(3) - 6\sin 30^\circ(6) - \overset{39240}{\cancel{4000}} \times 2 = 0$

$4.5(R_B) - 15.588 - 18 - \cancel{78480} = 0$

$4.5\,R_B = 78513.588$

$$R_B = \frac{78513.588}{4.5}$$

$$= 17447.464\ N$$

$$\boxed{R_B = 17.4\,kN}$$

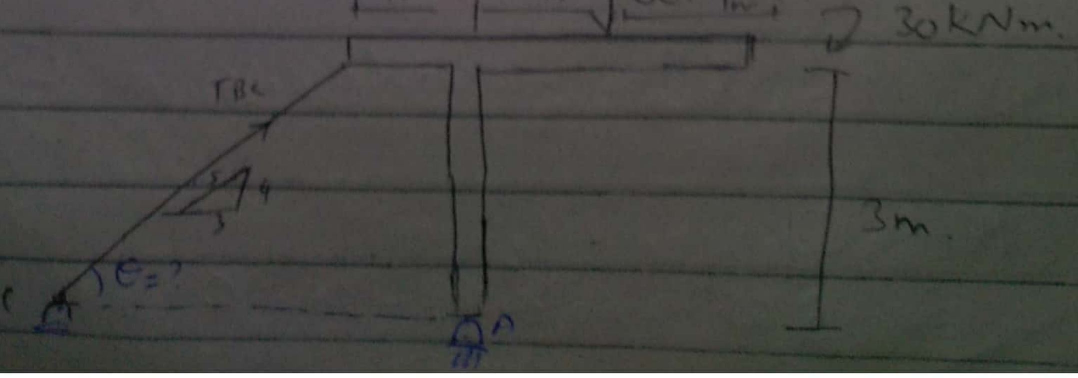
eq (1) ⇒ $R_A + R_B = 42240$

$$R_A = 42240 - 17447.464$$

$$R_A = 24792.536$$

$$\boxed{R_A = 24.8\,kN}$$ Ans

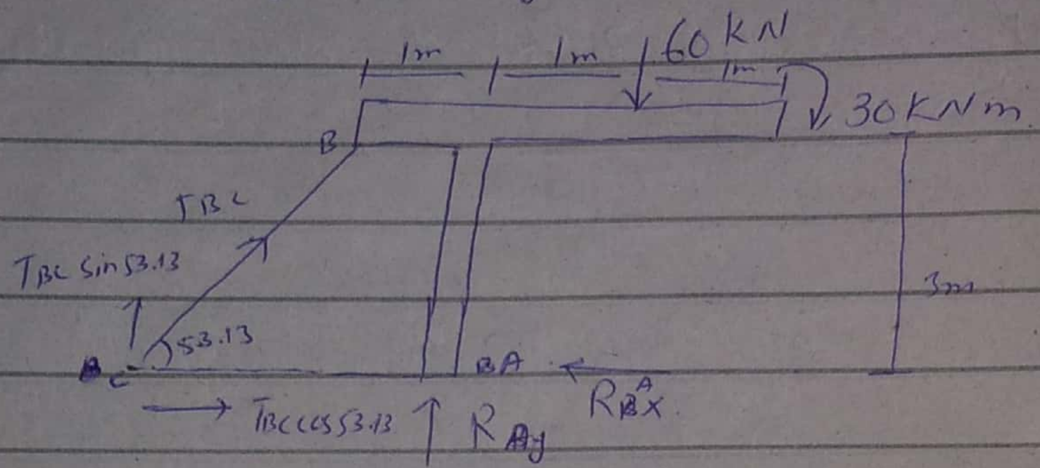
Q5:- Determine horizontal & vertical components of reactions at pin A and reaction develop at Cable BC used to support the steel frame.



Solution:-

$\sin\theta = 4/5 \Rightarrow \theta = 53.13°$.

Draw FBD of the system.

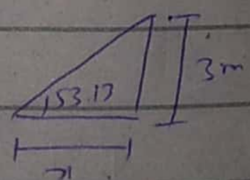


Apply condition of equilibrium.

$\Sigma F_x = 0$.

$T_{BC} \cos 53.13 - R_{Ax} = 0$.

$0.6\, T_{BC} = R_{Ax} \rightarrow (1)$



$\tan 53.13 = \frac{3}{x}$

$x = \frac{3}{1.33}$

$x = 2.25$

$\Sigma F_y = 0$

$T_{BC} \sin 53.13 + R_{Ay} - 60 = 0$.

$0.799\, T_{BC} - 60 + R_{Ay} = 0 \rightarrow (2)$

$x' = x + 1$

$= 3.25$

$\circlearrowleft +$

$\Sigma M_C = 0$.

$R_{Ay}(3.25) - 60(\overset{4}{3}.25) - 30 = 0$.

Scanned with CamScanner

$3.25(R_{Ay}) - 255 - 30 = 0$

$3.25\ R_{Ay} = 285$

$R_{Ay} = 87.69\ kN$

$\boxed{R_{Ay} = 87.7\ kN}$

eq ② ⇒ $0.799\ T_{BC} - 60 + 87.7 = 0$

$0.799\ T_{BC} - 27.7 = 0$

$0.799\ T_{BC} = \cancel{127.7}\ 27.7$

$\boxed{T_{BC} = 34.6\ kN}$

eq ① ⇒

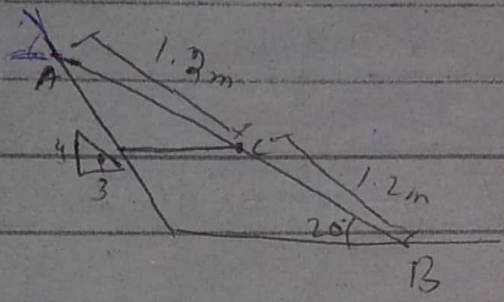
$T_{BC}(0.6) = R_{Ax}$

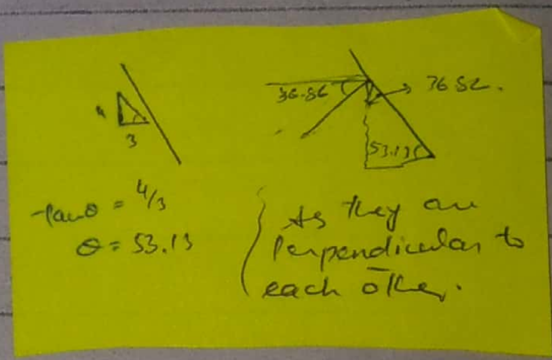
$R_{Ax} = 0.6(34.6)$

$R_{Ax} = 20.50\ kN$

$\boxed{R_{Ax} = 20.8\ kN}$

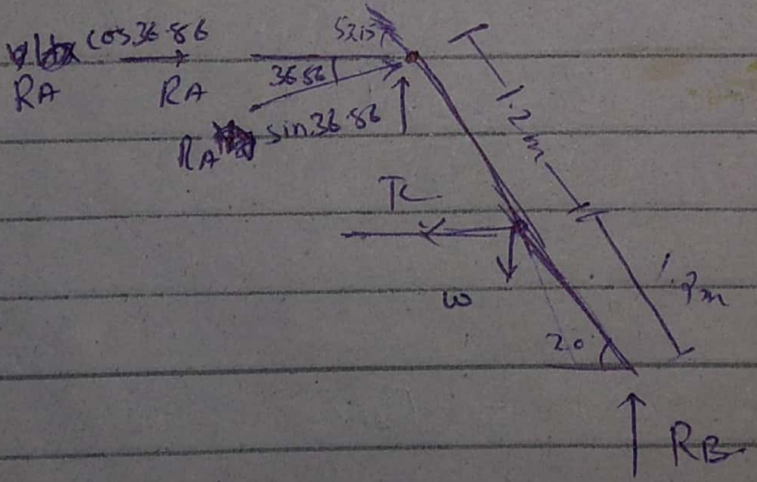
Q6:- Determine the mass of the heaviest uniform bar that can be supported in the position shown in fig if the breaking strength of the horizontal cable attached at C is 15 KN. Neglect friction.

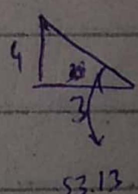




solution:-

Draw the FBD of the bar.





Apply Conditions of equilibrium.

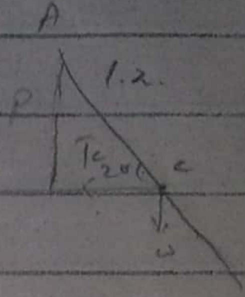
$\Sigma Fx = 0.$

$R_A \cos 36.86 - T_c = 0.$ $\quad \because T_c = 15\ KN$

$R_A (0.8) = 15$

$R_A = 18.75\,N \longrightarrow (1).$

$\Sigma F_y = 0.$



$R_A \sin 36.86 + R_B - w = 0.$

$\cos 20 = \frac{b}{1.2}$

$18.75(0.5998) + R_B - w = 0.$

$b = 1.127.$

$11.2474 + R_B - w = 0 \longrightarrow (2).$

$\sin 20 = \frac{P}{1.2}$

$P = 0.410$

$\circlearrowleft +$

$\Sigma M_A = 0.$

$-T_c \times (0.410424) - w(1.127) + R_B(2.25526) = 0.$

$-15 \times (0.410424) - 1.127w + 2.25526 R_B = 0$

$-6.156 - 1.127w + 2.25526 R_B = 0 \longrightarrow (3).$

Multipy 2.25526 to eq (2) & subtract from eq (3).

$-6.156 - 1.127w + 2.\cancel{25}526 R_B = 0$

$\underline{\pm 25.3658 \mp 2.25526w + \cancel{2.25526} R_B = 0}$

$-31.5218 + 1.12826w = 0$

$1.12826\, W = 31.5218$

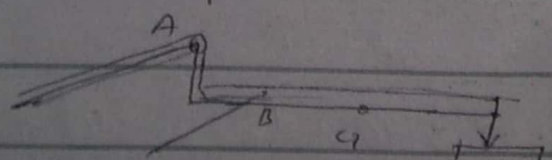
$W = 27.93$

$mg = \frac{27.93}{9.81}$

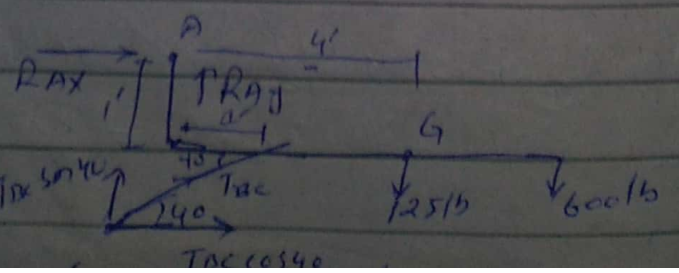
$m = 2.847$

$\boxed{m = 2847\ Kg}$

Q8:- The articulated crane boom has a weight of 125 lb and centre of gravity at G of it support a load of 600lb. Determine the force acting at the pin A and the force in hydraulic cylinder BC when the boom in the position shown.



Solution:- First draw FBD of the system.



Apply conditions of equillibrium.

$\Sigma F_x = 0.$

$R_{Ax} + T_{BC}\cos 40^\circ = 0 \rightarrow ①.$

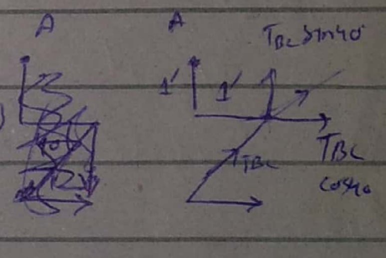
$\Sigma F_y = 0.$

$T_{BC}\sin 40^\circ + R_{Ay} - 125 - 600 = 0.$

$T_{BC}\sin 40^\circ + R_{Ay} - 725\,lb = 0 \rightarrow ②.$

$\overset{+}{\circlearrowright}\ \Sigma M_A = 0$

$T_{BC}\cos 40 \times 1 + T_{BC}\sin 40 - 125\times 4 - 600\times 9 = 0$



$0.7660\,T_B + 0.64278\,T_{BC} - 500 - 5400 = 0.$

$1.40878\,T_{BC} = 5900.$

$T_{BC} = 4188.02\,lb.$

$\boxed{T_{BC} = 4.19\,kip}$

eq ② ⇒

$$4188.02 \sin 40^\circ + R_{Ay} - 725 = 0.$$

$$2691.97 + R_{Ay} - 725 = 0.$$

$$R_{Ay} + 1966.97 = 0.$$

$$R_{Ay} = -1966.97 \text{ lb}.$$

or

$$\boxed{R_{Ay} = -1.97 \text{ kip}}$$

-ve sign shows our assume direction is wrong.

eq ① ⇒

$$R_{Ax} + T_{BC} \cos 40^\circ = 0$$

$$R_{Ax} + (4188.02)(0.7660) = 0.$$

$$R_{Ax} + 3208.02 = 0$$

$$R_{Ax} = -3208.02 \text{ lb}.$$

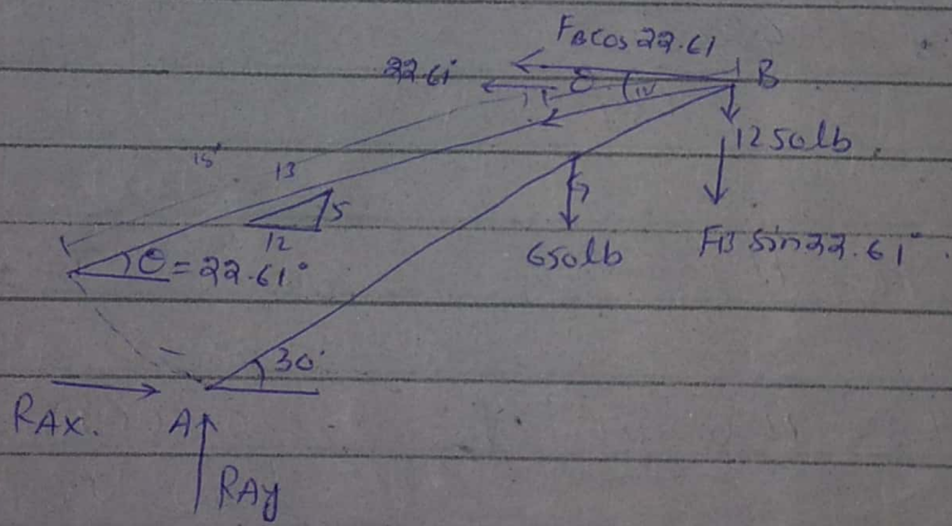
or

$$\boxed{R_{Ax} = -3.21 \text{ kip}}$$ Ans.

Negative sign show our assum direction is wrong

Q7:- Determine the horizontal and vertical components of reactions at A and tension in cable BC on the boom in given crane. Weight of boom (acting at G) is 650 lb & weight of crate suspended at B is 1250 lb

Sol:- Draw FBD of System (All crane)



Apply conditions of equillibrium

$\Sigma F_x = 0$

$R_{Ax} - F_B \cos 22.61 = 0$

$R_{Ax} - 0.9231 F_B = 0 \longrightarrow (1)$

$\Sigma F_y = 0$

$R_{Ay} - F_B \sin 22.61 - 650 - 1250$

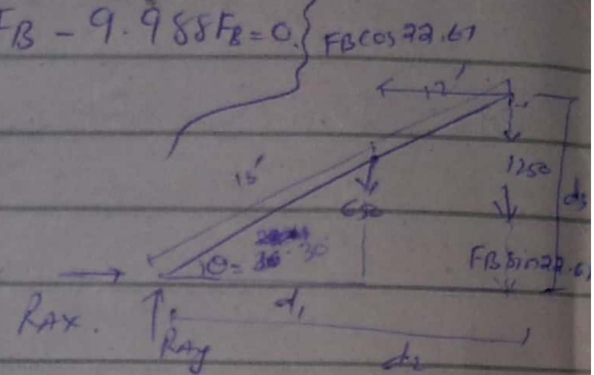
$R_{Ay} - 0.3844 F_B = 1900 \longrightarrow (2)$

$\circlearrowleft +$

$\Sigma M_A = 0$

$-650 \times 15.588 - 1250 \times 25.98 + F_B \cos 22.61 \times 15$
$- F_B \sin 22.61 \times 25.98 = 0$

$-10132.2 - 32475 + 13.85 F_B - 9.988 F_B = 0$



$(F_B)\, 3.861 = 42607.2$

$F_B = \dfrac{42607.2}{3.861}$

$F_B = 11035.27$

$\boxed{F_B = 11.0 \text{ kip}}$ Ans.

$\cos 30 = \dfrac{d_1}{18}$

$0.866(18) = d_1$

$\boxed{d_1 = 15.588'}$

eq (1) ⇒

$R_{Ax} - 0.9231(11) = 0$

$\boxed{R_{Ax} = 10.15 \text{ kip}}$

$\cos 30^\circ = \dfrac{d_2}{30}$

$\boxed{d_2 = 25.98'}$

eq (2) ⇒ $R_{Ay} - 0.3844(11) = 1900$

$R_{Ay} = 1900 + 4.2284$

$= 1.9 + 4.2284$

$\boxed{R_{Ay} = 6.12 \text{ kip}}$ Ans.

$\sin 30^\circ = \dfrac{d_3}{30}$

$\boxed{d_3 = 15}$

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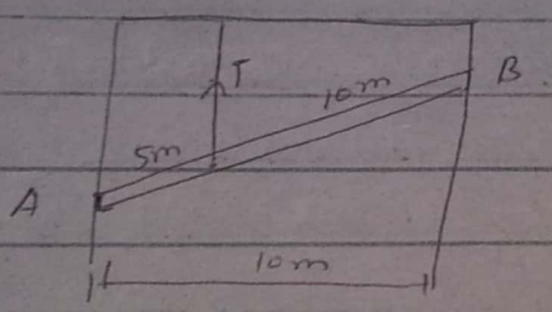
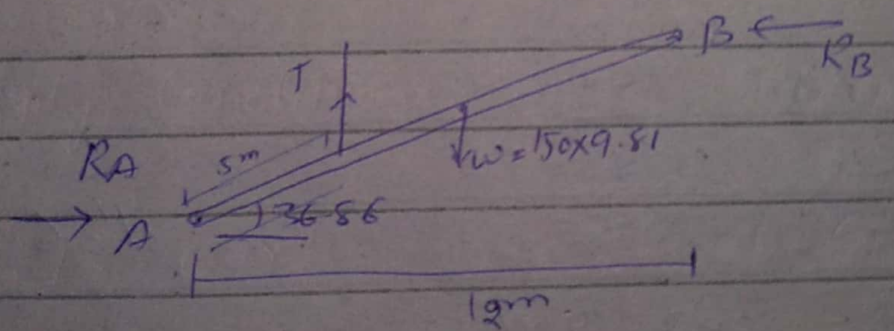
Q9:- The uniform 15m pole has a mass of 150 kg and is supported by its smooth ends against the vertical walls and by the tension T in vertical cable. Compute the reactions at A & B.



$w = 150 \times 9.81$

$= 1471.5N$

Solution:- Draw the FBD of the fig.



$\cos\theta = \frac{12}{15}$

$\theta = 36.86$

Apply conditions of equilibrium.

$\Sigma F_x = 0$

$R_A - R_B = 0 \rightarrow (1)$

&

$\Sigma F_y = 0$

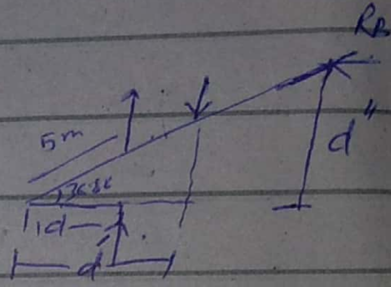
$T - W = 0$

$T - 150(9.81) = 0$

$\boxed{T = 1471.5N}$

$\overset{+}{\curvearrowleft} \Sigma M_A = 0$

$T \times d - W \times d' = 0$



$\cos 36.86 = \frac{b(d)}{5}$

$1471.5(4) - 1471.5(6) + R_B(8.996) = 0$

$d = 4$

$5886 - 8829 + 8.996 R_B = 0$

$\cos 36.86 = \frac{d'}{7.5}$

$8.996 R_B - 2943 = 0$

$R_B = \frac{2943}{8.996}$

$d' = 6$

$R_B = 327.14 \, N$

or $\boxed{R_B = 327N}$

~~$\cos 36.86 = \frac{d''}{15}$~~

$\tan 36.86 = \frac{d''}{12}$

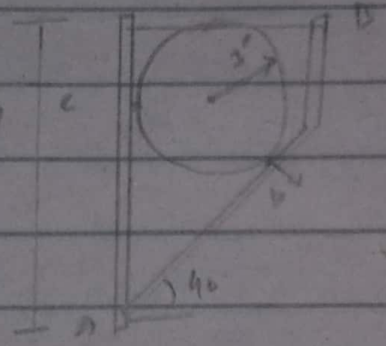
eq (1) $\Rightarrow$ $R_A - R_B = 0$

$d'' = 8.996$

$R_A = R_B$

$\boxed{R_A = 327N}$

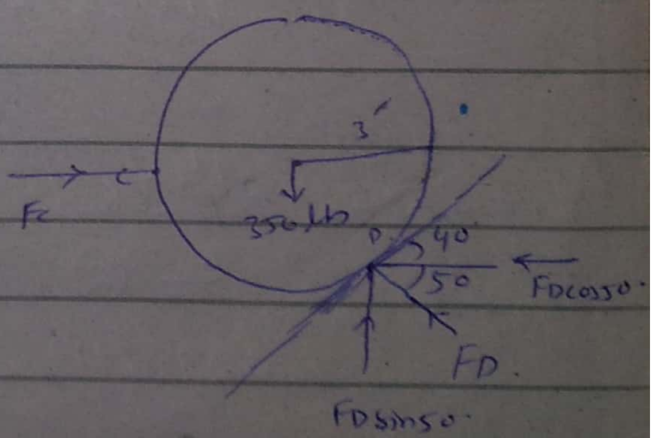
Q#8:- Determine the tension in the cable at B, given that uniform cylinder in figure weighs 350lb. Neglect friction and weight of bar AB.



Solution:-

First draw FBD of cylinder.

Apply conditions of equillibrium.



$\Sigma F_x = 0$

$F_C - F_D \cos 50° = 0 \rightarrow ①$

$\Sigma F_y = 0$

$F_D \sin 50° - 350 = 0$

$F_D (0.7660) = 350$

$\boxed{F_D = 456.89 \text{ lb}}$

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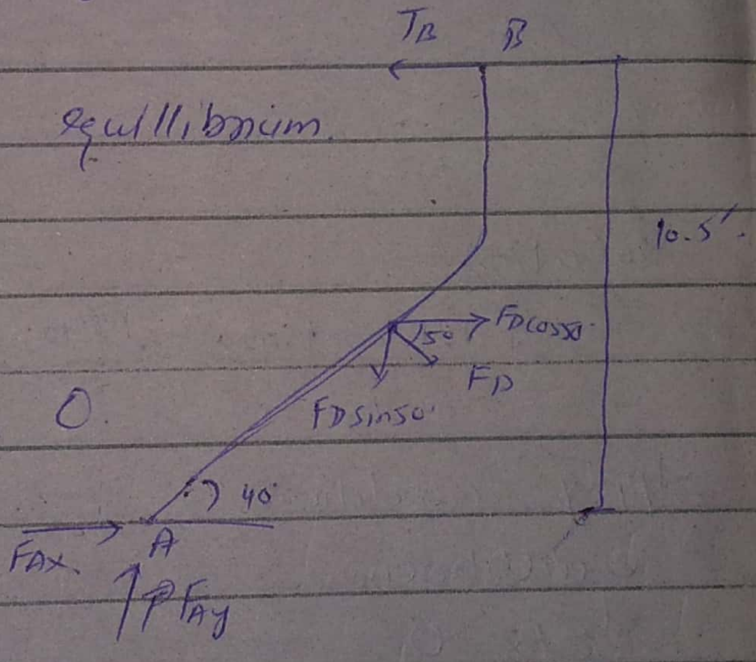
eq ① ⇒ $F_C - F_D \cos 50° = 0$

$F_C - 456.59(0.6427) = 0$

$\boxed{F_C = 293.68\ lb}$

Now draw FBD of bar AB.



Apply condition of equillibrium.

$\Sigma F_x = 0$

$F_{Ax} - T_B + F_D \cos 50° = 0$

$F_{Ax} - T_B = -293.68$ ↳ ②

$\Sigma F_y = 0$

$F_{Ay} - F_D \sin 50° = 0$

$\boxed{F_{Ay} = 349.99\ lb}$