Module # 09

Equillibrium Analysis of Statically determine Trusses.

Trusses:-

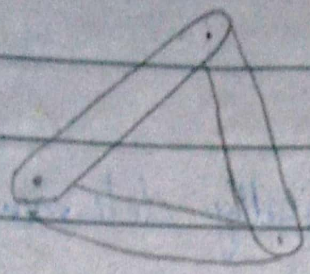
When member are connected through pin then the body is said to be trussed body.

All members of truss body (Two members) are subjected to two forces. These forces are equal in magnitude, collinear and opposite in direction in order to satisfy condition of equillibrium.

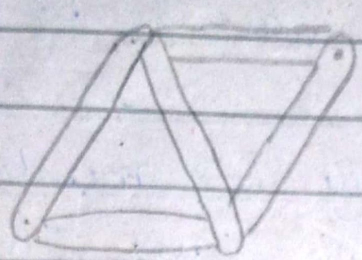
OR A type of structure formed by members in triangular form, the resulting fig is called truss.

↳ Each member of truss is two force member. Either they will subjected to tension or compression.

For Example:- The three members connected through pin to form a triangle with support as shown in fig.



More Complex Structure:-

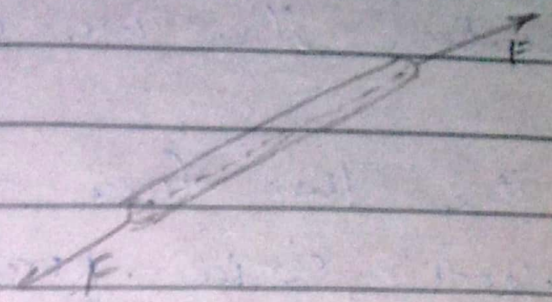


∵ Truss are not design to subject the binding Here when force is applied at joint so Just compression and tension occur.

→ The bars from which the structure is made is called member. & the point where they meet is called joint.

(*) Even these structure are simple but they used / resemble the structure to support bridge and roof of houses.

↳ When forces or load is applied at joint we neglet the weight of bars. Each bar is two force member as shown in fig.



Methods to find Internal forces :-

There are two methods to find the Int. forces.

① Method of Joint :-

⇒ In this method first we find the reactions at support from equilibrium equation. than find the int forces in members that compression and tension by this method. by ~~jo~~ taking joint.

2) Method of section :-

In this method we consider a a section and find the int. force by equillibrium condition.

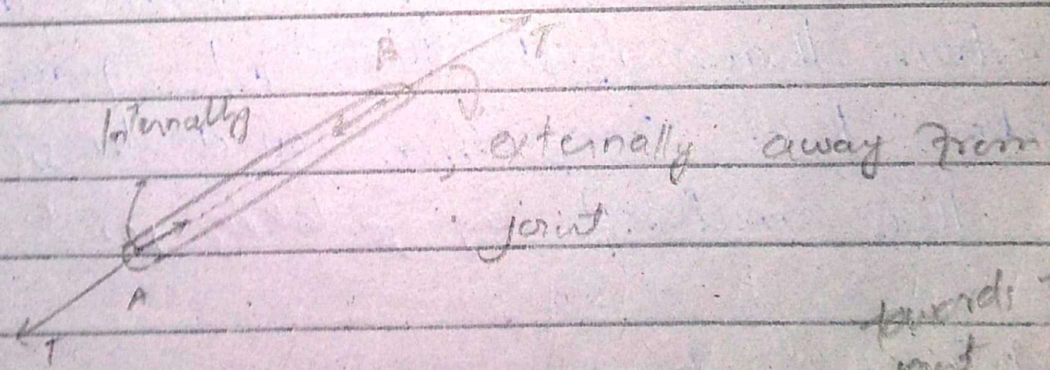
↳ Method of joint is preferable if the Int. forces are to be find in complete truss while method of section is preferable if it is find in specific member then we used method of section.

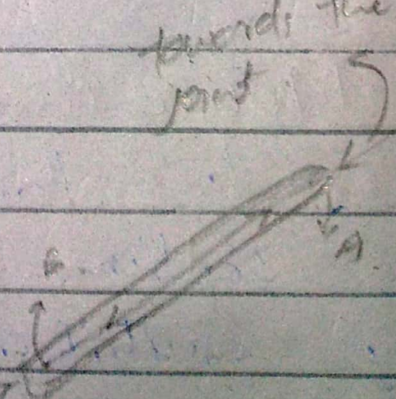
↳ The Two force member of truss is called axial force.

→ Tension:-

When the two member forces are away from each other, the member is in tension.

↳ OR when the forces are away from joint, the bar is in tension.





Compression:-

When the forces at towords each other, the bar is said to be in compression.

OR When the forces are towords the joint the bar is in compression.

Even when the member (bars) are connected by nut and bolts and may be weld, although there such structure also subject to bending, but when force is applied at joint, then the bar will not subject to binding, now the bar will just subject to tension and compression. So such structure can also act as truss.

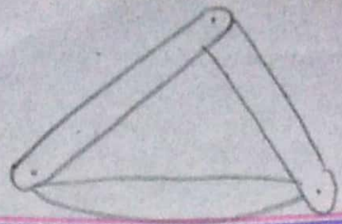
Statically Indeterminate Problem:-

When the number of unknown reactions are more than, the number of equation, then such problems are said to be statically indeterminate problems.

In case of trusses, there are two types of Indeterminacy, on basis of, internal and external forces.

Static Indeterminacy In case of truss:-

Consider the figure;



Here we have 3 joints, three unknown reaction and three bars.

We have; $U = b + r \rightarrow (1)$.

Where U is no. of joint, b is no. of bars and r is no. of unknown reaction.

↳ If $b + r > 2U$.
Then the problem will be statically indeterminate of that degree, and said to be reduntant. ~~member~~, there

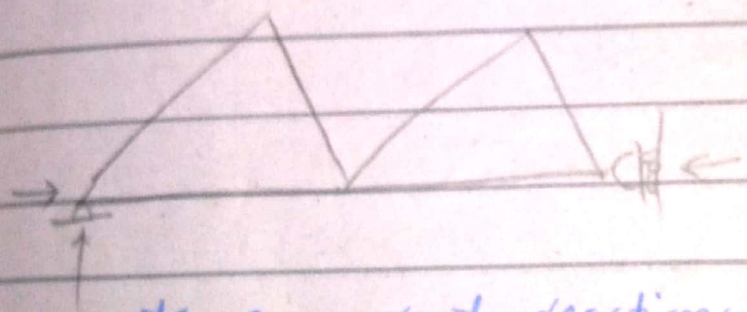
↳ If $b + r = 2U$.
Statically determinate and we can solve such problem.

If $b + r > 2U$ than it means there is reduntant member.
If $b + r < 2U$ It mean the structure is not possible So impossible constraint.

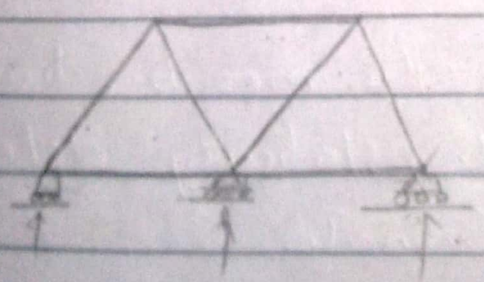
External & Internal Stability :-

A body is externally ~~indeterminate~~ unstable if either the reactions are parallel to each other or concurrent.

Consider the fig in which the reactions are ~~parallel~~ concurrent. So indeterminate and unstable.



As concurrent reactions so unstable externally.



Also here the reactions (all) are parallel.

Here if we ~~rot the s~~ translate the body horizontally, the body will become unstable and the body ~~lo~~ unstable externally.

Internally Instability -

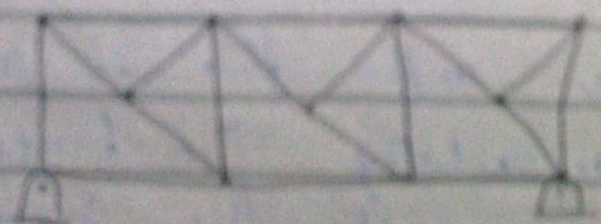
But when the number of member are (support) less than the number of eq. eqn. the body is said to instable internally.

e.g. means if a body is impossible constraint, the body will unstable internally

EXCERCISE 10.1

Determine each of the ~~member~~ trusses as stable, unstable, statically determinate or statically indeterminate.

(a)



Solution:- The structure is stable as Pin support are there and reactions are not parallel to each other.

Now to find whether determinate or Indeterminate;

Number of reactions = $r = 3$.

Number of members = bar = $b = 19$.

Now number of Joints:- $j = 11$.

So, $b + r \geq J$.

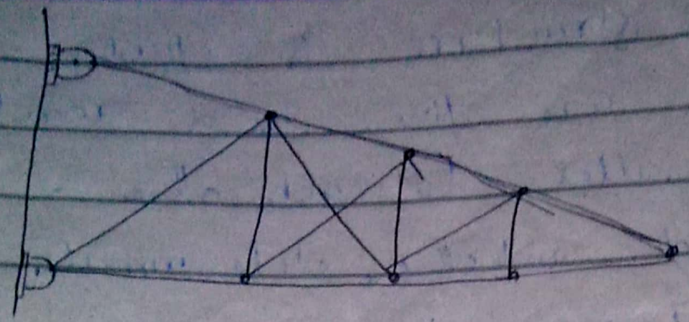
$3 + 19 = 2(11)$.

$22 = 22$.

As equal so this problem is statically determinate.

As $b + r = 2j$

(b)



Ans:- The structure is stable as reaction are not ~~parallel~~ concurrent.

Checking determinacy & Indeterminacy:-

Number of rxns = r = 4.

Number of bars = b = 15.

Number of Joints = j = 9.

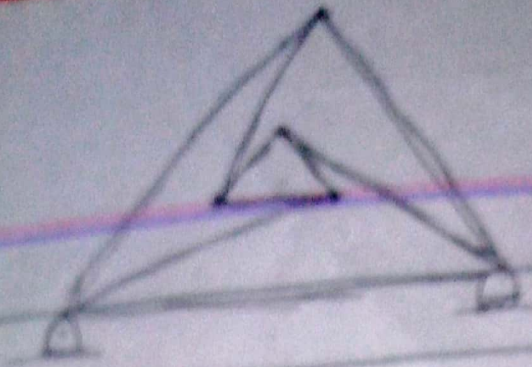
so; $b + r \geq j$.

$4 + 15 > 2(9)$

$19 > 18$.

So 1 degree Indeterminate and
one member redundant here.

(i)



Here reactions are concurrent

Solution:-

Stable or Unstable:-

The for given structure is unstable because if we give translation from right to left it will move and become unstable.

Statically determinate or Indeterminate:-

No. of bars = $b = 9$

No. of reactions = $r = 3$

No. of joints = $j = 6$

Since we have;

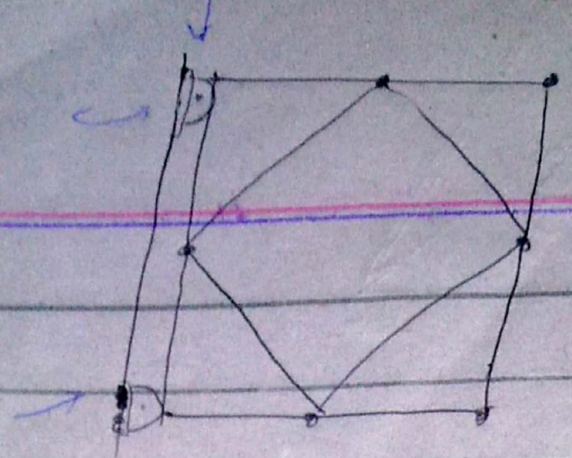
$b + r = 2j$ for determinate.

$9 + 3 = 2(6)$

$12 = 12$

The figure is statically determinate.

(d)



Sol:- The fig is unstable as the reactions are not ~~parallel~~. concurrent.

Statically determinate or Indeterminate :-

Number of reactions = $r = 3$.

Number of bars = 12.

Number of Joints = $J = 8$.

For Indeterminacy :-

$$b + r < 2(8).$$

$$3 + 12 < 16.$$

$$15 < 16.$$

Since the fig is of Impossible constraint. So such structure is unstable also from Int. forces. So this structure is neither determinate nor indeterminate.

Method of Joints for Analysis of statically determinate trusses.

Step#01

Draw FBD of Joints of truss one by one. Those Join should select having two unknown.

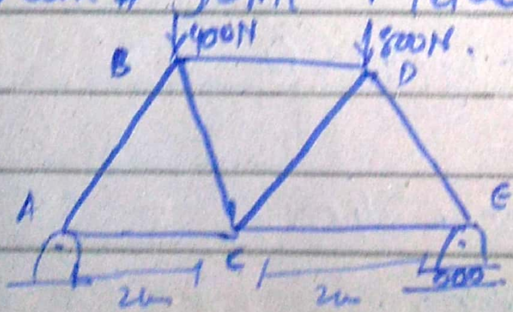
Step#02

Then use equillibrium equations to find the axial forces.

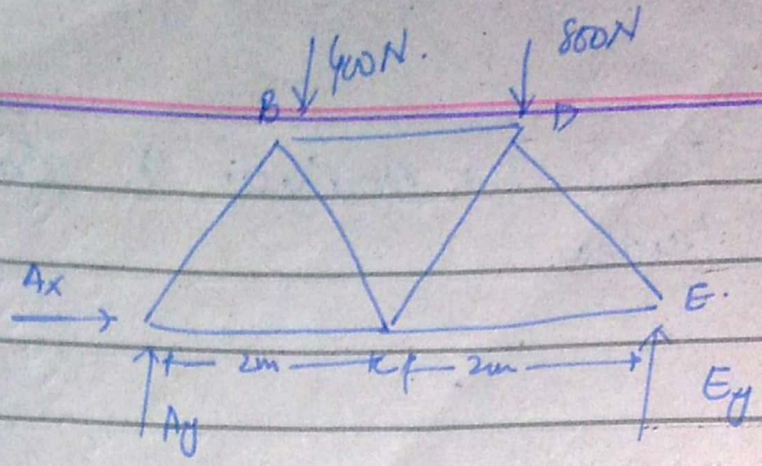
↳ Steps#.

First you should seperate all the points and show rxn forces. body from
It means first find reaction force.

Problem# Joint Method:-



Sol:- Draw FBD of the given fig.



Now apply condition of equilibrium to find reaction forces.

$\Sigma F_x = 0$.

$A_x = 0 \rightarrow (1)$

$\Sigma F_y = 0$.

$A_y + E_y - 400 - 800 = 0$.

$A_y + E_y = 1200 \rightarrow (2)$

$\circlearrowleft +$

$\Sigma M_A = 0$

$-400 \times 1 - 800 \times 3 + E_y(4) = 0$.

$(4) E_y = 400 + 2400$.

$E_y = \frac{2800}{4}$.

$\boxed{E_y = 700 N} \rightarrow (3)$

eq (3)

Put in eq ②.

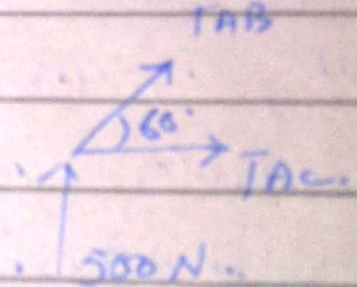
eq ② ⟹ $A_y + E_y = 1200$

$$A_y = 1200 - E_y$$
$$= 1200 - 700.$$

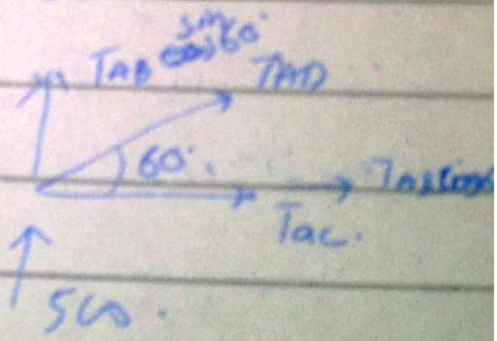
$$\boxed{A_y = 500N.}$$

Now by method of joints;

Select joint A :-
to find T_{AB} & T_{AC}.



Now apply condition of equillibrium at joint A.



$\Sigma F_x = 0.$

$T_{AC} + T_{AB} \cos 60° = 0. \rightarrow$ ④

$\Sigma F_y = 0.$

$0.866\, T_{AB} = -500.$

$$T_{AB} = -577N$$

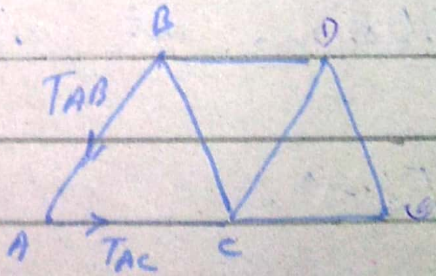
eq (4) ⇒

$$T_{AC} + T_{AB}\cos 60° = 0.$$

$$T_{AC} - 577(0.5) = 0.$$

$$T_{AC} = 289N$$

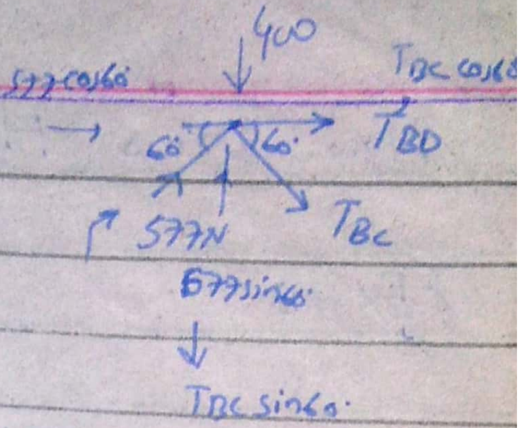
It mean the tension that we consider in AB are wrong there is compression. While in AC member our assumsion is right there is tension.



Now Consider joint B:-

As There is Two Unknown In contrast at joint C, where at C, There are 3 unknown.

Joint B is;



Apply conditions of equillibrium.

$\Sigma F_x = 0.$

$T_{BD} + T_{BC}\cos60^\circ + 577\cos60^\circ = 0.$

$T_{BD} + 0.5\,T_{BC} + 288.5 = 0. \rightarrow (6).$

$\Sigma F_y = 0.$

$577\sin60^\circ - T_{BC}\sin60^\circ - 400 = 0.$

$-T_{BC}(0.866) = -99.682.$

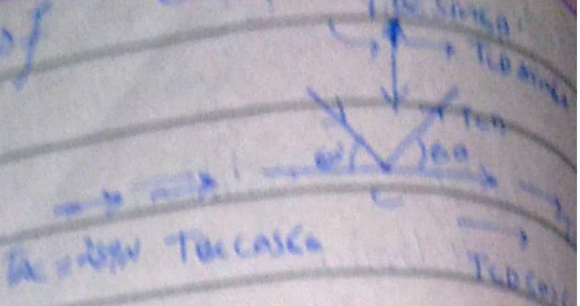
$\boxed{\therefore T_{BC} = 115\,N}$

eq (6) $\Rightarrow$ $T_{BD} + 0.5(115) + 288.5 = 0.$

$\boxed{T_{BD} = -346\,N}$

It mean the assume direction is wrong in TBD.

Now Draw FBD of C.



$\Sigma F_x = 0$

$$289 + T_{BC}\cos 60^\circ + T_{CD}\cos 60^\circ + T_{CE} = 0.$$

$$289 + 115(0.5) + T_{CD}(0.5) + T_{CE} = 0.$$

$$346.5 + 0.5\,T_{CD} + T_{CE} = 0 \longrightarrow (2)$$

$\Sigma F_y = 0$

$$T_{CD}\sin 60^\circ - T_{BC}\sin 60^\circ = 0.$$

$$T_{CD}(0.866) = 115(0.866).$$

$$T_{CD} = \frac{\cancel{57.5}\ 115N}{\cancel{0.866}}$$

$$\boxed{T_{CD} = \cancel{66.39N}\ 115N}$$

$\in (2) \Rightarrow$ $346.5 + 0.5(\cancel{66.39}\ 115) + T_{CE} = 0.$

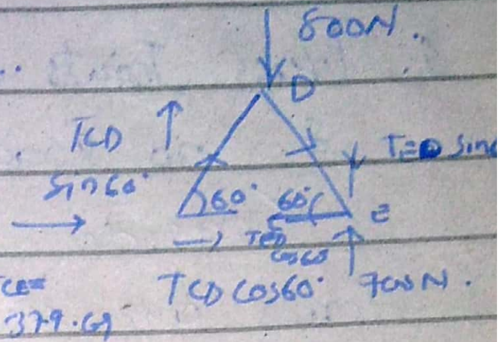
$T_{CE} + \cancel{379.69}\ 404 = 0.$

$$\boxed{T_{CE} = -\cancel{379.69}\ 404.5N}$$

opposite of assume direction

Now Draw FBD of D (Joint).

Apply condition of equillibrium.



$\Sigma F_x = 0$

$T_{CD}\cos 60° - T_{ED}\cos 60° - 379.69 = 0$

$66.39(0.5) - (0.5)T_{ED} - 379.69 = 0$

$33.195 - 379.69 - 0.5\,T_{ED} = 0$

$-0.5\,T_{ED} = 346.495$

$\boxed{T_{ED} = -692.99\,N}$

-ve sign our assume direction is wrong.

$\Sigma F_y = 0$

$-800 + 700 + T_{CD}\sin 60° - T_{ED}\sin 60 = 0$

$-100 + (66.39)(0.866) - T_{ED}(0.866) = 0$

$-100 + 57.49 - T_{ED}(0.866) = 0$

$-42.50 - T_{ED}(0.866) = 0$

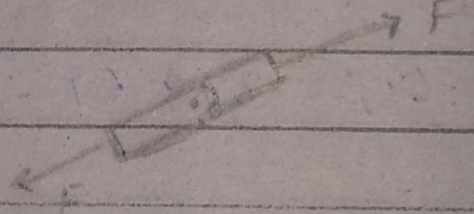
$T_{ED} = -\dfrac{42.5}{0.866} \Rightarrow T_{ED} = -49N$

-ve sign show our assume direction is wrong.

Types of Joint:-

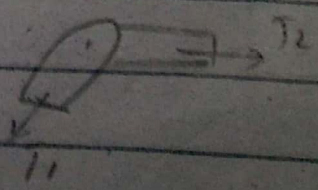
1. Truss Joints with 2 Collinear members & no load:-

Consider the joint in which two forces are acting, either they will cause tension or compression. these force will be collinear equal in magnitude but opposite in direction.



2. Truss Joint with 2 non collinear member & no load:-

If there is no applied load on joint as shown,
Then the force in each member will be zero.

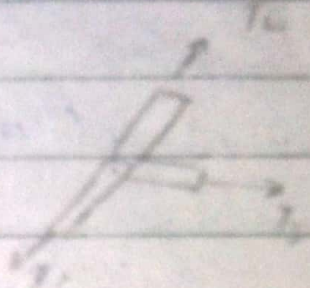


$T_2 = 0$
& $T_1 = 0$

$\Big\{ \begin{array}{l} \cancel{\Sigma F_x = 0} \\ T_2 = 0 \\ \Sigma F_y = 0 \end{array}$

3. Truss Joints with 3 members, 2 of which are collinear & no load:-

Consider the truss, consisting of 3 members as shown in fig.



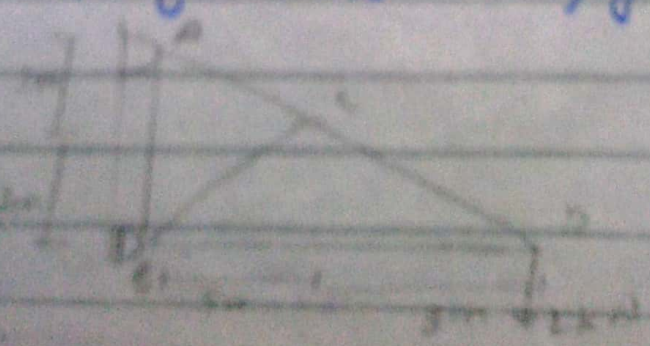
Consider there is no load acting. Then the members that are collinear having force (either tension or compression), have equal magnitude but opposite in direction. But in third member, there will be no force.

$$T_1 = T_2.$$

$$\& \ T_3 = 0.$$

Problem # 10.1

Determine the axial forces in the members of truss in fig. 1.



Solution:- First step is to find whether the given problem is determinate or Indeterminate so;

Number of reactions = r = 3.

Number of bars = b = 5.

Number of joints = J = 4.

$b + r = j$

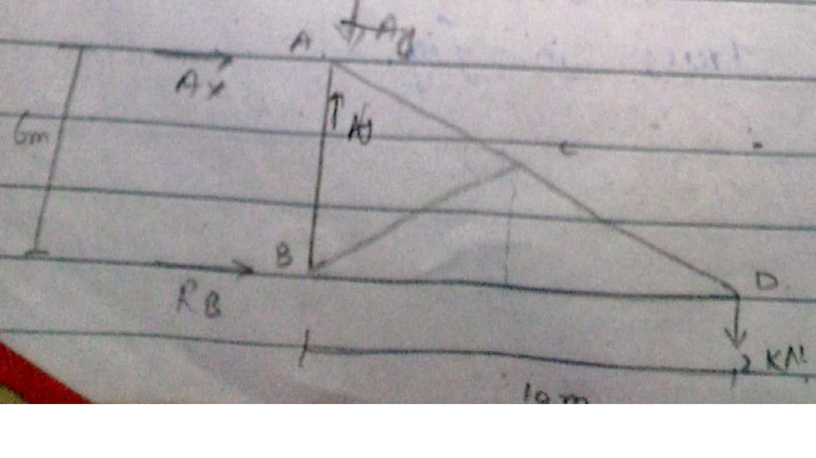
$3 + 5 = 2(4)$

$8 = 8$

Since determinate so we can find all the unknown.

Now step #02

Draw FBD of the whole truss bg to find unknown reactions.

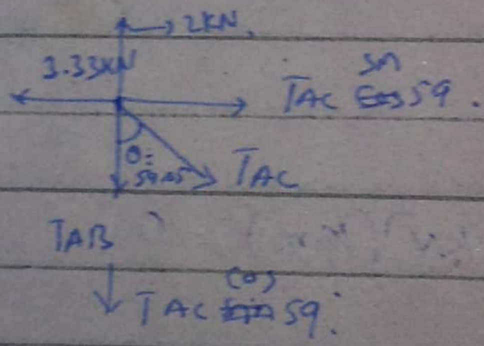


Step # 03 :-

Selecting Special joint :-

Select C as special joint as by selecting this we are able to find other unknown internal forces easily. The force in BC will be zero while T_{AC} & T_{CD} will be equal magnitude as collinear.

So draw FBD at joint A.



Now

Apply conditions of equillibrium.

$\Sigma F_x = 0.$

$T_{AC} \sin 59^\circ - 3.33 = 0.$

$T_{AC} = \dfrac{3.33}{\sin 59^\circ}$

$\boxed{T_{AC} = 3.89 \text{ kN}}$

$\Sigma F_y = 0.$

$2kN - T_{AC}\cos 59° - T_{AB} = 0.$

$2 - 3.89(0.515) - T_{AB} = 0.$

$-T_{AB} + 2 - 2 = 0.$

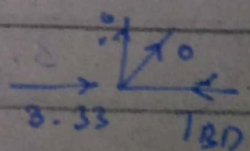
$\boxed{T_{AB} = 0}$

The force in T_{AC} & T_{CD} is tension of magnitude of 3.89 kN, while in $T_{AB} = 0$.

Now Draw FBD of point D to find force in BD.

So,

$\Sigma F_x = 0.$



$T_{BD} - 3.33 = 0.$

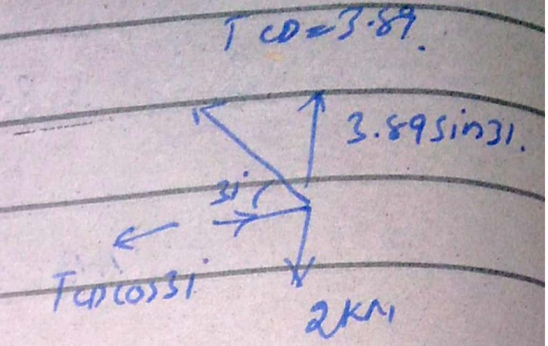
$\boxed{T_{BD} = 3.33KN}$

Thus the force in T_{BD} is also tension.

Verification:-

Take point D.

$\Sigma F_y = 0.$



$3.89 \sin 31° - 2 = 0.$

$\cancel{2} - \cancel{2} = 0 \Rightarrow 0 = 0.$

Result verified.

$\Sigma F_x = 0.$

$-3.89 \cos 31° + 3.33 = 0.$

$-3.\cancel{3}3 + 3\cancel{.}33 = 0.$

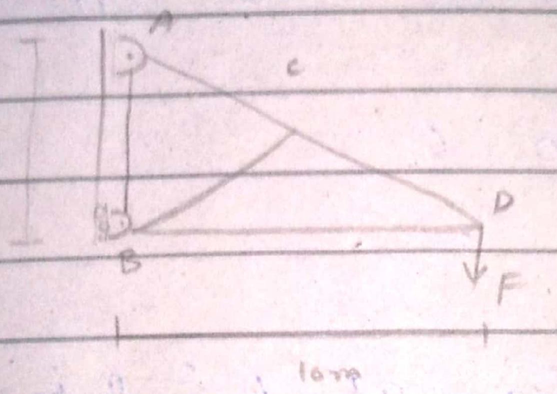
$0 = 0$

Hence result verified.

Problem 10.2

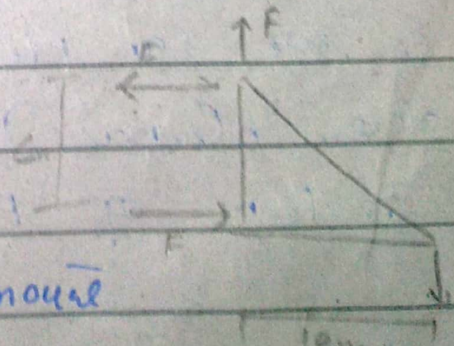
Determining the largest force a truss will support.

Each member of the fig will safely support a tensile force of 10kN & compressive force of 2kN. What is the largest downword load F that truss will safely support?



Solution:-

Since the force at D is F so another force at A having same magnitude will act upword, as shown in fig, to create clock wise torque.



Since the truss in equillibrium so same amount of torque will also produce due horizontal forces at A & B anticlockwise in order to keep the body in equillibrium.

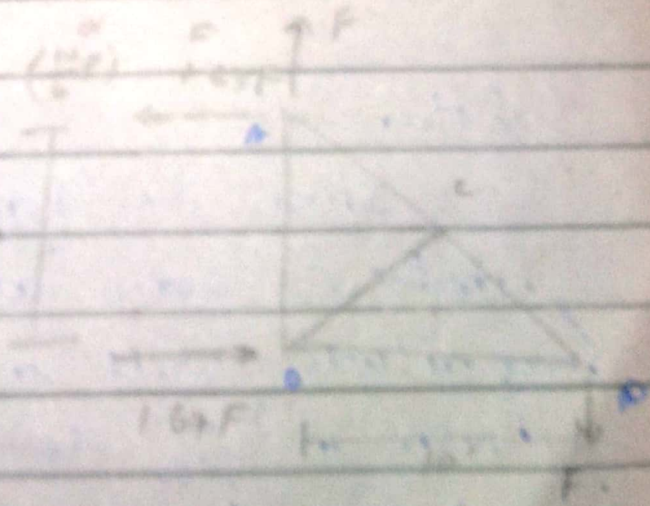
So; As the torque produce clockwise is;

$\tau = + F \times 10 = 10F$

So anticlockwise torque will be;

$$\tau = -\frac{F \times 10}{6} \times 6$$

So the reactions at A & B is.



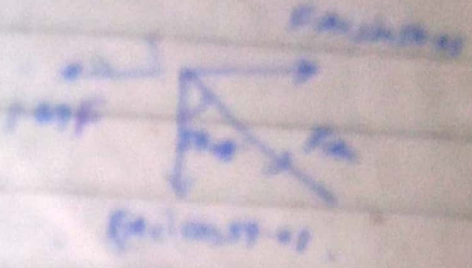
Now by same method as in problem 10.1 so we can find other Internal forces in term of F.

Take C as special joint.

At CB there is no force while

Force in $F_{AC} = F_{BD}$

So draw FBD of Joint A



Apply conditions of equilibrium.

$\sum F_x = 0$

$F_{AC}\sin 59.03° - 1.67F = 0 \Rightarrow 0.8574\,F_{AC} = 1.67F$

$F_{AC} = \dfrac{1.67F}{0.8574} \Rightarrow \boxed{F_{AC} = 1.947F}$

So at bar AC we have tension of magnitude $1.947F$.

Similarly the force in BCD is also tension of magnitude $1.947F$.

So we write:

$\sum F_y = 0$

$\boxed{F_{AD} = 0}$

Similarly $F_{CD} = 0$

& $F_{BD} = 1.67\,F\ (C)$.

$F_{AC} = 1.94F\ (T)$.

$F_{CD} = 1.94F\ (T)$.

As given that each member can take tension of 10kN & Compression of of 2KN.

As tension is in AC & CD. So; of magnitude 1.94F.

$1.94F = 10$.

$F = \dfrac{10}{1.94} \Rightarrow \boxed{F = 5.15\text{kN}}$

When F = 5.15 KN then there will 10KN Tension.

Now for compression.

$F(1.67) = 2$

$$F = \frac{2}{1.67} kN = 1.197 kN.$$

$$\boxed{F = 1.197 kN.}$$

So the safer load is 1.2kN but with this value we can limit the tension & compression.

∴ Max safer load in truss.

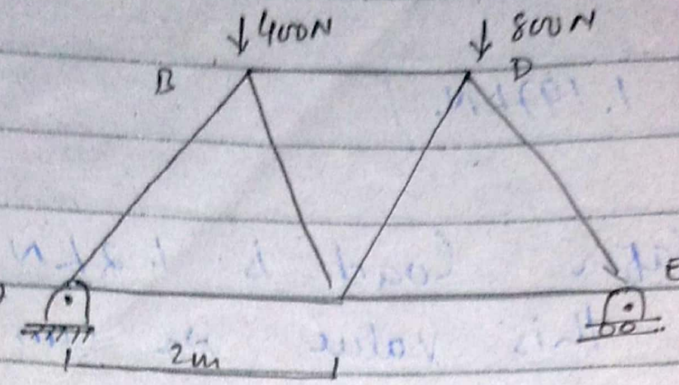
Here compression is governing the truss as we cannot add a value more than 2kN.

Method of section:-

In the entire truss if we are required to find the axial force at any member. Then it is easy to used method of section as compared to method of joint.

Problem 10.3:- Method of section.

Determine the axial force in member BC of Warren truss shown in fig. Length of each member is 2m.

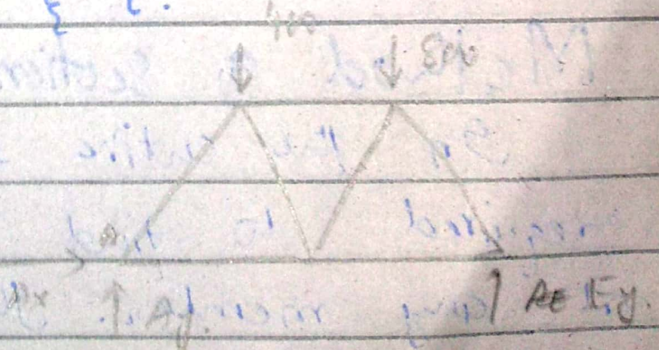


Solution:-

First we have to find whether the problem is determinate or Indeterminate. Since the problem is determinate.

Step # 02

Draw FBD of entire truss to find reaction at A & E.



Apply conditions of equilibrium:

$\Sigma F_x = 0$

$A_x = 0. \longrightarrow (1)$

$\Sigma F_y = 0.$

$$A_y + E_y - 400 - 800 = 0.$$

$$A_y + E_y = 1200 \rightarrow (2).$$

$\curvearrowleft + \Sigma M_A = 0$

$$-400 \times 1 - 800 \times 3 + E_y \times 4 = 0.$$

$$4E_y = 400 + 2400.$$

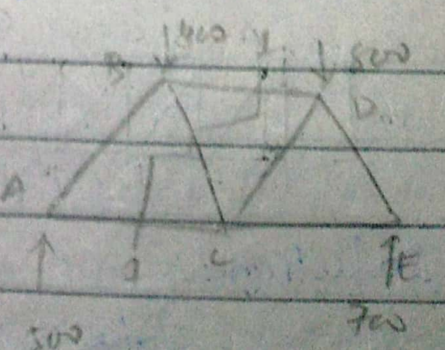
$$\boxed{E_y = 700\,N}$$

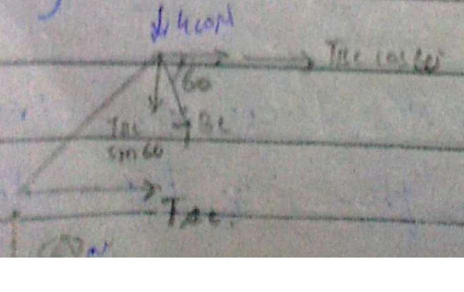
Eq (2) $\Rightarrow$ $A_y + 700 = 1200.$

$$\boxed{A_y = 500\,N}$$

Step #03.

Now cut the diagram such as shown in fig by 1-1. So that it passes through 3 members.





Now apply condition of equillibrium.

$\Sigma F_y = 0.$

$500 - 400 - T_{BC} \sin 60^\circ = 0.$

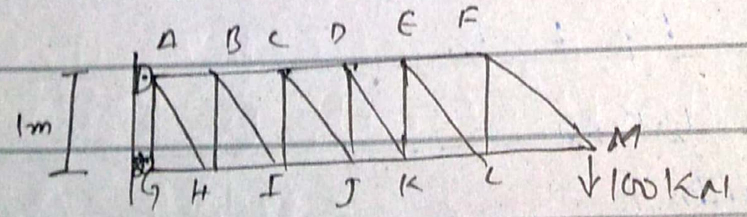
$+T_{BC}(0.866) = +100$

$\boxed{T_{BC} = 115.47\,N.}$

Which is our required axial force.

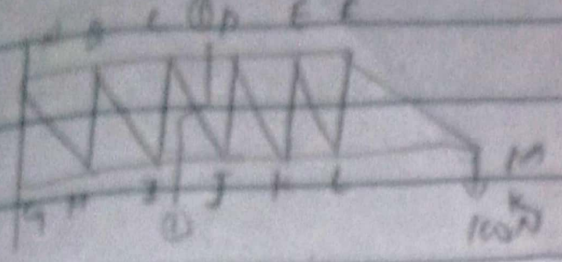
Problem 10.4 :-

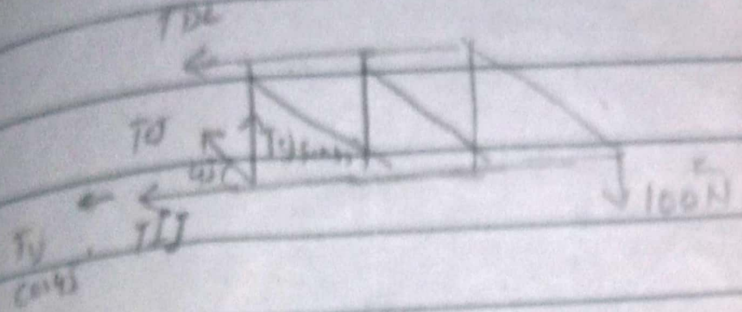
As in truss in fig Supports a 100KN load. The horizontal member are each 1m in length. Determine The axial force in member CJ & state whether Tension or compression.



Solution :-

Take R.H.S. and draw F.B.D of The fig. & by dividing the member including CJ by section i.e





Applying condition of equilibrium i.e

$\Sigma F_y = 0.$

$T_{cj} \sin 45° - 100 = 0.$

$(0.707) T_{cj} = 100$

$T_{cj} = \frac{100}{0.707}$

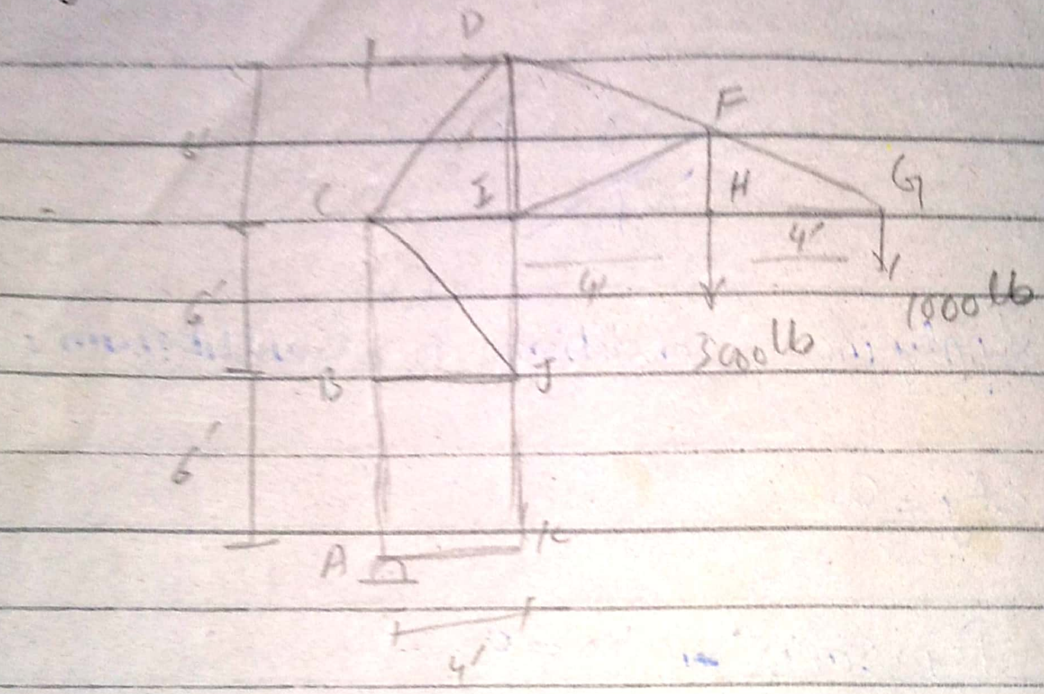
$\boxed{T_{cj} = 141.4\ kN.}$

Required axial i.e tension (away from joint) of mag. 141.4 kN. Ans.

Problem # 10.5

Using the method of sections, determine the forces in the following members of the truss in fig: FI & JC.

Figure:-

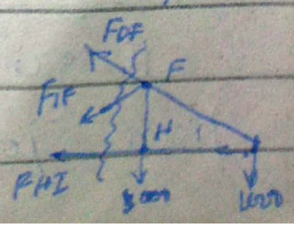


Solution:-

As the truss in equillibrium so all joints in truss along also in equillibrium. So, In order to find F_{IF}:-

$+\circlearrowleft M_G = 0$

Also cut the section as;



$3000\times4 + F_{IF}\sin \cancel{45}\, 36.86 = 0 \quad \times(8)$

$(8)\ F_{IF}\,(\cancel{0.707}\ 4.79) = -12000$

4 79°

~~Σ 6568~~ $F_{IF} = -12000$

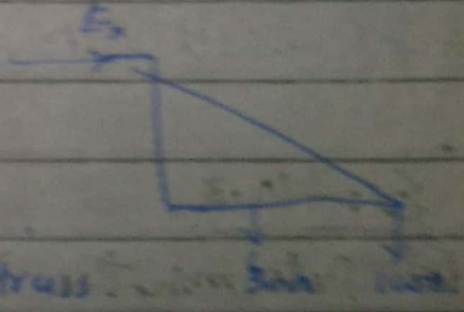
$$\boxed{\therefore F_{IF} = -\cancel{2281}.34\text{N.}}$$ Ans

As assume direction is wrong so
The force here will be compression
as towards the joint.

Now force at JC.

First find the reaction
at A & E.

Draw FBD of System;

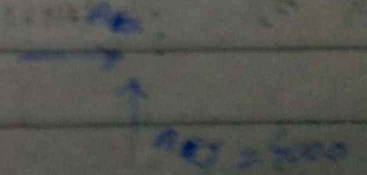


E_x

$\Sigma F_x = 0$

~~Ax~~ $E_x = 2000 \longrightarrow$ (1)

In order to balance the truss.



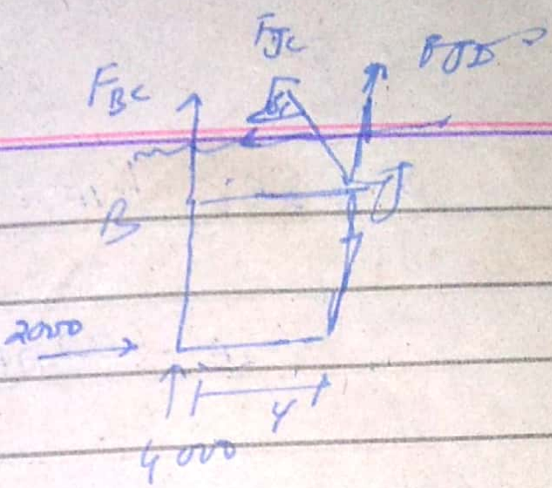
$\Sigma F_y = 0$

$A_y - 4000 = 0$

$A_y = 4000$.

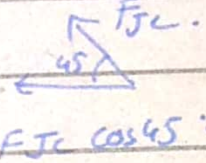
Now from down using method

Draw FBD.



$2000 - F_{JC} \cos \cancel{45°}\,56.3 = 0.$

$F_{JC} = \dfrac{2000}{\cos \cancel{45°}\,56.3}$



$F_{JC} = \dfrac{2000}{\cancel{0.7071}\ \cancel{0.01775}\ 0.5547} \Rightarrow$

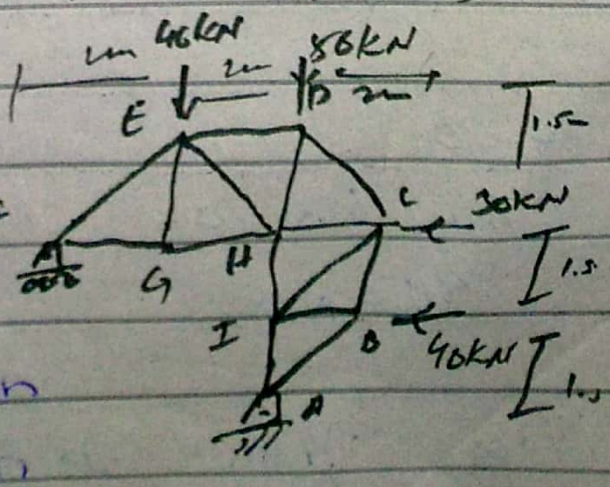
3604.6 N Ans.

$\boxed{F_{JC} = \cancel{228.45}\,8\ N.}$ Ans.

As it is away from joint so it will Tension.

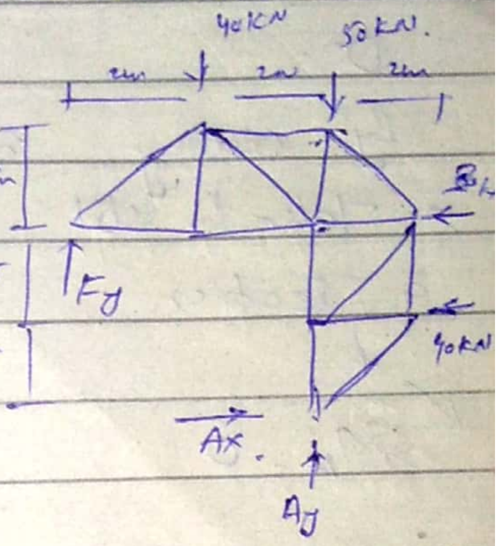
Ex:— 10.3

(b) Determine the force in member DC, HC & HI of truss



Sol:- First we draw FBD of whole fig to find unknown reactions at F & A.

Apply conditions of equilibrium.



$\Sigma F_x = 0.$

$A_x - 30 - 40 = 0$

$\boxed{A_x = 70\,kN} \longrightarrow ①$

$\Sigma F_y = 0$

$F_y + A_y - 40 - 50 = 0.$

$F_y + A_y - 90\,kN = 0 \longrightarrow ②$

$\circlearrowleft^{+}$ $\Sigma M_A = 0.$

$-F_y(4) + 40\times2 + 30\times3 + 40\times1.5 = 0.$

$-4F_y + 80 + 90 + 60 = 0$

$-4F_y = -230.$

$\boxed{F_y = 57.5\,kN}$

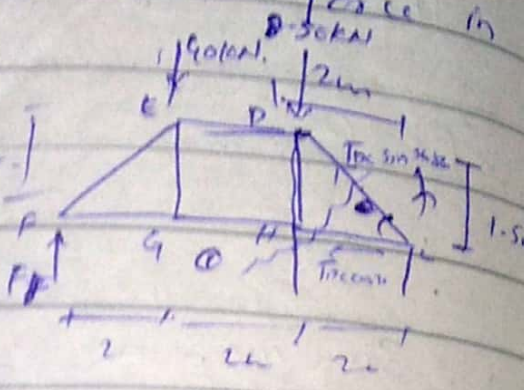
$\text{sub in } ② \Rightarrow \quad F_y + A_y - 90 = 0$

$A_y - 32.5 = 0$

$\boxed{A_y = 32.5\,kN}$

Now consider the Internal Force in DC.

By cutting section & Take to left of section.



$\tan\theta = \frac{1.5}{2}$

$\theta = 36.87$

$\circlearrowleft^{+} \Sigma M_H = 0$

$$T_{DC} \sin 36.87 (2) + 40 \times (2) - F_F (4) = 0$$

$$\cancel{T_{DC}(1.2) + 80 - 76.67(4) = 0}$$
57.

$$\cancel{T_{DC}(1.2) + 80 - 306.68 = 0}$$

$$\cancel{(1.2) T_{DC} - 226.68 = 0}$$

$$\cancel{\boxed{T_{DC} = 188.9\ kN}}$$

$$1.2 T_{DC} + 80 - 57.5(4) = 0$$

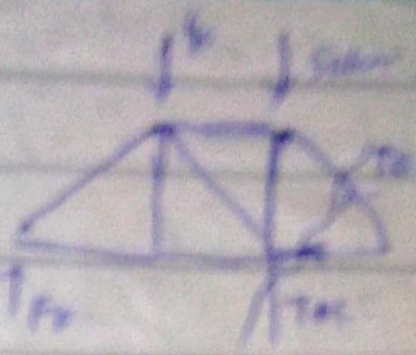
$$1.2 T_{DC} + 80 - 230 = 0$$

$$1.2 T_{DC} - 150 = 0$$

$$T_{DC} = \frac{150}{1.2}$$

$$T_{DC} = 125\ kN$$

Now Force in HC



$\Sigma F_x = 0$

-30

~~$T_{DC}\cos 36.86 - T_{HC} = 0$~~

~~$-0.8\, T_{HC} = 4/25 - 30$~~

~~$0.8\, T_{HC} = 95$~~

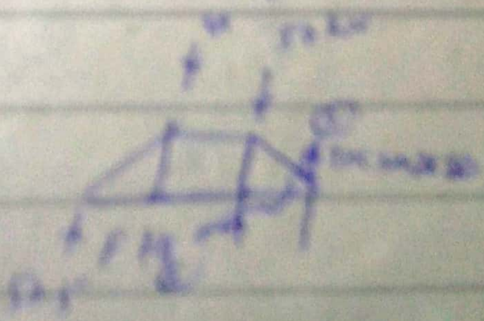
$-T_{DC}\cos 36.86 - T_{HC} = 0$

$-0.8(125) - T_{HC} = 0$

$-T_{HC} = 100 \Rightarrow T_{HC} = -100\,kN$

Tension

Now Force in HI



$\Sigma F_y = 0$ Section ①-①

$F_{HI} + T_{IK}\sin 36.86 - 50 - 50 + 57.5 = 0$

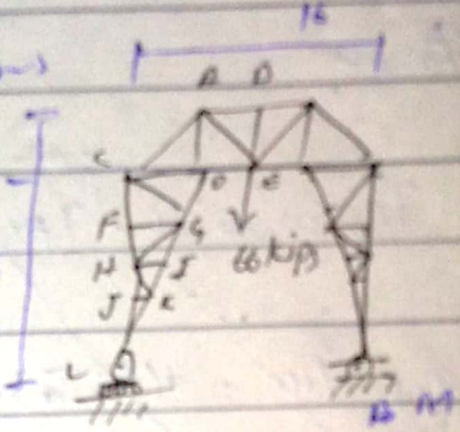
$T_{HI} = 42.48\,kN$

Ans

Excercise 10.3 (a).

Q:- Determine the forces in Member AC, AD & DE?

Sol:- First find reactions at L.



$\circlearrowleft +$

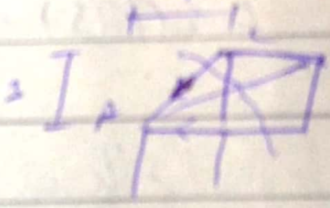
$\Sigma M_m = 0$

$-L(16) + 60(8) = 0$

$480 = 16L$

$\boxed{L = 30\,kN}$

Now Internal forces in member AC

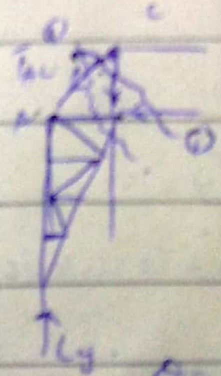


$\circlearrowleft +$

$\Sigma M_D = 0$

$T_{AC}\sin 36.86(4) - L_y(4).$

$T_{AC}(0.6)(4) = L_y(4).$



$T_{AC} = 30/0.6$

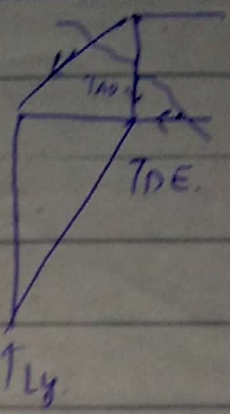
$\boxed{T_{AC} = 49.9^{k}N}$

(Compression)

Now In member AD.

T_{AC}

$\Sigma F_y = 0$



$-T_{AC} \sin 36.86 - T_{AD} + L_y = 0$

$-T_{AD} - 29.94 + 30 = 0$

$-T_{AD} = 0$

$T_{AD} = 0$

Now In member DE.

$\Sigma F_x = 0.$

$-T_{DE} - T_{AC} \cos 36.86 = 0$

$-T_{DE} = 49.9(0.8)$

$-T_{DE} = +39.9$

$T_{DE} = 39.9\,kN$

9t means here -Tension not compression as away from Joint