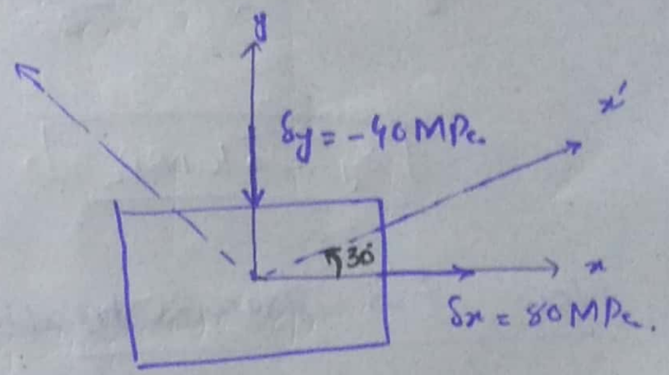
Pb# 923:- At a certain point in a stressed body, the principal stresses are $\sigma_x = 50MPa$ and $\sigma_y = -40MPa$. Determine σ & τ on the plane whose normals are at $+30^\circ$ and $+120^\circ$ with x-axis. Show your result on a sketch as a differential element.

As given that;

$\sigma_x = 50MPa$

$\sigma_y = -40MPa$



Required data;

$\sigma_x' = ?$

$\sigma_y' = ?$

$\tau_x' = ?$

$\tau_y' = ?$

Solution:-

As we know that;

$$\sigma_x' = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta \quad \to (1)$$

$\theta = 30^\circ$. By putting values.

$$\sigma_x' = \frac{80 - 40}{2} + \frac{80 + 40}{2} \cos 2(30) - 0$$

$$\boxed{\sigma_x' = 50MPa}$$

$$\delta_y' = \frac{\delta_x + \delta_y}{2} + \frac{\delta_x - \delta_y}{2}\cos 2\theta_1 - \cancel{\tau_{xy}\sin 2\theta_1} \rightarrow ②$$

Here

$\theta_1 = 90° + \theta$

$\theta_1 = 120°$.

∵ As τ_{xy} is not given

eq ② ⇒

$$\delta_y' = \frac{80-40}{2} + \frac{80+40}{2}\cos 2(120°) - 0.$$

$$\boxed{\delta_y' = \pm 10 MPa}$$

Now For shear stress:-

$$\tau_x' = \frac{\delta_x - \delta_y}{2}\sin 2\theta + \cancel{\tau_{xy}\cos 2\theta} \rightarrow ③$$

use $\theta = 30$

$$\tau_x' = \frac{80+40}{2}\sin 2(30)$$

$$\boxed{\tau_x' = 51.96\,psi}$$

$$\tau_y' = \frac{\delta_x - \delta_y}{2}\sin 2\theta_1 + \tau_{xy}\cos 2\theta_1 \rightarrow ④$$

Put $\theta_1 = \theta + 90 \Rightarrow 120°$.

$$\tau_y' = \frac{80+40}{2}\sin 2(120°)$$

$$\boxed{\tau_y' = -51.96\,Psi}$$

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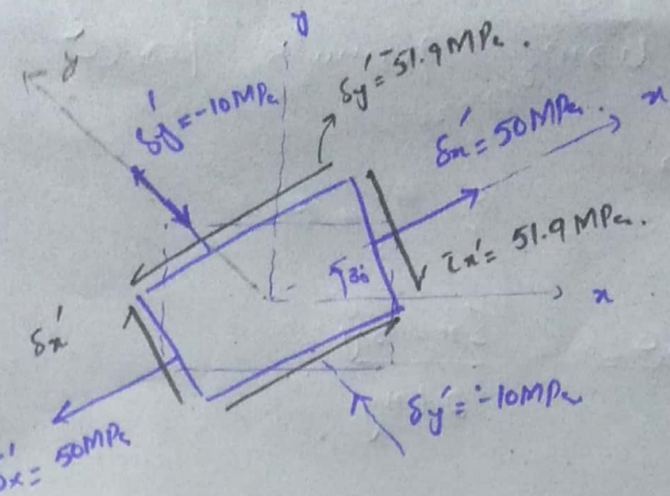
Figure:-



Pb# 924:- A state of stress is specified in fig. Determine the normal & shearing stress on

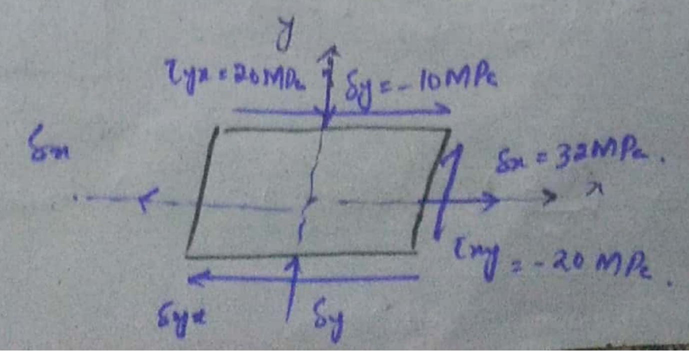
ⓐ Principal axis (plane), (b) the plane of maximum in plane shearing stress and ⓒ the plane whose normal at 36.8° & 126.8° with x-axis. Show result of ⓐ & ⓑ sketches of different element.

Solution:-

ⓐ For principal axis.

First we will find principal axis and than we will resolve stresses on that principal axis.

Figure:-



To find angle θ at which principal axis lies using formula for θ of normal stresses.

Since;

$$\cancel{\tan 2\theta = \frac{S_x - S_y}{2\tau_{xy}}}$$

~~By putting values.~~

$$\cancel{\tan 2\theta = \frac{32+10}{2(-20)}}$$

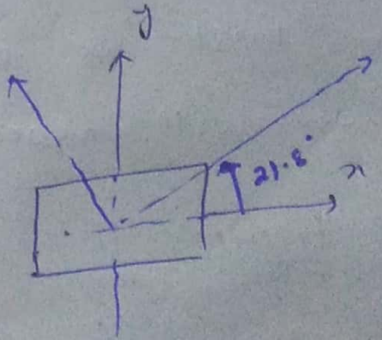
$$\cancel{\tan 2\theta =}$$

$$\tan 2\theta = -\frac{2\tau_{xy}}{S_x - S_y}$$

$$= \frac{-2(-20)}{32+10}$$

$$\tan 2\theta = 0.95 \Rightarrow \boxed{\theta = 21.8^\circ}$$

Now consider this value.



$$S_x' = \frac{S_x + S_y}{2} + \left(\frac{S_x - S_y}{2}\right)\cos 2\theta - \tau_{xy}\sin 2\theta.$$

$$= \frac{32-10}{2} + \left(\frac{42}{2}\right)\cos 2(21.8) - (-20)\sin 2(21.8)$$

$$\boxed{S_x' = 40\,\text{MPa}}$$

$$\delta_y' = \frac{\delta_x + \delta_y}{2} + \frac{\delta_x - \delta_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$\hookrightarrow$ ①.

Here put $\theta = 90° + \theta$.

$\theta = 90° + 21.8$.

$\theta = 111.8°$.

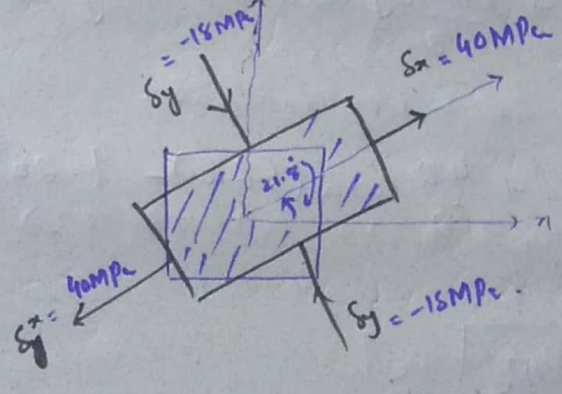
eq ① ⇒

$$\delta_y' = \frac{32-10}{2} + \frac{32+10}{2}\cos 2(111.8) - (-20)\sin 2(111.8)$$

$$\boxed{\delta_y' = -18\ MPa}$$

As on principal axis no shearing stress occur so shearing stresses are zero.

Figure:-



(b) The plane of max. shearing stress:-

First find angle for that plane Using eqs of $\tan 2\theta$ for shearing stresses

$$\tan 2\theta = \frac{\delta_x - \delta_y}{+2\tau_{xy}}$$

$$\tan 2\theta = \frac{32+10}{2(-20)}$$

$$\tan 2\theta = -1.05$$

$$\theta = -23.19$$

As -ve so;

$$\theta = 90 - 23.19$$

$$\boxed{\theta = 66.8^\circ}$$

Now use this value for stresses.

Normal stresses:-

$$\delta_x' = \frac{\delta x + \delta y}{2} + \frac{\delta x - \delta y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$= \frac{32-10}{2} + \frac{42}{2}\cos 2(66.8) - (-20)\sin 2(66.8).$$

$$\boxed{\delta x' = 11 MPa}$$

$$\delta y' = \frac{\delta x + \delta y}{2} + \frac{\delta x - \delta y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

use $\theta = 90 + 66.8$

$\theta = 156.8^\circ$.

$$\delta y' = \frac{32-10}{2} + \frac{42}{10}\cos 2(156.8) - (-20)\sin 2(156.8)$$

$$\boxed{\delta y' = 11 MPa}$$

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For shearing Stresses :-

$$\tau_{x'} = \frac{S_x - S_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta.$$

$$\theta = 66.8^\circ.$$

$$\tau_{x'} = \frac{32+10}{2} \sin 2(66.8) + (-20) \cos 2(66.8)$$

$$\boxed{\tau_x = 29\,MPa}$$

$$\tau_{y'} = \frac{S_x - S_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

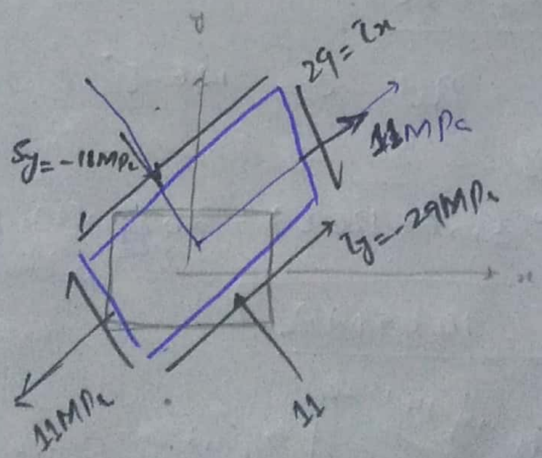
Here used $\theta = 90 + 66.8.$

$$\theta = 156.8^\circ$$

$$\tau_{y'} = \frac{32+10}{2} \sin 2(156.8) + (-20) \cos 2(156.8)$$

$$\boxed{\tau_y = -29\,MPa}$$

Figure :-



Now to find stresses at $\theta = 36.8^\circ$ & 126.8°

Normal stresses:-

$$\sigma_x' = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta \longrightarrow (A)$$

Put $\theta = 36.8^\circ$

$$= \frac{32-10}{2} + \frac{32+10}{2}\cos 2(36.8) - (-20)\sin 2(36.8)$$

$$\boxed{\sigma_x' = 36.1\,\text{MPa}}$$

For σ_y' :- use same formula Put $\theta = 126.8^\circ$.

eq (A) $\rightarrow$ $$\sigma_y' = \frac{32-10}{2} + \frac{32+10}{2}\cos 2(126.8) - (-20)\sin 2(126.8)$$

$$\boxed{\sigma_y' = -14.1\,\text{MPa}}$$

shearing stress:-

$$\tau = \frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\,\cancel{\sin}\cos 2\theta.$$

Put $\theta = 36.8^\circ$ ~~126.8~~

$$\tau_x' = \frac{32+10}{2}\sin 2(36.8) + (-20)\cos 2(36.8)$$

$$\boxed{\tau_x' = 14.49\,\text{MPa}}$$

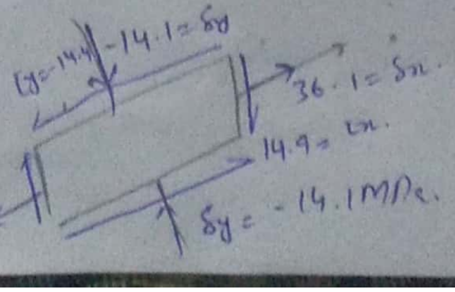
For τ_y' Put $\theta = 126.8^\circ$.

$\therefore$

$$\tau_y' = \frac{32+10}{2}\sin 2(126.8) - 20\,(\cos(2)(126.8))$$

$$\boxed{\tau_y' = -14.49\,\text{MPa}}$$

Figure:-



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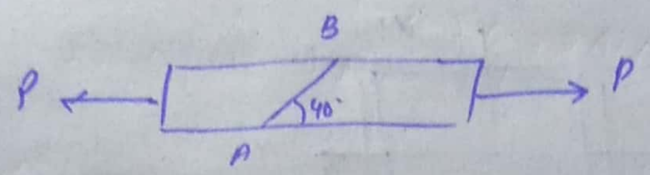
Pb # 925:- Two wooden joist 50mm x 100mm are glued together along the joint AB as show in fig. Determine the normal & shear stresses in glue if P = 200kN.

Figure:-

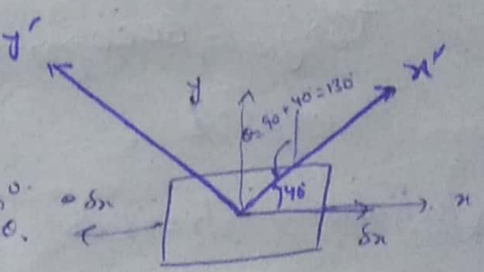


Solution:-

First find δ_x & δ_y.

$$\delta_x = \frac{P}{A} = \frac{200\times10^3 N}{50\times100} = 40MPa$$

$\delta_y = 0$ as no force acting along y-axis



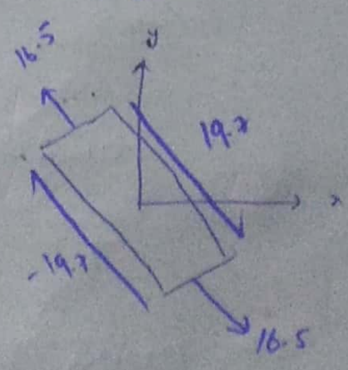
$$\delta_x' = \frac{\delta_x+\delta_y}{2} + \frac{\delta_x-\delta_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta.$$

Put $\theta = 130$, $\delta_y = 0$ & $\tau_{xy} = 0$

$$\delta_x' = \frac{40}{2} + \frac{40}{2}\cos 2(130).$$

$$\boxed{\delta_x' = 16.53}$$

Figure:-



Shear stress:-

$$\tau = \frac{\delta_x-\delta_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\tau = \frac{40}{2}\sin 2(130) \Rightarrow \boxed{\tau_y = -19.7MPa}$$

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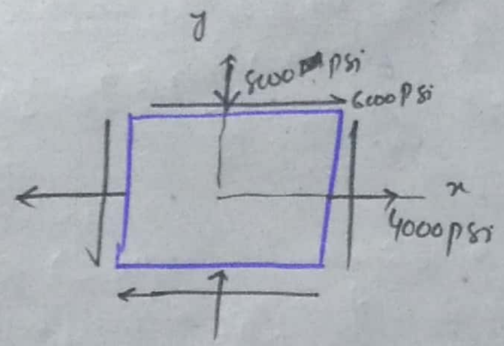
Pb # 926:- 9b. The element is subjected to state of stress shown in fig. Find the principal stresses. Also compute the stress component on plate at 30° counter clockwise from the x-face.

Figure:-



Given data;

$\sigma_x = 4000$ psi

$\theta = 30°$ c.c.w from x-axis.

$\sigma_y = -8000$ psi

$\tau_{xy} = -6000$ psi

Solution:- **Stresses at Principal axis.**

First find θ at which principal lies.

Using eq.

$$\tan 2\theta = \frac{-2\tau_{xy}}{\sigma_x - \sigma_y}$$

$$\tan 2\theta = \frac{-2(-6000)}{4000 + 8000} \Rightarrow \boxed{\theta = 22.5°}$$

So principal axis lies at $\boxed{\theta = 22.5°}$

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Normal stress:-

$$\delta_x' = \frac{\delta_x+\delta_y}{2} + \frac{\delta_x-\delta_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta \quad \to (1)$$

Put $\theta = 22.5$.

$$= \frac{4000-8000}{2} + \frac{4000+8000}{2}\cos 2(22.5) + 6000 \sin 2(22.5)$$

$$= -2000 + 8485.3$$

$$\boxed{\delta_x' = 6485.3 \text{ Psi}}$$

For $\delta_y' = ?$ Put $\theta = 90° + 22.5°$

$\theta = 112.5$.

eq (1) ⇒

$$\delta_y' = \frac{4000-8000}{2} + \left(\frac{4000+8000}{2}\right)\cos 2(112.5) + 6000\sin 2(112.5)$$

$$= -2000 + (-8485.28)$$

$$\boxed{\delta_y' = -10485.28 \text{ psi}}$$

Shearing stress:-

$$\tau = \frac{\delta_x-\delta_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$\theta = 22.5$

About x'.

$$\tau_x' = \left(\frac{4000+8000}{2}\right)\sin 2(22.5) - 6000\cos 2(22.5)$$

$$\boxed{\tau_x' = 0}$$

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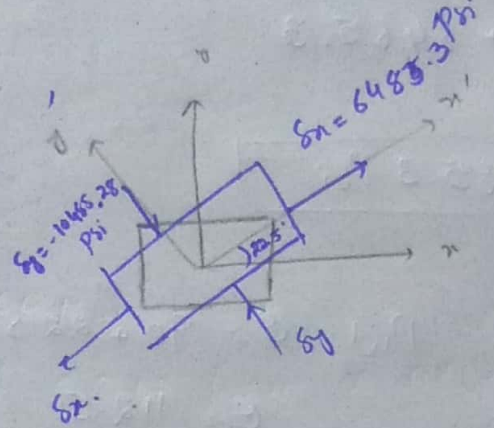
For $\tau_{y'} = ?$ Put $\theta = 112.5$

$$\tau_{y'} = \frac{4000+8000}{2}\sin 2(112.5) - 6000\cos 2(112.5)$$

$$\boxed{\tau_{y'} = 0}$$

Diagram:-



Now stresses on a plane:-

Since; $\theta = 30°$

So; using eqn.

$$\delta_{x'} = \frac{\delta_x + \delta_y}{2} + \frac{\delta_x - \delta_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$= \frac{4000-8000}{2} + \frac{4000+8000}{2}\cos 2(30) + 6000\sin 2(30)$$

$$\boxed{\delta_{x'} = 6196 \text{ psi}}$$

Put $\theta = 90° + 30° = 120°$

$$\delta_{y'} = \frac{\delta_x + \delta_y}{2} + \frac{\delta_x - \delta_y}{2}\cos 2(120) - \tau_{xy}\sin 2(120)$$

$$= \frac{4000-8000}{2} + \frac{4000+8000}{2}\cos 2(120) + 6000\sin 2(120)$$

$$\boxed{\delta_y = -10196.15 \text{ Psi}}$$

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Shearing stress:-

Along x'-axis $(\theta = 30^\circ)$

$$\tau_{x'} = \frac{\sigma_x - \sigma_y}{2} \sin 2(\theta) + \tau_{xy} \cos 2\theta$$

$$= \left(\frac{4000 + 8000}{2}\right) \sin^2(30^\circ) + (-6000) \cos(2)(30^\circ)$$

$$\boxed{\tau_{x'} = 2196 \text{ Psi}}$$

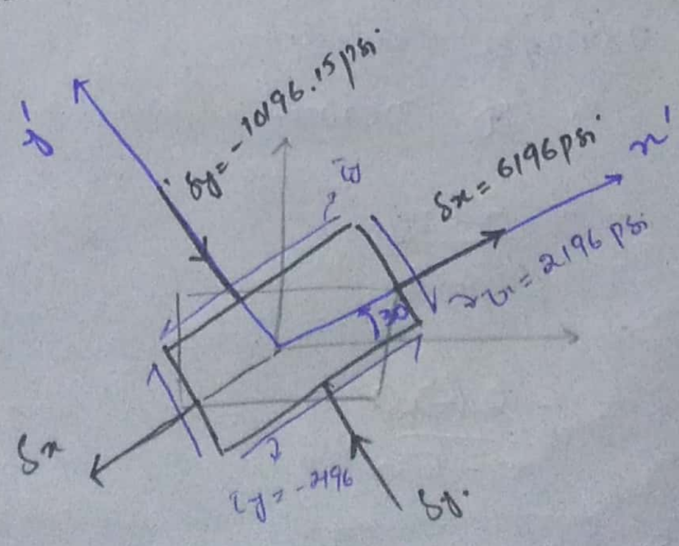
$$\tau_{y'} = \frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

Here $\theta = 120^\circ \because 30 + 90.$

$$= \left(\frac{4000 + 8000}{2}\right) \sin 2(120^\circ) - 6000 \cos 2(120^\circ)$$

$$\boxed{\tau_{y'} = -2196 \text{ psi}}$$

Diagram:-



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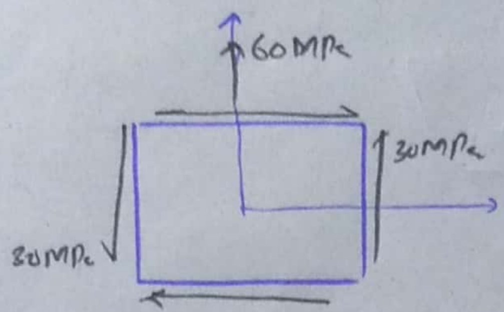
Pb # 927:- For the state of stress shown in fig determine the principal stresses & max in plane shearing stress. Show all results on complete sketches of a differential element.

Figure:-



Same as before question.

Given data;

$\sigma_x = 0$

$\sigma_y = 60 MPa$

$\tau_{xy} = -30 MPa$

(a) Stresses on principal axis.

(b) Max shearing stresses in plane.

Sol:- (a) For principal axis:-

using eq of normal

$$\tan 2\theta = -\frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

$$= -\frac{2(-30)}{0-60}$$

$$\boxed{\theta = -22.5^\circ}$$

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Since the $\theta = -ve$ so add $+90^\circ$ with it.

$\theta = -22.5 + 90^\circ \rightarrow \boxed{\theta = 67.5^\circ}$

So normal stresses will be;

$\sigma' = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta . \rightarrow (A)$

Put $\theta = 67.5^\circ$ For σ_x'.

$\sigma_x' = \frac{60}{2} + \frac{(-60)}{2} \cos 2(67.5) + 30 \sin 2(+67.5)$

$\boxed{\sigma_x' = 72.4 MPa}$

For $\sigma_y' =$ Put $\theta = 67.5 + 90^\circ = 157.5^\circ$.

$\xi (A) \Rightarrow$

$\sigma_y' = \frac{60}{2} + \frac{(-60)}{2} \cos 2(157.5) + 30 \sin 2(157.5)$

$\boxed{\sigma_y' = -12.42 MPa}$

<u>For shearing stress:-</u>

Since; $\tau = \frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$.

For $\tau_x = ?$ use $\theta = 67.5$.

$\tau_x' = \frac{-60}{2} \sin 2(67.5) + 30 \cos 2(67.5)$

$\boxed{\tau_x' = 0}$

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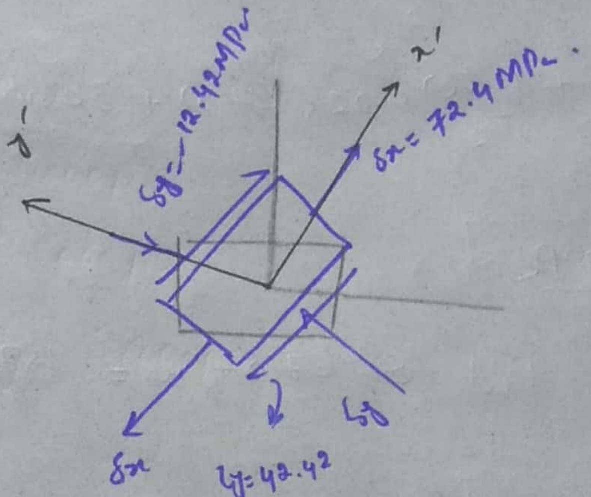
For $\tau_{y}' = ?$

Put $\theta = 157.5^\circ$

$$\tau_{y}' = -\frac{60}{2}\sin 2(157.5) + 30\cos 2(157.5)$$

$$\boxed{\tau_y = 42.42\ \text{MPa}}$$

<u>Figure</u>:- (For P.A)



(b) For Max. shearing stress in plane.

By using eq:-

$$\tan 2\theta = \frac{\delta_x - \delta_y}{2\tau_{xy}} = \frac{-60}{2(-30)}$$

$$\boxed{\theta = 22.5^\circ}$$

Now use this angle to find stresses.

Normal stresses:-

$$\delta_x' = \frac{\delta_x + \delta_y}{2} + \frac{\delta_x - \delta_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

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$$= \frac{60}{2} - \frac{60}{2} \cos 2(22.5) + 30 \sin 2(22.5).$$

$$\boxed{\sigma_x' = 30\,MPa}$$

use $\theta = 157.5°$

$$\sigma_y' = \frac{60}{2} - \frac{60}{2} \cos 2(157.5) + 30 \sin 2(157.5)$$

$$\boxed{\sigma_y' = -12.42\,MPa}$$

Shearing Stress:-

$$\tau' = \frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta.$$

For $\tau_x' = ?$ $\quad \theta = 22.5°$.

$$\tau_x' = -\frac{60}{2} \sin 2(22.5) + 30 \cos 2(22.5)$$

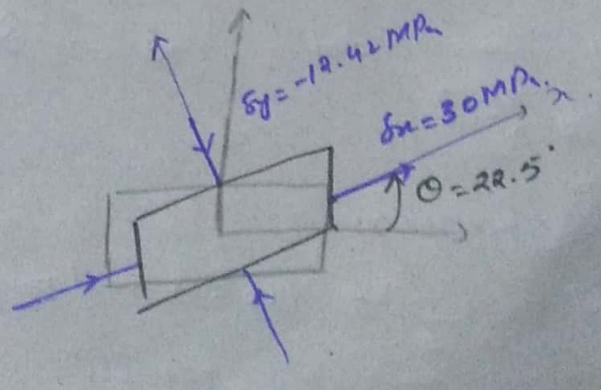
$$\boxed{\tau_x' = 0}$$

For $\tau_y' = ?$
Put $\theta = 112.5°$

$$\tau_y' = -\frac{60}{2} \sin 2(112.5) + 30 \cos 2(112.5).$$

$$\boxed{\tau_y' = 0.}$$

Figure:-



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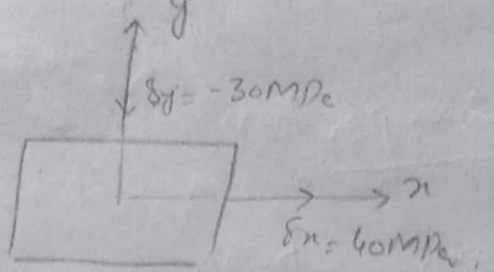
Pb#928:- An element is subjected to the principal stresses $\sigma_1 = \sigma_x = 40\,MPa$ & $\sigma_2 = \sigma_y = -30\,MPa$. Compute the stresses components on plane whose normals are at $+30°$ & $+120°$ with x-axis. Show a complete sketch.

Figure:-

$\tau_{xy} = 0$
$\sigma_x = 40\,MPa$.
$\sigma_y = -30\,MPa$



Solution:-

As we know for normal stresses.

$$\sigma' = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \cancel{\tau_{xy}}\sin 2\theta. \quad \to (1)$$

For σ_x' put $\theta = 30°$.

$$\sigma_x' = \frac{40 - 30}{2} + \frac{40 + 30}{2}\cos 2(30)$$

$$\boxed{\sigma_x' = 22.5\,MPa.}$$

For $\sigma_y' = ?$ Put $\theta = 90 + 30 = 120°$.

eq (A) ⇒

$$\sigma_y' = \frac{40 - 30}{2} + \frac{40 + 30}{2}\cos 2(120°).$$

$$\boxed{\sigma_y' = -12.5\,MPa}$$

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Shearing stress:- (For new axis)

$$\tau' = \frac{\delta_x - \delta_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta \rightarrow ②$$

For τ_x' Put $\theta = 30°$.

eq ② ⇒ $\tau_x' = \frac{40+30}{2} \sin 2(30°) + 0$

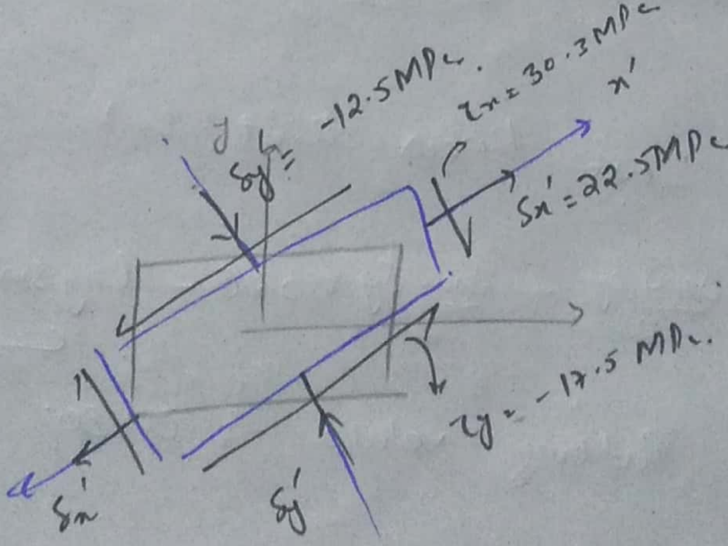
$$\boxed{\tau_x' = 30.3 \text{ MPa}}$$

$\tau_y' = ?$ Put $\theta = 120°$.

eq ② ⇒ $\tau_y' = \frac{40+30}{2} \cos 2(120°)$

$$\boxed{\tau_y' = -17.5 \text{ MPa}}$$

Figure:-



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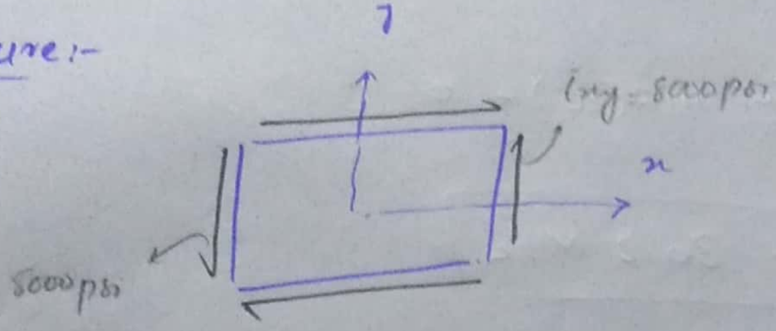
Pb # 929:- For the state of pure shear shown in fig Find the stress components on plane whose normals are at 30° & 120° with x_axis as shown. Draw a complete sketch.

Figure:-



Given data

As pure shear stress

$S_x = 0$.

$S_y = 0$.

$\tau_{xy} = -5000psi$.

$\theta = 30°$.

Solution:-

Since we know that;

$$S'_x = \frac{S_x + S_y}{2} + \frac{S_x - S_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta . \longrightarrow (A)$$

Putting values & $\theta = 30°$

$$S_x' = 0 + 0 - (-5000) \sin 2(30°).$$

$$\boxed{S_x' = 6925.2 psi}$$

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$$\sigma_y' = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

Put $\theta = 90 + 30$

$\theta = 120$

$$\sigma_y' = 0 + 0 - (8000)\sin 2(120^\circ)$$

$$\boxed{\sigma_y' = -6928.2\,psi}$$

Now for shear stress:-

$$\tau' = \cancel{\frac{\sigma_x-\sigma_y}{2}\sin 2\theta} + \tau_{xy}\cos 2\theta \longrightarrow (B)$$

for $\tau_x' = ?$ Put $\theta = 30$

$$\tau_x' = (-8000)\cos 2(30^\circ)$$

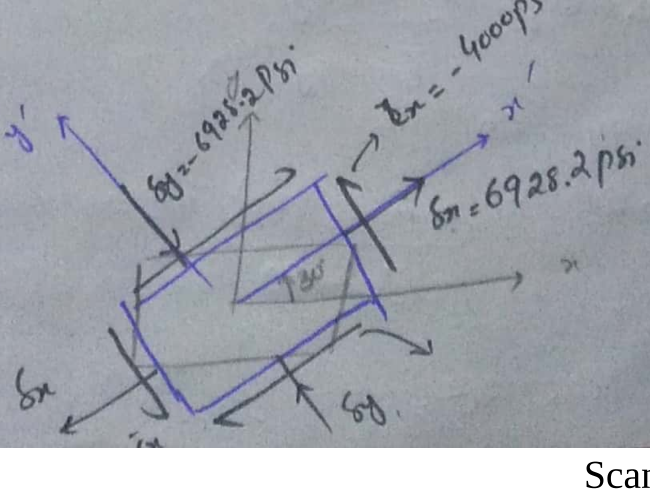
$$\boxed{\tau_x' = -4000\,psi}$$

For $\tau_y' = ?$ Put $\theta = 120^\circ$.

$$\tau_y' = (-8000)\cos 2(120^\circ)$$

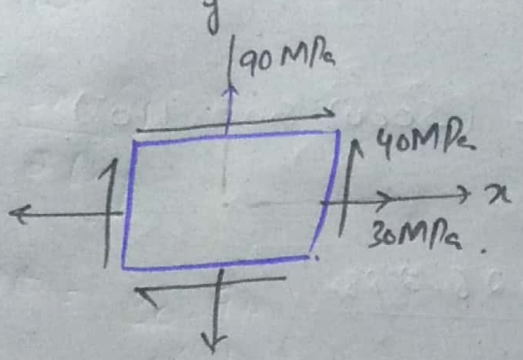
$$\boxed{\tau_y = 4000\,psi}$$

Figure:-



Pb # 930:- If a point is subjected to state of stress shown in fig. Determine the principal stresses and the maximum in plane shearing stress. Draw sketches complete.

Figure:-



Solution:-

Given data.

$S_y = 90\ MPa$.

$S_x = 30\ MPa$.

$\tau_{xy} = -40\ MPa$.

For Principal axis:-

First find $\theta = ?$ at which principal axis lies w.r.t x-axis

Since we know

$$\tan 2\theta = \frac{-2\tau_{xy}}{S_x - S_y}$$

$$\tan 2\theta = \frac{-2(-40)}{30-90}$$

$$\tan 2\theta = -4/3.$$

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$\theta = -26.56°$

$\sin a$ -ve value so add with 90.

$\theta = 90° - 26.56$.

$$\boxed{\theta = 63.43°}$$

For normal stress:-

Put $\theta = 30°$ & $120°$

$$\sigma' = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta.$$

$$\sigma_x' = \frac{90+30}{2} + \frac{30-90}{2}\cos 2(30°) + 40\sin 2(30°).$$

$$\boxed{\sigma_x' = 79.64 \text{ MPa}}$$

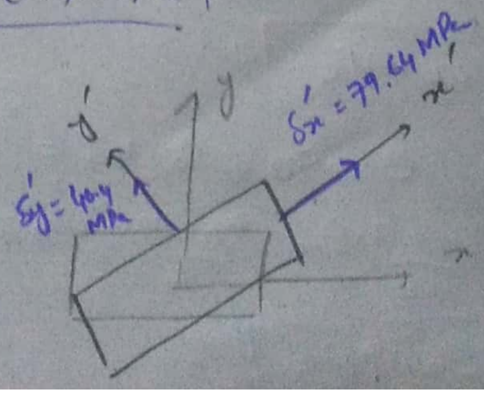
For $\sigma_y' = ?$ Put $\theta = 120°$.

$$\sigma_y' = \frac{90+30}{2} + \frac{30-90}{2}\cos 2(120°) + 40\sin^2(120°).$$

$$\boxed{\sigma_y' = 40.4 \text{ MPa}}$$

Note:-
on principal plane we have no shear stresses

Figure:-



Now for

Maximum In plane shearing stress:-

First find $\theta = ?$

Since we know that;

$$\tan 2\theta = \frac{\delta_x - \delta_y}{2\tau_{xy}}$$

$$\tan 2\theta = \frac{30-90}{2(-40)}$$

$$\boxed{\theta = 18.43^\circ}$$

Now use this angle and find normal

Normal stresses:-

Since;

$$\delta_x' = \frac{\delta_x + \delta_y}{2} + \frac{\delta_x - \delta_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

By putting values & $\theta = 18.43^\circ$.

$$\delta_x' = \frac{30+90}{2} + \frac{30-90}{2}\cos 2(18.43) + 40\sin 2(18.43)$$

$$\boxed{\delta_x' = 60\,MPa}$$

$\delta_y' = ?$ Put $\theta = 90 + 18.43 = 108.43^\circ$.

$$\delta_y' = \frac{30+90}{2} + \frac{30-90}{2}\cos 2(108.43) + 40\sin 2(108.43)$$

$$\boxed{\delta_y' = 60\,MPa}$$

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For shearing stress:-

$$\tau = \frac{\delta x - \delta y}{2} \sin 2\theta + \tau xy \cos 2\theta \rightarrow (A).$$

$\theta = 18.43$.

$$\tau_x' = \frac{30-90}{2} \sin 2(18.43) - 40 \cos 2(18.43)$$

$$\boxed{\tau x' = -50MPa}$$

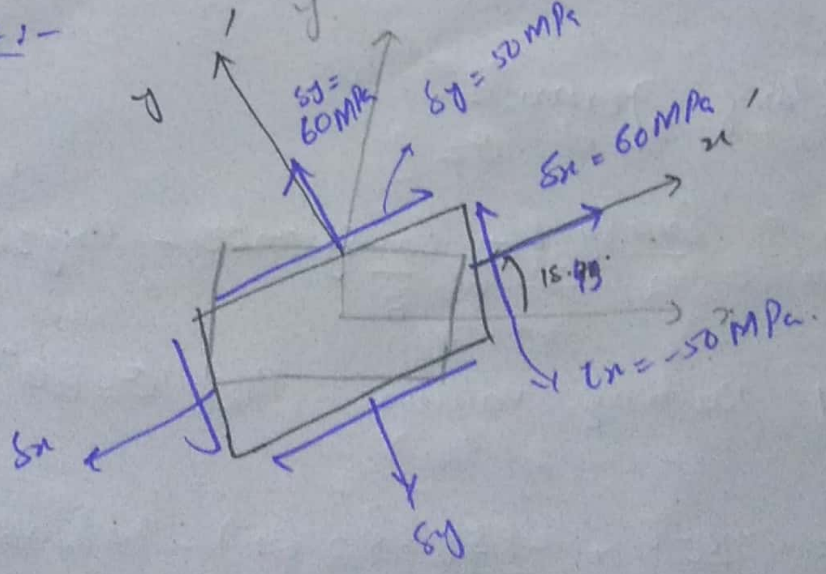
$\tau y' = ?$ Put $\theta = 108.43°$.

eq(A) ⇒

$$\tau y' = \frac{30-90}{2} \sin 2(108.43°) - 40 \cos 2(108.43°)$$

$$\boxed{\tau y' = 50MPa}$$

Figure:-



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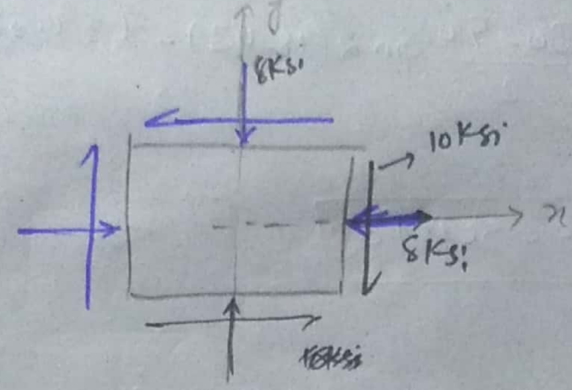
Pb# 931:-

For the state of stresses shown in fig. determine the normal and shearing stresses on planes whose normals are at +60° & +150° with the x-axis. Sketch complete.

Figure:-



Given data;

$\sigma_x = -8 Ksi$

$\sigma_y = -8 Ksi$

$\tau_{xy} = 10 Ksi$

Solution:-

Using formula i-e

$$\sigma_x' = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta \rightarrow (A)$$

By putting values. Put $\theta = 60°$

$$\sigma_x' = \frac{-8-8}{2} + \cancel{\frac{-8+8}{2}}\cos 2(60°) - 10\sin 2(60°)$$

$$= -8 - 10(0.866)$$

$$\boxed{\sigma_x = -16.66 Ksi}$$

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$\delta y' = ?$ Put $\theta = 150°$

Eq

$$\delta y' = \frac{-8-8}{2} \; \frac{-8+8}{2} \cos 2\theta - 10 \sin 2(150)$$

$$\boxed{\delta y' = 0.66\, Ksi}$$

For shearing Stress:-

$$\tau' = \frac{\delta x - \delta y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta \longrightarrow (B)$$

Put $\theta = 60°$

$$\tau x' = \cancel{\frac{-8+8}{2}}^{\,0} \sin 2\theta - 10 \cos 2(60°)$$

$$\boxed{\tau x' = 5\, Ksi}$$

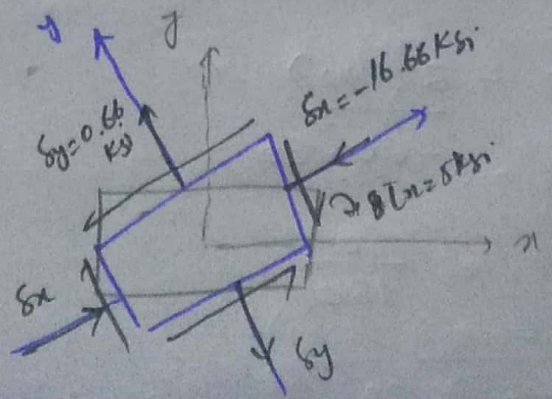
For $\tau y = ?$ Put $\theta = 150°$

Eq (B) $\Rightarrow$

$$\tau y' = \cancel{\frac{-8+8}{2}}^{\,0} \sin 2\theta - 10 \cos 2(150°)$$

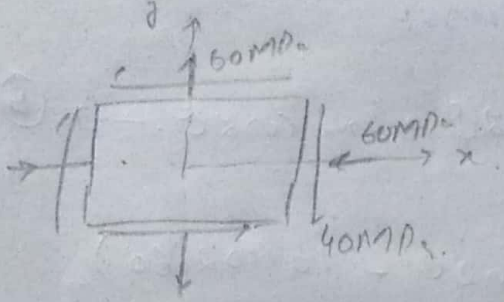
$$\boxed{\tau y' = -5\, Ksi}$$

Figure:-



Pb # 933:- If an element is subjected to state of stress shown in fig, find the principal stresses and the max. in plane shearing stress. Also determine the stress components on plane whose normals are at 45° & 135° with x-axis. Draw sketch?

Figure:-



Given data;

$\sigma_y = 60 MPa$

$\sigma_x = -60 MPa$

$\tau_{xy} = 40 MPa$

Solution:- For Principal axis:-

First find $\theta = ?$

$$\tan 2\theta = -\frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

$$= -\frac{2(40)}{-60-60}$$

$$= \frac{80}{120}$$

$\boxed{\theta = 16.84°}$ (Principal axis angle)

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Now Normal stress on P.A.

$$\sigma' = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta \quad \hookrightarrow (A)$$

Put $\theta = 16.84^\circ$

$$= \cancel{\frac{-60+60}{2}}^{0} + \frac{(-60)-60}{2}\cos 2(16.84) - 40\sin 2(16.84)$$

$$\boxed{\sigma_x' = -72.11\,MPa}$$

For $\sigma_y' = ?$ Put $\theta = 90^\circ + 16.84$

$= 106.84^\circ$

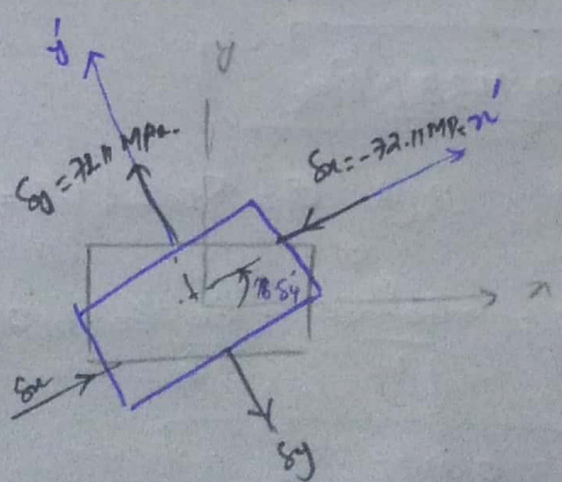
eq (A) $\Rightarrow$

$$\sigma_y' = \cancel{\frac{-60+60}{2}}^{0} \frac{-60-60}{2}\cos(2(106.84)) - 40\sin 2(106.84)$$

$$\boxed{\sigma_y = 72.11}$$

As on P.A no shearing stresses are produce. So

Figure :-



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For max. In plane shearing stress:-

First $\theta = ?$

Using eq $\Rightarrow$

$$\tan 2\theta = \frac{\delta x - \delta y}{2\tau xy} \Rightarrow \frac{-60-60}{2(40)}$$

$$\theta = -28.15^\circ$$

Add 90° with θ to get +ve value

$$\theta = 90^\circ - 28.15^\circ$$

$$\boxed{\theta = 61.8^\circ}$$

<u>Normal stresses:-</u>

$$\delta' = \frac{\delta x + \delta y}{2} + \frac{\delta x - \delta y}{2}\cos 2\theta - \tau xy \sin 2\theta . \longrightarrow (B)$$

<u>For $\delta x'$:-</u> Put $\theta = 61.8^\circ$.

$$\delta x' = \frac{\cancel{-60+60}}{2}^{0} + \frac{(-60)-60}{2}\cos 2(61.8) - 40\sin 2(61.8).$$

$$\boxed{\delta x' = -0.1133 \text{ MPa}}$$

For $\delta y' = ?$ Put $\theta = 90^\circ + 61.8 \Rightarrow 151.8^\circ$.

eq (B) $\Rightarrow$ $\delta y' = \frac{\cancel{-60+60}}{2}^{0} - \frac{60-60}{2}\cos 2(151.8) - 40\sin 2(151.8).$

$$\boxed{\delta y' = 0.1133 \text{ MPa}}$$

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Shearing Stresses:-

Since;

$$\tau' = \frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta . \rightarrow \text{(C)}$$

Put $\theta = 61.8^\circ$ for τ_x'.

$$\tau_x' = \frac{-60-60}{2} \sin 2(61.8) + 40 \cos 2(61.8)$$

$$\boxed{\tau_x = -72.11\,MPa}$$

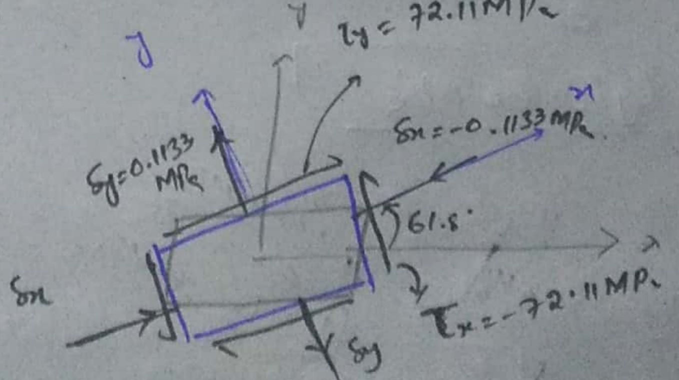
$\tau_y' = ?$ Put $\theta = 151.8^\circ$.

& (C) =>

$$\tau_y' = -\frac{60-60}{2} \sin 2(151.8) + 40 \cos 2(151.8).$$

$$\boxed{\tau_y' = 72.11\,MPa}$$

Figure:-



Now for $\theta = 45°$ & $135°$.

<u>Normal stresses:-</u>

using eq (B) & put $\theta = 45°$

$$\sigma_x' = \cancel{\frac{-60+60}{2}}^{0} - \frac{60-60}{2}\cancel{\cos 2(45°)}^{0} - 40\sin 2(45°)$$

$$\boxed{\sigma_x' = -40\,MPa}$$

For $\sigma_y' = ?$ Put $\theta = 135°$.

$$\sigma_y' = \cancel{\frac{-60+60}{2}}^{0} - \frac{60-60}{2}\cancel{\cos 2(135)}^{0} - 40\sin 2(135°)$$

$$\boxed{\sigma_y' = 40\,MPa}$$

<u>Shearing stresses:-</u>

using eq (C) Put $\theta = 45°$.

$$\tau_x' = \frac{-60-60}{2}\sin 2(45°) + 40\cancel{\cos 2(45°)}^{0}$$

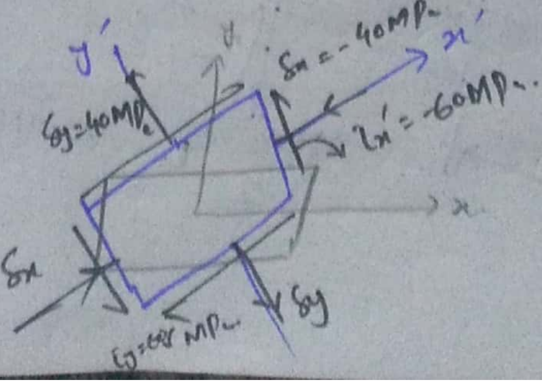
$$\boxed{\tau_x' = -60\,MPa}$$

For $\tau_y' = ?$ Put $\theta = 135°$.

$$\tau_y' = \frac{-60-60}{2}\sin 2(135°) + 40\cancel{\cos 2(135°)}^{0}$$

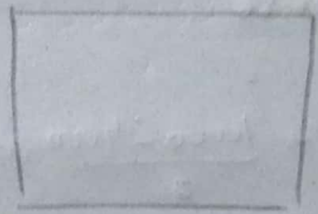
$$\boxed{\tau_y' = 60\,MPa}$$

<u>Figure:-</u>



Pb#934:- A small block is 1.6 in long, 1.2 in. high and 0.2 in thick. It is subjected to uniformly distributed load (Tensile forces) having resultant values shown in fig. Compute the stress components develop along diagonal AB.

Figure:-



Solution:-

$$S_x = \frac{P}{A} = \frac{2400}{1.2 \times 0.2}$$

$$\boxed{S_x = 10{,}000 \text{ psi}}$$

$$S_y = \frac{P}{A} = \frac{1280}{1.6 \times 0.2}$$

$$\boxed{S_y = 4000 \text{psi}}$$

Now to find $\theta = ?$

$$\tan\theta = \frac{1.2}{1.6} \Rightarrow \boxed{\theta = 36.86}$$

But we measured angle with y-axis to

Crack counter clock wise.

So add 90°.

$\theta = 36.86 + 90^\circ$.

$$\boxed{\theta = 126.86^\circ}$$

Normal stresses:-

As;
$$\delta' = \frac{\delta_x + \delta_y}{2} + \frac{\delta_x - \delta_y}{2} \cos 2\theta - \cancel{\tau_{xy}} \sin 2\theta.$$

For δ_x': Put $\theta = 126.86^\circ$.

$$\delta_x' = \frac{10000 + 4000}{2} + \frac{10000 - 4000}{2} \cos 2(126.86).$$

$$\boxed{\delta_x' = 6160 \text{ psi}}$$

$\delta_y' = ?$ Put $\theta = 126.86 + 90^\circ = 216.86$.

$$\delta_y' = \frac{10000 + 4000}{2} + \frac{10000 - 4000}{2} \cos(2(216.86))$$

$$= 7000 + 3000(0.2803).$$

$$\boxed{\delta_y' = 7840.99 \text{ Psi}}$$

Shearing stress:-

As shearing stress is;

$$\tau = \frac{\delta_x - \delta_y}{2} \sin 2\theta + \cancel{\tau_{xy}} \cos 2\theta. \longrightarrow (1)$$

For $\tau_x' = ?$ put $\theta = 126.86^\circ$.

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$$\tau_{x'} = \frac{10000 - 4000}{2} \sin 2(126.86)$$

$$\boxed{\tau_{x'} = -2879.7 \text{ psi}}$$

$\tau_{y'} = ?$ Put $\theta = 126.86 + 90°$

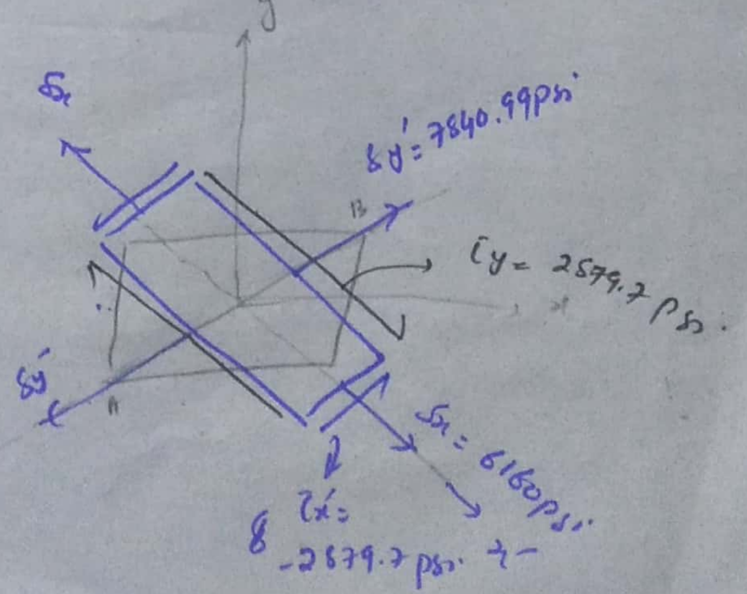
$\theta = 216.86°$.

eq ⓒ $\Rightarrow$

$$\tau_{y'} = \frac{10000 - 4000}{2} \sin 2(216.86)$$

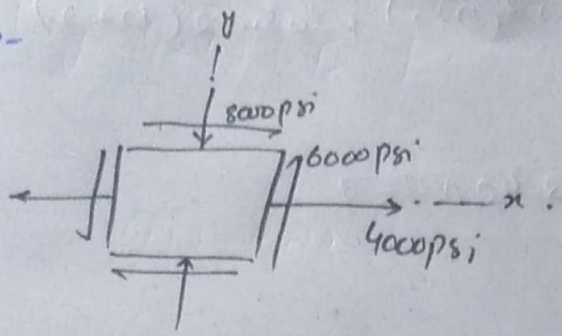
$$\boxed{\tau_{y'} = 2879.7 \text{ psi}}$$

Figure:-



Pb #932:- For the state of stress shown in fig. Find the maximum in plane shearing stress and show it on complete sketch of a differential element.

Figure:-



Given data;

$\sigma_x = 4000 psi$.

$\sigma_y = -8000 psi$.

& $\tau_{xy} = -6000 psi$.

Solution:- First find $\theta = ?$

$$\tan 2\theta = \frac{\sigma_x - \sigma_y}{2\tau_{xy}}$$

$$= \frac{4000 + 8000}{2(6000)} \Rightarrow \boxed{\theta = -22.5^\circ}$$

As -ve so add 90 $\Rightarrow \theta = -22.5 + 90$.

$\boxed{\theta = 67.5^\circ}$

For Normal stresses:-

$$\sigma' = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta.$$

By putting values. For σ_x' put $\theta = 67.5^\circ$.

$$\sigma_x' = \left(\frac{4000 - 8000}{2}\right) + \left(\frac{4000 + 8000}{2}\right)\cos 2(67.5) + 6000 \sin 2(67.5)$$

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$$\boxed{\delta x' = -2000\,psi}$$

$\delta y' = ?$ Put $\theta = 90^\circ + 67.5 = 157.5^\circ$

$$\delta y' = \frac{4000 - 8000}{2} + \frac{4000 + 8000}{2} \cos 2(157.5) + 6000 \sin 2(157.5)$$

$$\boxed{\delta y' = -2000\,psi}$$

For shearing stress:-

$$\tau' = \frac{\delta x - \delta y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

For $\tau x' = ?$
Put $\theta = 67.5^\circ$.

$$\tau x' = \left(\frac{4000 + 8000}{2}\right) \sin 2(67.5) + (-6000) \cos 2(67.5).$$

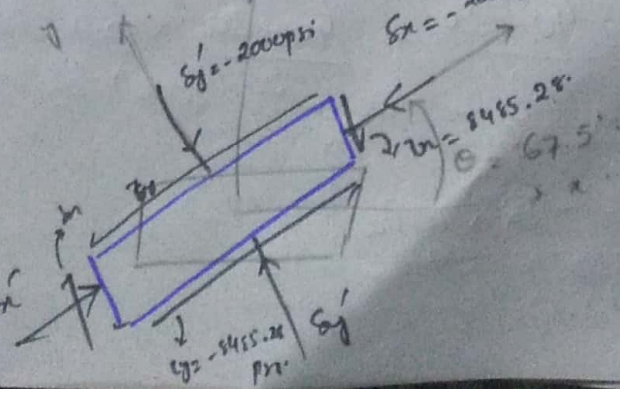
$$\boxed{\tau x' = 8485.28\,psi}$$

$\tau y = ?$ Put $\theta = 90 + 67.5 = 157.5^\circ$.

$$\tau y' = \left(\frac{4000 + 8000}{2}\right) \sin 2(157.5) - 6000 \cos 2(157.5)$$

$$\boxed{\tau y' = -8485.28\,psi}$$

Fig:-



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