Examples.

(a) Binomial Distribution.

(i) Example # 01

There is probability of 0.7 that the already buildi structure to be collapsed in a colony. If the colony contain 40 buildings (structures). Then find what distribution it is & find the expected value & variance also find probability that 6 building (structures) will collapsed?

Ans:- The distribution is binomial because we have two results either the building will collapse or not. & also the number of building $n = 40$ & success that is in this case 6 is given.

so;

As we know that

$$P(X=x) = {}^{n}C_{x}\, p^{x}\, q^{n-x}$$

Here $P = 0.7$.

By putting values

$$p(X=6) = {}^{40}C_{6}\,(0.7)^{6}\,(1-0.7)^{40-6}$$

$$p(X=6) = \frac{40!}{6!\,(34)!}\,(0.117649)(1.6677\times10^{-18})$$

Scanned by CamScanner

$$= \frac{\cancel{40}^{20} \times 39 \times 38 \times 37 \times \cancel{36}^{2\,12} \times 35 \times \cancel{34!}}{\cancel{6} \times \cancel{3} \times \cancel{2}\,(\cancel{34!})} (0.117649)(1.6677 \times 10^{-1?})$$

$$\boxed{P(X=6) = 1.5 \times 10^{-11}}$$

As

$E(x) = np$.

$E(x) = 40(0.7)$.

$$\boxed{E(x) = 28}$$

Variance.

As

$V(X) = npq$.

$= np(1-p)$.

$= 40(0.7)(1-0.7)$

$= 40(0.7)(0.3)$

$= 28(0.3)$

$$\boxed{V(x) = 8.4}$$ Ans

Scanned by CamScanner

Example #02

The UTM (universal testing Machine) that is used to how much the cylinder can bear the load. If 10 cylinders are checked & the probability of acceptance of strength is 0.3. So what is probability that exactly 4 will discarded after checking them in UTM, also find expected value & variance.

Solution:- The distribution is Binomial as we have results either the cylinder will accept if the required strength is achieve or discarded.

So,

at $X = 4 = ?$ Here $p = (1 - 0.3)$

$q = 0.3$

$P(X=4) = ?$

Since;

$$P(X=x) = {}^{n}C_{x}\, p^{x} \cdot q^{n-x},$$

$$= {}^{10}C_{4}\, (1-a)^{4} \cdot (0.3)^{10-4}$$

$$= {}^{10}C_{4}\, (0.7)^{4} \cdot (0.3)^{6}$$

$$= \frac{10!}{4!\,6!} (0.2401)(7.29 \times 10^{-4})$$

$$= \frac{\cancel{10}^{5} \times \cancel{9}^{3} \times \cancel{8}^{2} \times 7 \times \cancel{6!}}{\cancel{4} \times \cancel{3} \times \cancel{2} \times \cancel{6!}} (1.75 \times 10^{-4})$$

$$\boxed{P(X=4) = 0.03675}$$

Expected Value

As $E(x) = np.$

$= 10(0.7)$

$= 7.$

Variance:-

As $V(x) = npq.$

$= n(1-q)(q)$

$= 10(1-0.3)(0.3)$

$= 7(0.3)$

$\boxed{V(x) = 2.1}$

(b) Geometric Distribution:-

Example #01

In University of Engineering & Technology, Peshawar an experiment is performing on Soil for foundation in soil mechanics lab. If the probability that the soil is good found foundation is 0.6. So what is the probability that at 4 testing of soil we get the that soil is good for foundation (success) also find expected value & variance.

Solution:- As This is geometric distribution but we are required the probability of first succes after some experiments on soil.

So, As; $P(X=x) = pq^{x-1}$

$p = 0.6$

$q = 1 - 0.6$

$P(X=4) = p(1-p)^{x-1}$

$= 0.6(1-0.6)^{4-1}$

$= 0.6(0.4)^3$

$\boxed{P(X=4) = 0.0384}$

Expected Value:-

As $E(X) = 1/p$.

$E(X) = \frac{1}{0.6}$

$\boxed{E(X) = 1.67}$

Variance:-

As $V(X) = \frac{q}{p^2}$

$V(X) = \frac{(1-0.6)}{(0.6)^2} = \frac{0.4}{0.6^2}$

$\boxed{V(X) = 1.11}$ Ans

Example # 02 (Urban Planning related)

According to survey, In Pakistan 80% as Construction & town or city made without planning. If four cities are Selected what is the probability that city selected is made with planning like Islamabad? Also find expected value & variance

Solution:- As geometric as first success is going to discuss & its probability.

$P(X=4) = ?$

As; $P(X=x) = p\,q^{x-1}$

Here $p = 1-q$ $\quad q = 0.8$ (80%)

$p = 1-0.8$

$\boxed{p = 0.2}$

$P(X=4) = 0.2(0.8)^{4-1} \Rightarrow \boxed{P(X=4) = 0.1024}$

Variance:-

As $var(x) = q/p^2$

$= \dfrac{0.8}{(0.2)^2}$

$\boxed{Var(x) = 20}$

Expected value:-

$E(x) = \dfrac{1}{p}$

$= \dfrac{1}{0.2}$

$\boxed{E(x) = 5}$

(c). Negative Binomial Distribution:-

Example #01

In a city buildings were inspected after earth quake. If the probability of damage buildings are 0.8. What is probability of 3rd building that is to be damaged in 10 buildings that were inspected? Also find expected value & variance.

Solution:-

As here probability of 3rd success (damage) building is require so it is negative binomial distribution.

So, As

$$P(X=x) = \binom{x-1}{r-1} p^r q^{x-r}$$

Here $x = 10$, $r = 3$, $p = 0.8$ & $q = 1-p$.

$$P(X=10) = \binom{9}{2} (0.8)^3 (1-0.8)^{10-3}$$

$$P(X=10) = \frac{9!}{2!\,7!} (0.512)(1.28\times10^{-5})$$

$$= \frac{9 \times \overset{4}{\cancel{8}} \times \cancel{7!}}{\underset{1}{\cancel{2}} \times \cancel{7!}} (0.512)(1.28\times10^{-5})$$

$$\boxed{P(X=10) = 2.36\times10^{-4}}$$

As Expected Value is,

$E(X) = r/p = 3/0.8 \Rightarrow \boxed{E(X) = 3.75}$

$Var(x) = ?$

As $Var(x) = \frac{rq}{p^2}$

$= \frac{3(1-0.8)}{(0.8)^2}$

$$\boxed{Var(x) = 0.93}$$

Example #02

In dam, if the probability of starting or rotating turbine is 0.1 that it will not rotate when sufficient water is falling in spill way. So what is the probability of 3rd failure (not start) at 8 trail when sufficient water flowing? Also find $E(x)$ & $V(x)$.

Solution:- As negative binomial because we are required probability of 3rd success in 8 trails.

So, As

$$P(X = x) = \binom{x-1}{r-1} p^r q^{x-r}$$

Put $x = 8$, $r = 3$, $p = 0.1$, $q = 0.9$

$$P(X=8) = \binom{8-1}{3-1} (0.1)^3 (1-0.1)^{8-3}$$

$$= \binom{7}{2} (0.1)^3 (0.9)^5$$

$$= \frac{7!}{2!\,5!} (1\times10^{-3})(0.59)$$

$$P(x=8) = \frac{7 \times 6 \times 51}{2 \times 51}(5.9 \times 10^{-4})$$

$$= 21(5.9 \times 10^{-4})$$

$$\boxed{P(x=8) = 0.0124}$$

Expected Value:-

As $E(x) = \frac{r}{p}$

$$E(x) = \frac{3}{0.1}$$

$$\boxed{E(x) = 30}$$

Variance:-

As $Var(x) = \frac{rq}{p^2}$

$$= \frac{r(1-0.1)}{(0.1)^2}$$

$$= \frac{3(0.9)}{(0.1)^2}$$

$$= \frac{2.7}{0.01}$$

~~E(x)~~ $\boxed{Var(x) = 270}$ Ans

Scanned by CamScanner

(d). Hyper Geometric Distribution:-

Example #01

Ten civil engineers for job of construction manager are to be selected. Five have applicants apply for job, 7 of them done M.Sc & 3 did B.Sc. If 3 are randomly selected what is probability that all three are B.Sc one?

Solution:- As hyper geometric bcz there is no repetition occur

So;

As $P(X=x) = ?$

$$P(X=3) = \frac{{}^7C_0 \; {}^3C_3}{{}^{10}C_3}$$

$$= \frac{1 * \frac{3!}{3!(3-3)!}}{\frac{10!}{3!\,7!}}$$

$$= \frac{3!\,7!}{10!}$$

$$= \frac{\cancel{3} \times \cancel{2} \times \cancel{7!}}{\cancel{10} \times 8 \times \cancel{9} \times \cancel{7!}}$$

$$\boxed{P(X=3) = 0.00833}$$

Expected Value:- As we know

$\therefore E(X) = \frac{nr}{N}$

$= \frac{3 \times 3}{10}$

$E(X) = \frac{9}{10}$

$\boxed{E(X) = 0.9}$

& Variance:-

As $V(X) = \frac{N-n}{N-1} * \frac{nr}{N} * \left(1 - \frac{r}{N}\right)$

By putting values.

$= \frac{10-3}{10-1} * \frac{3 \times 3}{10} * \left(1 - 3/10\right)$

$= \frac{7}{9} * 0.9 * 0.7$

$\boxed{V(X) = 0.49}$

Example #02

If series of ~~10~~ experiments are performed on 10 concrete blocks out of them 6 concrete blocks are not satisfying the demand (Low strength). If 4 blocks are choosen what is the probability that it contain exactly 2 not workable concrete blocks?

Solution:- As hypergeometric distribution with

$N = 10, \quad n = 4, \quad r = 6 \quad x = 2$

As we know that;

$$P(X=x) = f(x) = \frac{\binom{r}{x}\binom{n-r}{n-x}}{\binom{N}{n}}$$

$$= \frac{({}^{6}C_{2})\binom{10-6}{6-4}}{{}^{10}C_{4}}.$$

$$= \frac{\frac{6!}{2!\,4!}\left(\frac{4!}{2!\,2!}\right)}{10!/4!\,6!} = \frac{\frac{6\times5\times4!}{2\times4!} \times \frac{4\times3\times2!}{2\times2!}}{\frac{10\times9\times8\times7\times6!}{6!\times4\times3\times2}}.$$

$$P(X=2) = \frac{15\times6}{30\times7} \Rightarrow \boxed{P(X=2) = 0.43}$$

$$E(x) = \frac{nr}{N} = \frac{4(6)}{10} = \boxed{E(x) = 2.4}$$

$$V(x) = \frac{N-n}{N-1} \times \frac{nr}{N} \times \left(1-\frac{r}{N}\right)$$

$$= \frac{10-4}{9} \times 2.4\times\left(1-\frac{6}{10}\right)$$

$$= 6/9 \times 2.4\times(0.4)$$

$$\boxed{V(x) = 0.64}$$

(e). Pisson Distribution:-

Example # 01

The number of cracks in aramic tóile (tile) has a poisson distribution with a mean of $\lambda = 2.4$. What is probability that tile has no crack also find expected value & varience

Solution:-

As we know that;

$$P(X=x) = f(x) = \frac{e^{-\lambda} \lambda^{x}}{x!}$$

As $X = 0$
& $\lambda = 2.4$

$$P(X=0) = f(x) = \frac{e^{-2.4} \lambda^{0}}{0!} \Rightarrow e^{-2.4}$$

$$\boxed{P(X=0) = 0.091}$$

As expected value & Variance of Pisson distribution is λ.

so,

$$E(X) = \lambda$$

$$\boxed{E(X) = 2.4}$$

Also

$$var(X) = \lambda$$

$$\boxed{Var(x) = 2.4}$$

Example #02

A wall contain 1000 bricks, each one has the probability of 0.005 as being defective. What is the probability that wall contain exactly 4 defective bricks. Also find expected value & variance?

Solution:- As we know that.

$$P(X=x) = \frac{e^{-\lambda}\lambda^x}{x!}$$

As $x = 4$. To find $\lambda = ?$

As $\lambda =$ Average or expected value

which $np = \lambda$.

So, $\lambda = 1000 \times 0.005$

$\lambda = 5$

By putting values

$$P(X=4) = \frac{e^{-5}(5)^4}{4!}$$

$$= \frac{6.738 \times 10^{-3}(625)}{4 \times 3 \times 2}$$

$$\boxed{P(X=4) = 0.175}$$

The expected value;

As $E(X) = \lambda$

So, $E(x) = \lambda$

$$\boxed{E(x) = 5}$$

& $Var(x) = \lambda$

$$\boxed{Var(x) = 5}$$

Ans