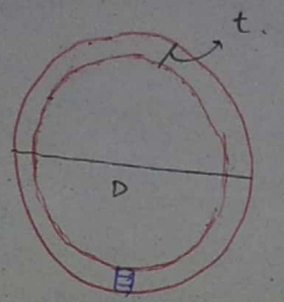
Thick & thin wall Cylinder:-

There are two different types of Cylinder depending on diameter and thickness.

i. Thin wall cylinder.

The cylinder will be consider thin walled if $D/t < 20$.



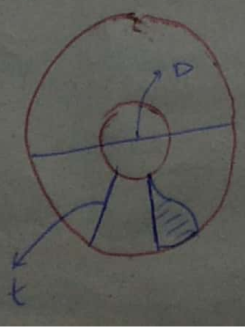
These cylinders are used to transmit some fluids under pressure, in piping system, in hydraulic, nuclear power plant, earth quake engineering centre where fluid are kept & transferred in pressure

For thin wall cylinder, the stresses are consider to be uniformly distributed

ii. Thick wall Cylinder:-

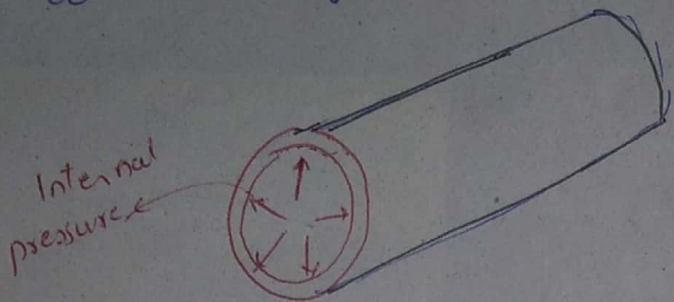
The cylinder will be consider thick wall if $D/t > 20$

For Thick walled Cylinder, the stresses are not consider to be uniformly distributed.

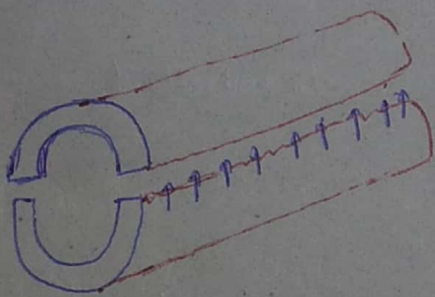


Analyze of Thin Walled Cylinder:-

Let consider a thin walled cylinder which is subjected to internal pressure P. The cylinder can be fail ~~into~~ by two different ways.



Failures:- Failure in longitudinal direction.

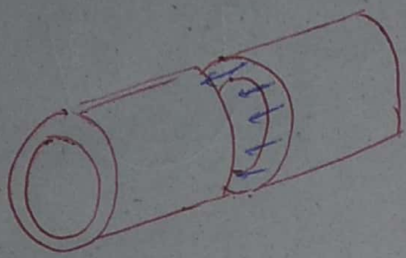


This type of failure is called longitudinal failure. longitudinal failure takes place when the stress components which is acting on the surface of cylinder greater than strength of material. As these stress components are parallel to surface of cylinder (tangent to surface of cylinder so these stress components are called tangential stresses, ~~circumferential~~, girth or hoop stresses.

• Note :-

Tangential stresses produce or cause longitudinal failure.

② Circumferential or tangential failure :-

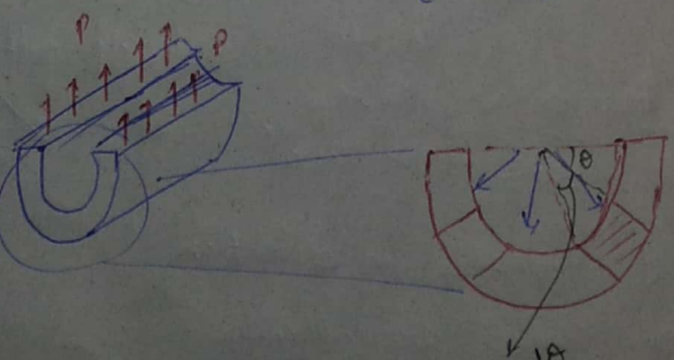


This failure takes place when internal longitudinal stresses are more than strength of material. As the stresses are parallel in direction of longitudinal axis so the stresses are called Longitudinal stresses & these stresses produced Circumferential or tangential failure.

Derivation of Equation:-

(a). For Tangential stresses:-

Let consider the FBD of bottom part.



where P is internal pressure

As; $P = dF/dA$ ∴ For differential element

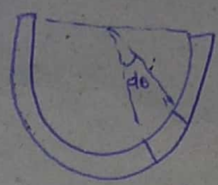
$dF = P.dA \longrightarrow (A)$

∴ S = Arc length

To find $dA = ?$

As,

$dA = S \times L \longrightarrow (1)$

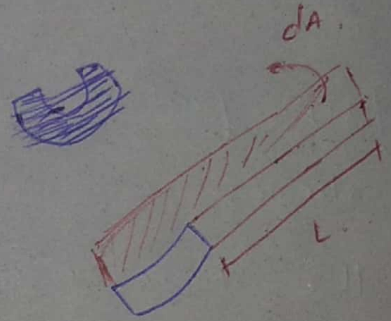


where

$S = r\theta$.

$S = D/2 \cdot d\theta$.

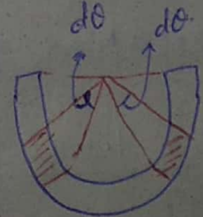
& (1) ⇒ $dA = D/2 \, d\theta \times L$



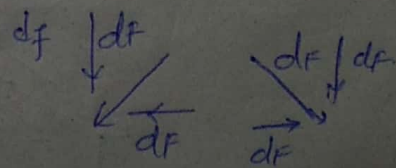
& (A) ⇒

$dF = P \, D/2 \, d\theta \longrightarrow (B)$

If we consider another differential element which also makes $d\theta$ but on opposite side.



∴ The force on both differential element is also dF.



As for horizontal case (component) both have equal magnitude & opposite in direction so they will cancel effect of each other. So the only force in vertical direction

Contribute to stresses.

so; $dF_y = P_i (D/_2 L) \sin\theta \, d\theta$

Apply $\int$ on b/s.

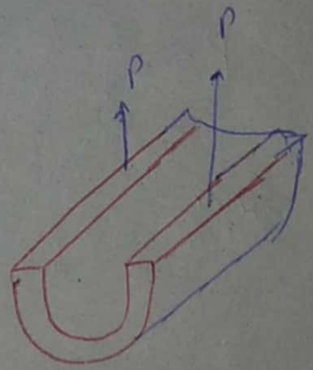
$$\int dF_y = \int_0^{\pi} P_i (D/_2) L \sin\theta \, d\theta$$

$$F_y = P_i (D/_2 L) \int_0^{\pi} \sin\theta \, d\theta$$

$$= -P_i (D/_2 L) (\cos\theta) \Big|_0^{\pi}$$

$$= -P_i \frac{D}{2} L (-1-1)$$

$$\boxed{F_y = P_i D L}$$



For equillibrium.

$\Sigma F_y = 0$

$F_y = 2p$

$2p = P_i D L$

$$P = \frac{P_i \cdot D L}{2}$$

$\delta_T = P/A$

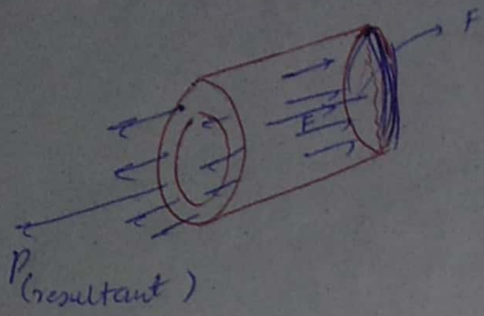
$\because A = t \times L$

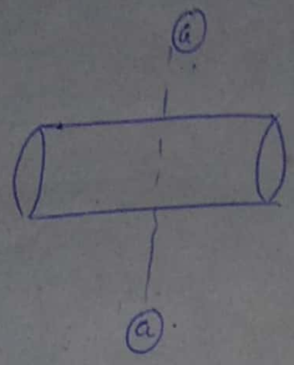
As P is applied through thickness of cylinder along length.

$$= \frac{P_i \cdot D \cancel{L}}{\cancel{2}(2t \times \cancel{L})}$$

$$\boxed{\delta_T = \frac{P_i D}{2t}}$$

(b) Longitudinal stresses:-





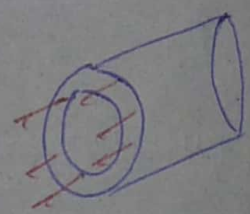
Consider left side

$F = P_i \times A$

$= P_i \times \pi \dfrac{D^2}{4}$

$\Sigma F_x = 0$

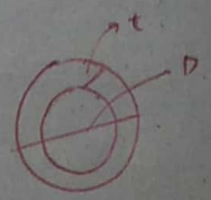
$P = F = \dfrac{P_i \pi D^2}{4}$



Since;

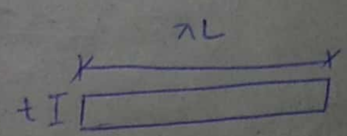
$\sigma_L = P/A$

$= \dfrac{P_i \cancel{\pi} D^{\cancel{2}}}{4 \cancel{\pi} \cancel{D} t}$



So $A = \pi D t$

$$\boxed{\sigma_L = \frac{P_i D}{4t}}$$

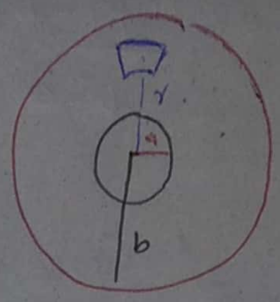


Tangential stresses are larger than longitudinal stresses.
If we have cylinder than it will fail due to tangential stresses & longitudinal failure will take place.

Thick walled Cylinder:-

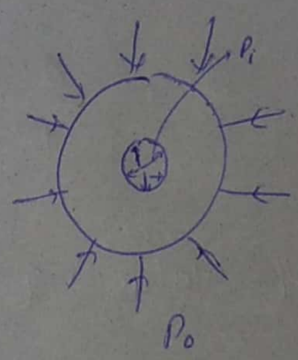
In thick walled cylinder, the stresses varies across the thickness

Consider thick walled cylinder of external diameter "b" & Internal diameter "a".



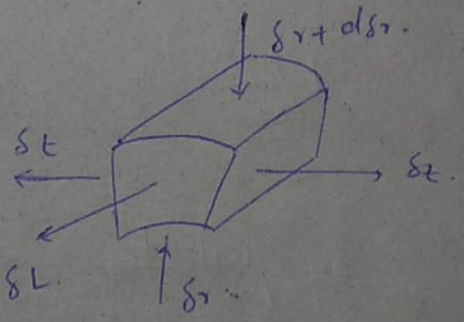
→ It is subjected to Internal pressure P_i & also to external pressure P_o as shown.

Consider a differential element at r from centre.



· Tangential stresses are assume to varies along the thickness.

· Radial stresses are compressing the thickness of cylinder.



→ Also there is longitudinal stresses in direction of board.

For longitudinal stresses.

$$\boxed{\delta_L = \frac{P_i D}{4t}}$$

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$$\delta_r = \frac{a^2 p_i - b^2 p_o}{b^2 - a^2} - \frac{a^2 b^2 (p_i - p_o)}{(b^2 - a^2) r^2}$$

$$\delta_t = \frac{a^2 p_i - b^2 p_o}{b^2 - a^2} + \frac{a^2 b^2 (p_i - p_o)}{(b^2 - a^2) r^2}$$

Special Cases:-

when $P_o = 0$. (external pressure = 0).

$$\delta_r = \frac{a^2 p_i}{b^2 - a^2}\left(1 - \frac{b^2}{r^2}\right) \longrightarrow ①$$

$$\delta_T = \frac{a^2 p_i}{b^2 - a^2}\left(1 + \frac{b^2}{r^2}\right) \longrightarrow ②$$

If value of r is either a & b.

$r \to a \to b$.

$b^2/r^2 \geq 1$.

& ① => δ_r will be always compression.

δ_r will be minimum at external diameter.

δ_r will be maximum at internal diameter.

For tensile stresses:-

δ_T will be always tensile.

δ_T → Min - ext surface

→ Max - Internal surface

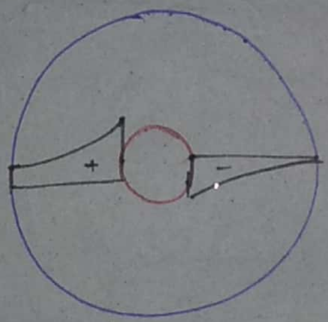
δ_t & δ_r both will be maximum at Internal diameter.

$$\tau_{max} = \frac{\delta_1 - \delta_2}{2} = \frac{(\delta_t)_{max} - (\delta_r)_{max}}{2}$$

__Note:-__ Tangential stresses will be always larger than radial stresses.

__Stress profile:-__

Radial stresses are compressive.



__Case # 02__ When $P_i = 0$.

$$\delta_r = \frac{-b^2 P_o}{b^2 - a^2}\left(1 - a^2/r^2\right)$$

$$\delta_T = \frac{-b^2 P_o}{b^2 - a^2}\left(1 + a^2/r^2\right)$$

$r = a \to b$.

Than
value of r will be;

$$\frac{a^2}{r^2} \leq 1.$$

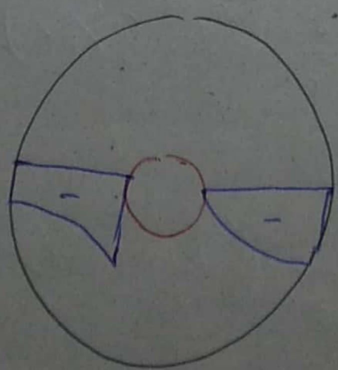
Radial stresses will be always Compressive.

σ_r — Minimum at Internal diamete.
— Max at ext diamete.

σ_T will always Compression.

σ_T — Min at ext surface.
— Max. at Internal surface

<u>Profile stress:-</u>

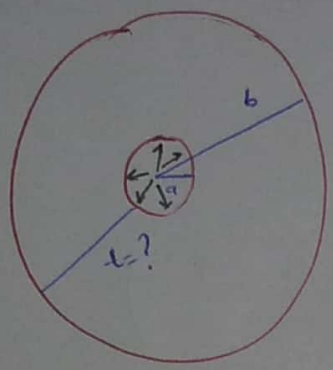


Lecture #12

Pbs on thick & Thin walled Cylinder.

Pb#01 There is cylinder which contain certain fluid inside it, the internal pressure is 60MPa. The inside diameter is 300mm. Determine the thickness of cylinder if the max. shearing stresses are 90MPa.

Figure:-



Given data

$D_i = 300mm$

$P_i = 60MPa$

$a = 150mm$

$t = ?$

$\tau_{max} = 90MPa.$

Solution:- As we know that;

$$t = b - a \longrightarrow (1)$$

To find $b = ?$

As $\tau_{max} = \frac{\sigma_1 - \sigma_2}{2}.$

$$\tau^{max} = \frac{(\sigma_t)^{max} - (\sigma_r)^{max}}{2} \longrightarrow ②.$$

As;

$$\sigma t = \frac{a^2 P_i}{b^2 - a^2}\left(1 + \frac{b^2}{r^2}\right).$$

$\sigma t \to max$ when $r \to a$.

$$(\sigma_t)^{max} = \frac{a^2 P_i}{b^2 - a^2}\left(1 + \frac{b^2}{a^2}\right) \longrightarrow ③.$$

$$\sigma_r = \frac{a^2 P_i}{b^2 - a^2}\left(1 - \frac{b^2}{r^2}\right).$$

$\sigma r \to max$ when $r \to a$.

$$(\sigma_r)^{max} = \frac{a^2 P_i}{b^2 - a^2}\left(1 - \frac{b^2}{a^2}\right) \longrightarrow ④.$$

Put eq ③ & ④ in ②.

$$\tau^{max} = \frac{\frac{a^2 P_i}{b^2 - a^2}\left(1 + \frac{b^2}{a^2}\right) - \frac{a^2 P_i}{b^2 - a^2}\left(1 - \frac{b^2}{a^2}\right)}{2}.$$

$$= \frac{a^2 P_i}{2(b^2 - a^2)}\left[\cancel{1} + \frac{b^2}{a^2} - \cancel{1} + \frac{b^2}{a^2}\right)$$

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$$\tau_{max} = \frac{a^2 P_i^{\cancel{2}}}{\cancel{2}(b^2-a^2)}\left(\frac{\cancel{2}\,b^2}{\cancel{a^2}}\right)$$

$$\boxed{\tau_{max} = \frac{P_i b^2}{b^2-a^2}}$$

By putting values.

$$90 = \frac{b^2 P_i}{b^2-a^2}.$$

$$90b^2 - 90a^2 = b^2 P_i. \quad \text{(Re-arranging)}$$

$$b^2[90 - P_i] = 90a^2.$$

$$b = \sqrt{\frac{90a^2}{90-P_i}}.$$

$$b = \sqrt{\frac{90 \times 150^2}{90-60}}.$$

$$\boxed{b = 259.8\,mm}$$

Using eq ①.

$$t = b - a.$$

$$t = 259.8 - 150.$$

$$\boxed{t = 109.8\,mm}\ \underline{\underline{Ans}}$$

Compound Cylinder:-

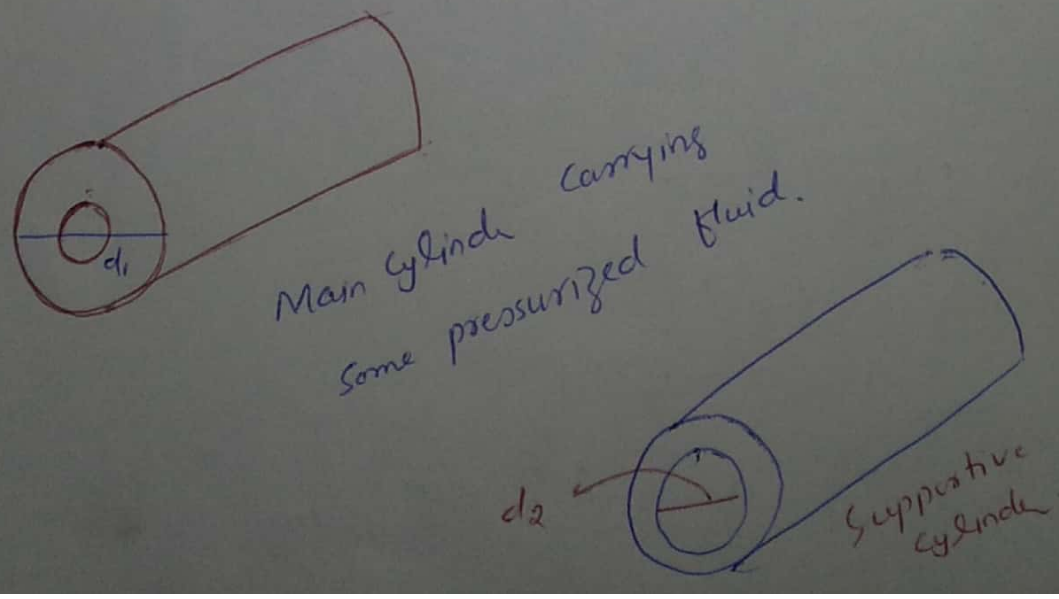
The cylinder which is made of number of cylinders to carry pressurized fluid.

This can made by we have main cylinder which transfer pressurized fluid and it is than wraped by another cylinder.

→ In this way we are strengthening the internal cylinder

- The external cylinder trying to compress the int cylinder it is like prestressing.

→ By doing so, we strengthen it and it than carry more pressurized fluid.



$d_2 < d_1$.

We put the main cylinder into supportive cylinder by heating the supportive cylinder so that it gets expand and than placed it in so when cool down it shrinks and exert some pressure on smaller cylinder.

So some prestress are develop inside the smaller cylinder. & make it strengthen.

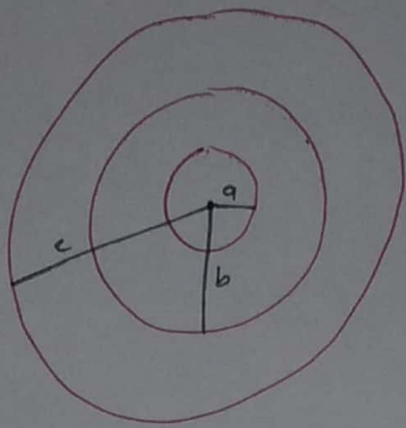
Pb# A compound thick cylinder is fabricated by shrinkage and fitting a steel jacket of 300mm dia onto a steel tube of 100mm internal diameter & 200mm of external diameter.

A contact pressure of 49.2MPa is generated on the contact surface of junction.

1. Calculate maximum tangential stresses in tube.
2. Calculate max. tangential stresses in jacket.

(3). If the internal pressure of 280MPa is applied what will be max. tangential stresses in the tube?

Figure:-



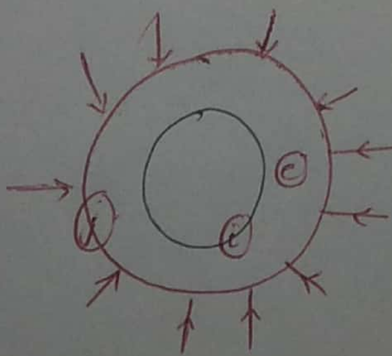
$a = 50\,mm.$

$b = 100\,mm.$

$c = 150\,mm.$

(a) Calculate max. tangential stresses in tube.

Consider.



Pressure act by Jacket on Inside Cylinder.

So the stresses after shrinkage.

As $P_i = 0$.

so; $$\sigma''_{t(i)} = \frac{-b^2 P_o}{b^2 - a^2}\left(1 + \frac{a^2}{r^2}\right) \longrightarrow (1)$$

(ii) → shrinkage.

σ_t (c)(i) → surface $= \dfrac{-b^2 P_o}{b^2-a^2}\left(1+\dfrac{a^2}{r^2}\right) \longrightarrow (i)$

↓ cylinder

Tensile stress

$a = 50mm$.

$b = 100mm$.

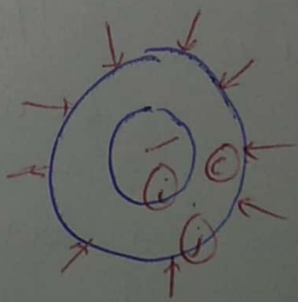
$P_o = 49.2 MPa$.

$r_i = 50mm$.

$\xi (i) \Rightarrow \sigma''_{tci} = \dfrac{-(100)^2(49.2)}{(100)^2-(50)^2}\left(1+\dfrac{(50)^2}{(50)^2}\right)$

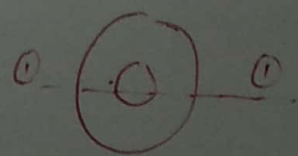
$$\boxed{\sigma''_t ci = -131.2 MPa}$$

For surface J.



$\sigma''_t cj = -\dfrac{b^2 P_o}{b^2-a^2}\left(1+\dfrac{a^2}{r^2}\right)$

$\hookrightarrow (2)$

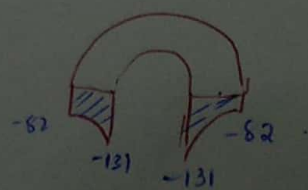


$a = 50 mm$.

$b = 100mm$.

$r_j = 100mm$

$P_o = 49.2$

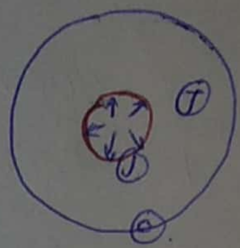


$\xi (2) \Rightarrow \boxed{\sigma''_t cj = -82MPa}$

Now Consider pressure act on jacket.

when Internal pressure acts

$$\sigma''_{tJi} = \frac{a^2 P_i}{b^2 - a^2}\left(1 + b^2/r^2\right) \rightarrow ③.$$

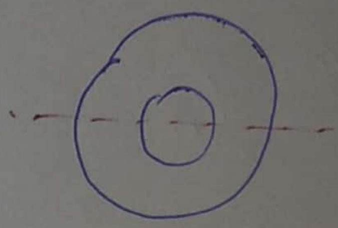


$a = 100mm.$

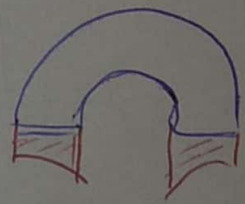
$b = 150mm.$

$r_j = 100mm.$

$P_i = 49.2\ MPa.$



eq ③ $\Rightarrow$ $\boxed{\sigma''_{tJi} = 127.92MPa}$



For Surface O :-

$$\sigma''_{tJO} = \frac{a^2 P_i}{b^2 - a^2}\left(1 + \frac{b^2}{r^2}\right) \longrightarrow ④.$$

$a = 100mm.$

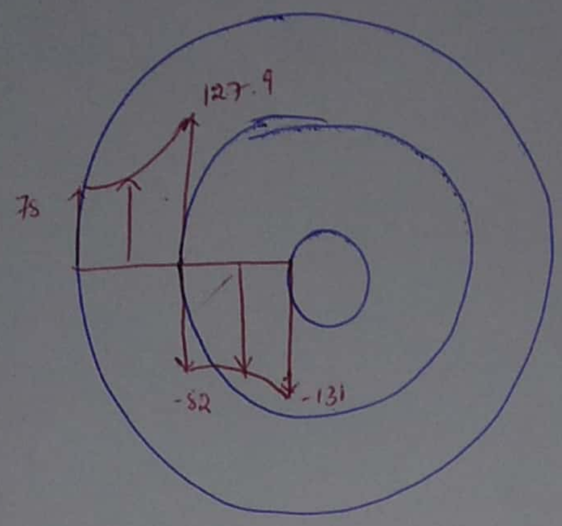
$b = 150mm.$

$r_o = 150mm.$

$P_i = 49.2MPa$

eq ④ $\Rightarrow$ $\boxed{\sigma''_{tJo} = 75.72MPa}$

Stress Profile:-



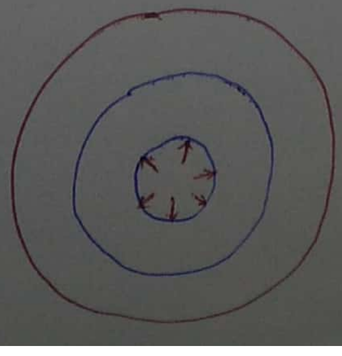
(a) Max. tangential stress in tube.

$$\boxed{\sigma^{t}_{max} = 131 MPa}$$ (Compression)

(2) Max. tangential stresses in jacket.

$$\boxed{\sigma^{t}_{max} = 127.8 MPa}$$ (Tension).

(3) Stresses Due to Internal Pressure Only.

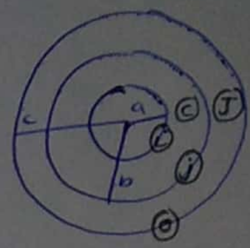


In this case stresses are produce due to internal pressure of fluid also due to tangential and axial stresses which is due to contact pressure b/w tube cylinder & Jacket. So first calculate stresses due to Int pressure & than add with these

$a = 50mm$

$b = 100mm$

$c = 150mm.$



For tube Cylinder :-

$$\sigma'_{tci} = \frac{a^2 P_i}{b^2 - a^2}\left(1 + b^2/r^2\right).$$

(5) → show stresses due to Internal pressure only.

$r_i = 50mm.$

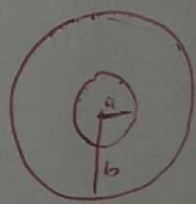
$$\boxed{\sigma'_{tci} = 350MPa}$$

$$\sigma'_{tcj} = \frac{a^2 P_i}{b^2 - a^2}\left(1 + b^2/r^2\right)$$

$r_j = 100mm.$

$$\boxed{\sigma'_{tcj} = 113.75MPa}$$

Tube



$a = 50mm$

$b = 100mm.$

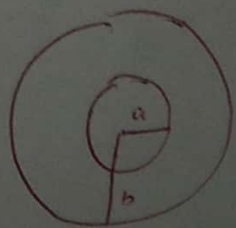
For jacket :-

$$\sigma'_{tJi} = \frac{a^2 P_i}{b^2 - a^2}\left(1 + b^2/r^2\right).$$

$r_i = 100mm.$

$$\boxed{\sigma'_{tJi} = 113.75MPa}$$

For Jacket



$a = 100mm$

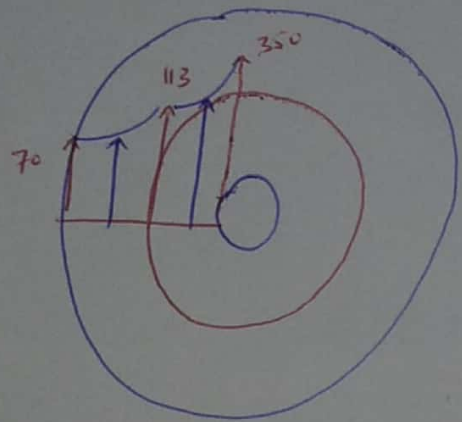
$b = 150mm.$

$$\sigma_t'{}_{ro} = \frac{a^2 P_i}{b^2 - a^2}\left(1 + \frac{b^2}{r^2}\right)$$

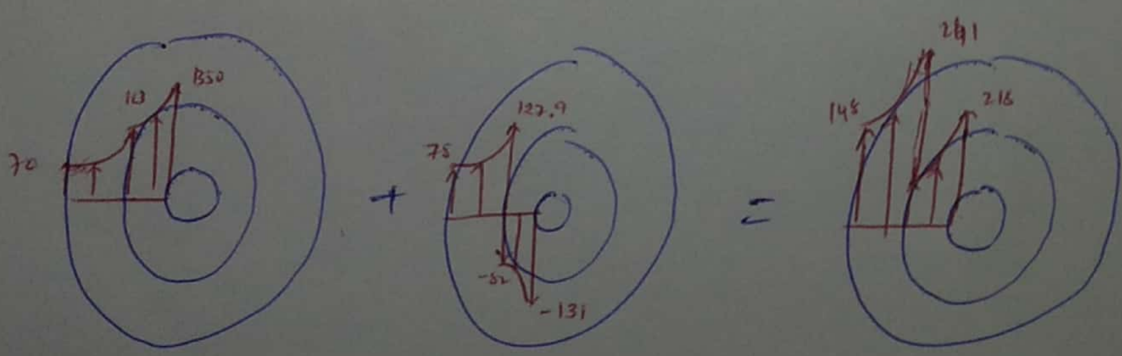
$r_o = 150mm.$

$$\boxed{\sigma_t'{}_{ro} = 70MPa}$$

Stress Profile:-



Now For max. tangential stresses In tube



$\sigma_{tci} = \sigma''_{tci} + \sigma'_{tci} = 218.5 MPa$

$\sigma_{tj} = \sigma''_{tcj} + \sigma'_{tcj} = 3175.$

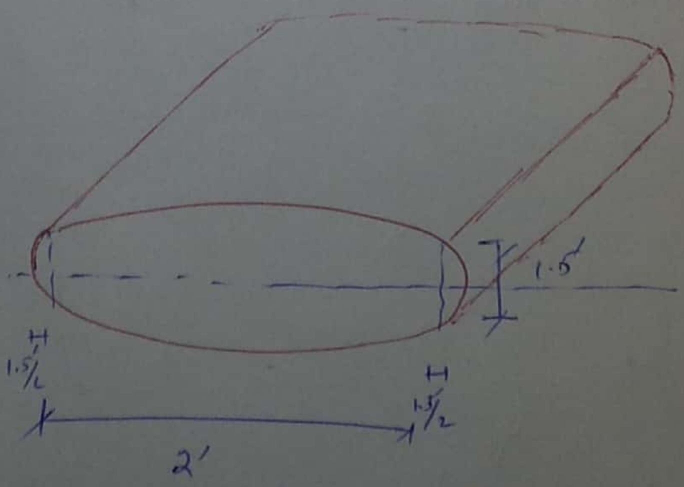
$\sigma_{tJi} = \sigma''_{tJj} + \sigma'_{tJj} = 241.7\,MPa.$

$\sigma_{tJi} = \sigma''_{tJo} + \sigma'_{tJo} = 148.72\,MPa.$

So the maximum tangential stresses in tube are 218 MPa. Ans. (Also known from stress profile).

Pb#141:- The tank shown in fig is fabricated from 1/8" steel plate. Calculate the maximum Longitudinal & Circumferential stress caused by an Internal pressure of 125 psi.

Figure:-

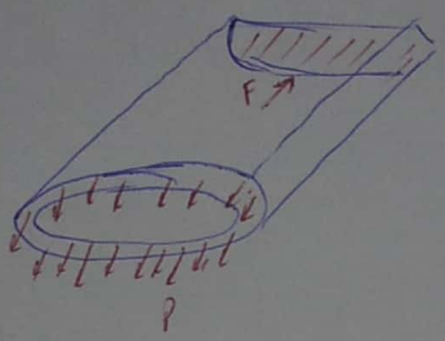


Solution:-

For longitudinal stresses:-

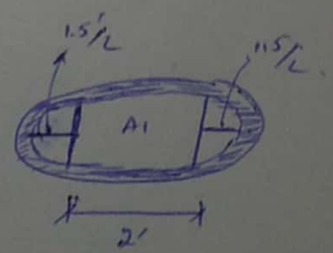
Consider the tangential / circumferential failure

Draw FBD at section 1 – 1.



F = Pressure (Area over it is acting).

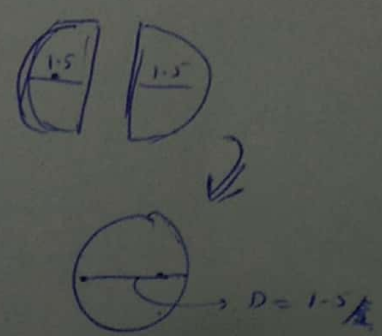
$$F = P_i (A_1 + A_2) \longrightarrow (1).$$



$A_1 = 2' \times 1.5'$

$A_1 = 3\ ft^2.$

$$A_2 = \frac{\pi D^2}{4} = \frac{\pi (1.5)^2}{4}$$



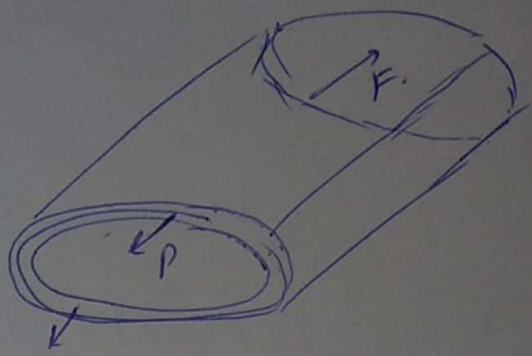
$A_2 = 1.77\ ft^2.$

eq (1) $\Rightarrow F = P_i (3 + 1.77)\ ft^2.$

$F = P_i(4.77)\,ft^2.$

As $\Sigma F_x = 0.$

$P = F.$



$\sigma_l = \dfrac{P}{A}.$

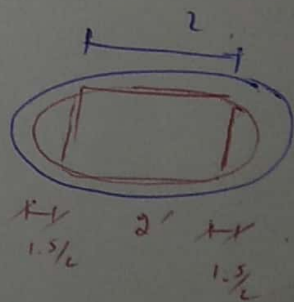
$\sigma_l = F/A$ As $P = F.$

$= \dfrac{P_i(4.77)\,\cancel{\ }\,ft^2}{A}. \rightarrow (2)$ $A =$

$A = A_1 + A_2.$

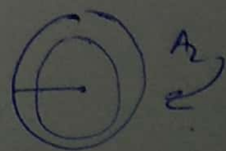
$A = 2 \times 2 \times (1/8) \times 12^2.$ t.

$+ \pi(1.5) * 12 \times (1/8).$



$\{ (2) \Rightarrow$

$\sigma_l = \dfrac{P_i(4.77) \times 12^2}{4 \times (1/8) \times 12^2 + \pi(1.5) * 12(1/8)}.$ 125



$A_2 = \pi(D)(t)$

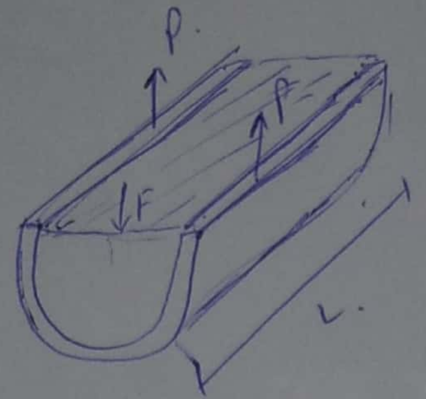
$= \pi(1.5)(1/8) \times 12$

$\boxed{\sigma_l = 1085.9\,Psi.}$

For longitudinal failure.

The longitudinal failure will cause by tangential stresses.

Consider longitudinal failure.



For tangential stress case.

$\because F = P_i \times D \times L.$

For P

$A = 2 \times t \times L.$

As; $F = P_i\, A.$

$= P_i (2+1.5) \times 12 \times L.$

$\sum F_y = 0.$

$2P = F.$

$P = F/2.$

$\sigma t = P/A.$

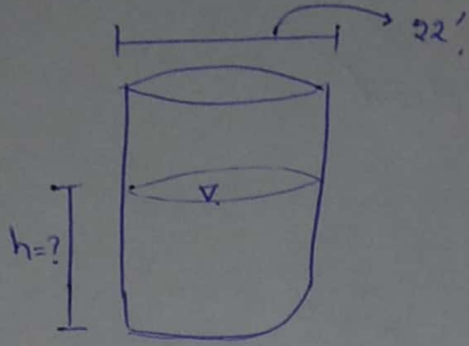
$= \dfrac{F}{2A} = \dfrac{P_i (2+1.5) \times \cancel{12}^{6} \times \cancel{L}}{\cancel{2}_{1} \times t \times \cancel{L}}.$

$= \dfrac{125(3.5) \times 6}{1/8}$

$\boxed{\sigma_t = 21\,\text{Ksi}}$ Ans 2.

Pb # 137:- A water tank 22 ft in diameter is made from steel plate that is 1/2" thick. Find the max. height the tank may be filled if the circumferential stresses are limited to 6000 psi. The specific weight of water is $62.4 \frac{lb}{ft^3}$.

Figure:-



$t = 1/2"$

$\gamma = 62.4\ lb/ft^3$.

$\sigma_{circumferential} \ngtr 6000 psi$.

Solution:- As $\sigma_t = \frac{P}{A} \rightarrow (1)$.

But $P = \rho g h \Rightarrow P = \gamma h$.

eq (1) $\Rightarrow$ $\sigma_t = \frac{\gamma h D}{2t} = \frac{62.4 \times 22 \times h \times 12}{12^3 \times 2(1/2)}$.

$\sigma\ 6000 = 9.53h$.

$h = 629.37"$

or $h = \left(\frac{629.37}{12}\right)'$.

$\boxed{h = 52.4'}$ Ans