Flow and Storage of Powders

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Quick review

- Flow problems
  - Ratholing
  - Arching

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- Flow problems
  - Ratholing
  - Arching
- Flow patterns
  - Mass flow
  - Funnel flow

Quick review

- Flow properties
  - Cohesive strength
  - Internal friction
  - Compressibility
  - Wall friction
  - Permeability

Quick review

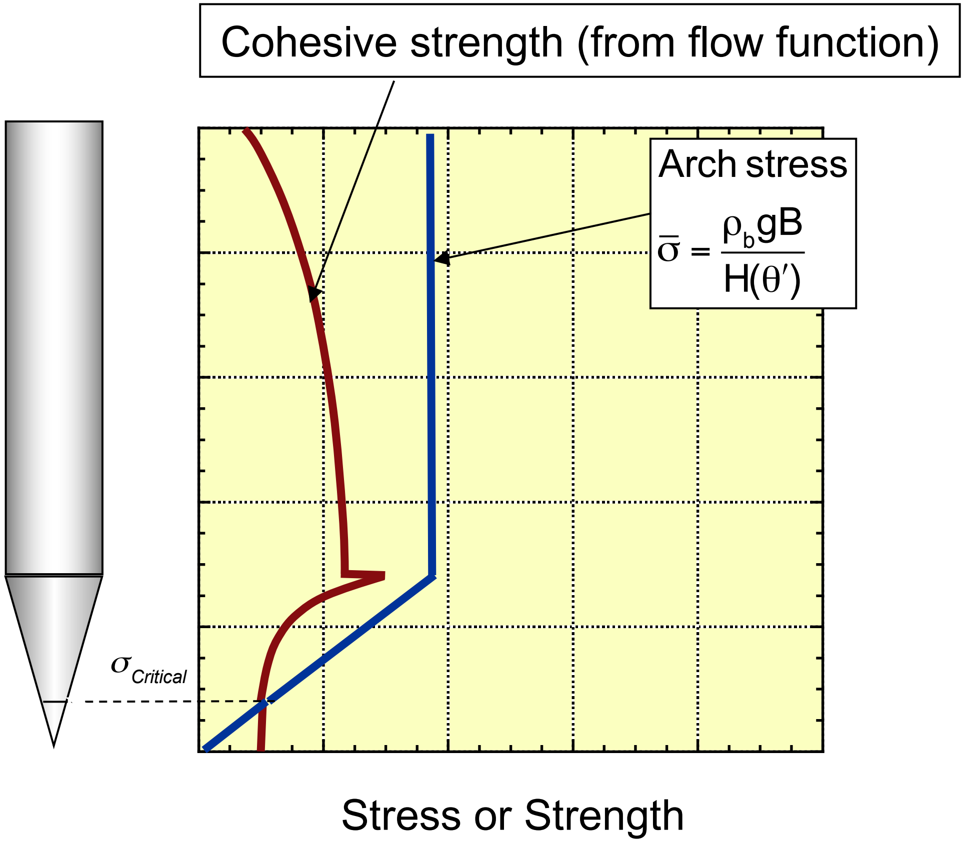
- Flow properties
  - Cohesive strength
  - Internal friction
  - Compressibility
  - Wall friction
  - Permeability
- Mass flow hopper design
  - Outlet size
  - Hopper angle

Mass flow hopper design

$$B_{min} = \frac{H(\theta')\sigma_{Critical}}{\rho_b g}$$

Critical stress

Stress = Strength

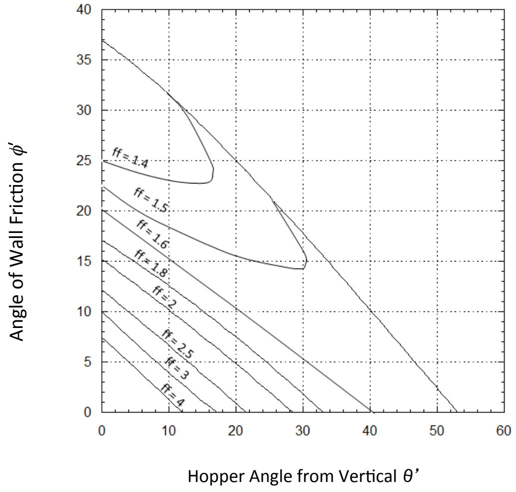


Cohesive strength (from flow function)

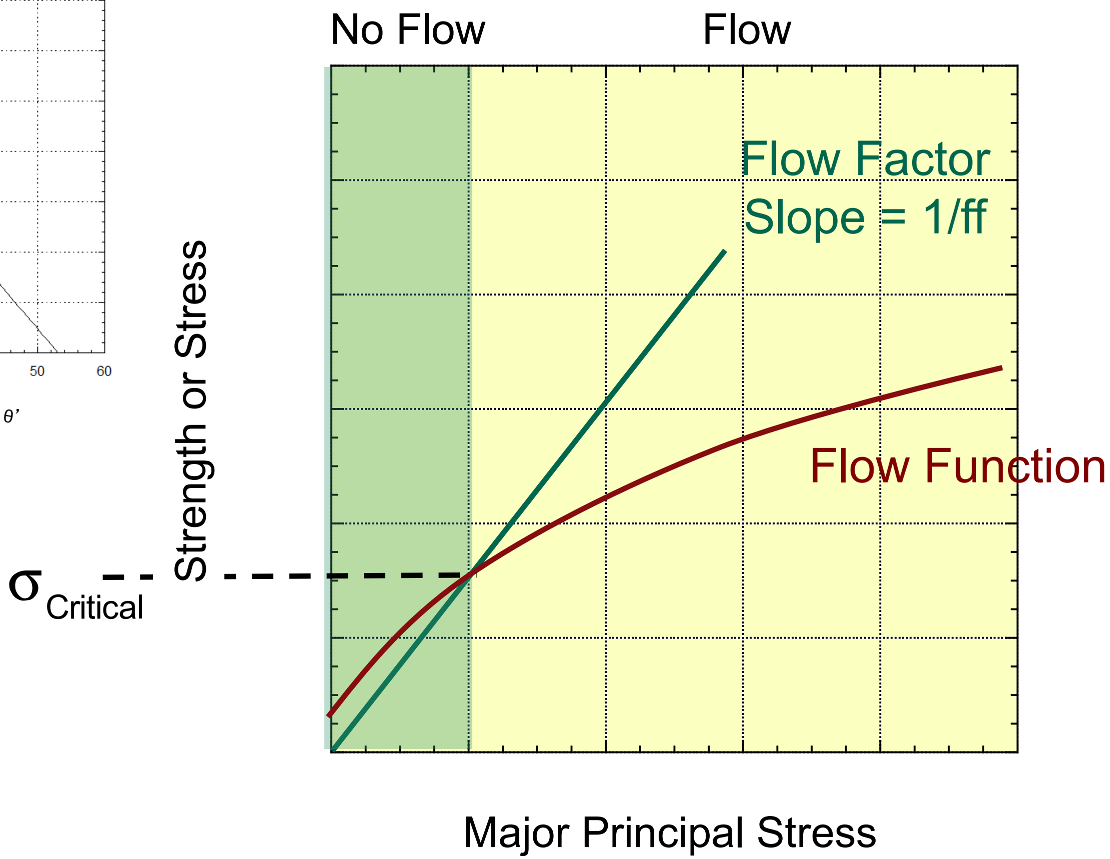
Arch stress

$$\bar{\sigma} = \frac{\rho_b g B}{H(\theta')}$$

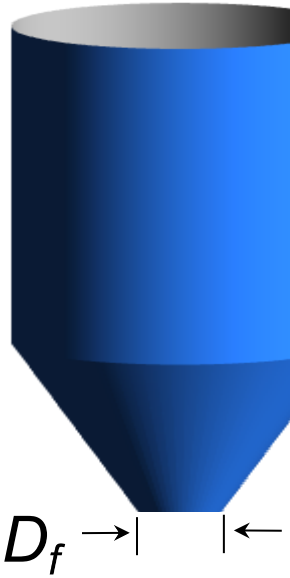
Mass flow hopper design



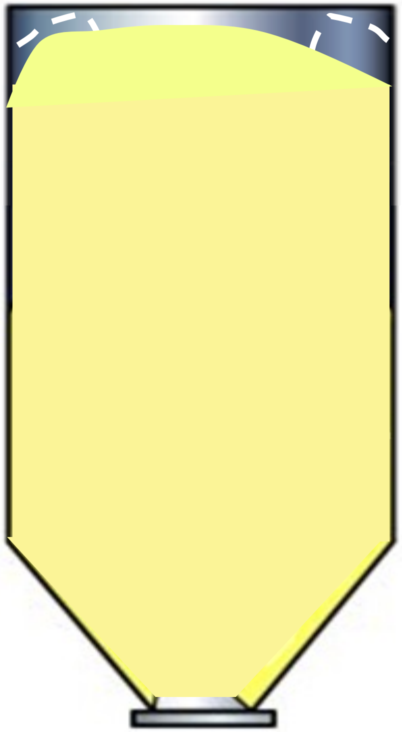
$$B_{min} = \frac{H(\theta')\sigma_{Critical}}{\rho_b g}$$

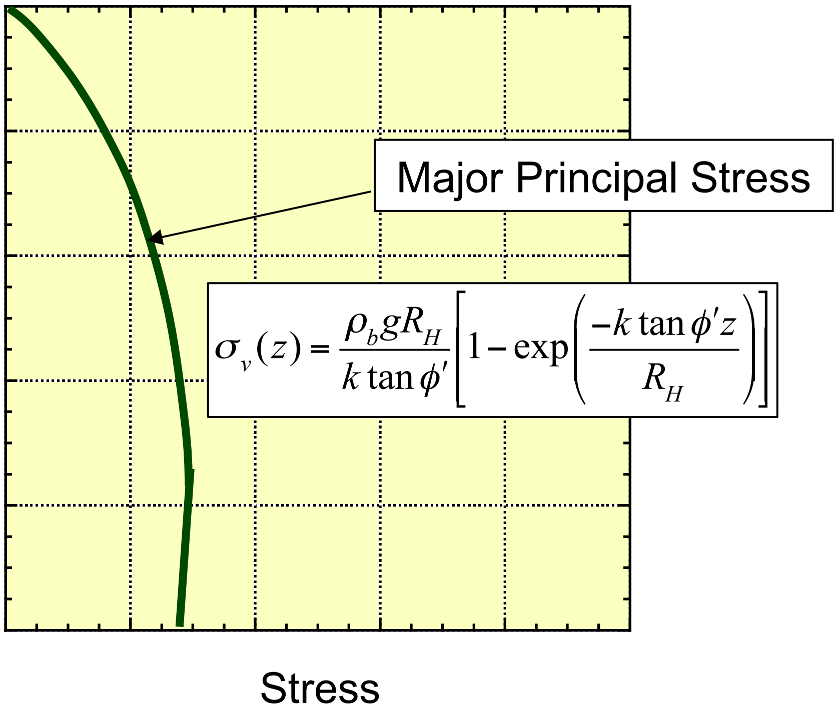


Funnel flow hopper design



Funnel flow bins



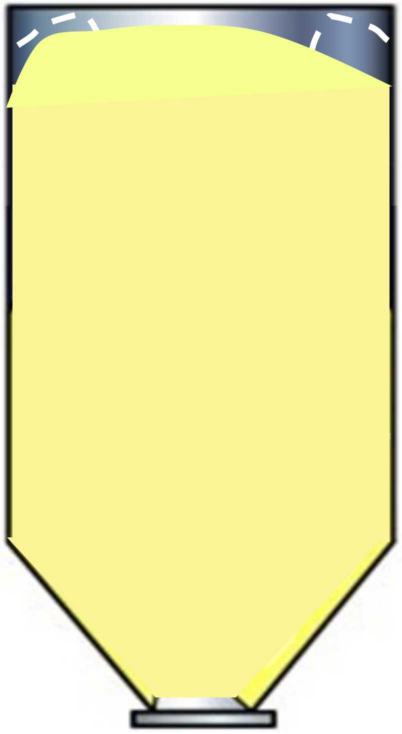


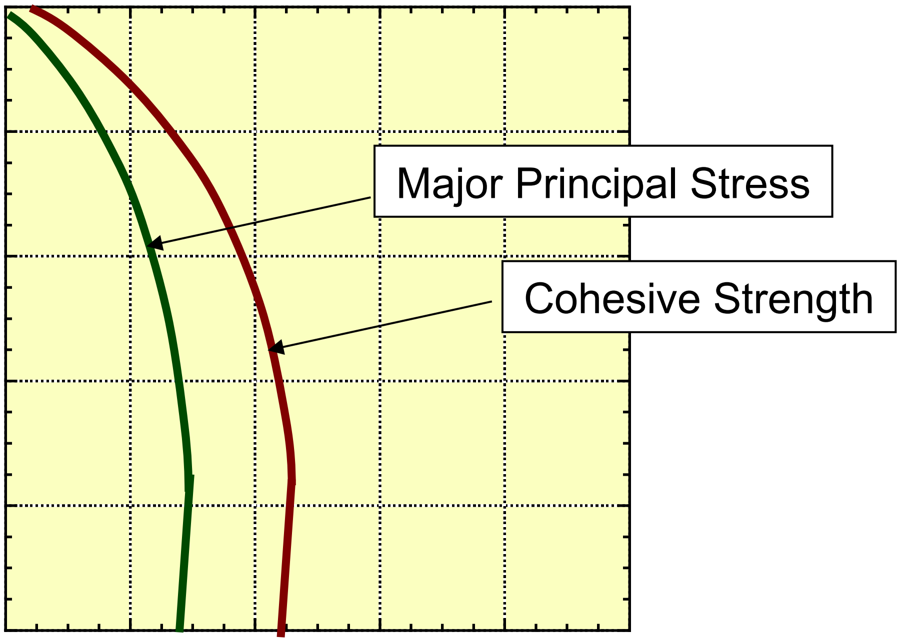
Major Principal Stress

$$\sigma_v(z) = \frac{\rho_b g R_H}{k \tan\phi'}\left[1 - \exp\left(\frac{-k\tan\phi' z}{R_H}\right)\right]$$

D = diameter of round outlet or
diagonal of slotted outlet

Funnel flow bins





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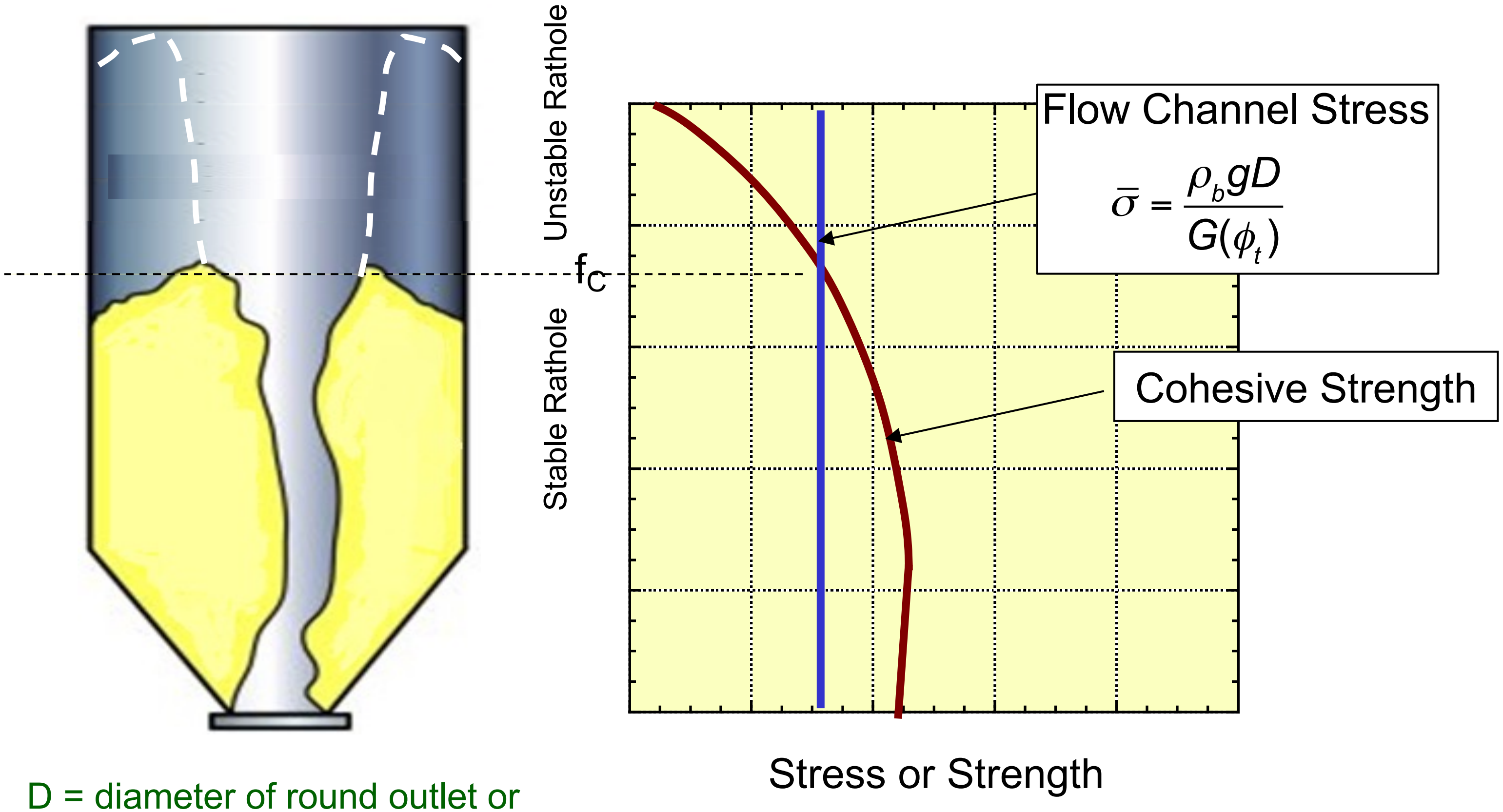
Funnel flow bins



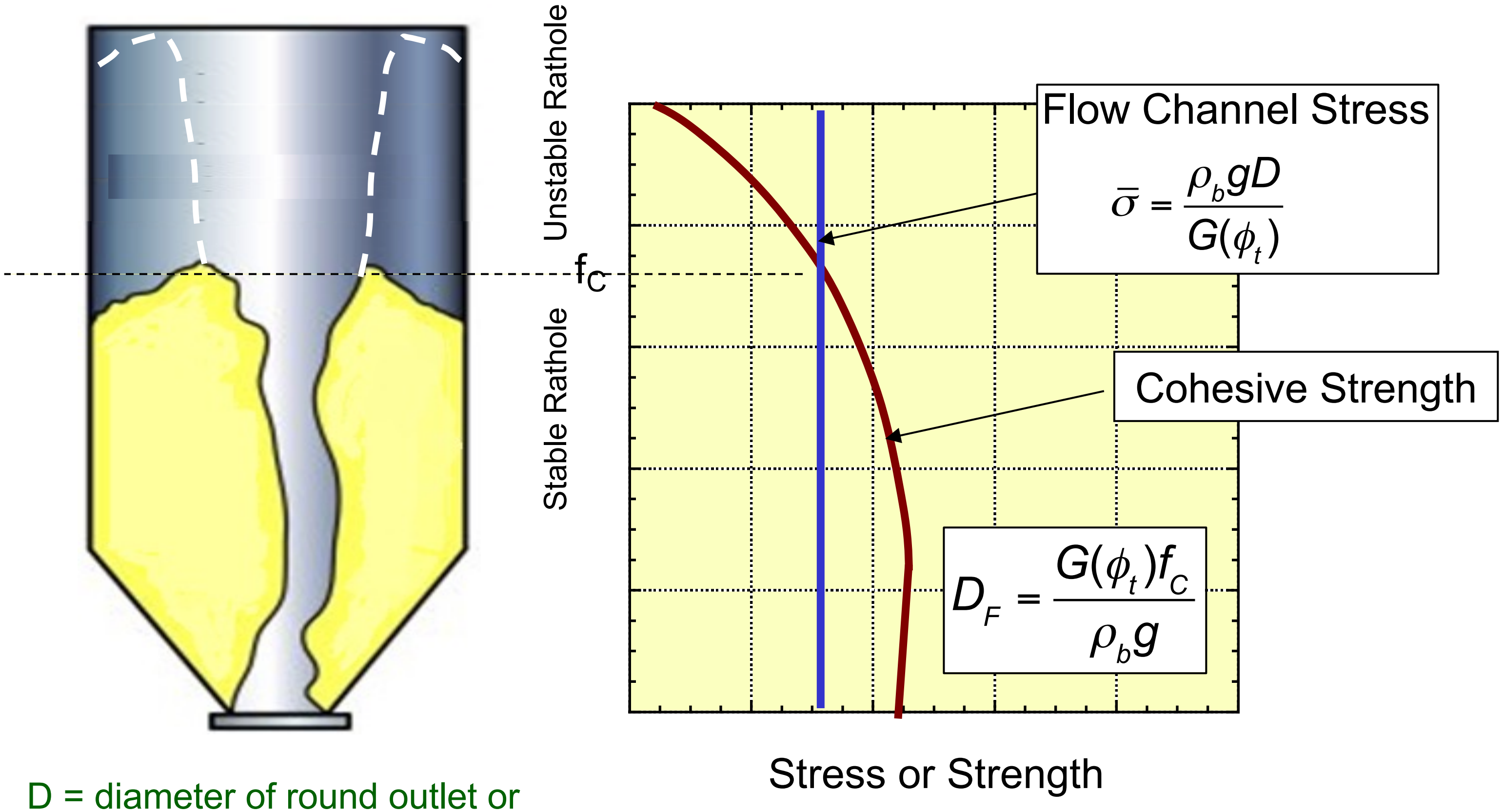
Flow Channel Stress

$$\bar{\sigma} = \frac{\rho_b g D}{G(\phi_t)}$$

Cohesive Strength

D = diameter of round outlet or
diagonal of slotted outlet

Funnel flow bins



Flow Channel Stress

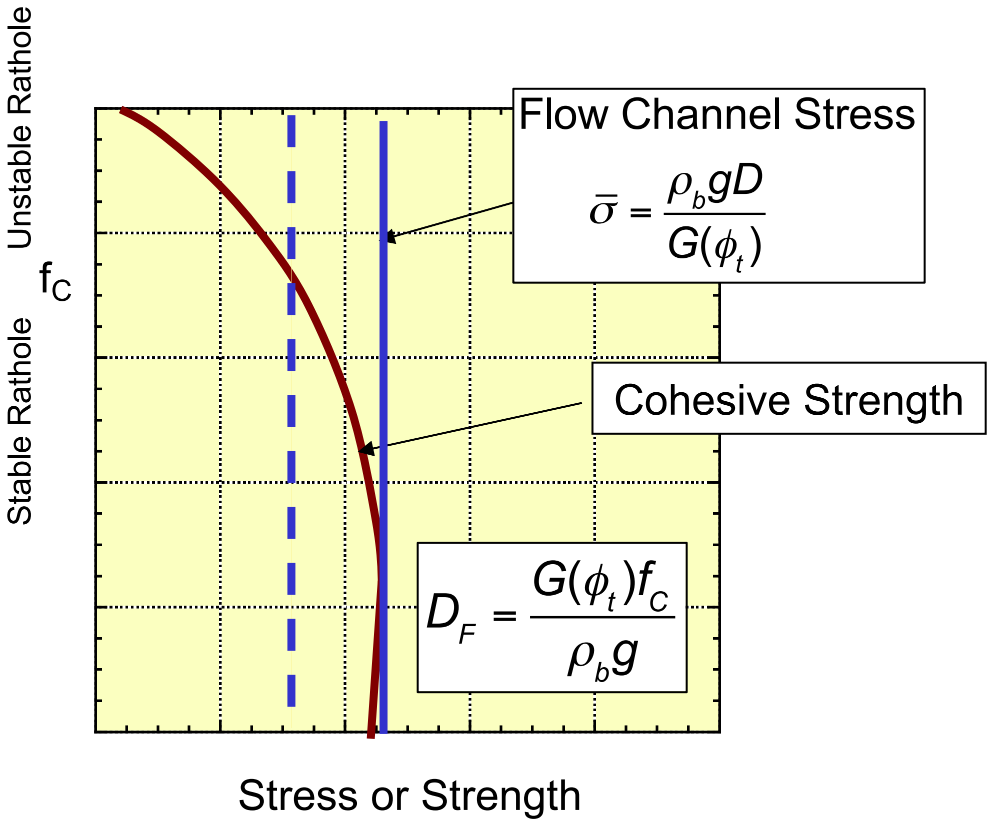
$$\bar{\sigma} = \frac{\rho_b g D}{G(\phi_t)}$$

Cohesive Strength

$$D_F = \frac{G(\phi_t) f_C}{\rho_b g}$$

D = diameter of round outlet or *diagonal* of slotted outlet

Funnel flow bins



Flow Channel Stress

$$\bar{\sigma} = \frac{\rho_b g D}{G(\phi_t)}$$

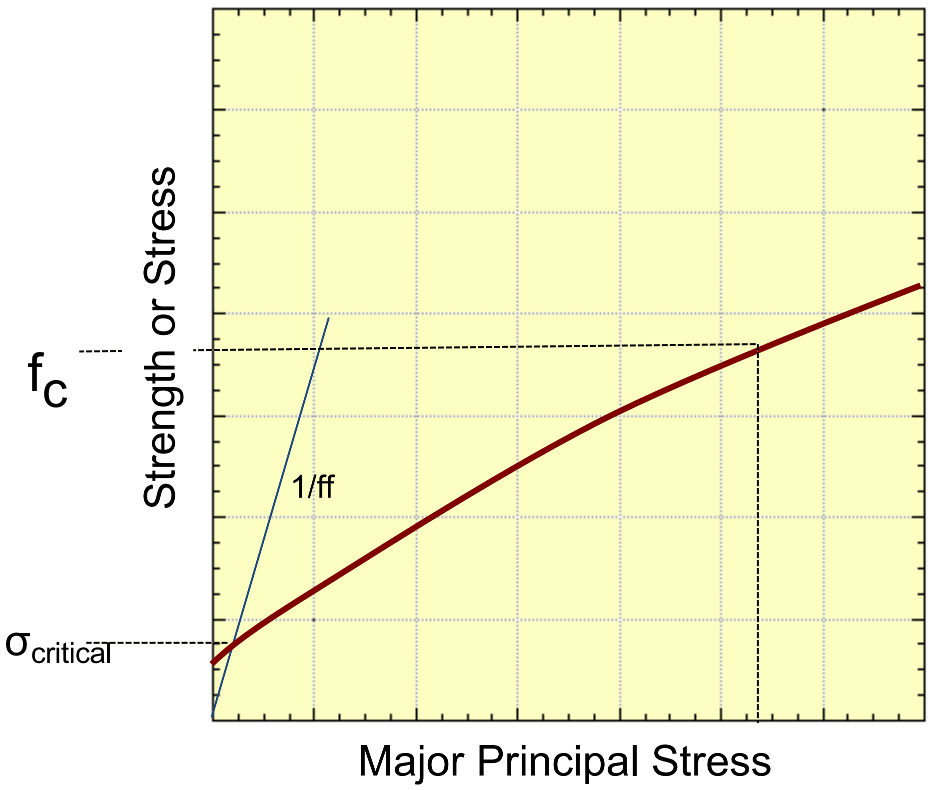
Cohesive Strength

$$D_F = \frac{G(\phi_t) f_C}{\rho_b g}$$

D = diameter of round outlet or
diagonal of slotted outlet

GM&A

Funnel flow bins



To prevent arching in a mass flow bin:

$$B_{min} = \frac{H(\theta')\sigma_{critical}}{\rho_b g}$$

To prevent a stable rathole in a funnel flow bin:

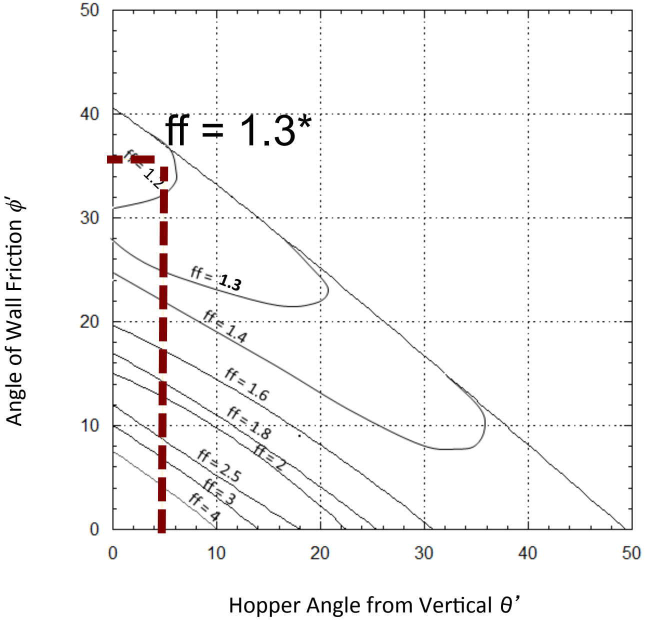
$$D_F = \frac{G(\phi_t) f_C}{\rho_b g}$$

$$\sigma_v(z) = \frac{\rho_b g R_H}{k \tan\phi'}\left[1 - \exp\left(\frac{-k\tan\phi' z}{R_H}\right)\right]$$

Flow factor for funnel flow bins

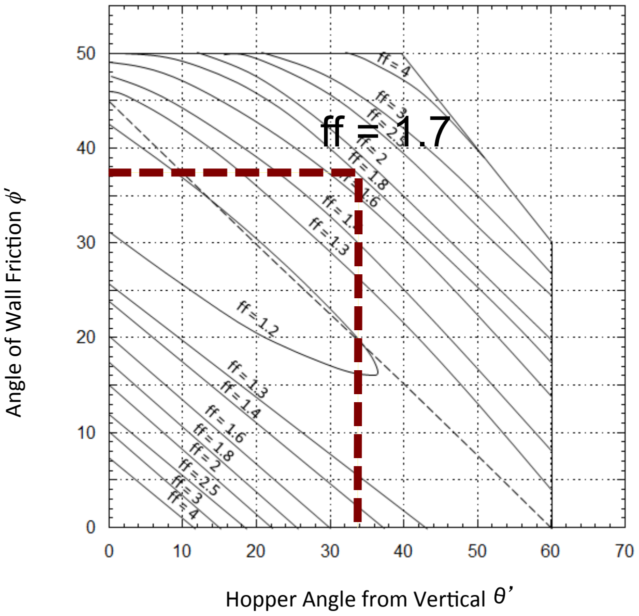
$$\tan \phi' = \sin \delta$$

Conical



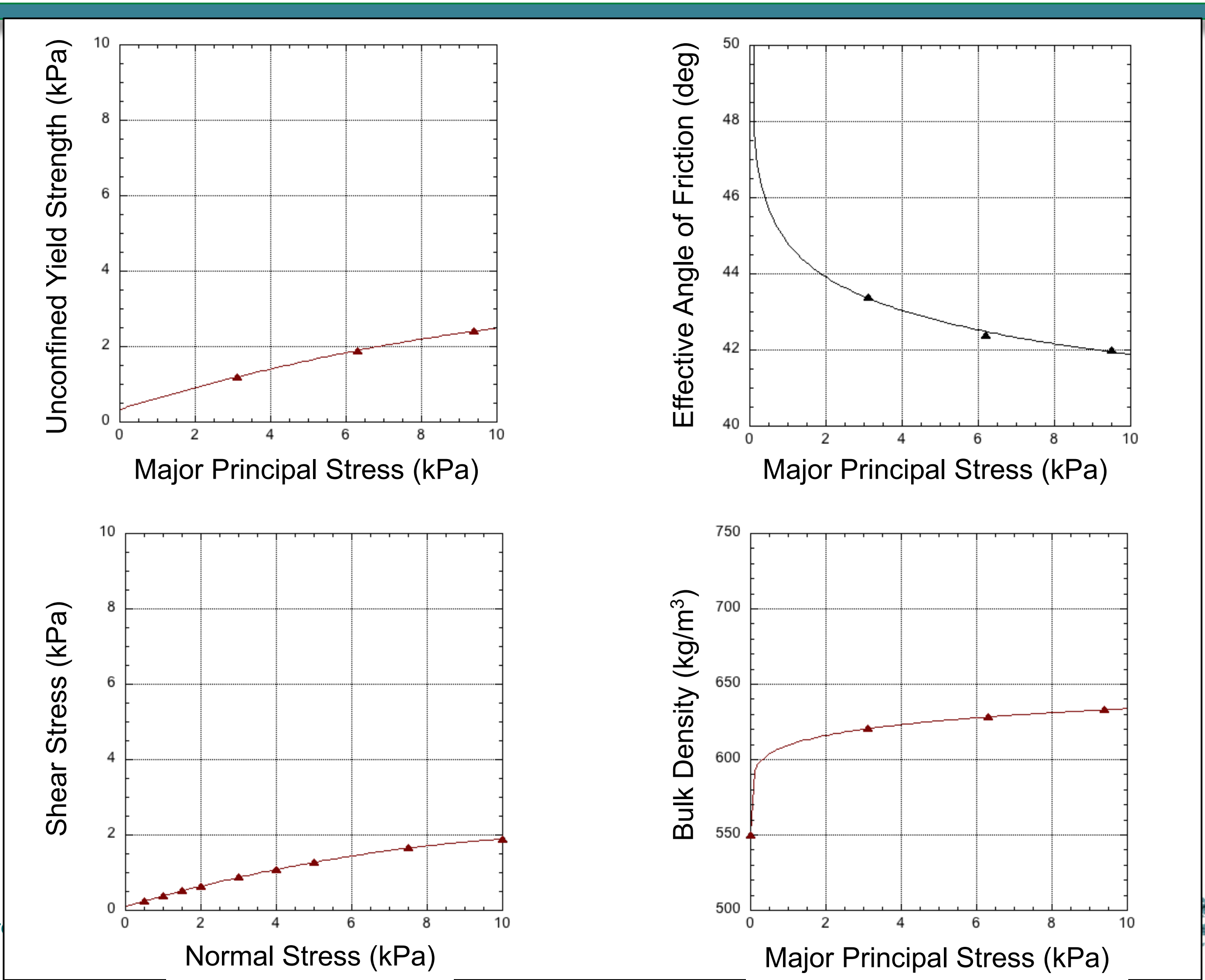
δ = 50°

Planar

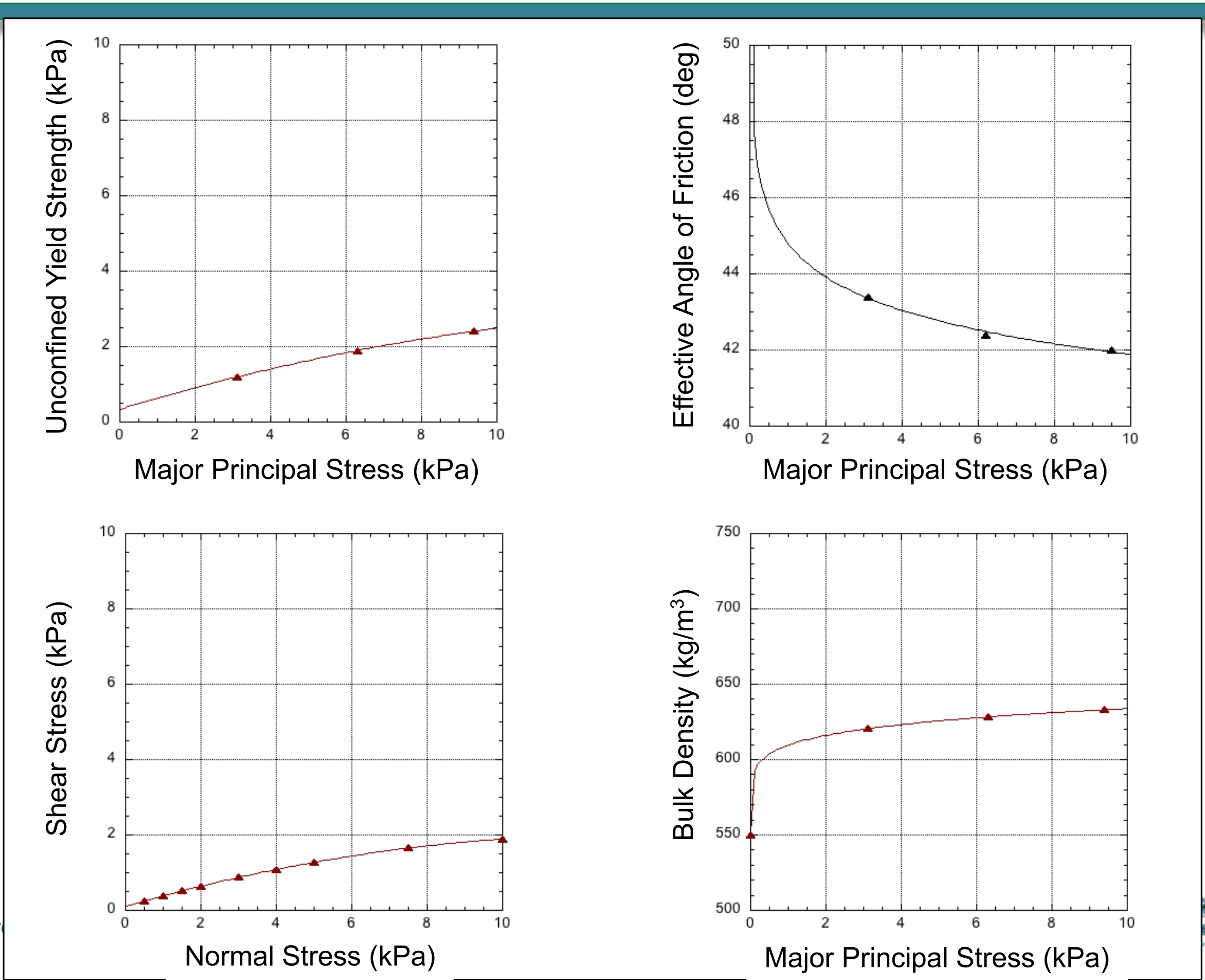


Use flow channel value for hopper angle

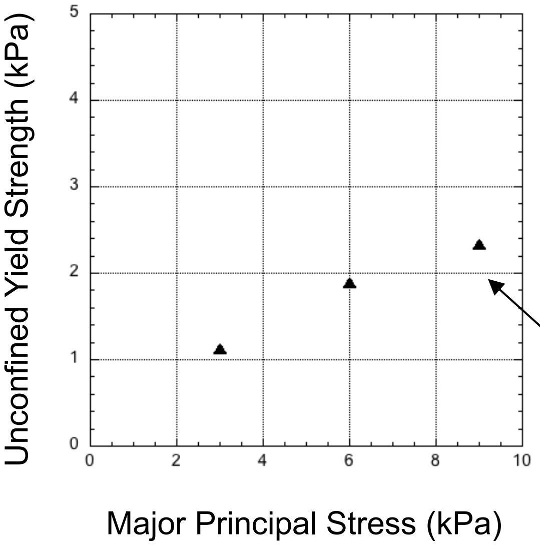
Shear cell test results

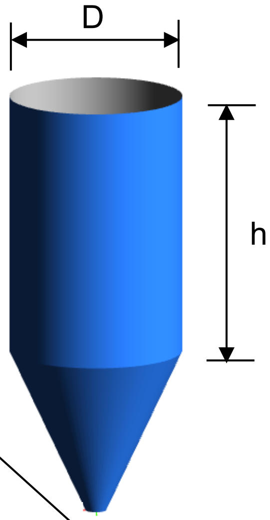


How do we choose the stresses?



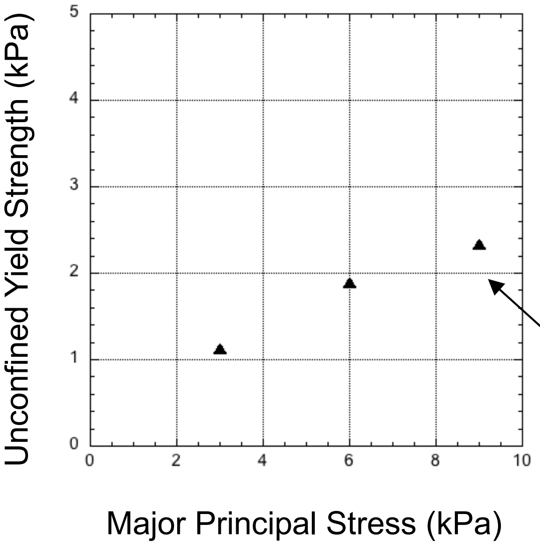
Flow function

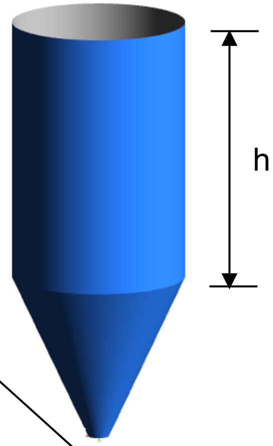




$$\sigma_1 = \frac{\rho_b g D}{4k \tan\phi'}\left[1 - \exp\left(-\frac{4k \tan\phi'}{D} h\right)\right]$$

Flow function

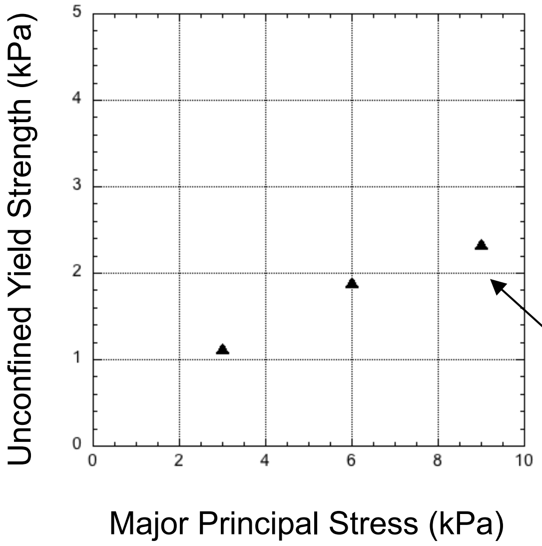


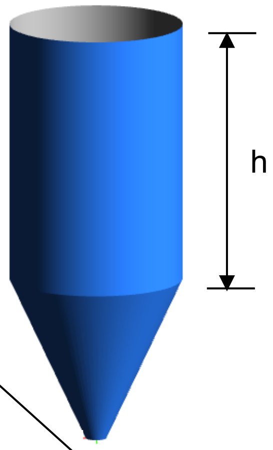


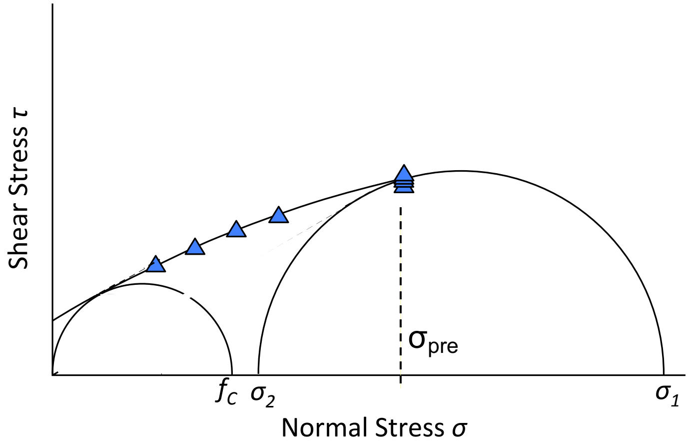
$$\sigma_1 = \frac{\rho_b g D}{4k \tan\phi'}\left[1 - \exp\left(-\frac{4k\tan\phi'}{D}h\right)\right]$$

$$\approx \tfrac{1}{2}\rho_b g h$$

Flow function



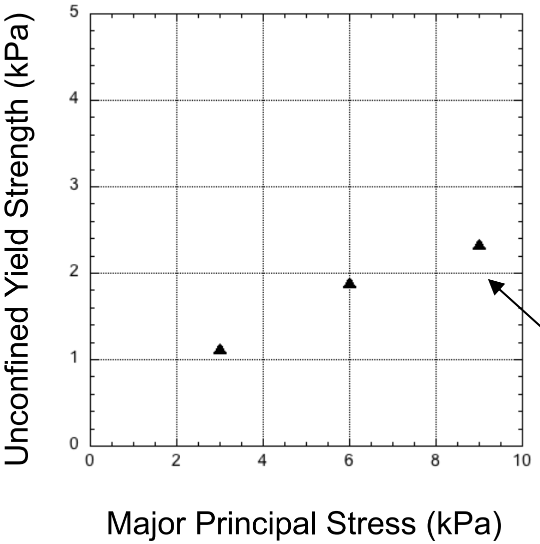


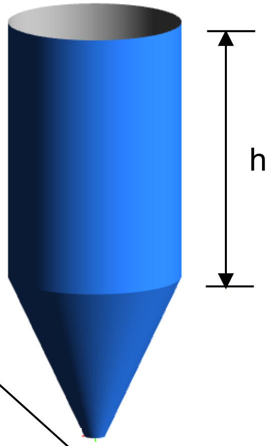


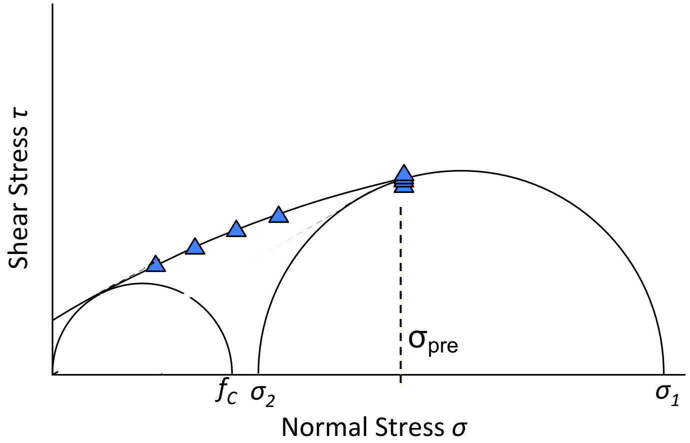
$$\sigma_1 \approx \tfrac{1}{2}\rho_b g h$$

$$\sigma_{pre} \approx \tfrac{1}{2}\sigma_1$$

Flow function





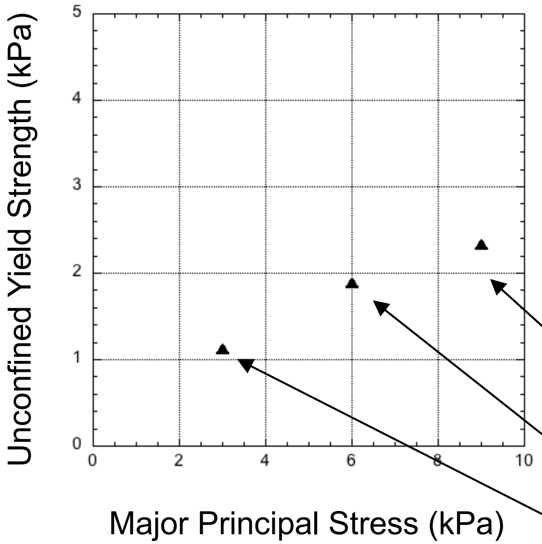


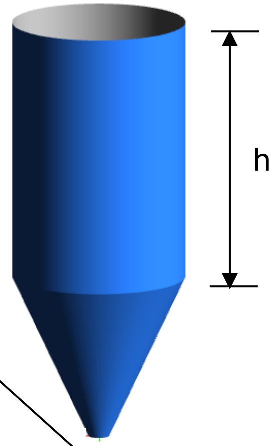
$$\sigma_1 \approx \tfrac{1}{2}\rho_b g h$$

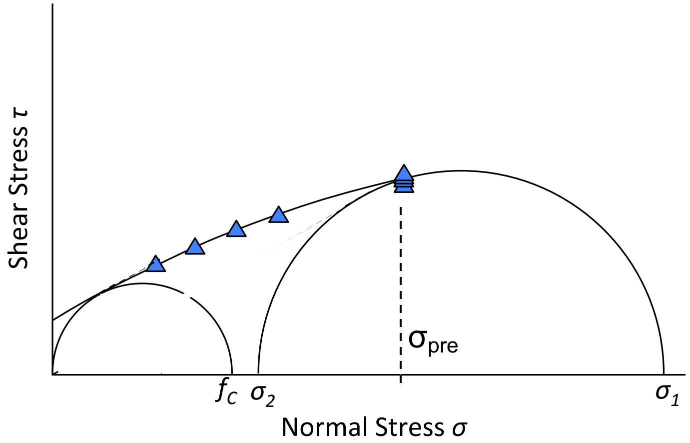
$$\sigma_{pre} \approx \tfrac{1}{2}\sigma_1$$

$$\sigma_{pre} = \tfrac{1}{4}\rho_b g h$$

Flow function



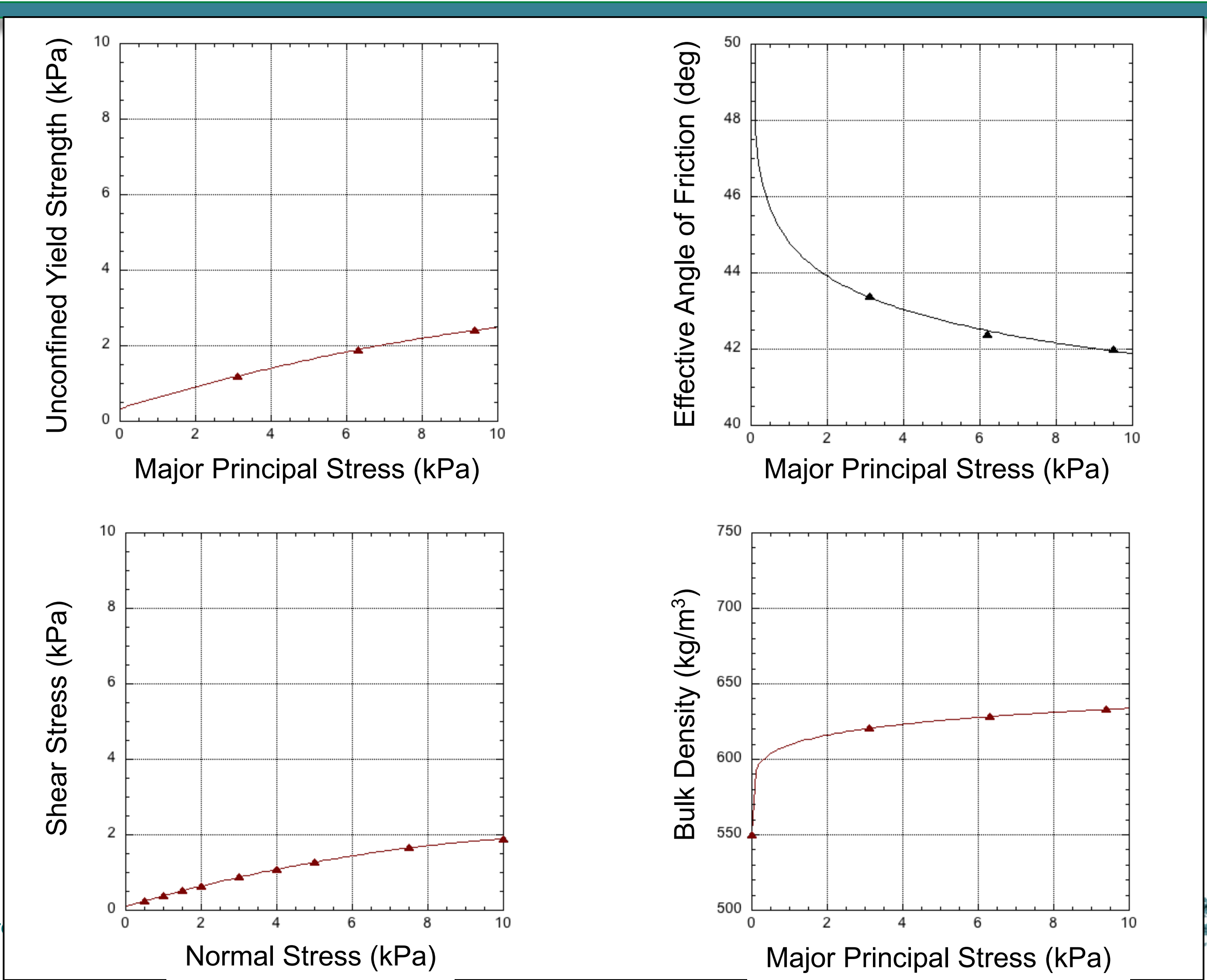




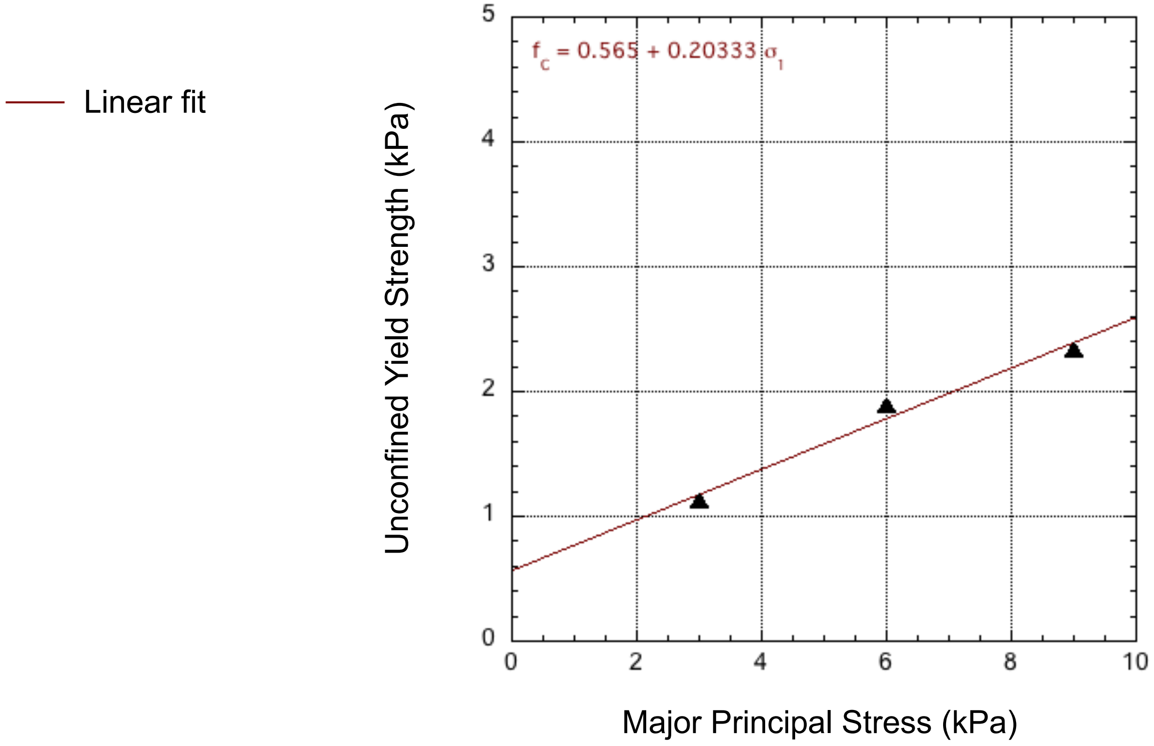
$$\sigma_{pre} = \tfrac{1}{4}\rho_b g h$$

Lowest allowable by tester
and one in the middle

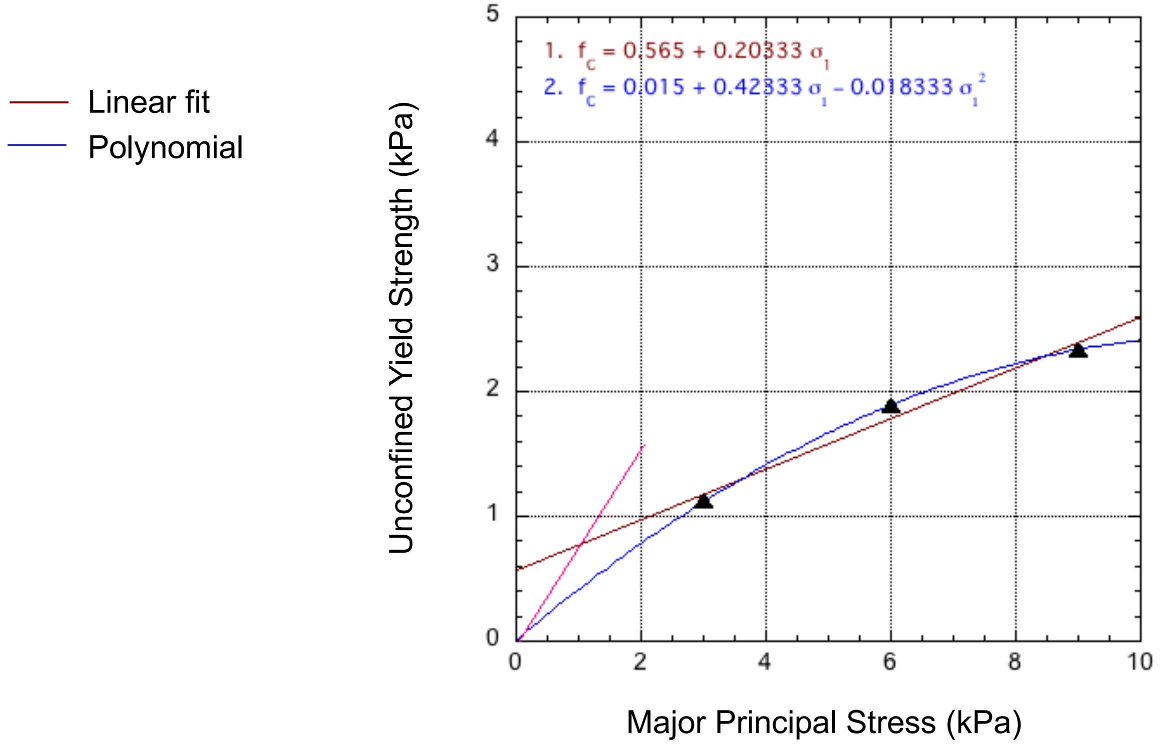
Regression of test results



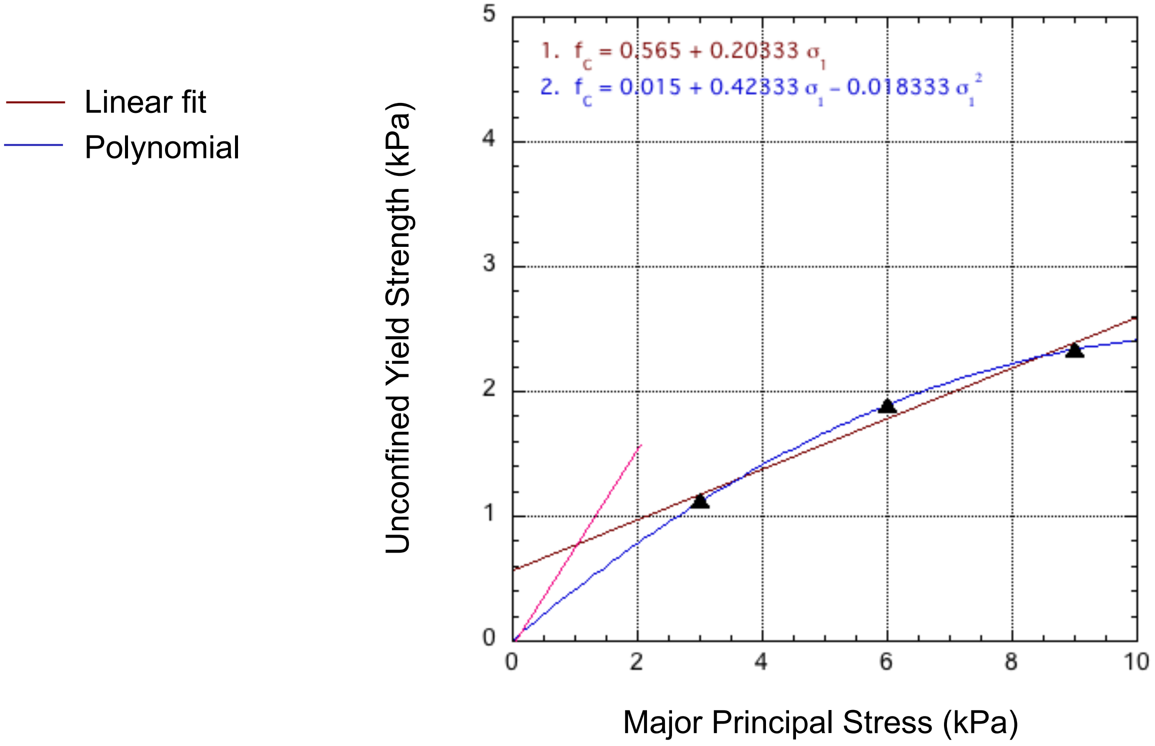
Flow function



Flow function

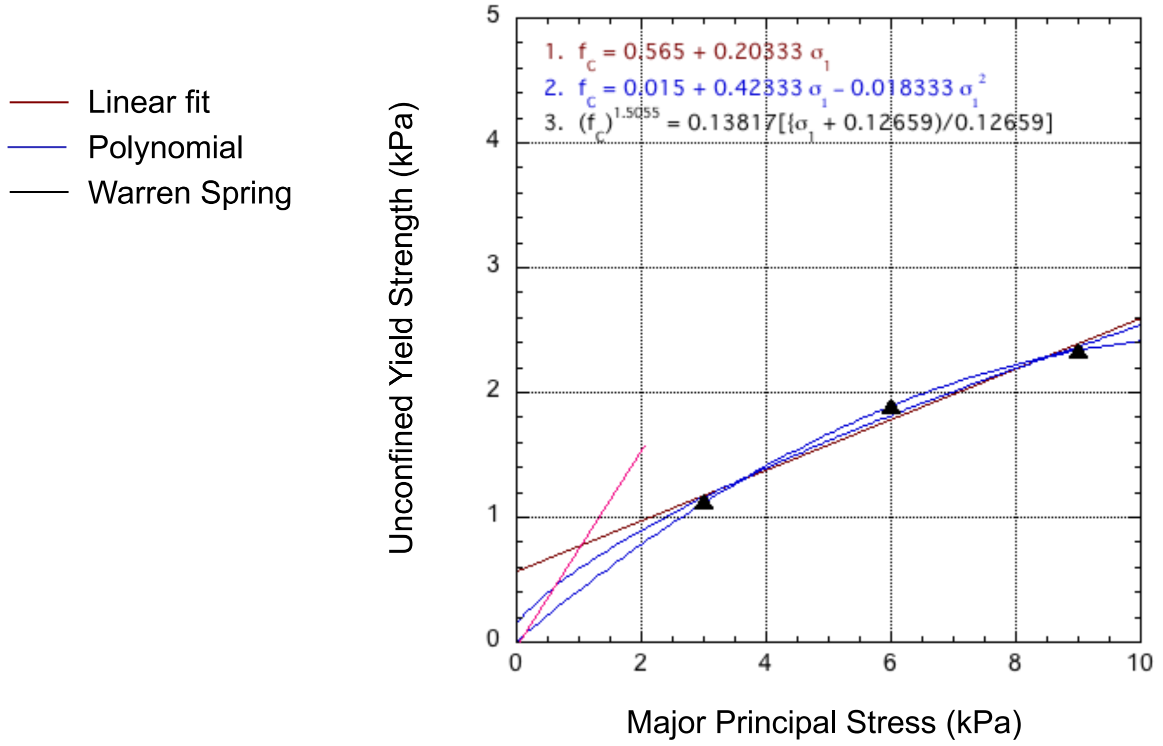


Flow function



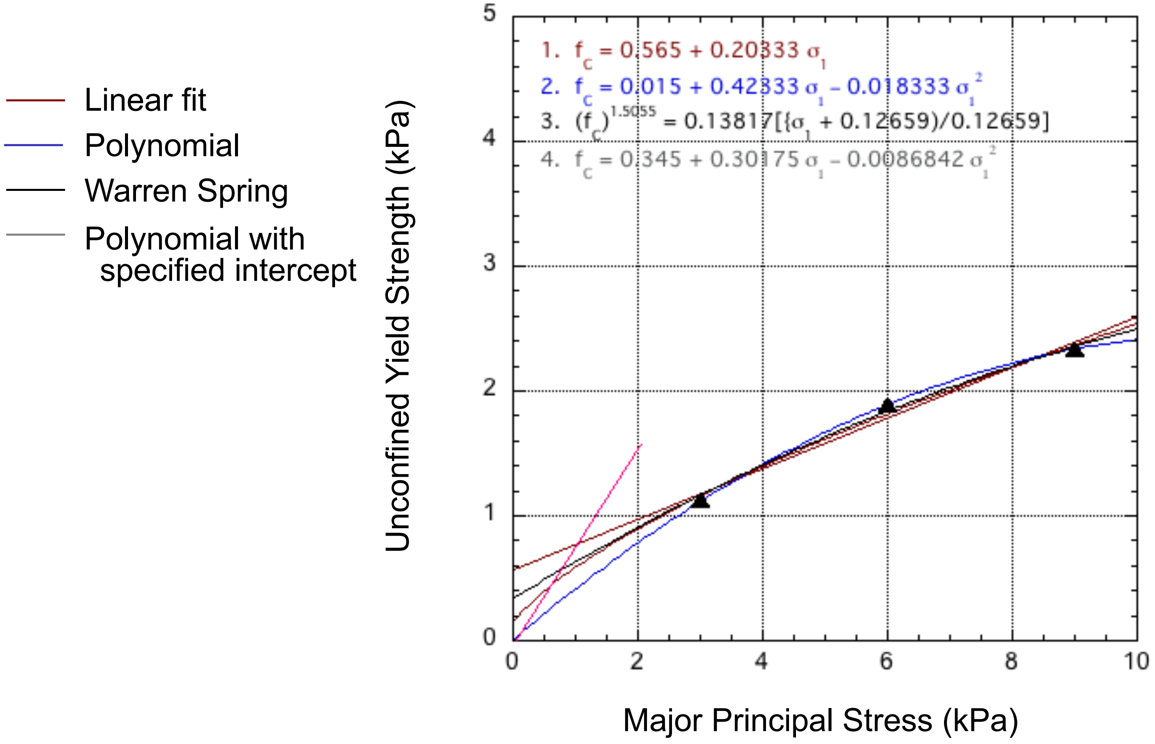
$$\boxed{B_{min} = \frac{H(\theta')\sigma_{Critical}}{\rho_b g}}$$

Flow function



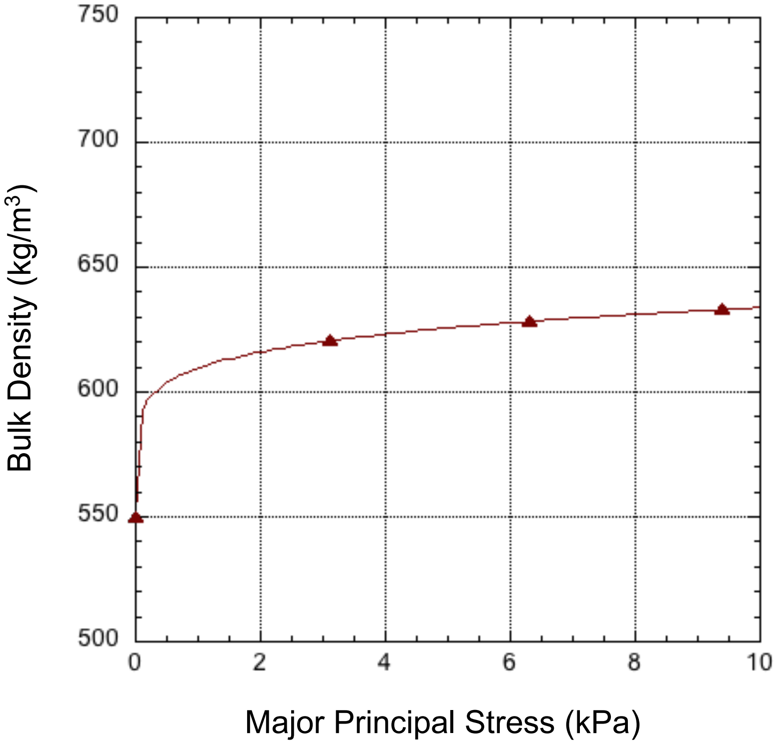
$$B_{min} = \frac{H(\theta')\sigma_{Critical}}{\rho_b g}$$

Flow function



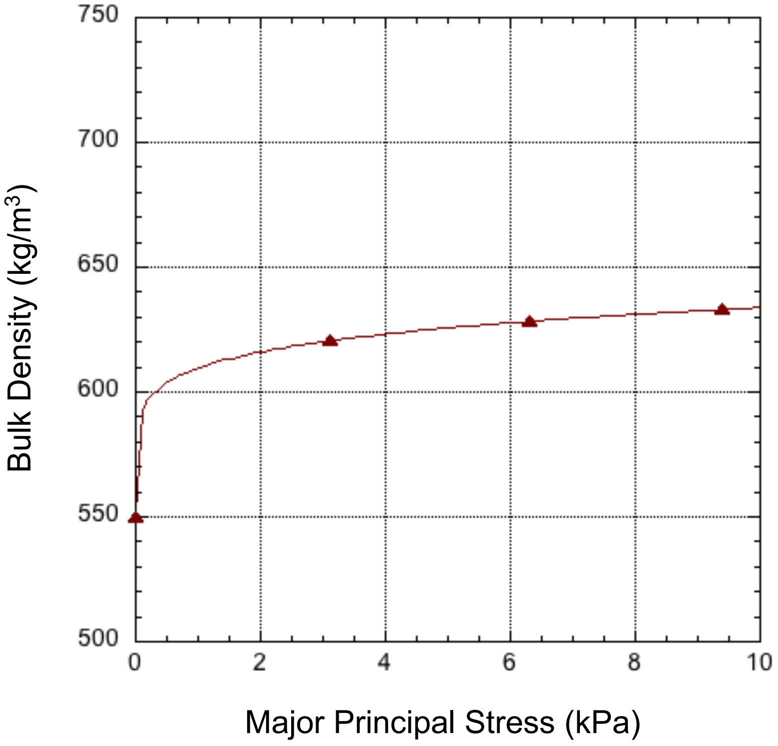
$$\boxed{B_{\text{min}} = \frac{H(\theta')\sigma_{Critical}}{\rho_b g}}$$

Compressibility



$$\boxed{\rho_b = \rho_{b\min} + \alpha\sigma_1^{\beta}}$$

Compressibility

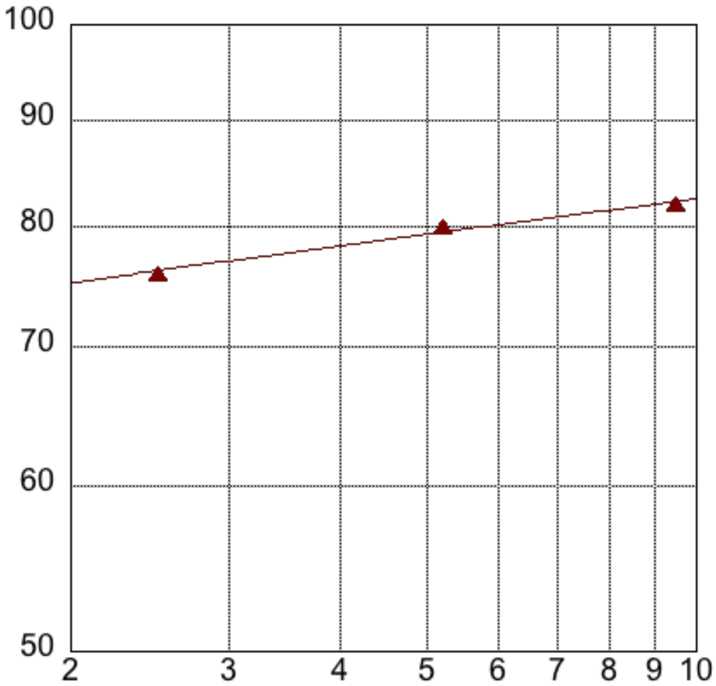


$$\boxed{\rho_b = \rho_{b\min} + \alpha\sigma_1^{\beta}}$$

$$\rho_b - \rho_{b\min} = \alpha\sigma_1^{\beta}$$

$$\ln(\rho_b - \rho_{b\min}) = \ln\alpha + \beta\ln\sigma_1$$

Compressibility



$\rho_b - \rho_{b\min}$

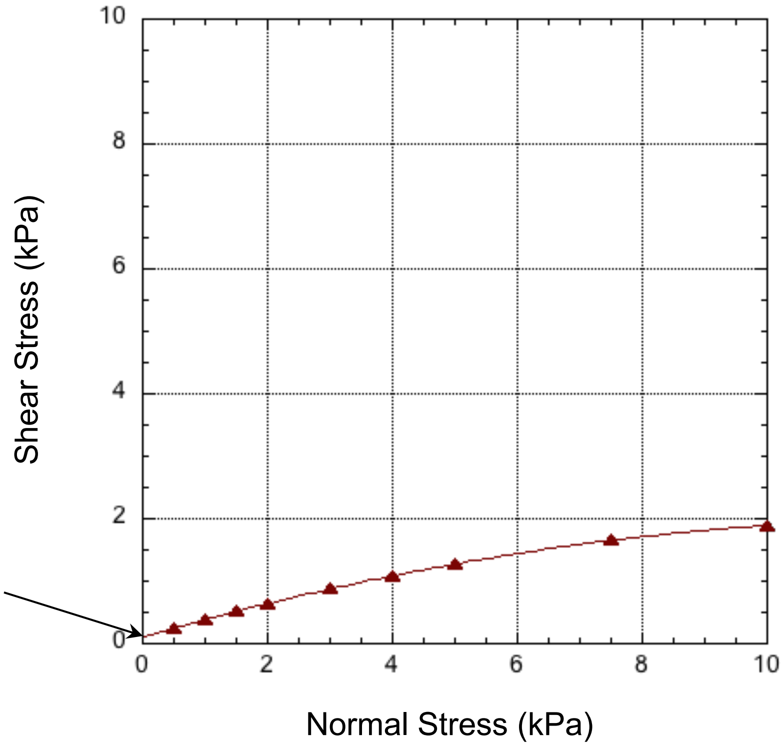
σ_1

$$\boxed{\rho_b = \rho_{b\min} + \alpha\sigma_1^{\beta}}$$

$$\rho_b - \rho_{b\min} = \alpha\sigma_1^{\beta}$$

$$\ln(\rho_b - \rho_{b\min}) = \ln\alpha + \beta\ln\sigma_1$$

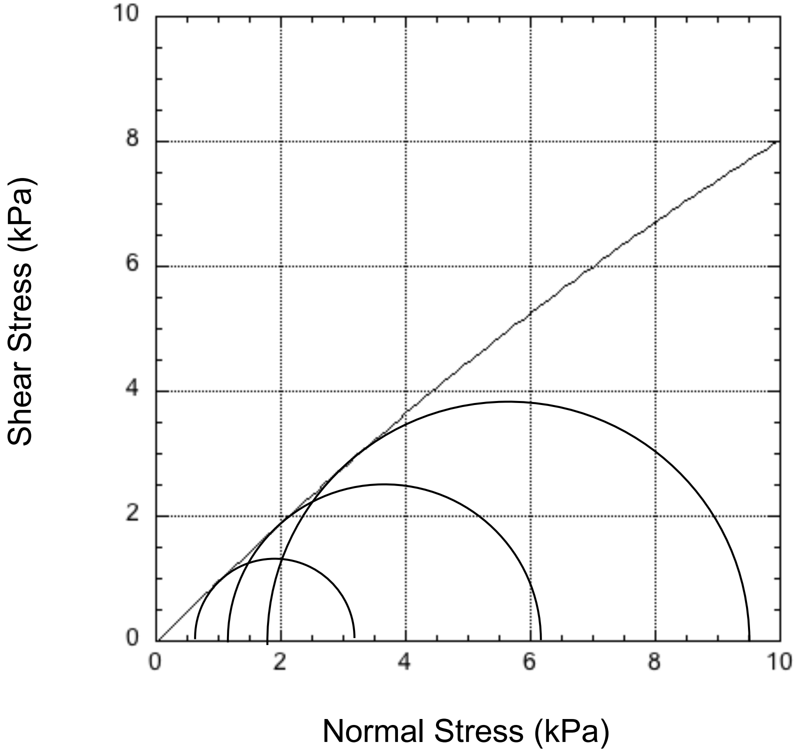
Wall friction

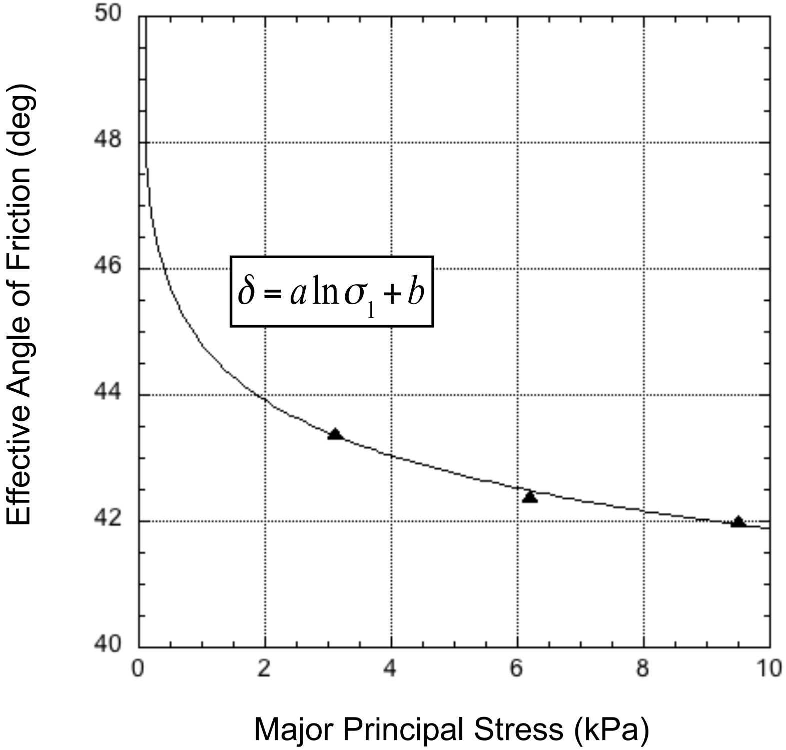


Intercept ≠ 0

$$\tau' = a\left(\sigma'\right)^2 + b\sigma' + c$$

Effective yield locus

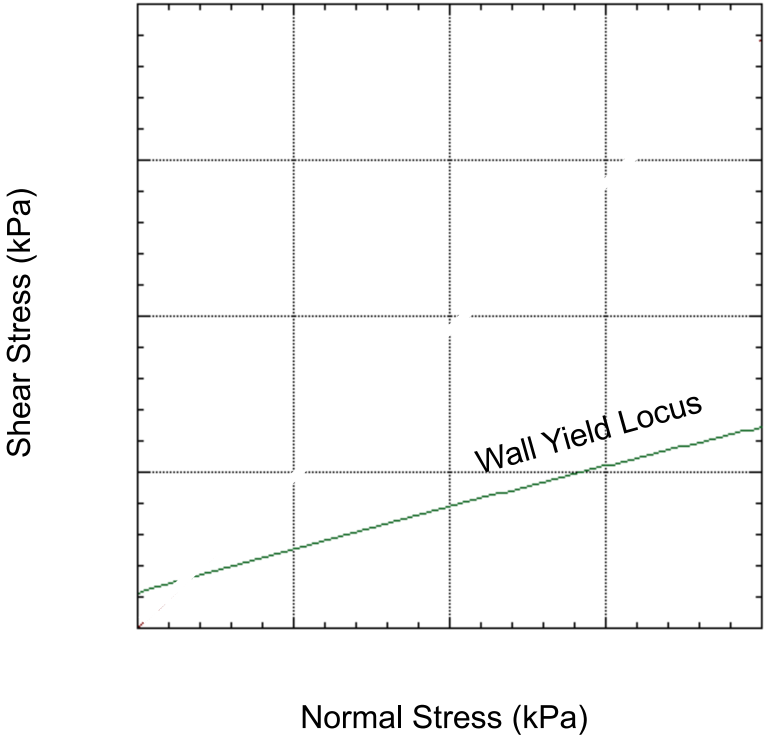




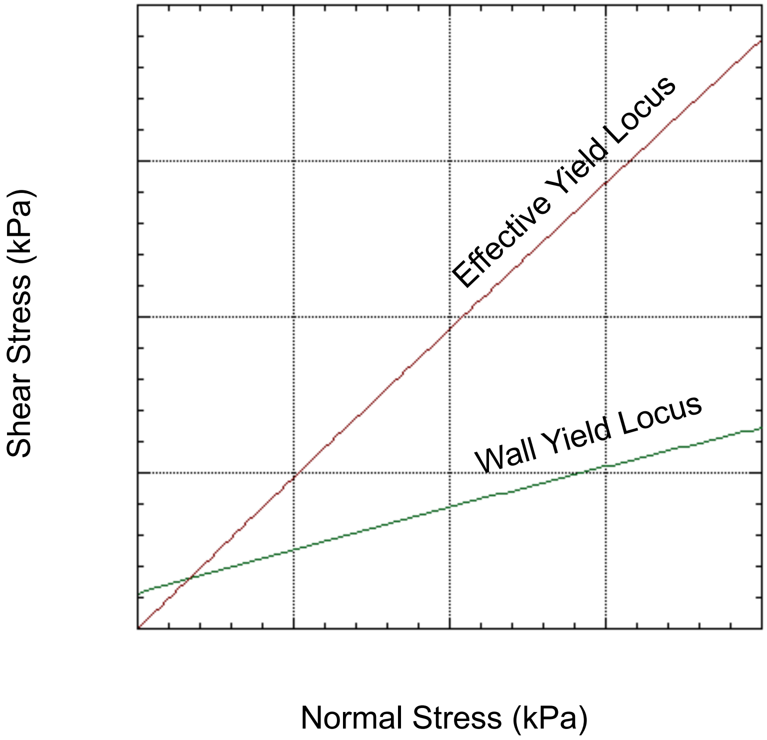
$$\delta = a \ln \sigma_1 + b$$

$$\sin\delta = \frac{\sigma_1 - \sigma_2}{\sigma_1 + \sigma_2}$$

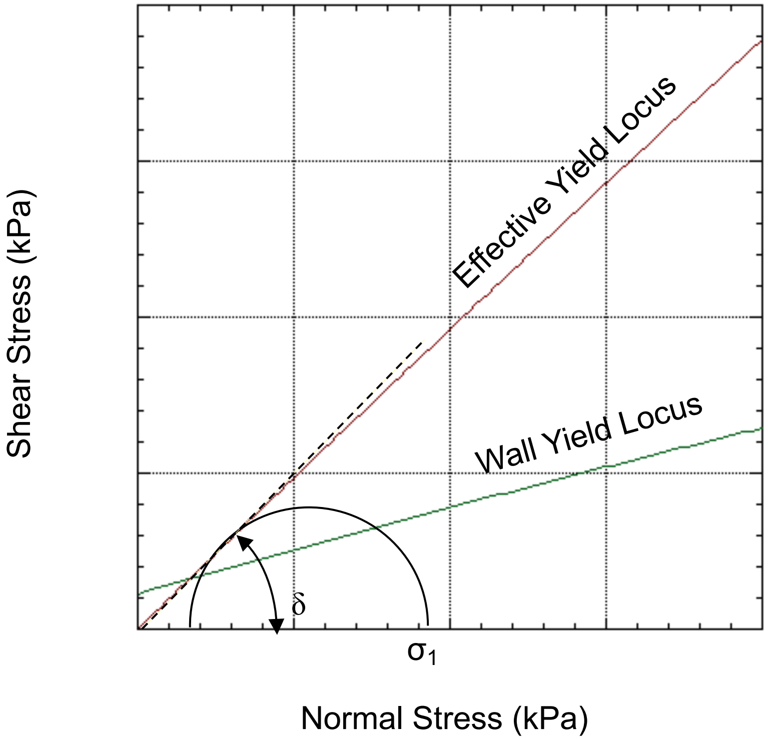
Wall stress



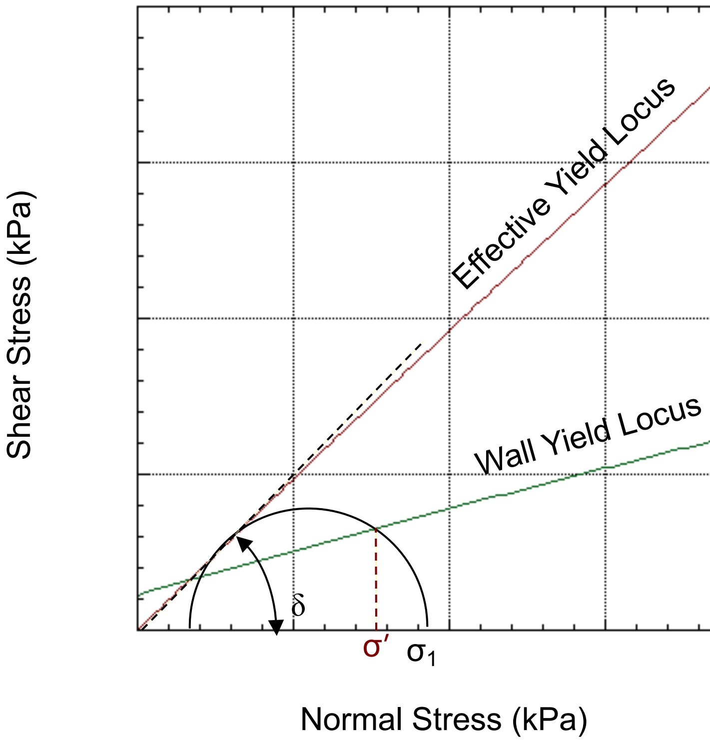
Wall stress


Effective Yield Locus
Wall Yield Locus
Shear Stress (kPa)
Normal Stress (kPa)

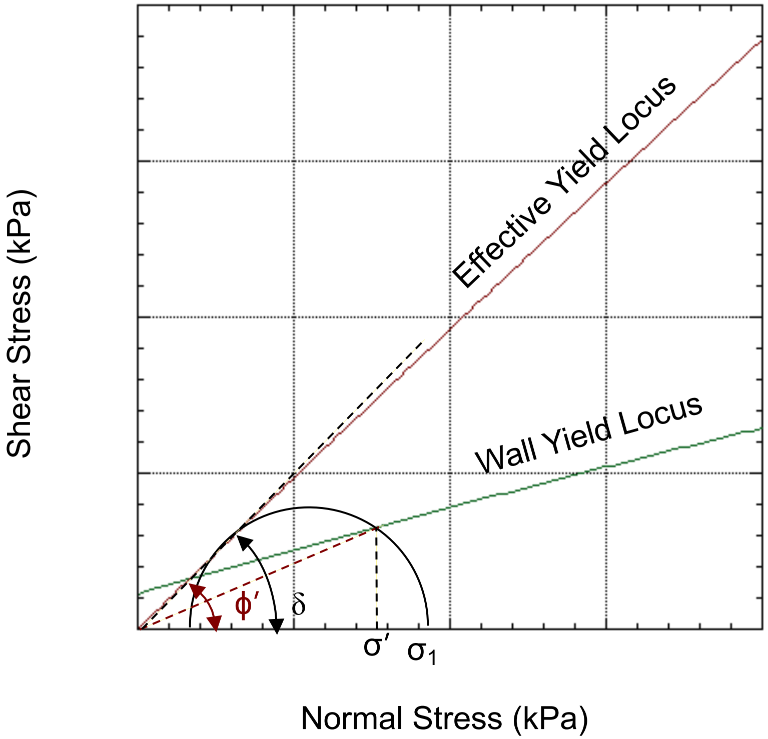
Wall stress


Effective Yield Locus
Wall Yield Locus
δ
σ1
Shear Stress (kPa)
Normal Stress (kPa)

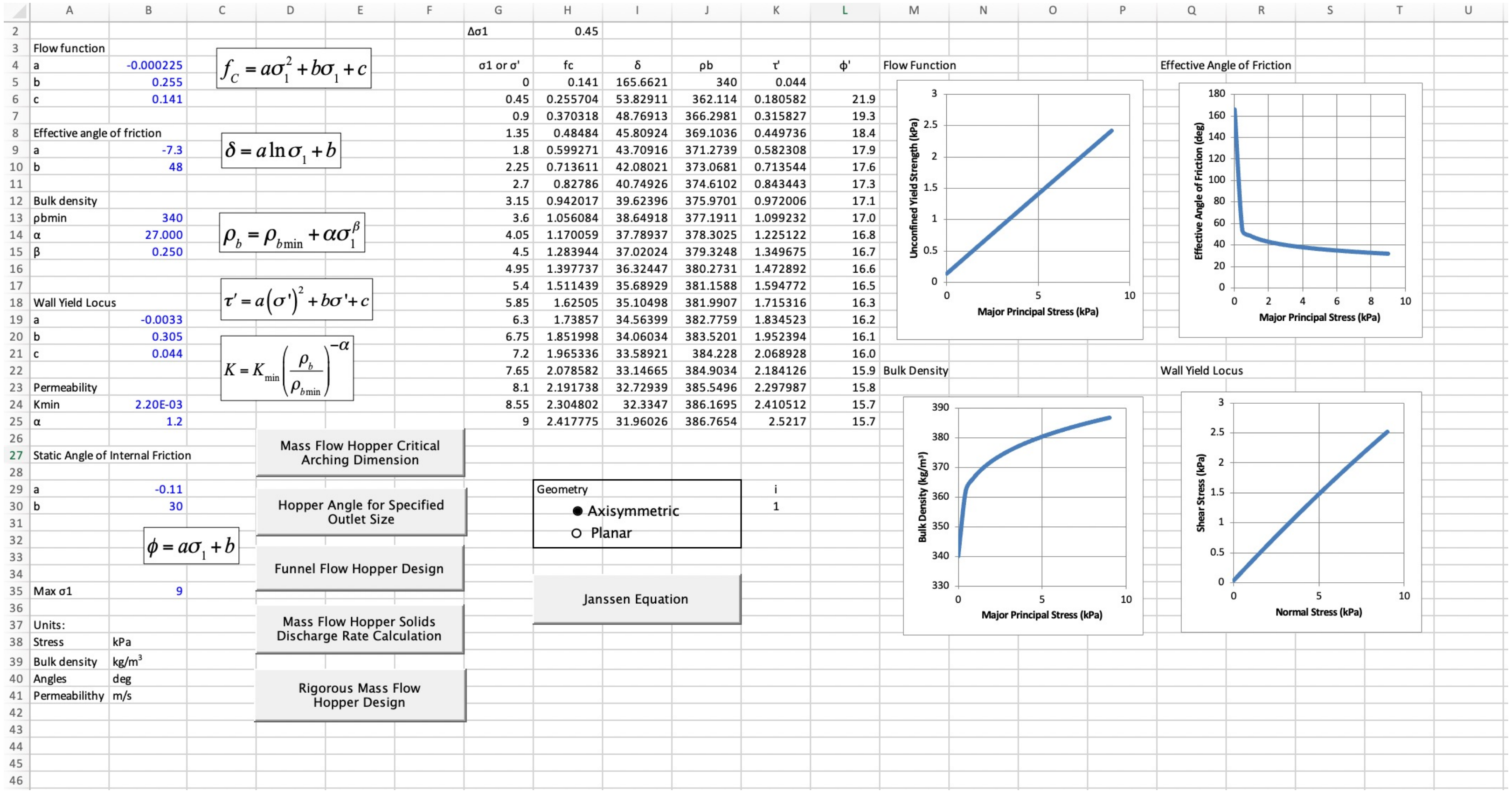
Wall stress


Effective Yield Locus
Wall Yield Locus
Shear Stress (kPa)
δ
σ′
σ_1
Normal Stress (kPa)

Wall stress


Effective Yield Locus
Wall Yield Locus
Shear Stress (kPa)
Normal Stress (kPa)
ϕ′
δ
σ′
σ_1

Hopper design



Flow function	
a	-0.000225
b	0.255
c	0.141
Effective angle of friction	
a	-7.3
b	48
Bulk density	
ρbmin	340
α	27.000
β	0.250
Wall Yield Locus	
a	-0.0033
b	0.305
c	0.044
Permeability	
Kmin	2.20E-03
α	1.2
Static Angle of Internal Friction	
a	-0.11
b	30
Max σ1	9
Units:	
Stress	kPa
Bulk density	kg/m^3
Angles	deg
Permeability	m/s

$$f_C = a\sigma_1^2 + b\sigma_1 + c$$

$$\delta = a\ln\sigma_1 + b$$

$$\rho_b = \rho_{b\min} + \alpha\sigma_1^{\beta}$$

$$\tau' = a(\sigma')^2 + b\sigma' + c$$

$$K = K_{\min}\left(\frac{\rho_b}{\rho_{b\min}}\right)^{-\alpha}$$

$$\phi = a\sigma_1 + b$$

Δσ1 0.45

σ1 or σ'	fc	δ	ρb	τ'	ϕ'
0	0.141	165.6621	340	0.044	
0.45	0.255704	53.82911	362.114	0.180582	21.9
0.9	0.370318	48.76913	366.2981	0.315827	19.3
1.35	0.48484	45.80924	369.1036	0.449736	18.4
1.8	0.599271	43.70916	371.2739	0.582308	17.9
2.25	0.713611	42.08021	373.0681	0.713544	17.6
2.7	0.82786	40.74926	374.6102	0.843443	17.3
3.15	0.942017	39.62396	375.9701	0.972006	17.1
3.6	1.056084	38.64918	377.1911	1.099232	17.0
4.05	1.170059	37.78937	378.3025	1.225122	16.8
4.5	1.283944	37.02024	379.3248	1.349675	16.7
4.95	1.397737	36.32447	380.2731	1.472892	16.6
5.4	1.511439	35.68929	381.1588	1.594772	16.5
5.85	1.62505	35.10498	381.9907	1.715316	16.3
6.3	1.73857	34.56399	382.7759	1.834523	16.2
6.75	1.851998	34.06034	383.5201	1.952394	16.1
7.2	1.965336	33.58921	384.228	2.068928	16.0
7.65	2.078582	33.14665	384.9034	2.184126	15.9
8.1	2.191738	32.72939	385.5496	2.297987	15.8
8.55	2.304802	32.3347	386.1695	2.410512	15.7
9	2.417775	31.96026	386.7654	2.5217	15.7

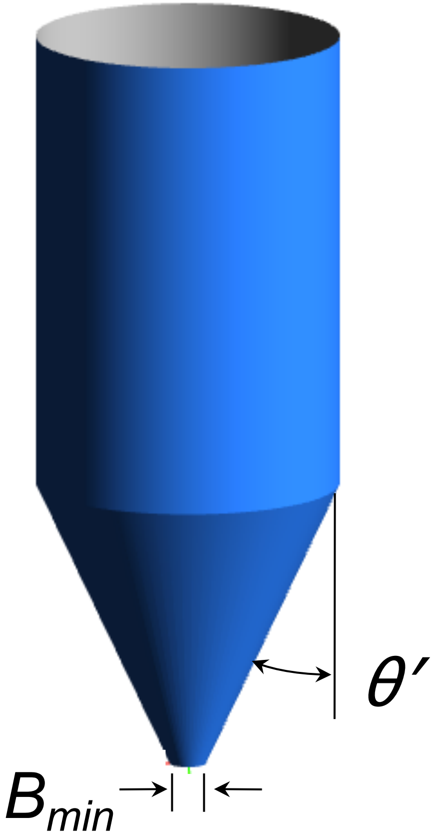
Geometry

● Axisymmetric

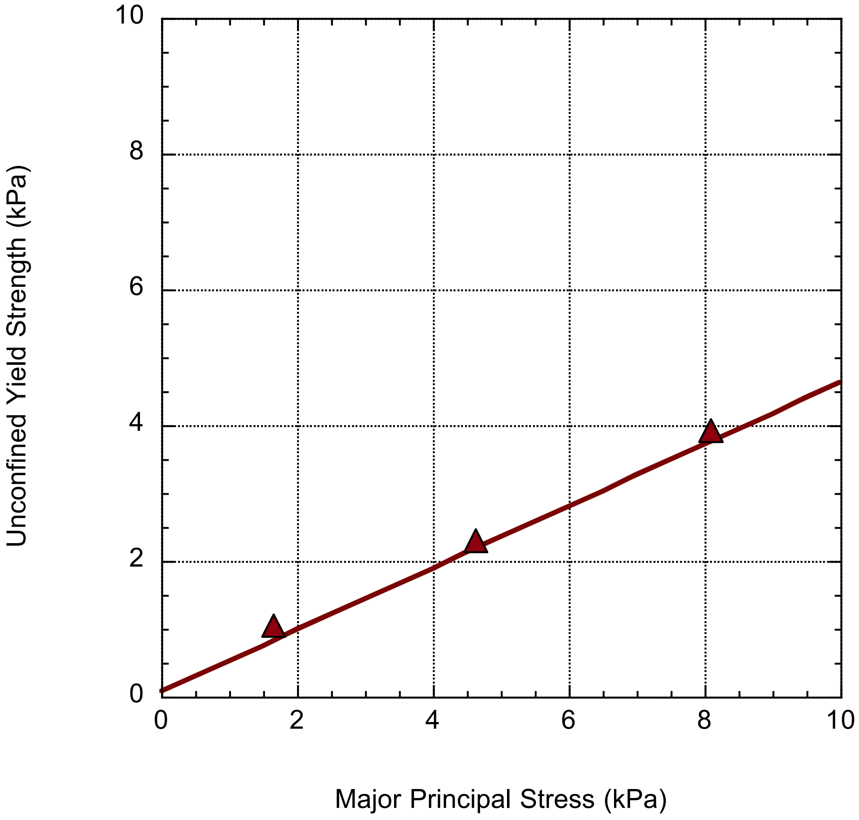
○ Planar

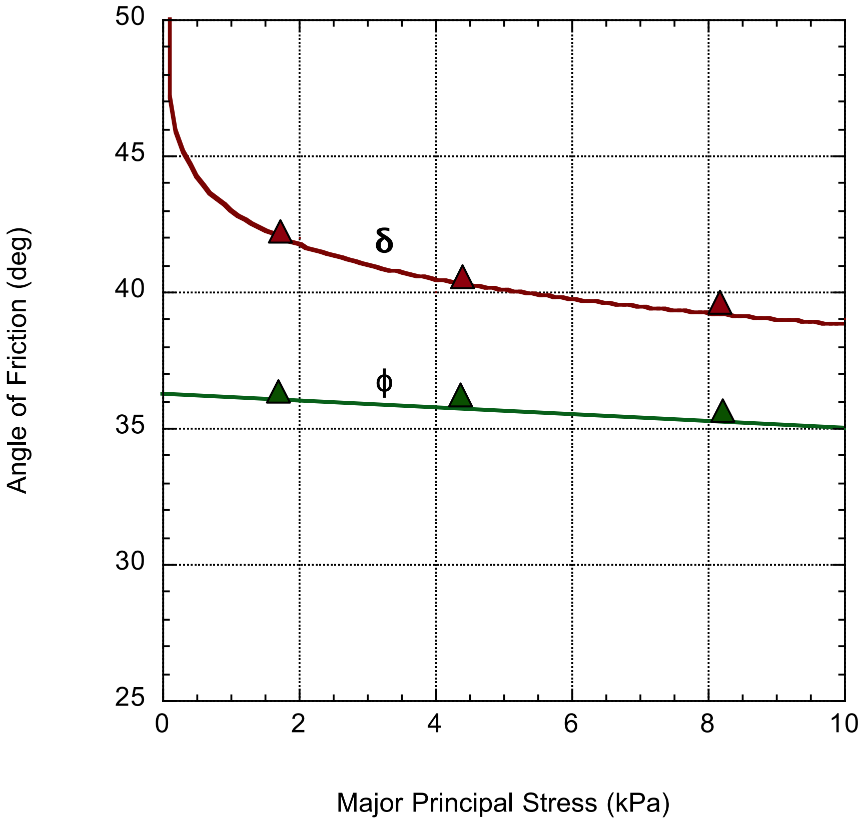
i 1

Mass flow hopper design

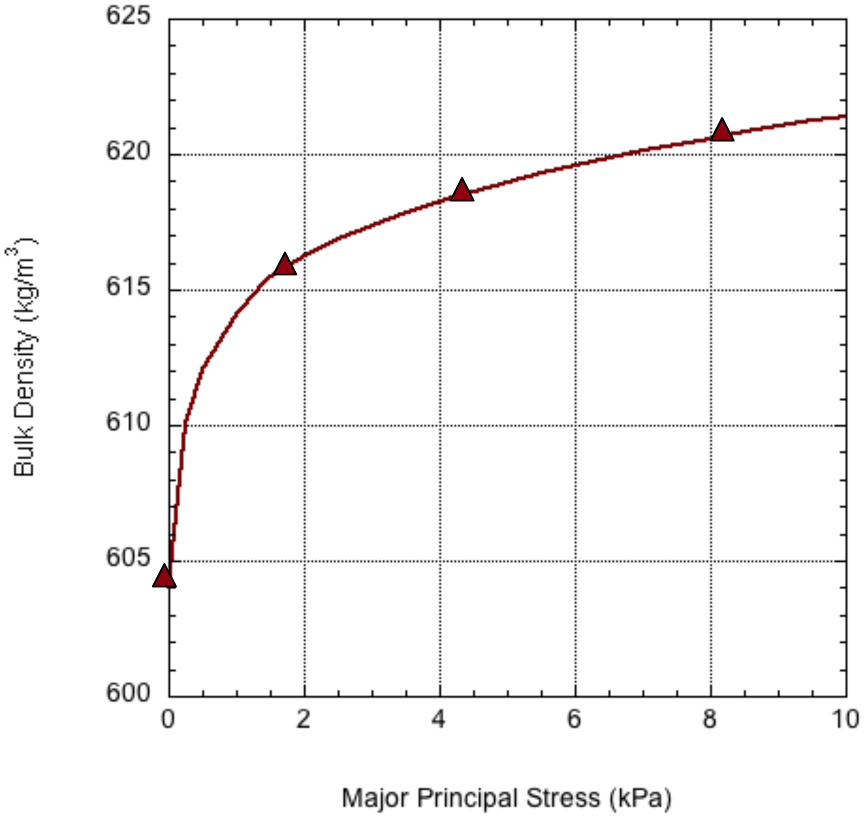


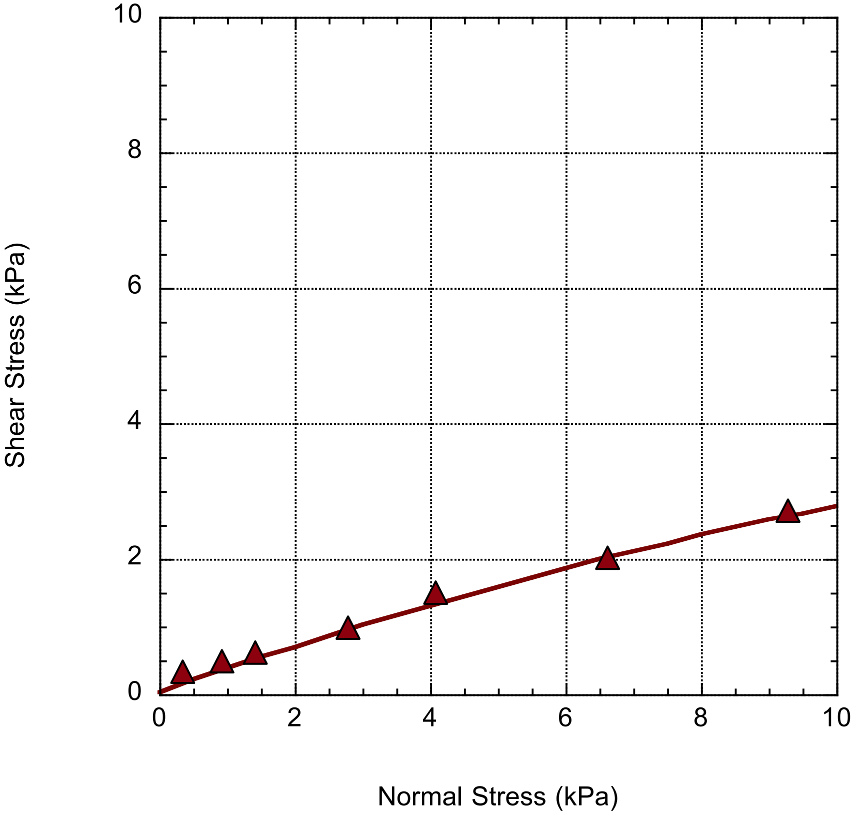
Flow properties



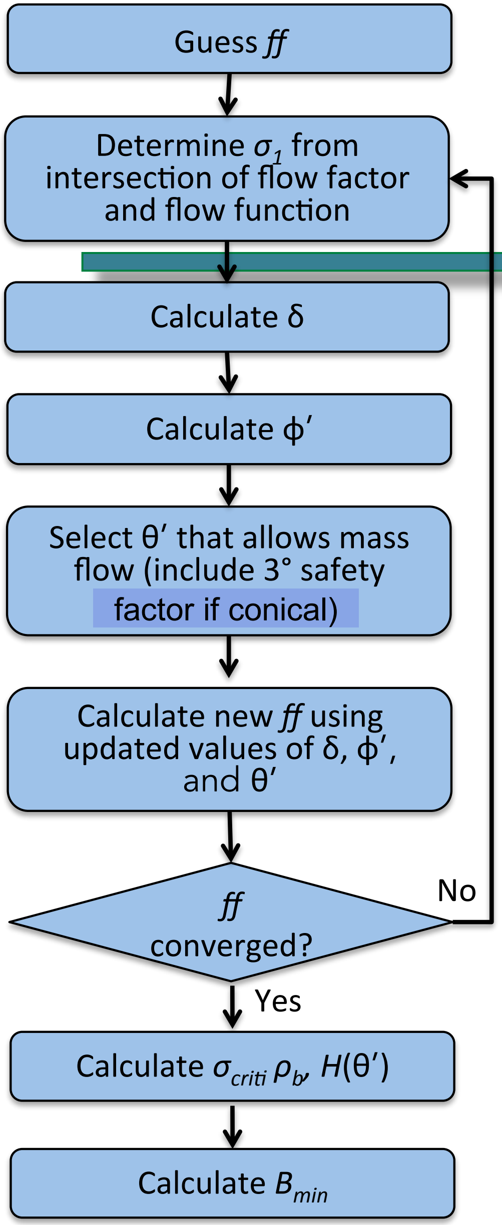


Flow properties





Mass flow hopper design



Guess *ff*

Determine σ_1 from intersection of flow factor and flow function

Calculate δ

Calculate ϕ′

Select θ′ that allows mass flow (include 3° safety factor if conical)

Calculate new *ff* using updated values of δ, ϕ′, and θ′

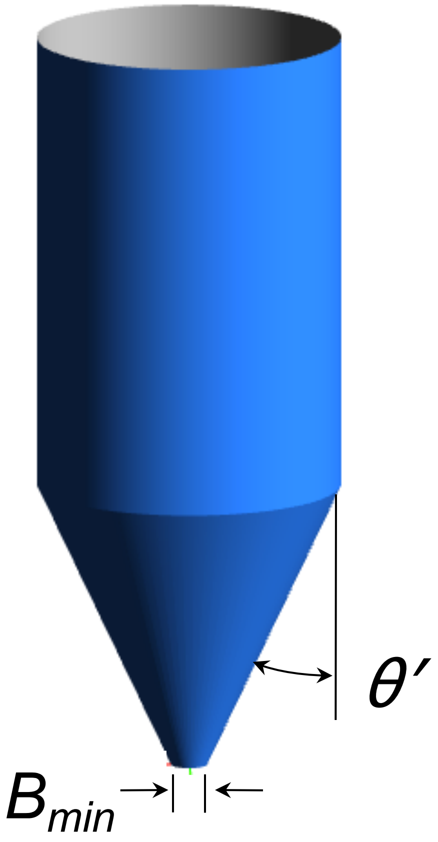
ff converged?

No

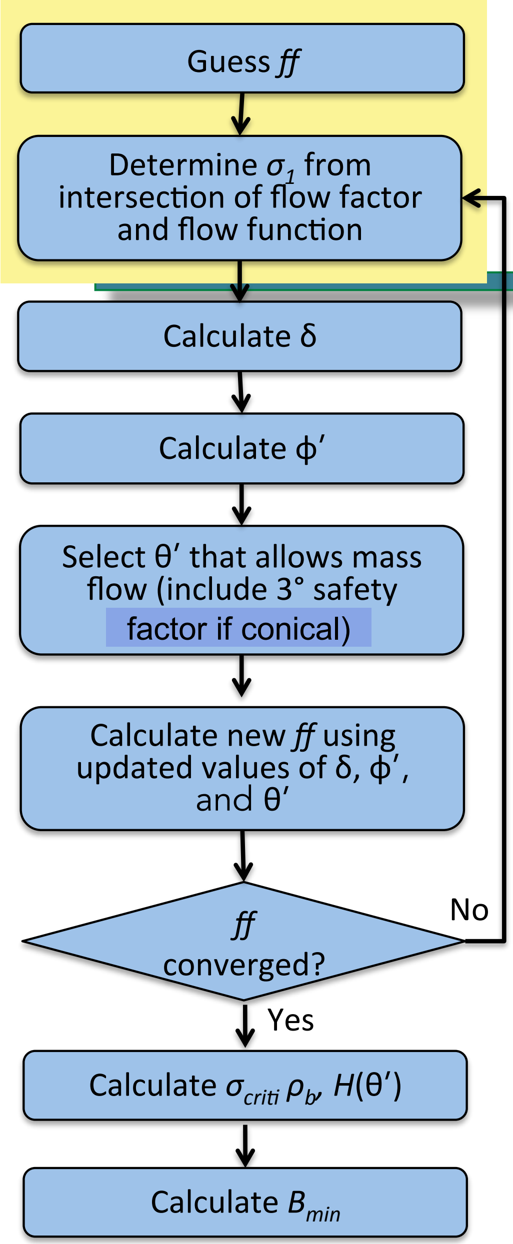
Yes

Calculate σ_{crit}, ρ_b, $H(\theta')$

Calculate B_{min}



Mass flow hopper design



Guess *ff*

Determine σ_1 from intersection of flow factor and flow function

Calculate δ

Calculate ϕ′

Select θ′ that allows mass flow (include 3° safety factor if conical)

Calculate new *ff* using updated values of δ, ϕ′, and θ′

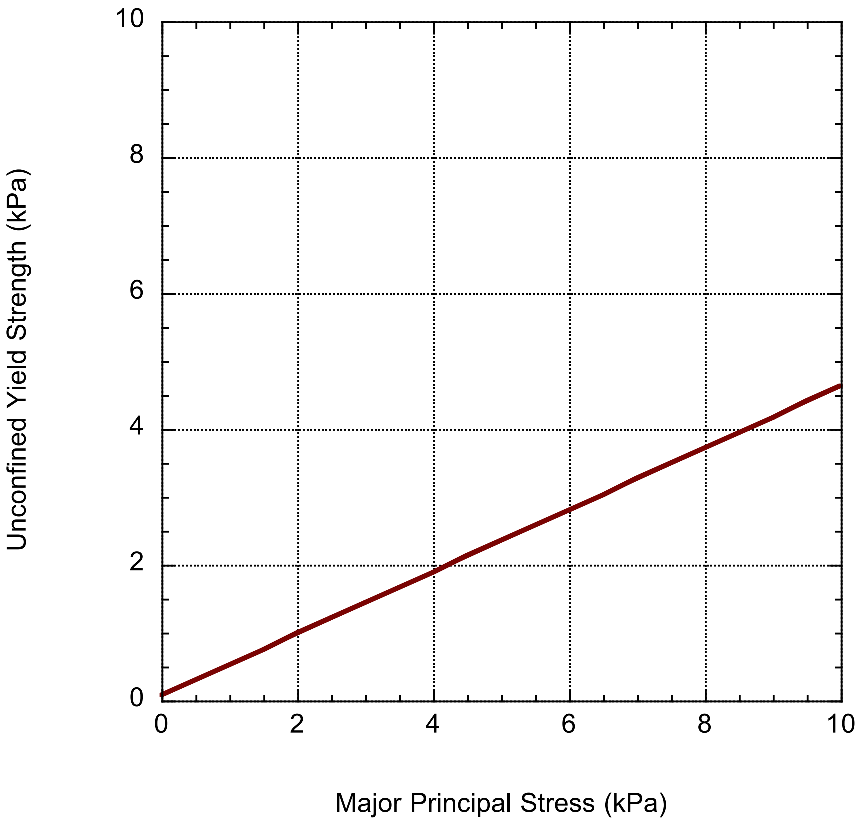
ff converged?

No

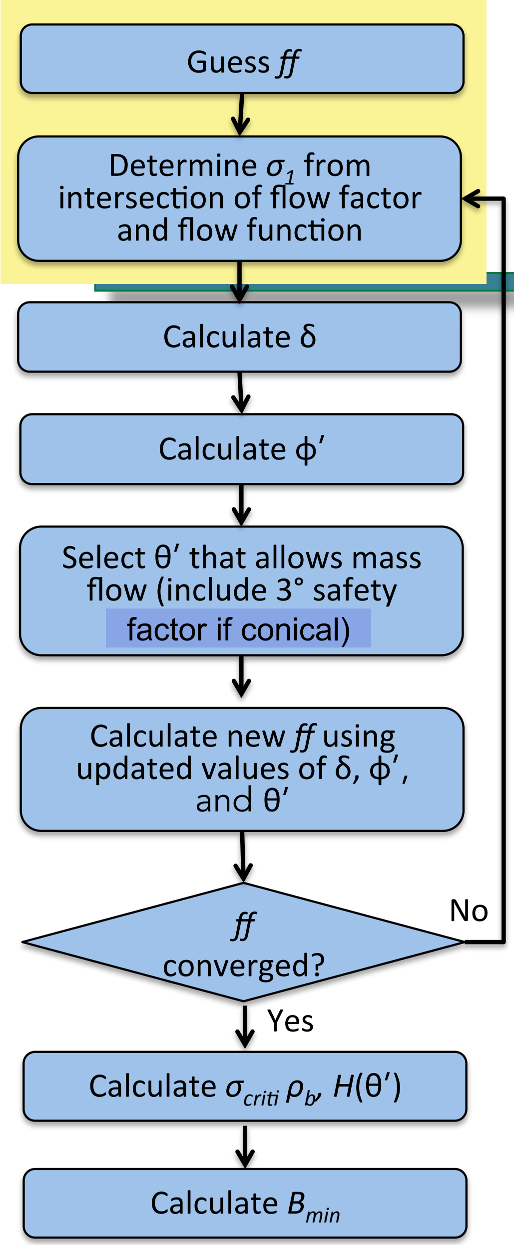
Yes

Calculate σ_{crit}, ρ_b, $H(\theta')$

Calculate B_{min}



Mass flow hopper design



Guess *ff*

Determine σ_1 from intersection of flow factor and flow function

Calculate δ

Calculate ϕ'

Select θ' that allows mass flow (include 3° safety factor if conical)

Calculate new *ff* using updated values of δ, ϕ', and θ'

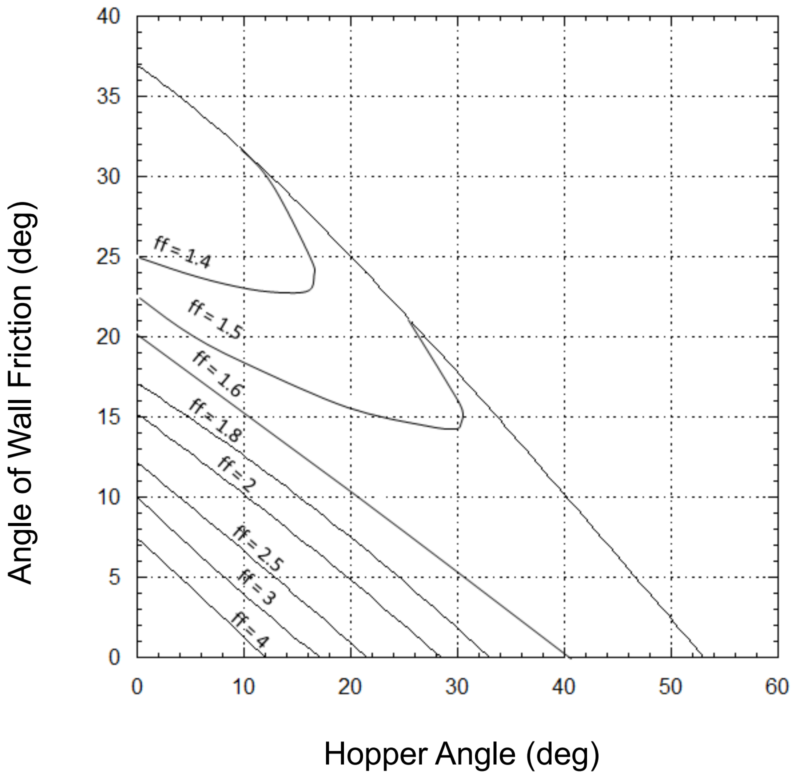
ff converged?

No

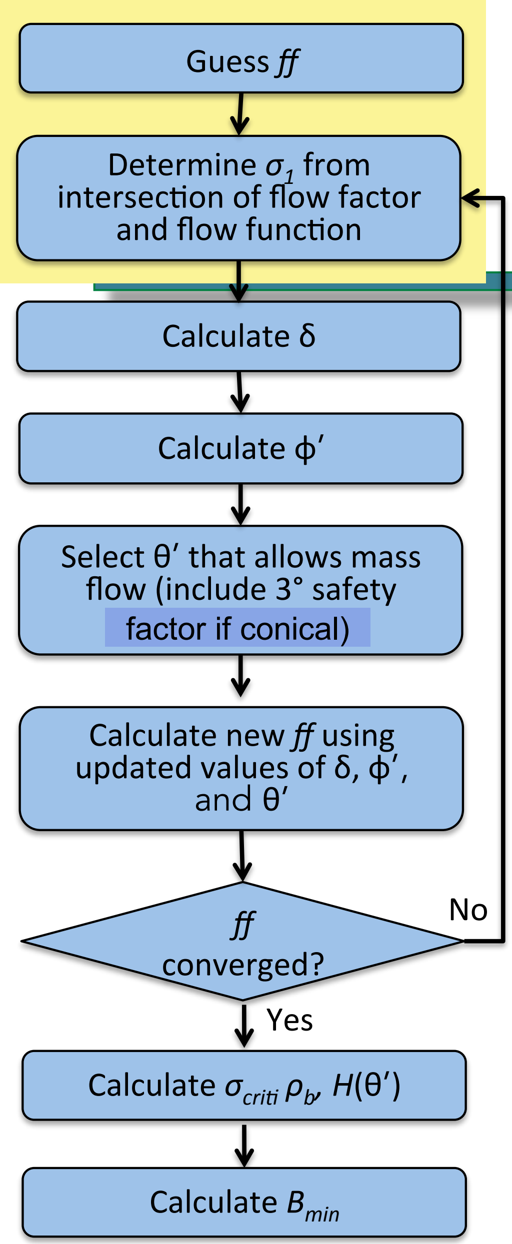
Yes

Calculate σ_{criti} ρ_b, $H(\theta')$

Calculate B_{min}



Mass flow hopper design


Guess *ff*

Determine σ_1 from intersection of flow factor and flow function

Calculate δ

Calculate ϕ′

Select θ′ that allows mass flow (include 3° safety factor if conical)

Calculate new *ff* using updated values of δ, ϕ′, and θ′

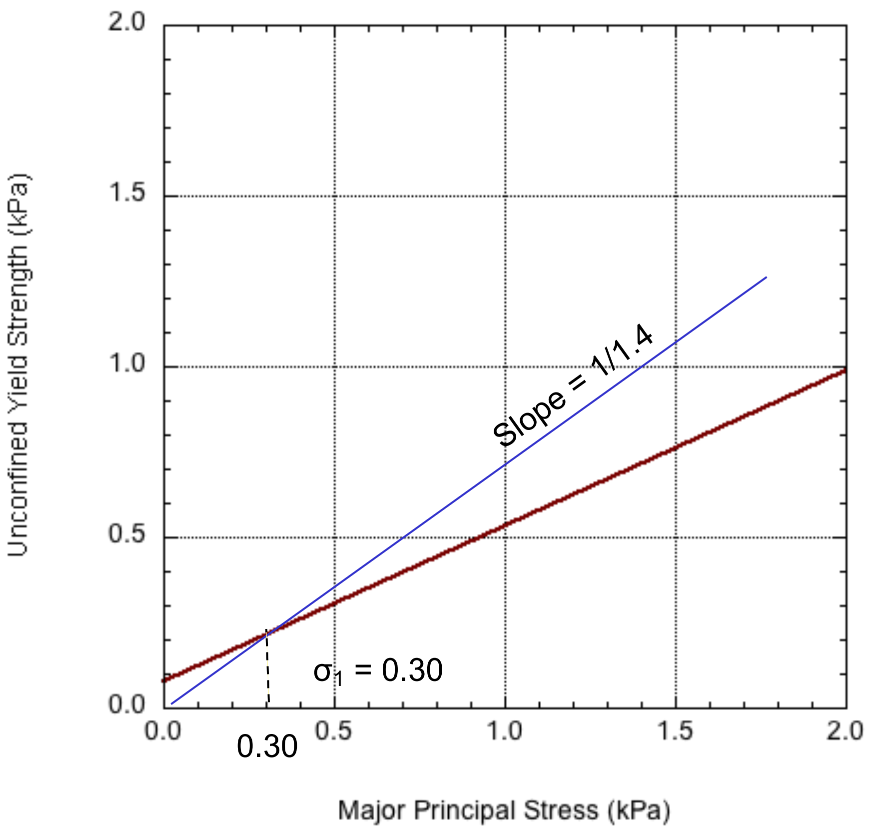
ff converged?

No

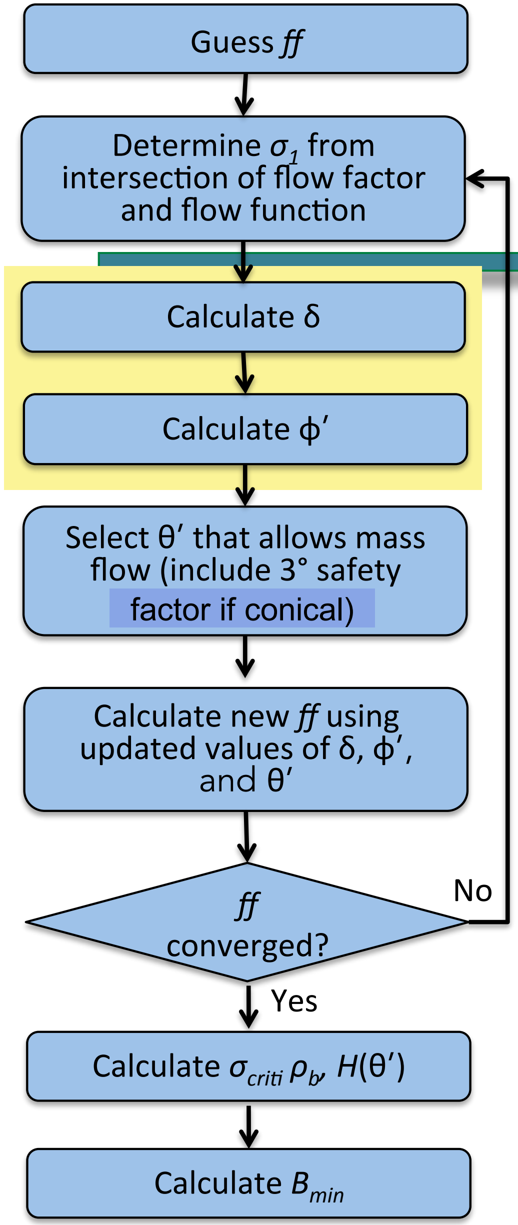
Yes

Calculate σ_{criti} ρ_{b}, $H(\theta')$

Calculate B_{min}



Mass flow hopper design



Guess *ff*

Determine σ_1 from intersection of flow factor and flow function

Calculate δ

Calculate ϕ′

Select θ′ that allows mass flow (include 3° safety factor if conical)

Calculate new *ff* using updated values of δ, ϕ′, and θ′

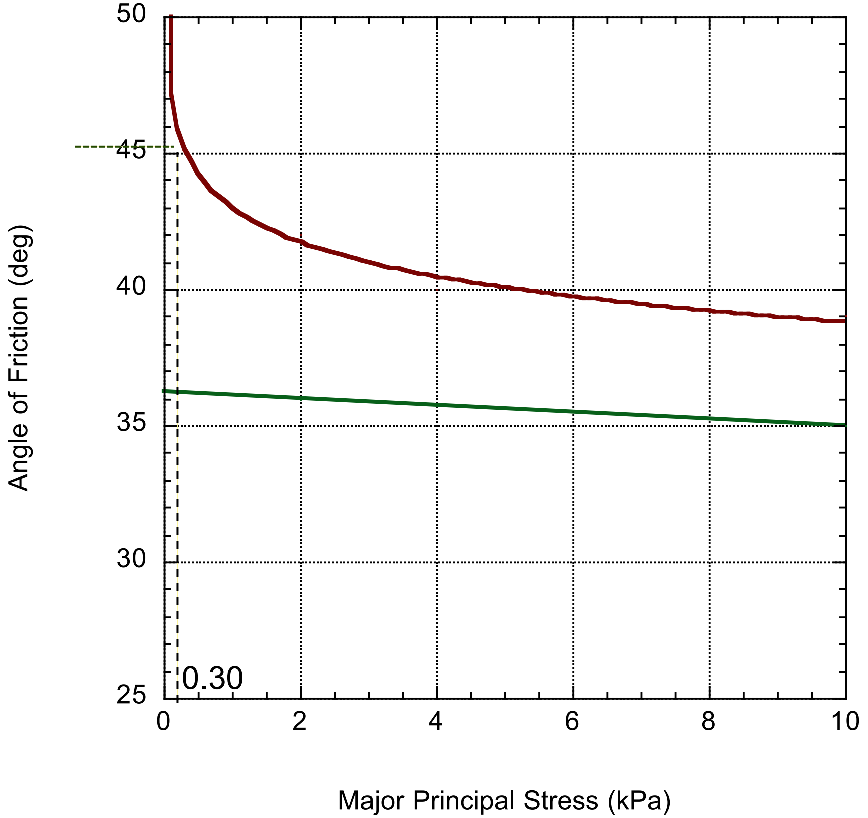
ff converged?

No

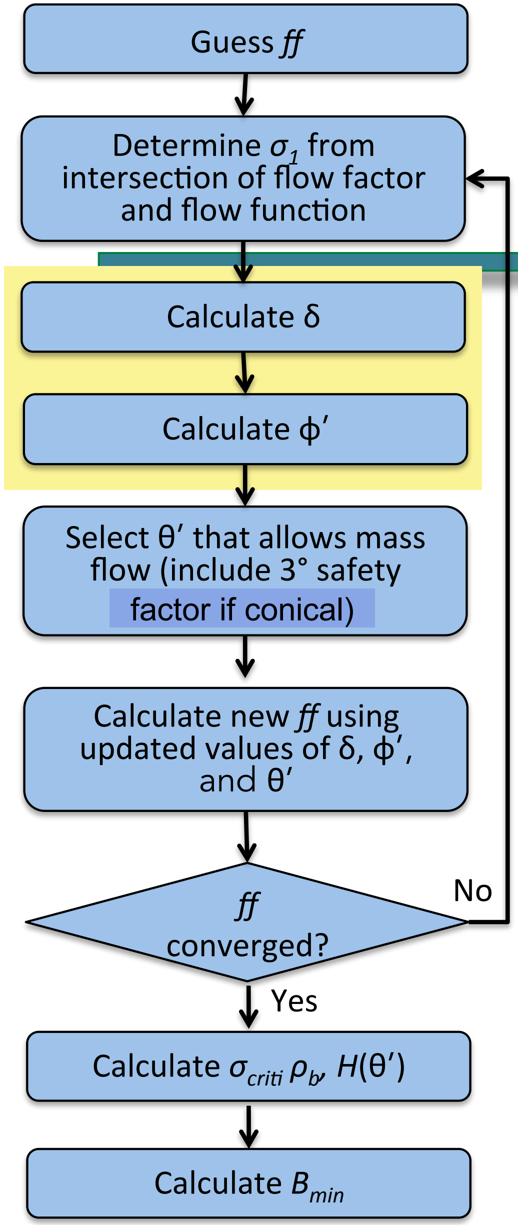
Yes

Calculate σ_{crit}, ρ_b, $H(\theta')$

Calculate B_{min}



Mass flow hopper design



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Select θ′ that allows mass flow (include 3° safety factor if conical)

Calculate new *ff* using updated values of δ, ϕ′, and θ′

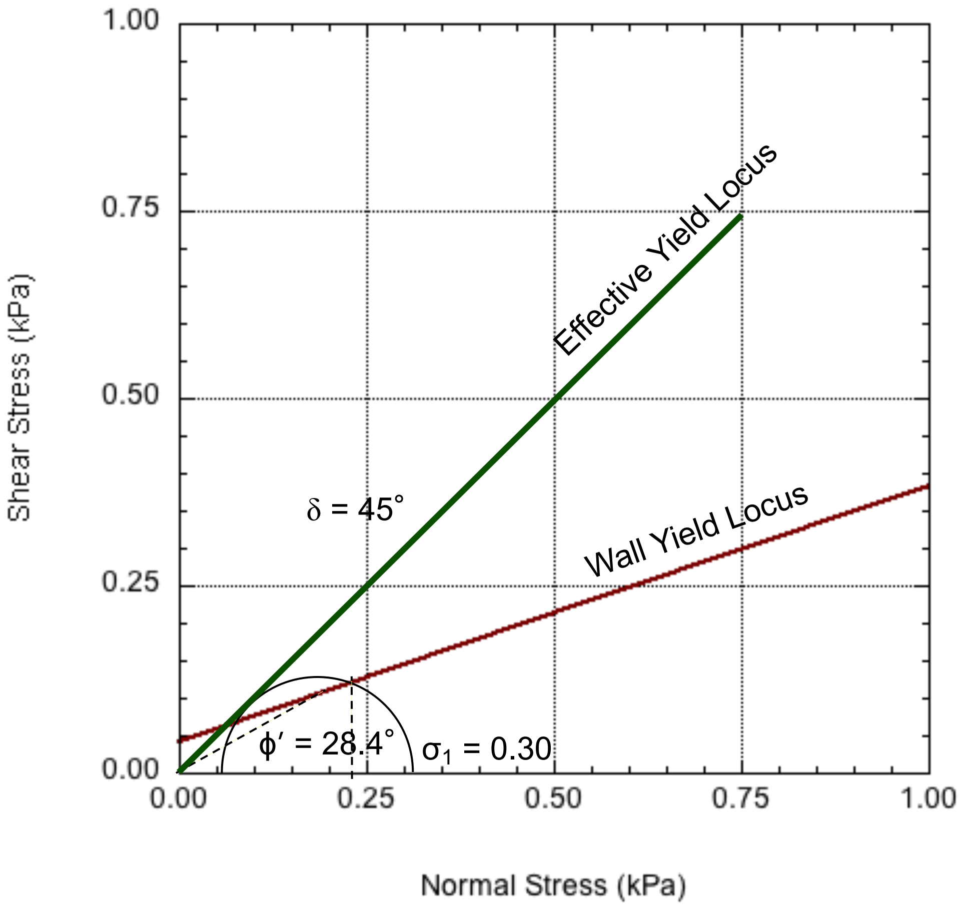
ff converged?

No

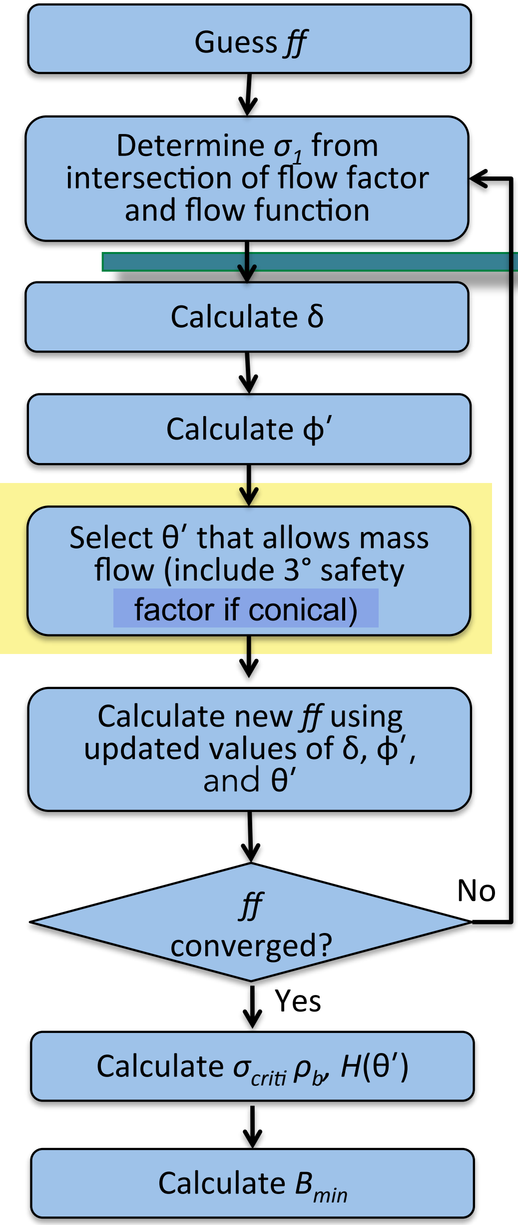
Yes

Calculate σ_{crit}, ρ_b, $H(\theta')$

Calculate B_{min}



Mass flow hopper design



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Select θ′ that allows mass flow (include 3° safety factor if conical)

Calculate new *ff* using updated values of δ, ϕ′, and θ′

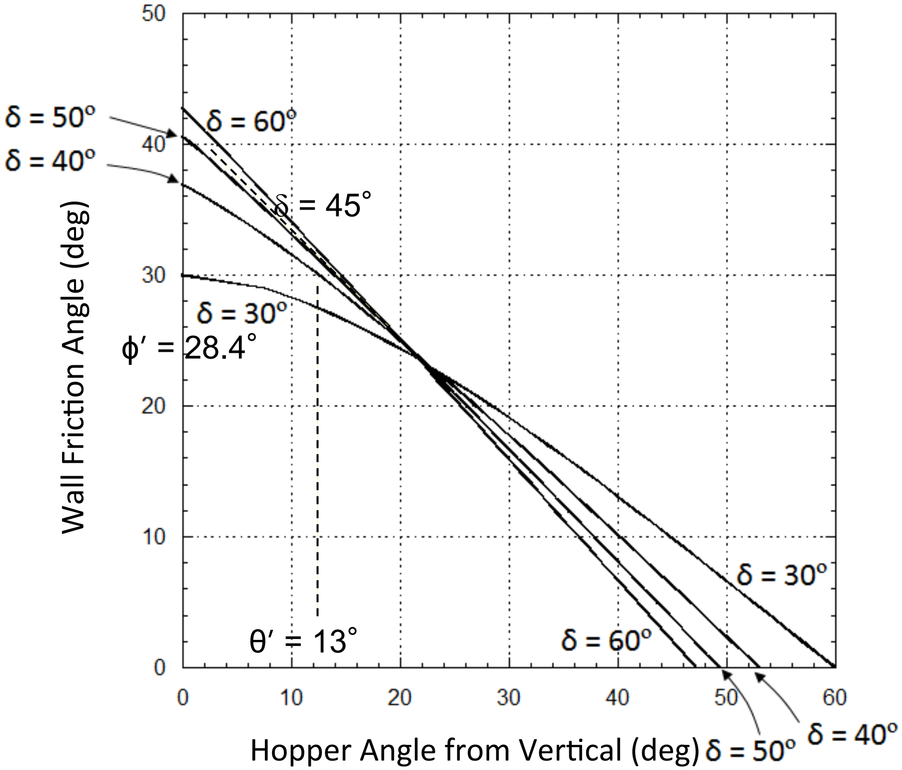
ff converged?

No

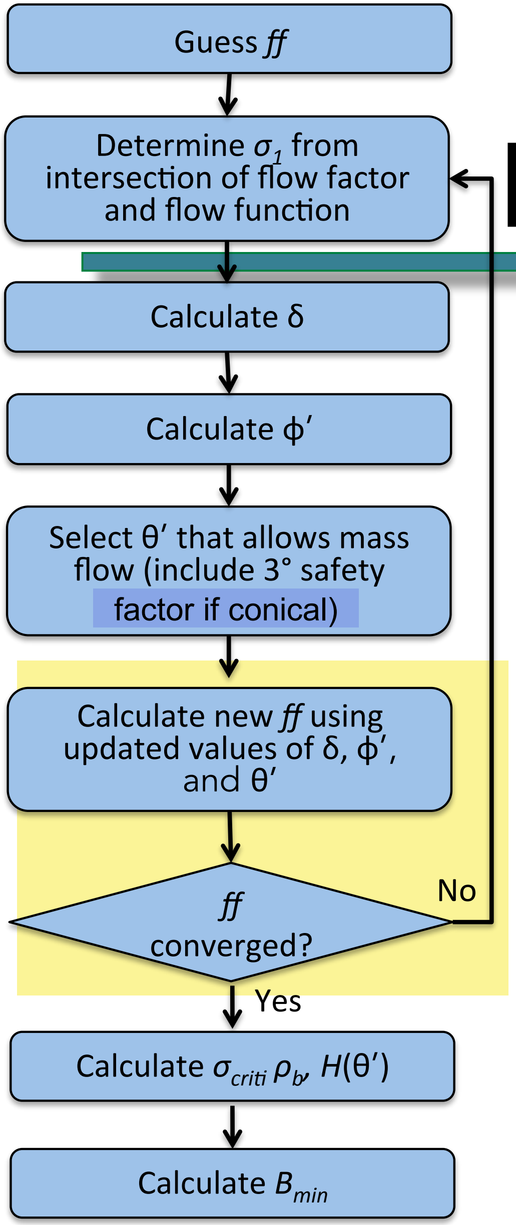
Yes

Calculate σ_{crit}, ρ_b, $H(\theta')$

Calculate B_{min}



Mass flow hopper design



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Select θ′ that allows mass flow (include 3° safety factor if conical)

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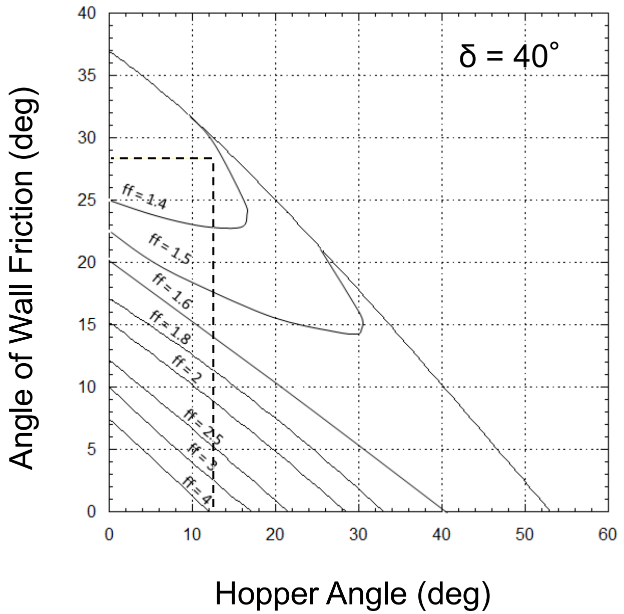
ff converged?

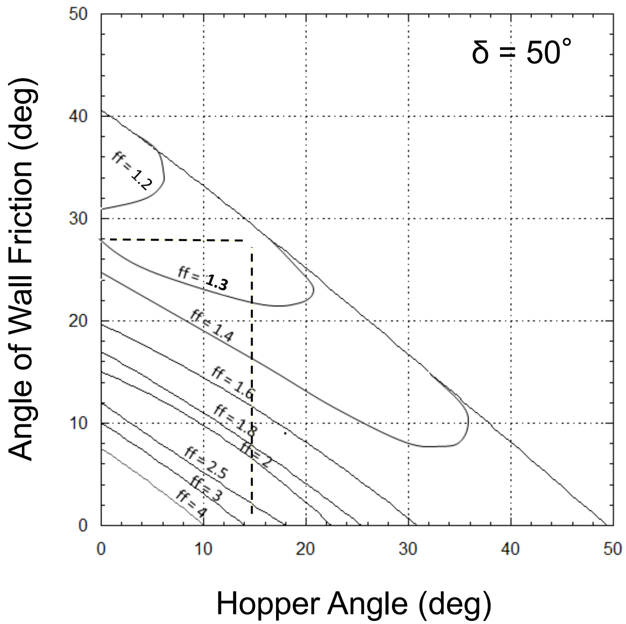
No

Yes

Calculate σ_{crit}, ρ_b, $H(\theta')$

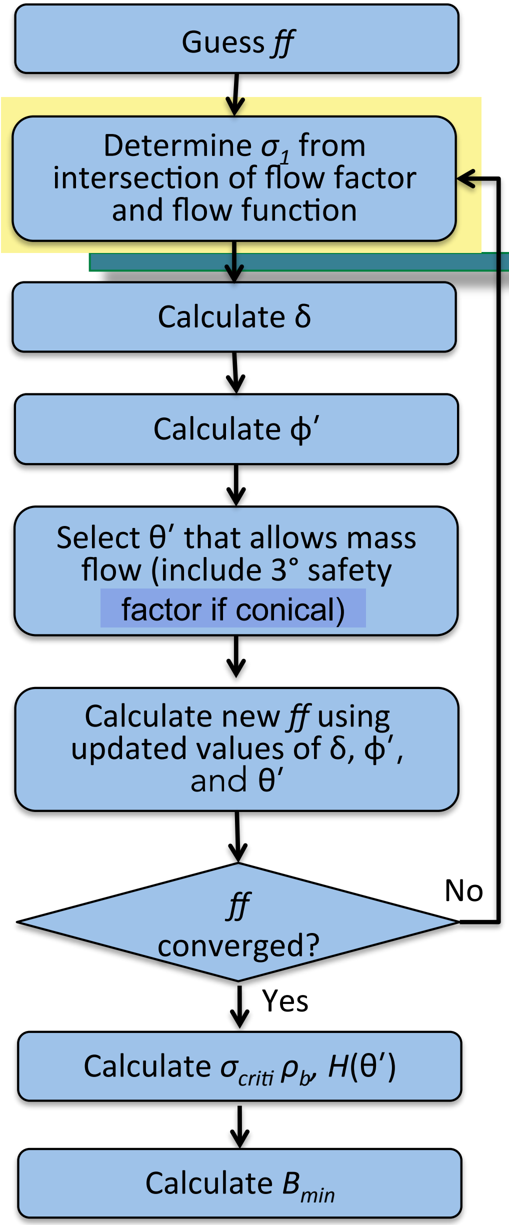
Calculate B_{min}





@ δ = 45° , ϕ′ = 28.4° , θ′ = 13° , *ff* = 1.28

Mass flow hopper design



Guess *ff*

Determine σ_1 from intersection of flow factor and flow function

Calculate δ

Calculate ϕ′

Select θ′ that allows mass flow (include 3° safety factor if conical)

Calculate new *ff* using updated values of δ, ϕ′, and θ′

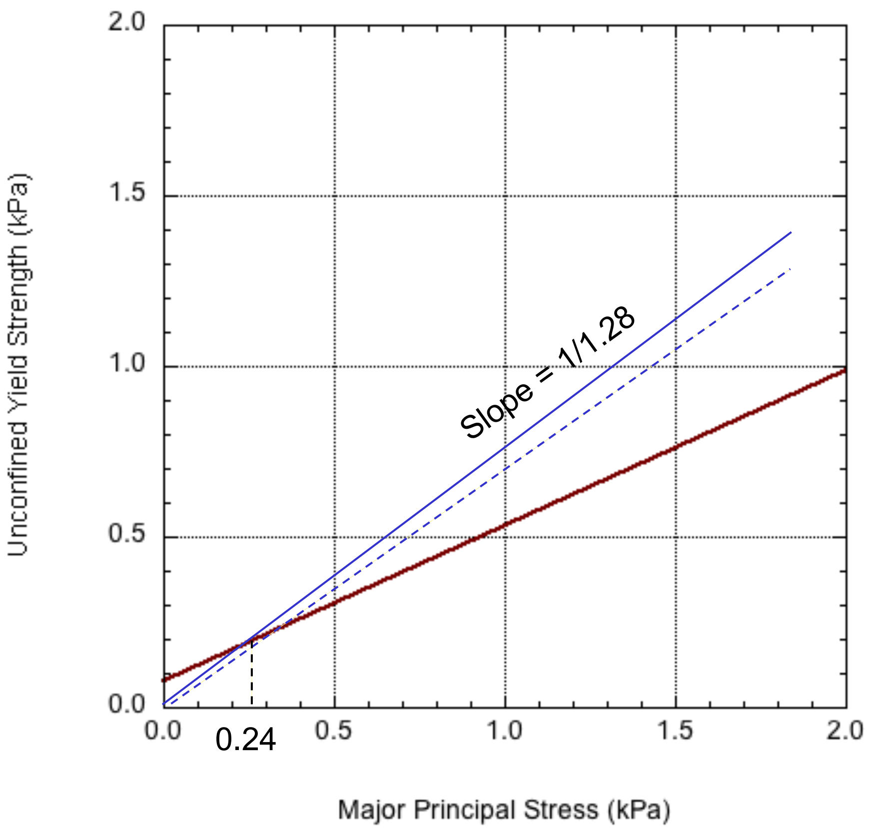
ff converged?

No

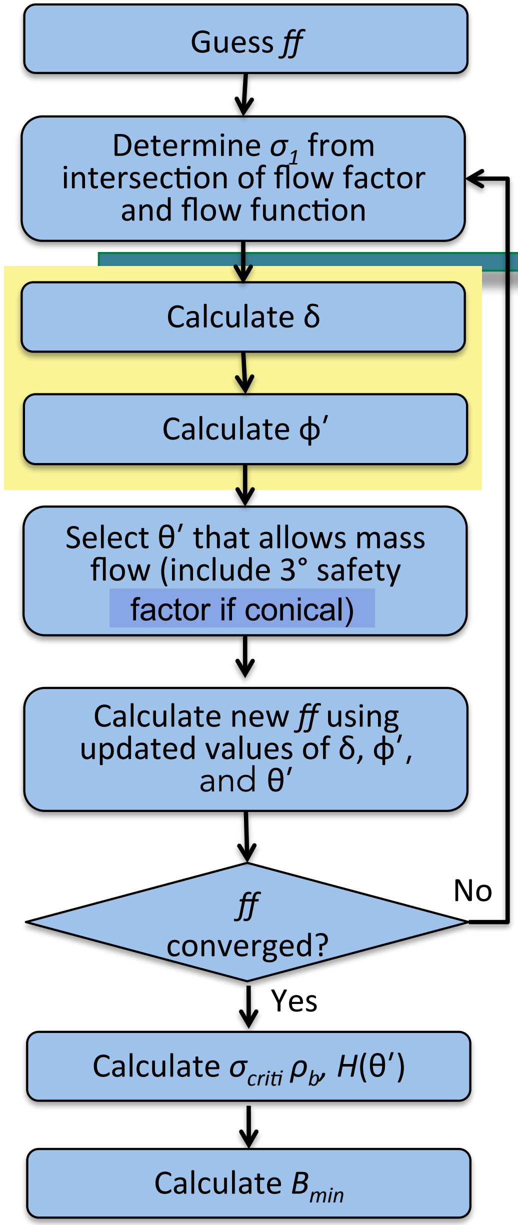
Yes

Calculate σ_{crit}, ρ_b, $H(\theta')$

Calculate B_{min}



Mass flow hopper design



Guess *ff*

Determine σ_1 from intersection of flow factor and flow function

Calculate δ

Calculate ϕ′

Select θ′ that allows mass flow (include 3° safety factor if conical)

Calculate new *ff* using updated values of δ, ϕ′, and θ′

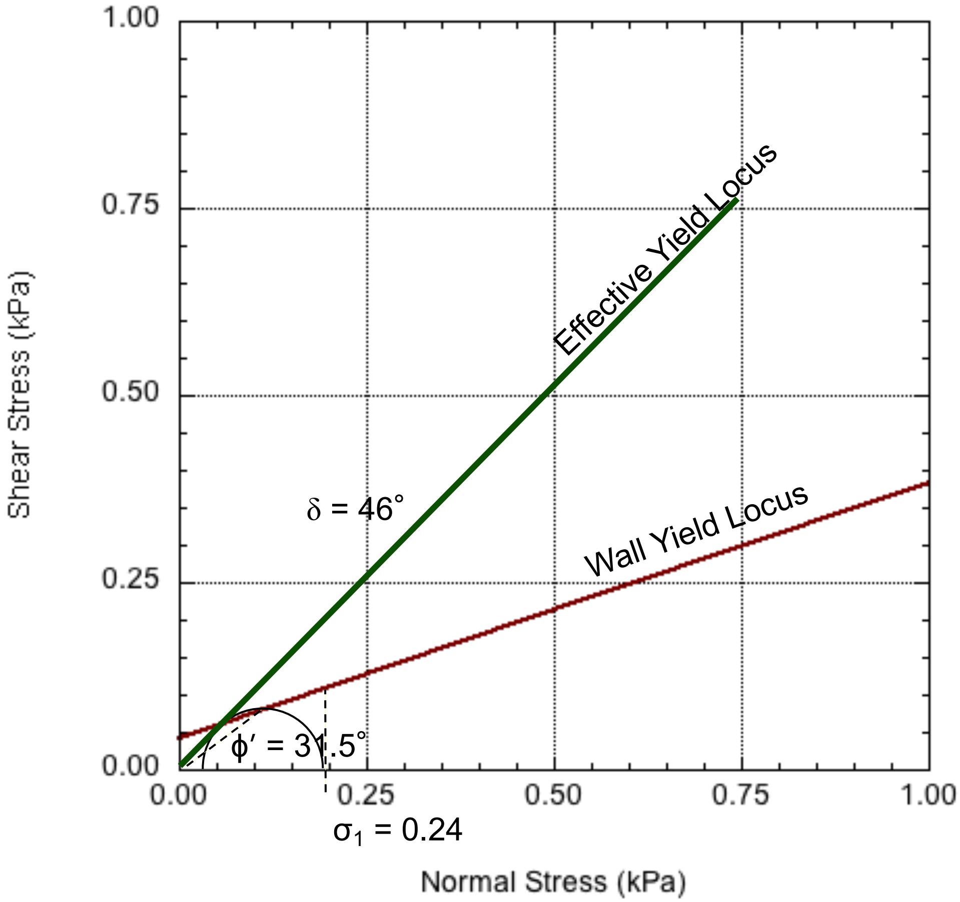
ff converged?

No

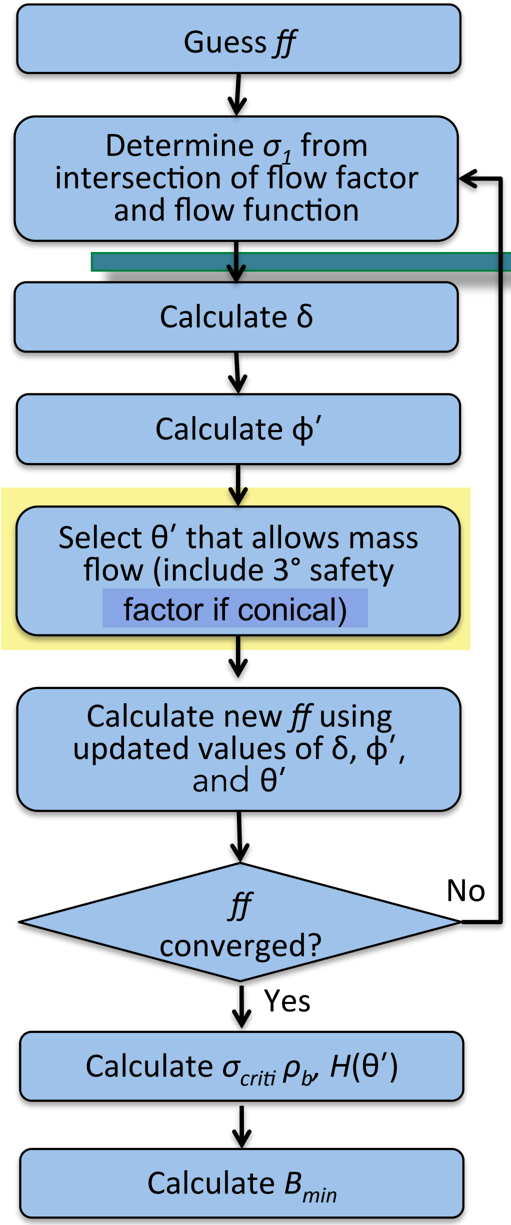
Yes

Calculate σ_{criti} $\rho_{b,}$ $H(\theta')$

Calculate B_{min}



Mass flow hopper design



Guess *ff*

Determine σ_1 from intersection of flow factor and flow function

Calculate δ

Calculate ϕ′

Select θ′ that allows mass flow (include 3° safety factor if conical)

Calculate new *ff* using updated values of δ, ϕ′, and θ′

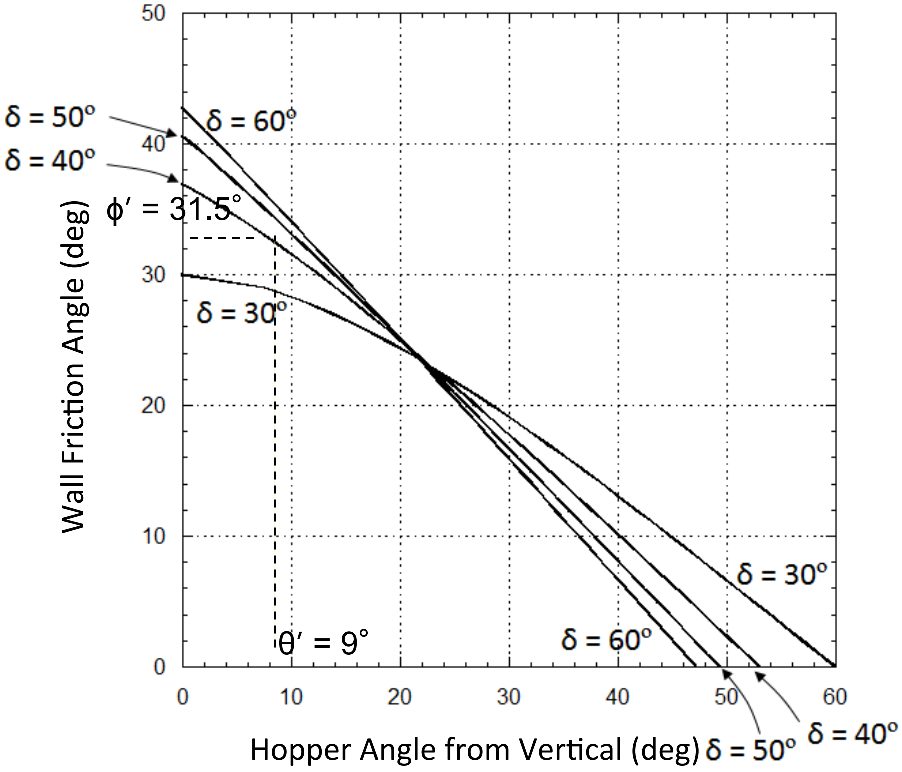
ff converged?

No

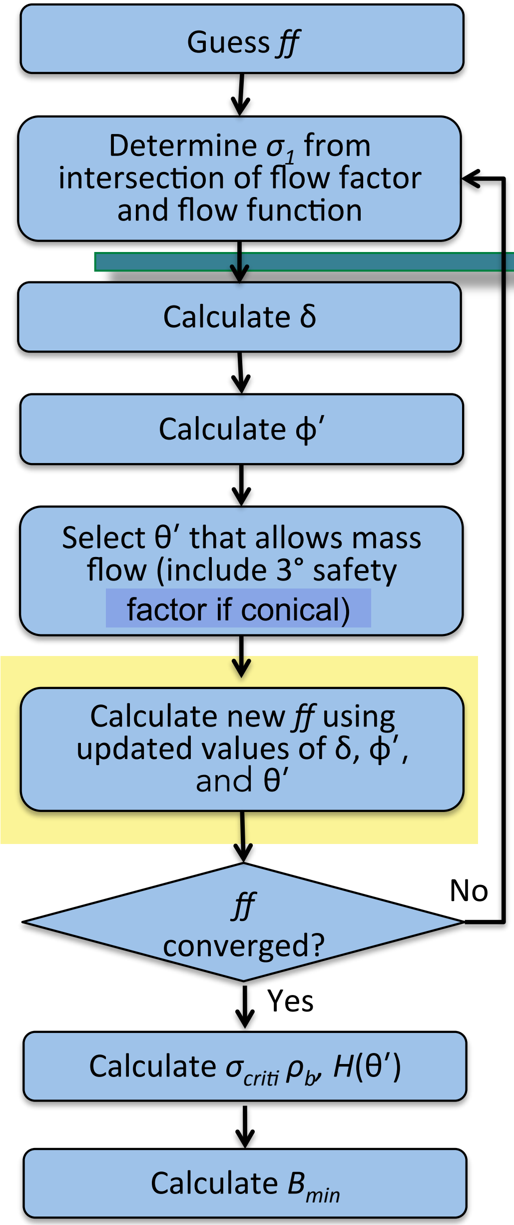
Yes

Calculate σ_{crit}, ρ_b, $H(\theta')$

Calculate B_{min}



Mass flow hopper design



Guess *ff*

Determine σ_1 from intersection of flow factor and flow function

Calculate δ

Calculate ϕ′

Select θ′ that allows mass flow (include 3° safety factor if conical)

Calculate new *ff* using updated values of δ, ϕ′, and θ′

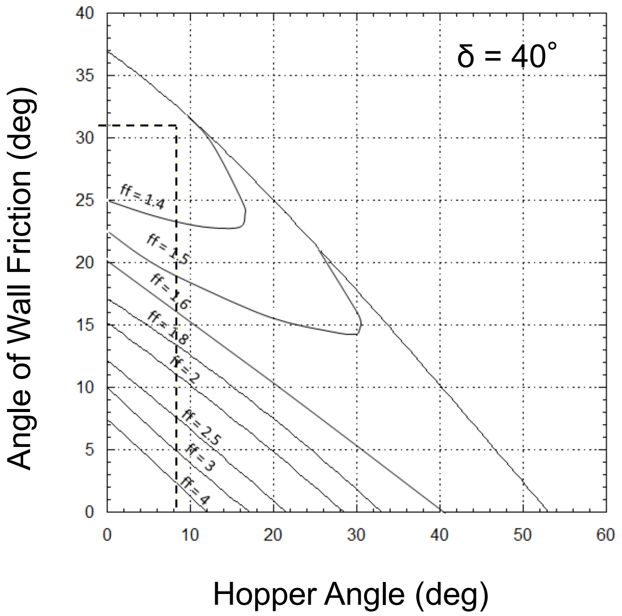
ff converged?

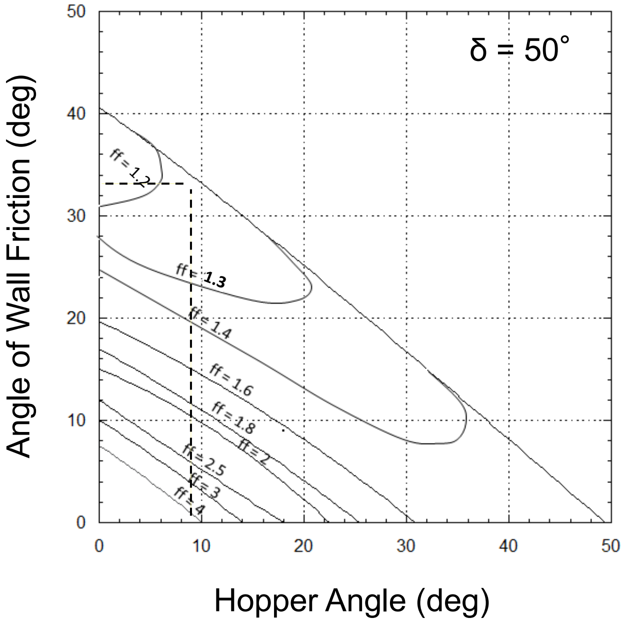
No

Yes

Calculate σ_{crit}, ρ_b, $H(\theta')$

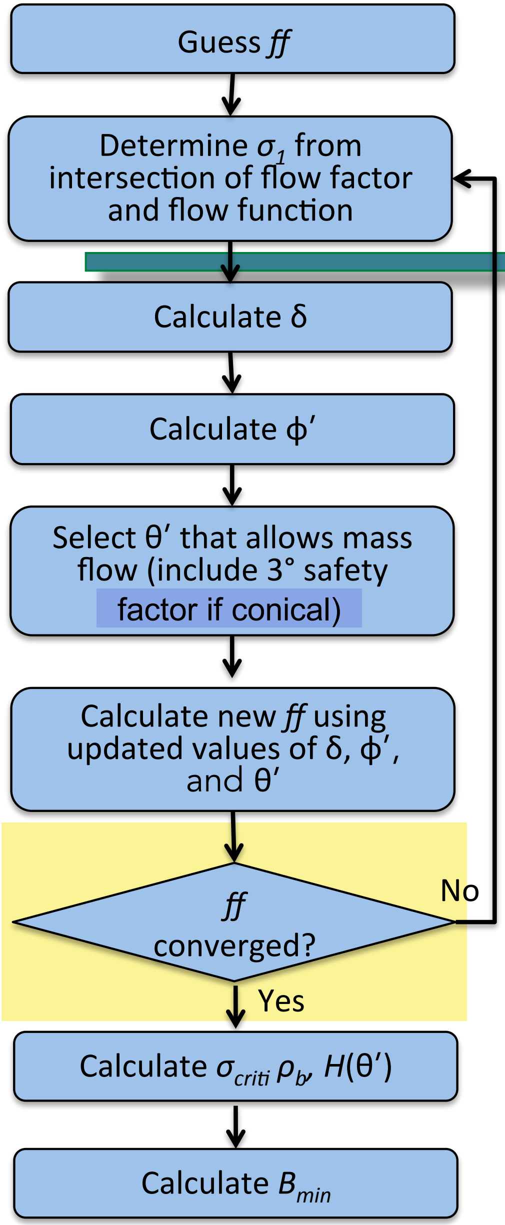
Calculate B_{min}





@ δ = 46° , ϕ′ = 31.5° , θ′ = 9° *ff* = 1.25

Mass flow hopper design



Guess *ff*

Determine σ_1 from intersection of flow factor and flow function

Calculate δ

Calculate ϕ′

Select θ′ that allows mass flow (include 3° safety factor if conical)

Calculate new *ff* using updated values of δ, ϕ′, and θ′

ff converged?

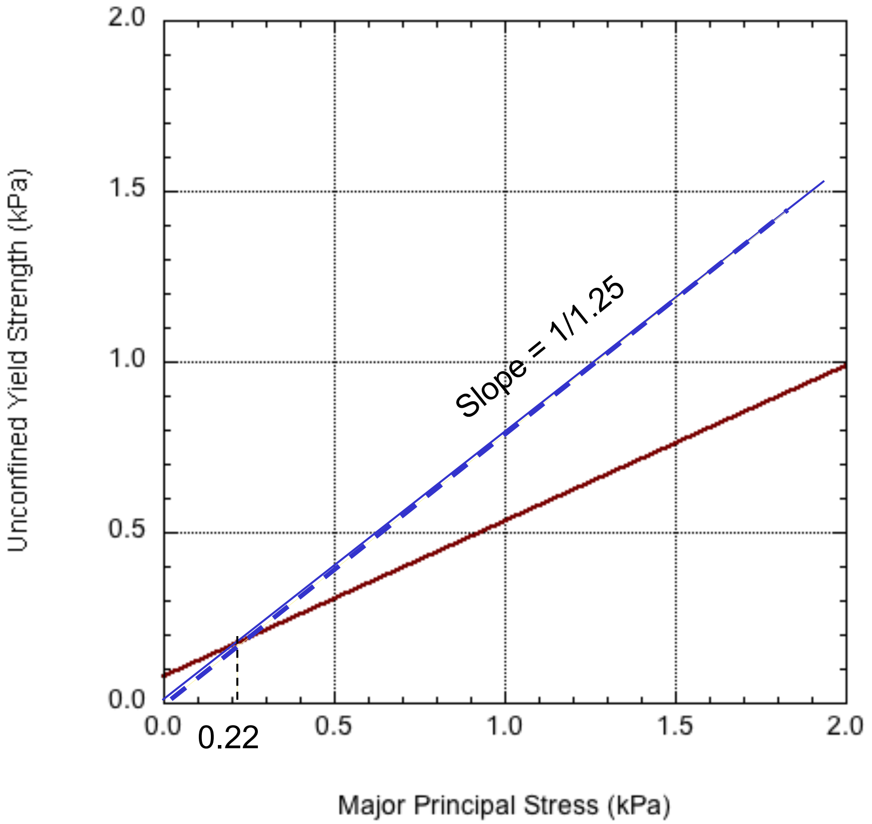
No

Yes

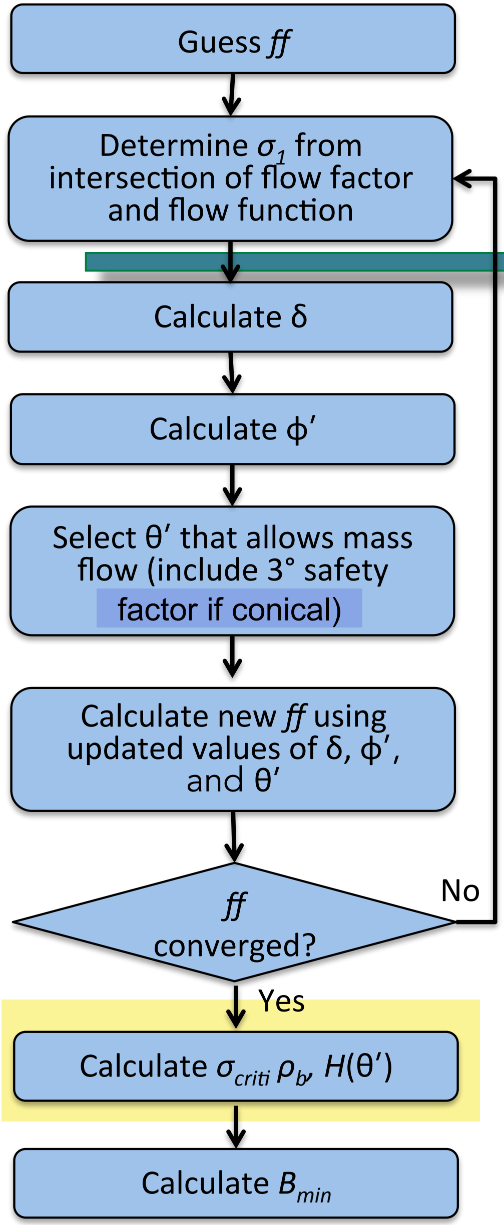
Calculate σ_{crit}, ρ_b, $H(\theta')$

Calculate B_{min}

Yes!



Mass flow hopper design



Guess *ff*

Determine σ_1 from intersection of flow factor and flow function

Calculate δ

Calculate ϕ′

Select θ′ that allows mass flow (include 3° safety factor if conical)

Calculate new *ff* using updated values of δ, ϕ′, and θ′

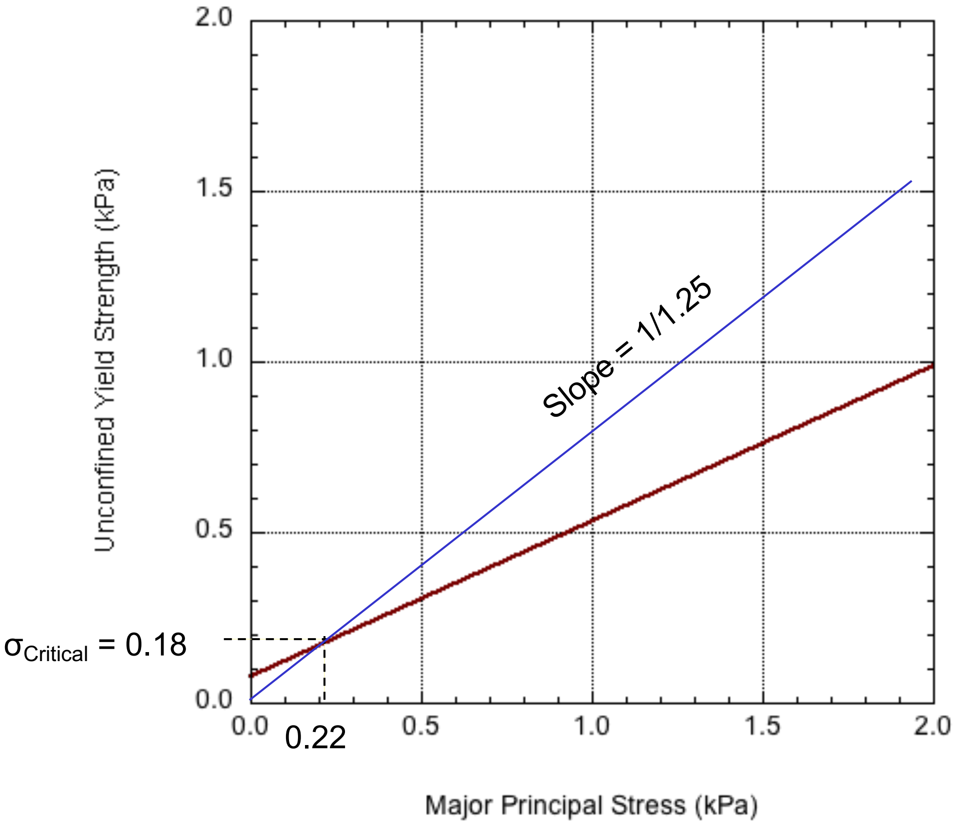
ff converged?

No

Yes

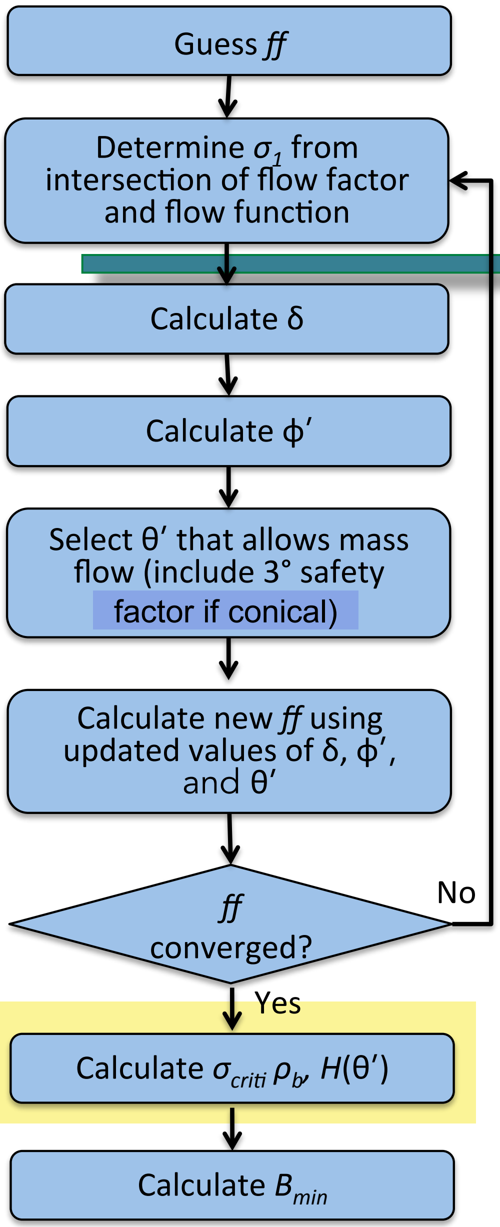
Calculate σ_{criti} ρ_b, H(θ′)

Calculate B_{min}



$$B_{\min} = \frac{H(\theta')\sigma_{Critical}}{\rho_b g}$$

Mass flow hopper design



Guess *ff*

Determine σ_1 from intersection of flow factor and flow function

Calculate δ

Calculate ϕ′

Select θ′ that allows mass flow (include 3° safety factor if conical)

Calculate new *ff* using updated values of δ, ϕ′, and θ′

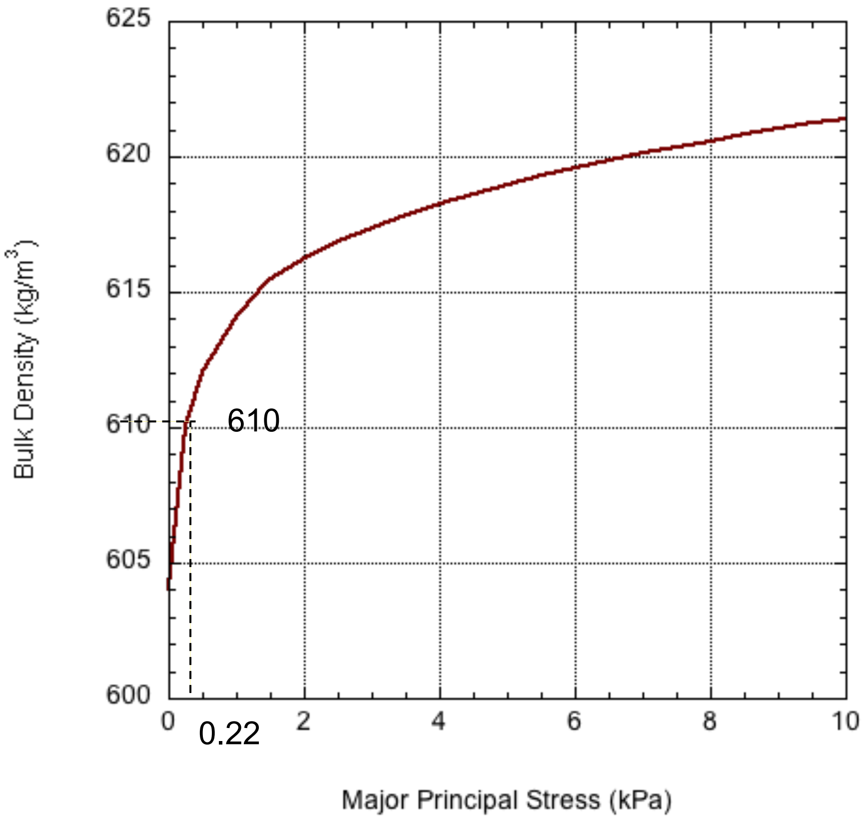
ff converged?

No

Yes

Calculate σ_{crit}, ρ_b, $H(\theta')$

Calculate B_{min}



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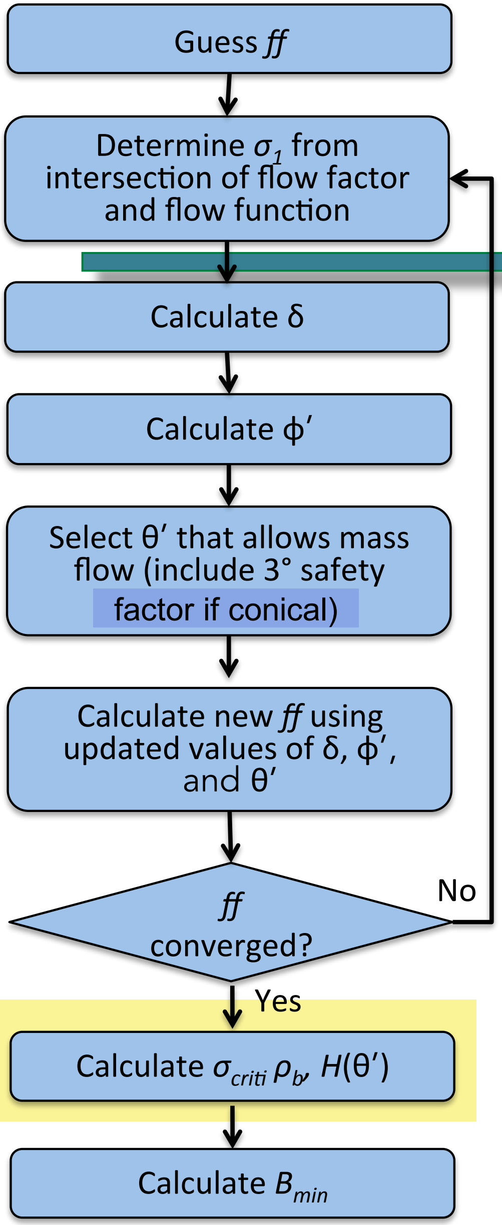
Mass flow hopper design



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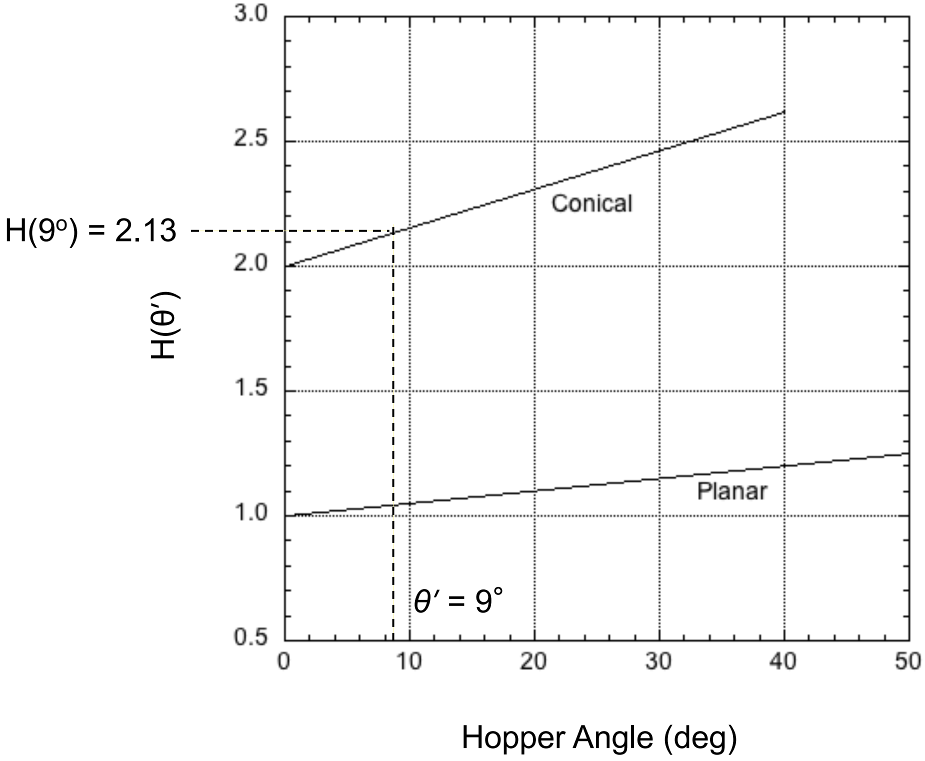
No

Yes

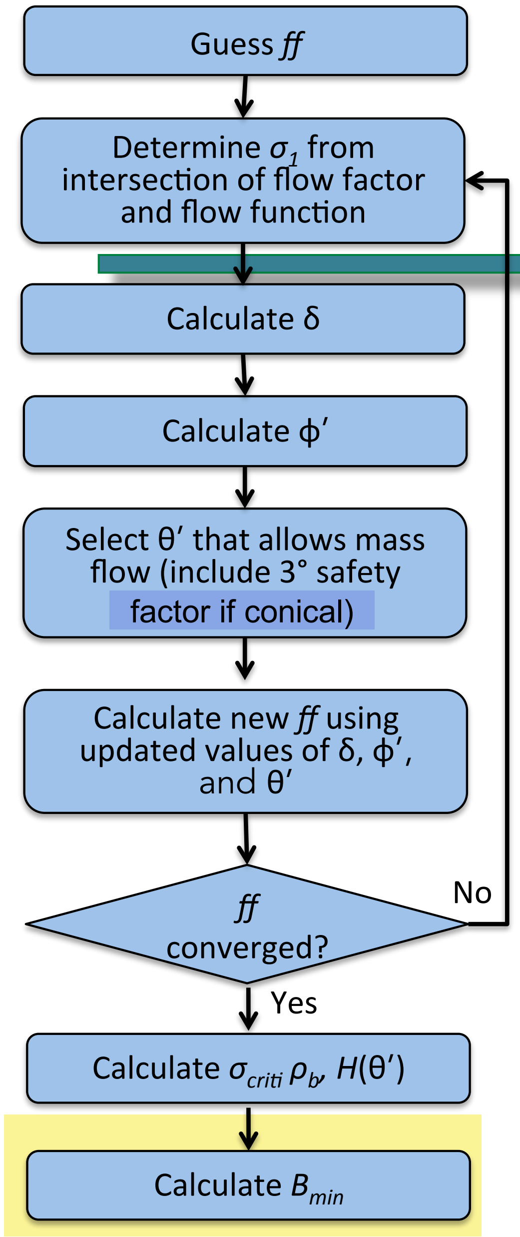
Calculate σ_{criti} ρ_b, $H(\theta')$

Calculate B_{min}



$$B_{\min} = \frac{H(\theta')\sigma_{Critical}}{\rho_b g}$$

Mass flow hopper design


Guess *ff*

Determine σ_1 from intersection of flow factor and flow function

Calculate δ

Calculate ϕ′

Select θ′ that allows mass flow (include 3° safety factor if conical)

Calculate new *ff* using updated values of δ, ϕ′, and θ′

ff converged?

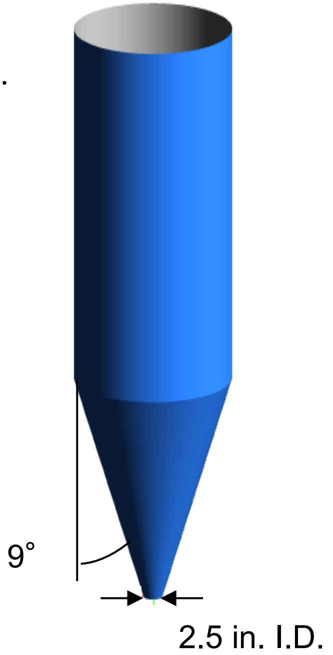
No

Yes

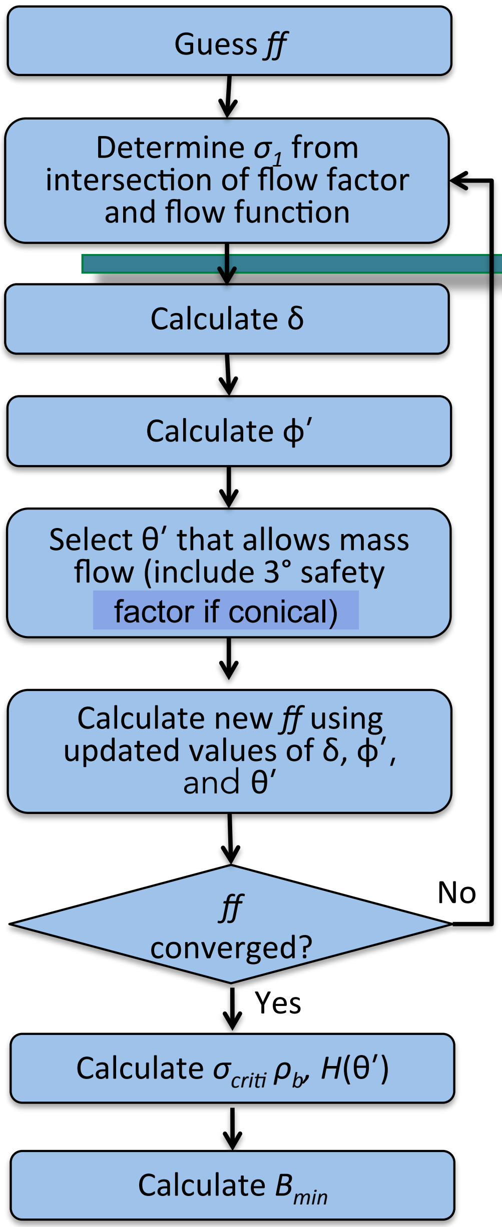
Calculate σ_{criti} ρ_b, $H(\theta')$

Calculate B_{min}

$$B_{\min} = \frac{H(\theta')\sigma_{Critical}}{\rho_b g} = \frac{(2.19)(0.18)(1000)}{(9.8)(610)} = 0.064 \text{ m} = 2.5 \text{ in.}$$



Mass flow hopper design



Guess *ff*

Determine σ_1 from intersection of flow factor and flow function

Calculate δ

Calculate ϕ′

Select θ′ that allows mass flow (include 3° safety factor if conical)

Calculate new *ff* using updated values of δ, ϕ′, and θ′

ff converged?

No

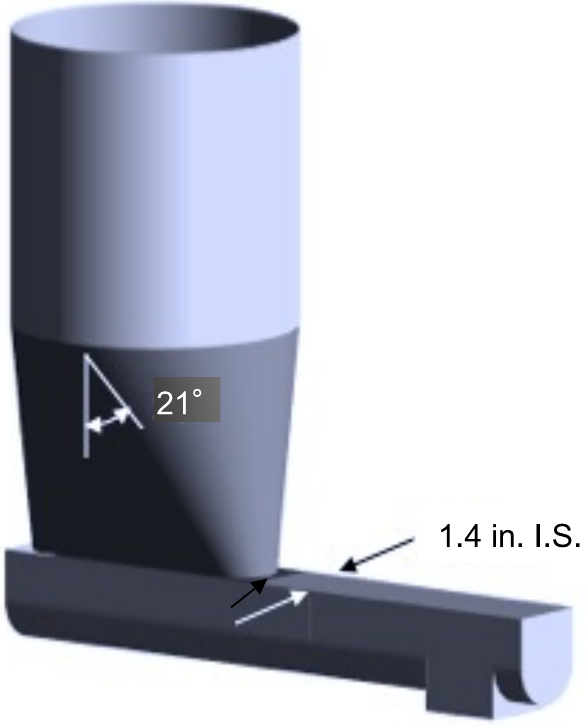
Yes

Calculate σ_{crit}, ρ_b, $H(\theta')$

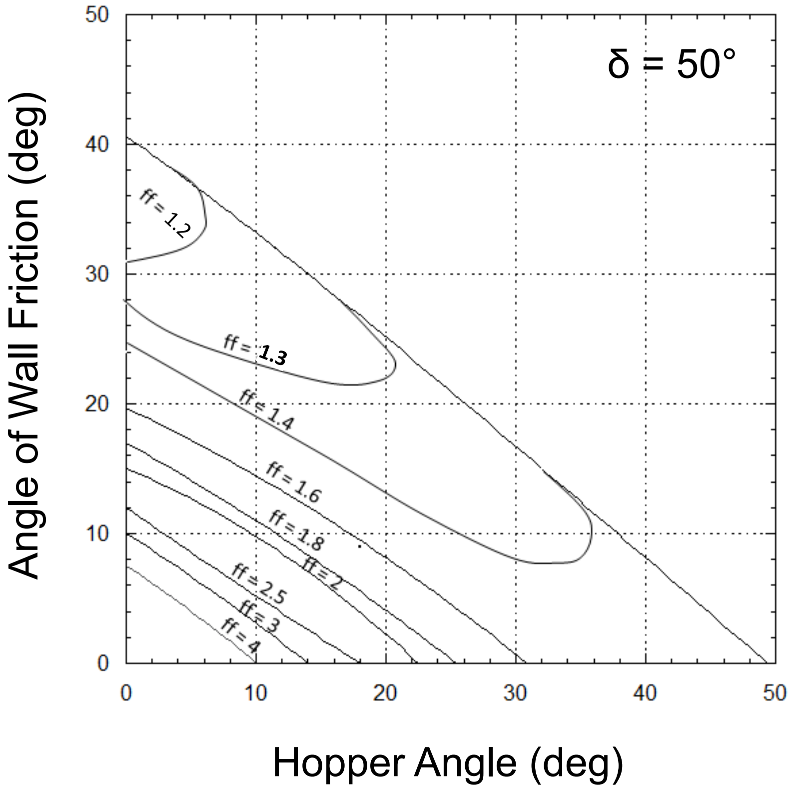
Calculate B_{min}

$$B_{\min} = \frac{H(\theta')\sigma_{Critical}}{\rho_b g}$$

$$= \frac{(1.110)(0.20)(1000)}{(9.8)(611)} = 0.036 \text{ m} = 1.4 \text{ in.}$$



Short-cut method



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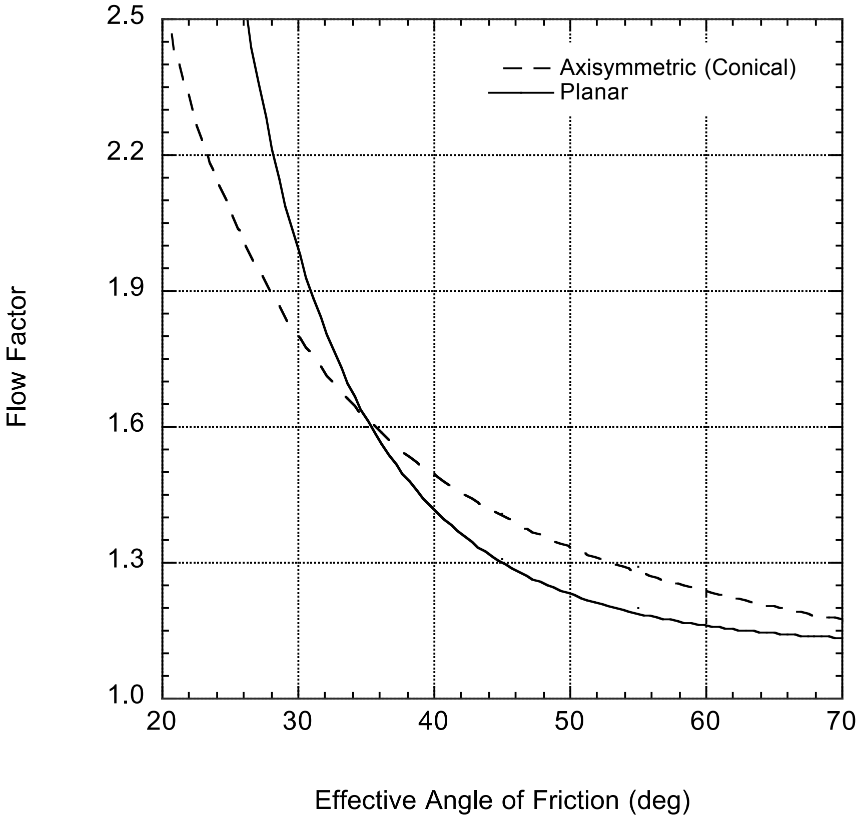
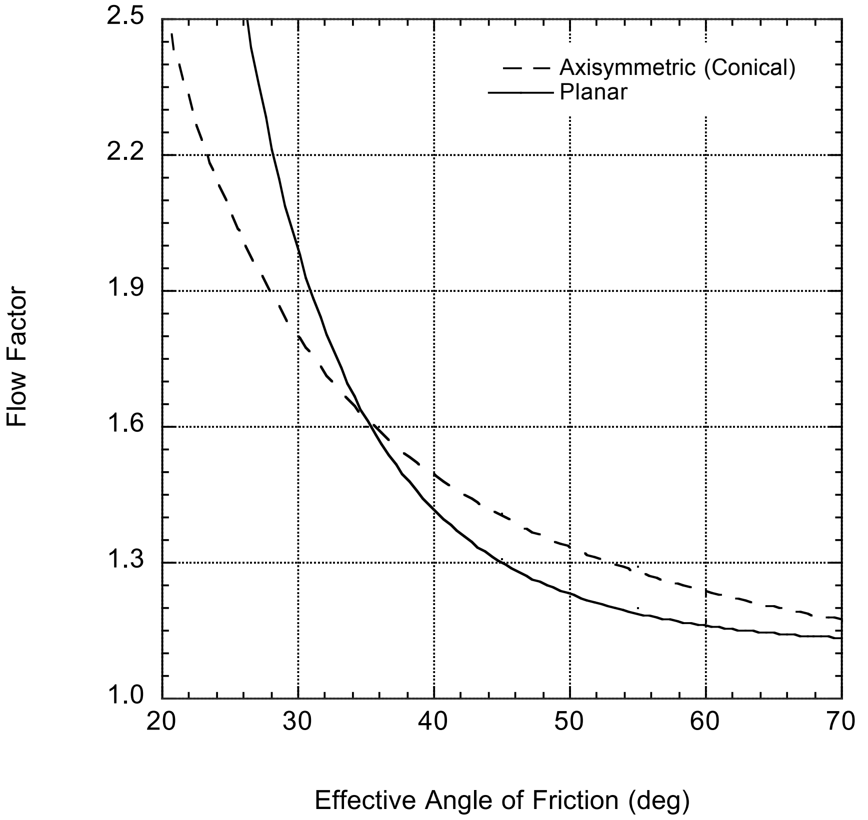


Johanson (1985)

Short-cut method



Johanson (1985)

Mehos (2019)

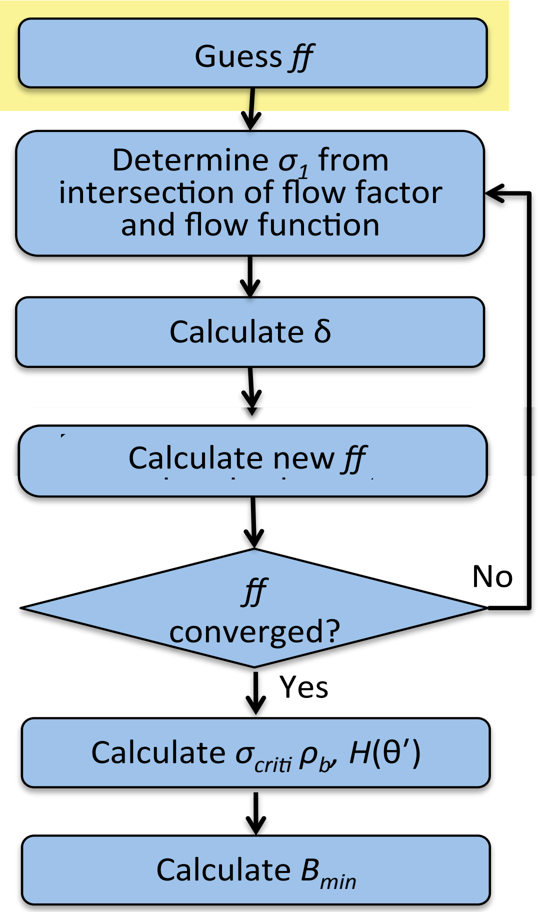
Axisymmetric

$$ff = 1.118 + \frac{0.285}{(\tan\delta)^{1.59}}$$

Planar

$$ff = 1.125 + \frac{0.176}{(\tan\delta)^{2.90}}$$

Short-cut method



Guess ff

Determine σ_1 from intersection of flow factor and flow function

Calculate δ

Calculate new ff

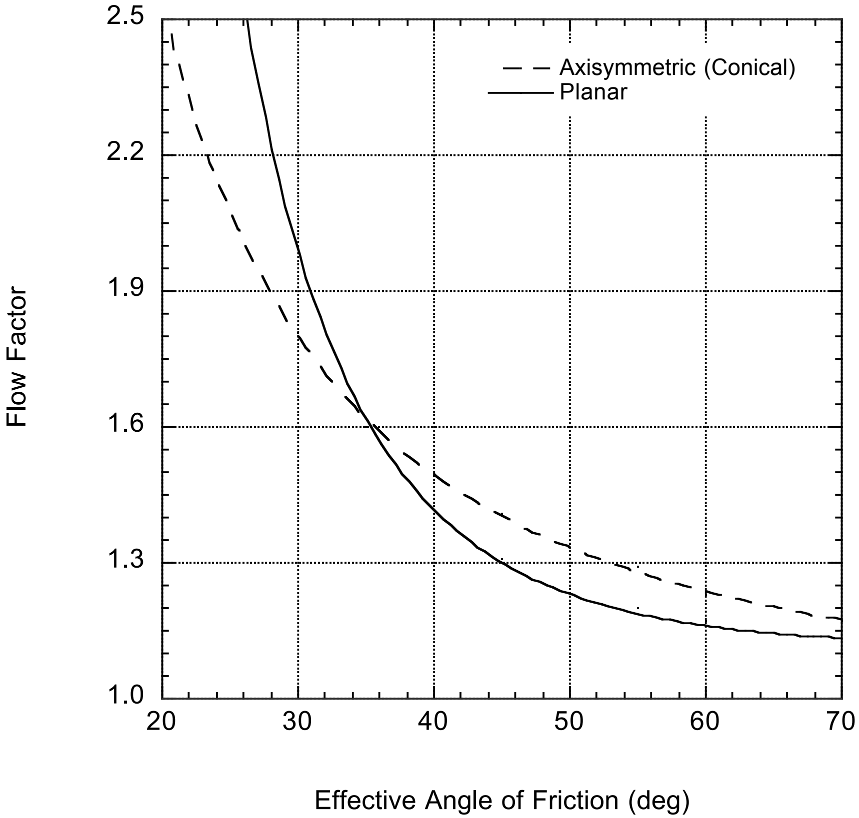
ff converged?

No

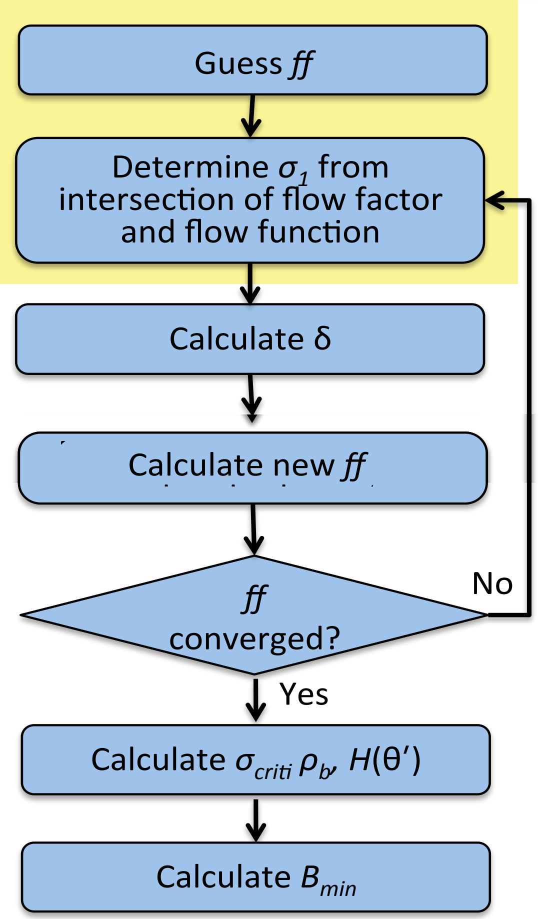
Yes

Calculate σ_{crit}, ρ_b, $H(\theta')$

Calculate B_{min}



Short-cut method



Guess *ff*

Determine σ_1 from intersection of flow factor and flow function

Calculate δ

Calculate new *ff*

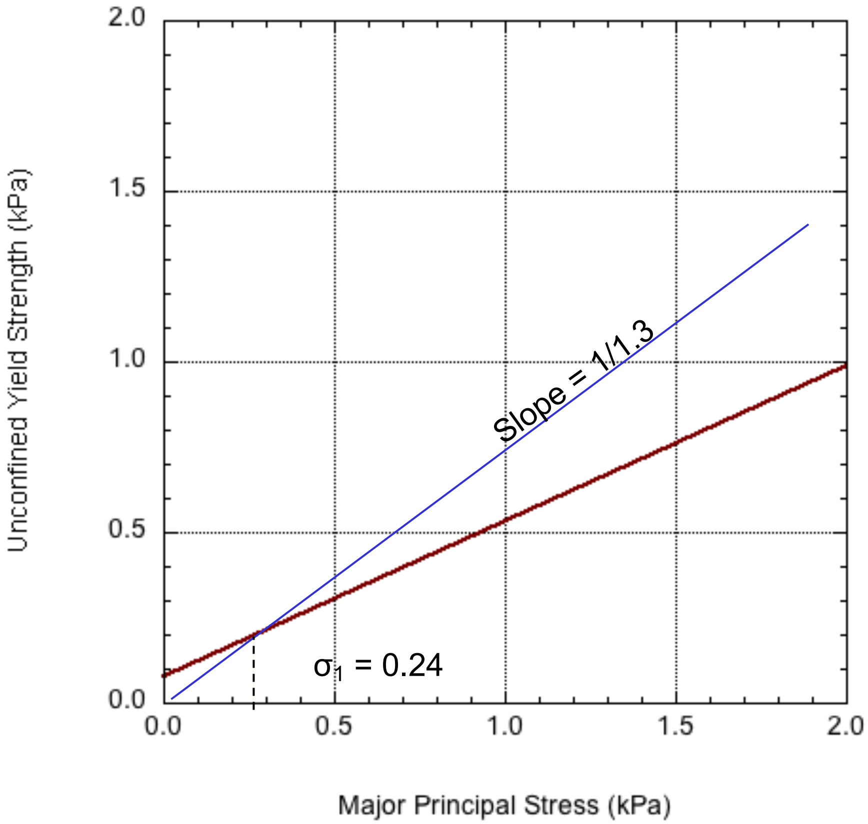
ff converged?

No

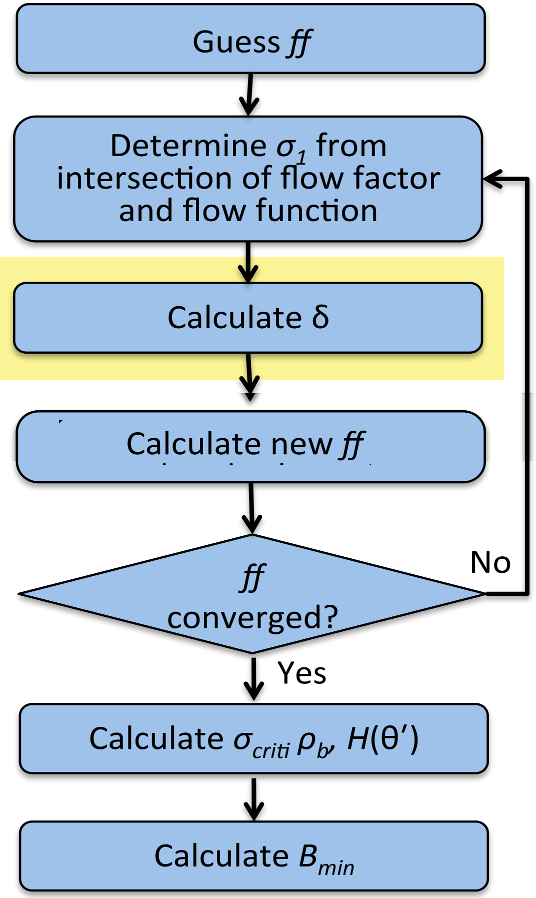
Yes

Calculate σ_{criti} ρ_{b}, $H(\theta')$

Calculate B_{min}



Short-cut method



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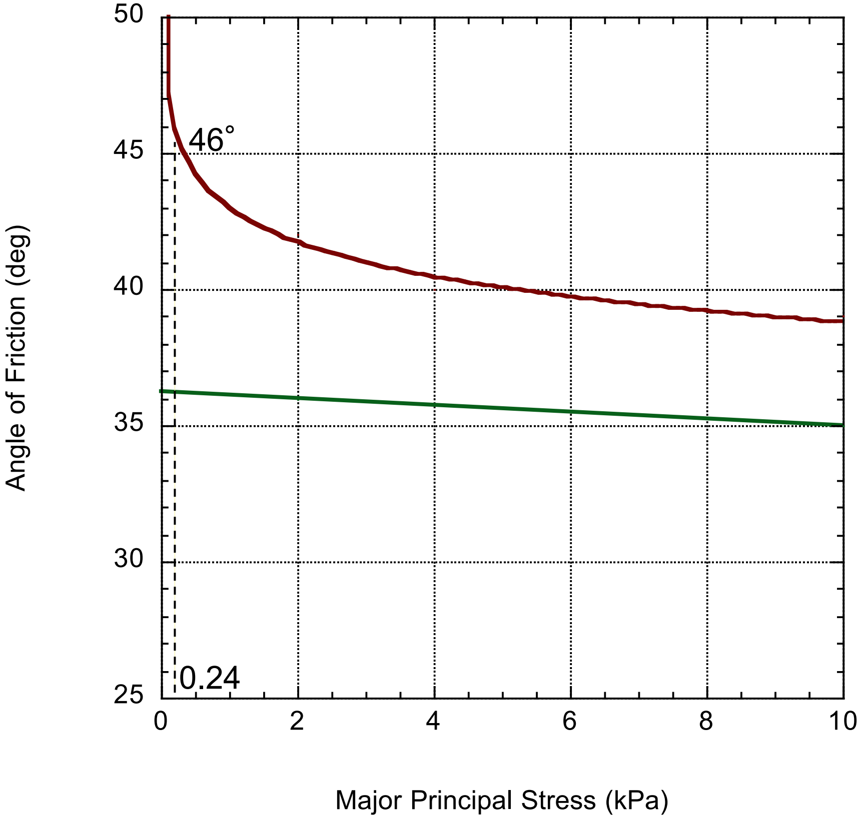
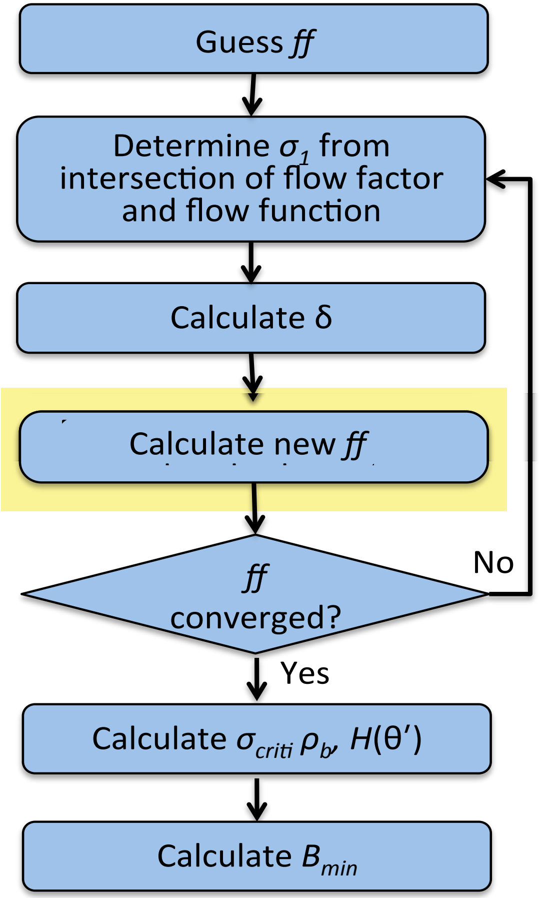


Short-cut method



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Calculate σ_{crit}, ρ_b, $H(\theta')$

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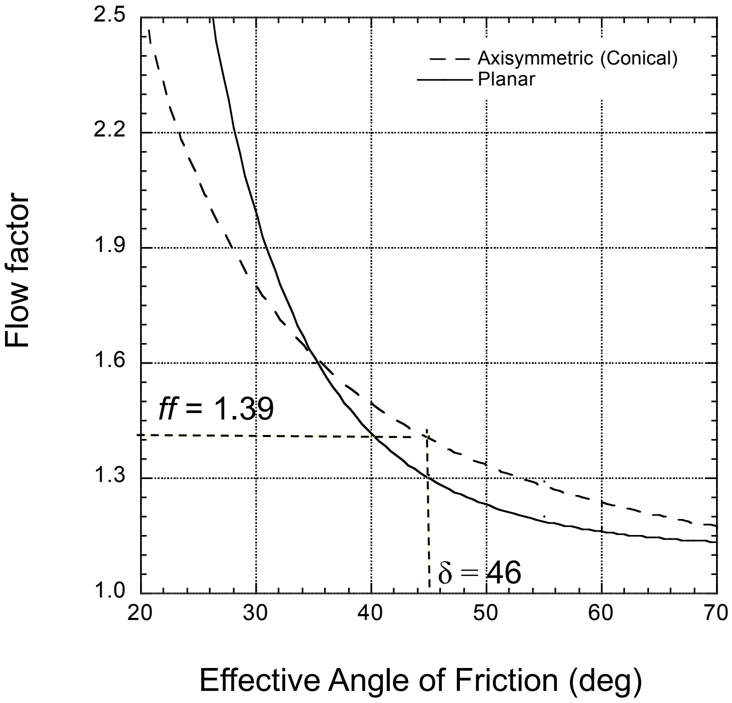
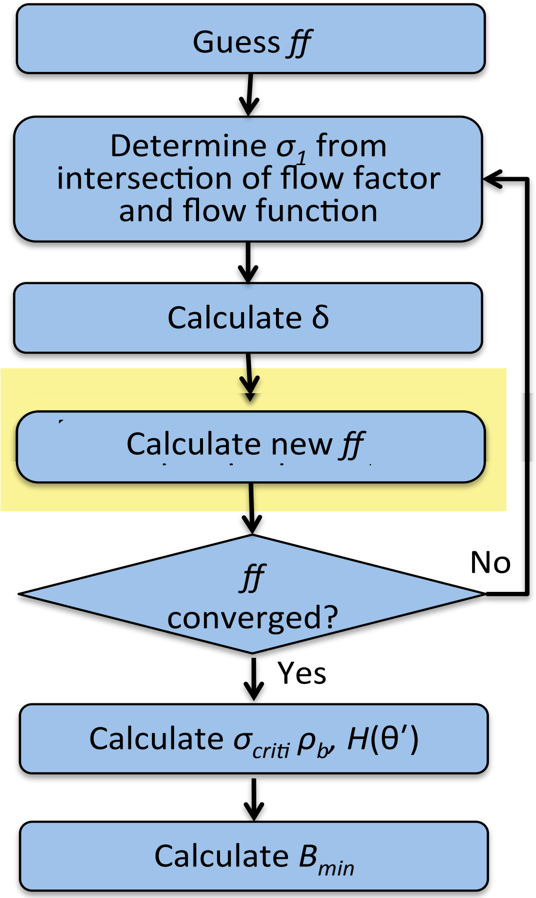


Short-cut method



Guess *ff*

Determine σ_1 from intersection of flow factor and flow function

Calculate δ

Calculate new *ff*

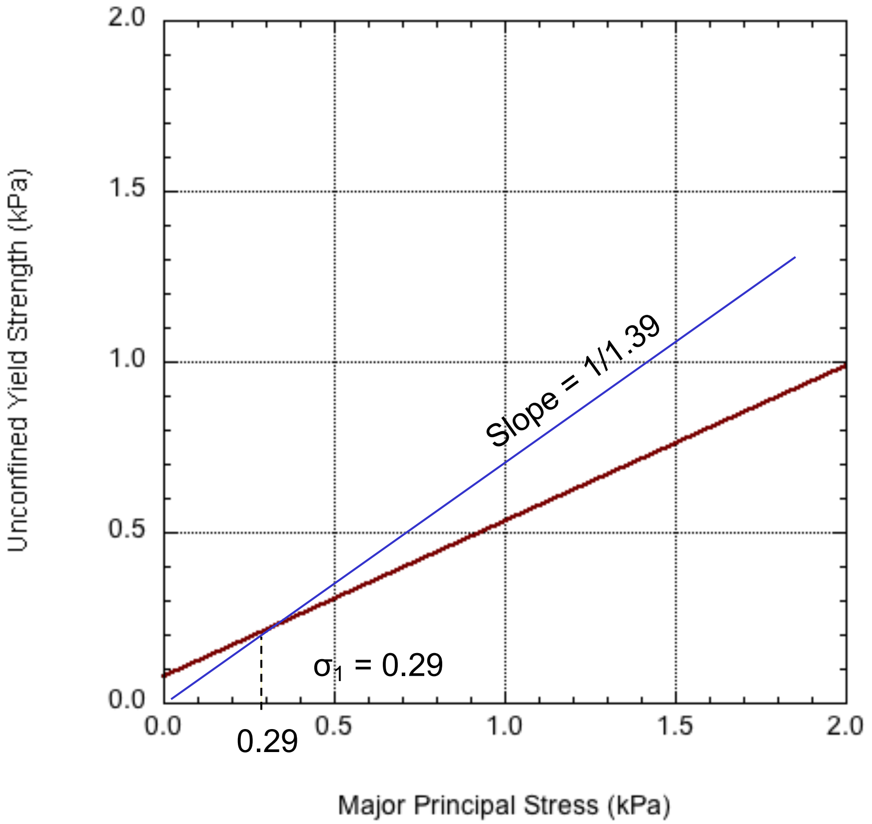
ff converged?

No

Yes

Calculate σ_{crit}, ρ_b, $H(\theta')$

Calculate B_{min}



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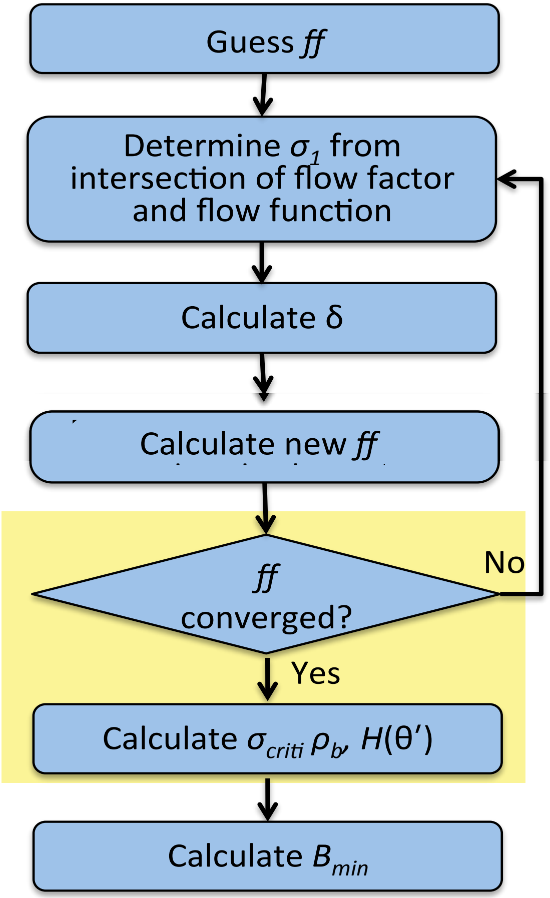

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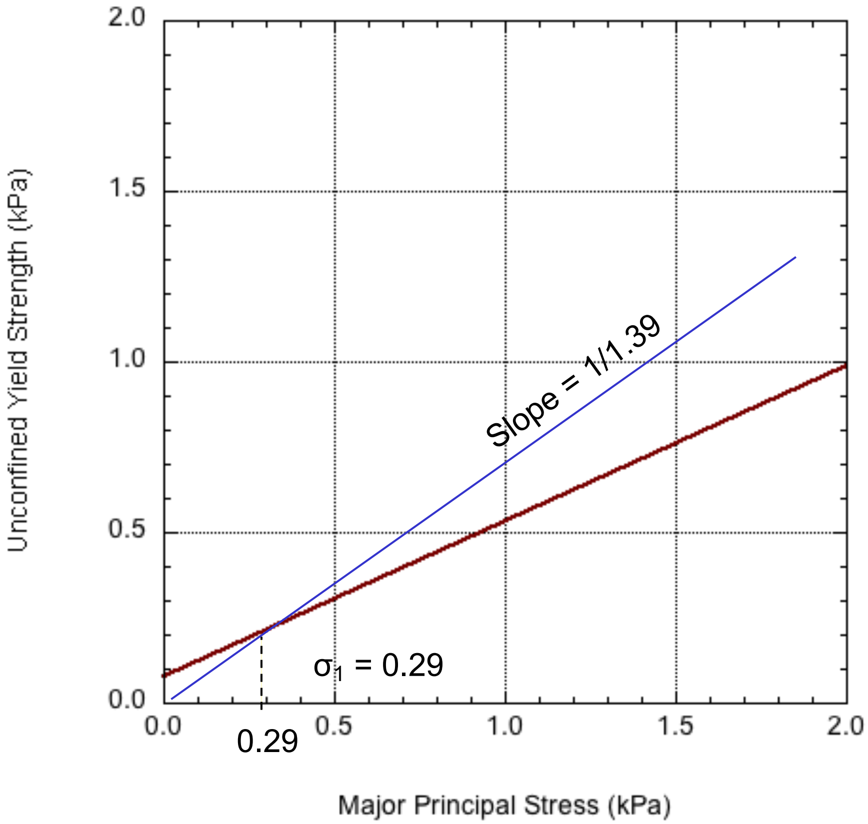
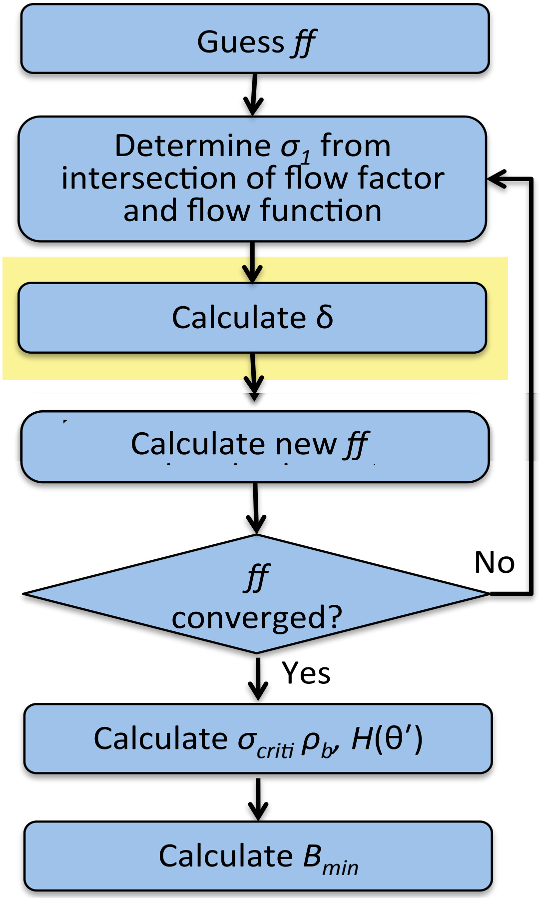


Short-cut method



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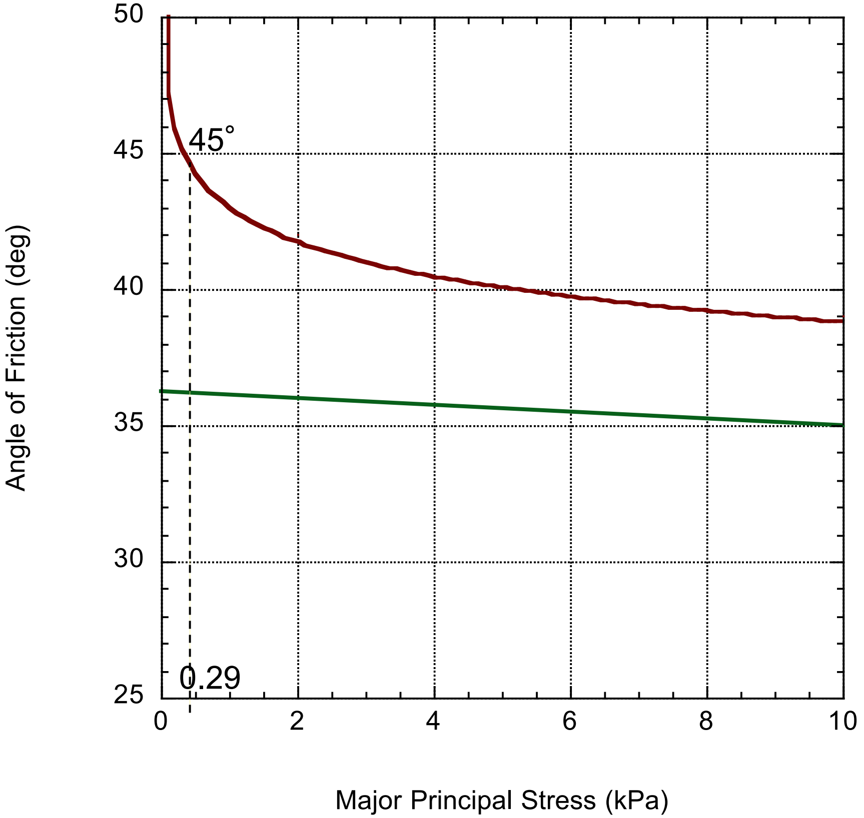

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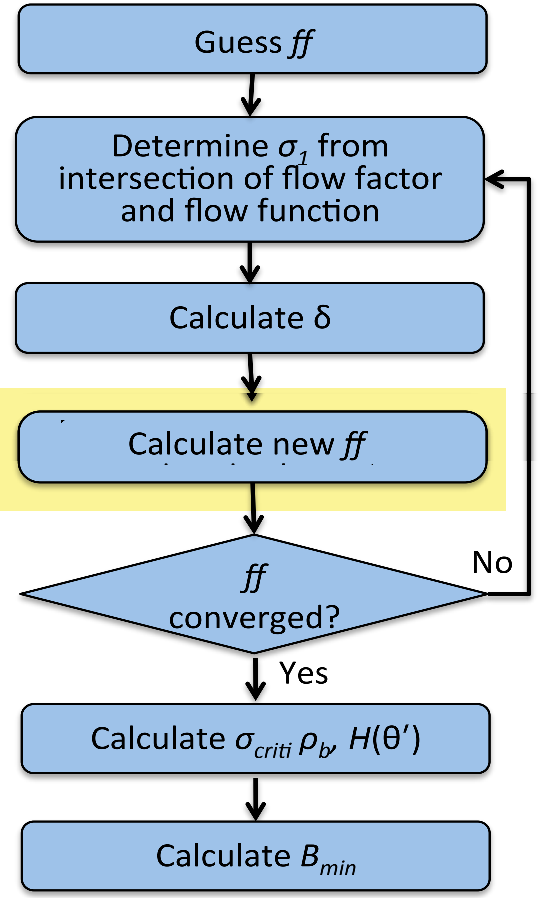

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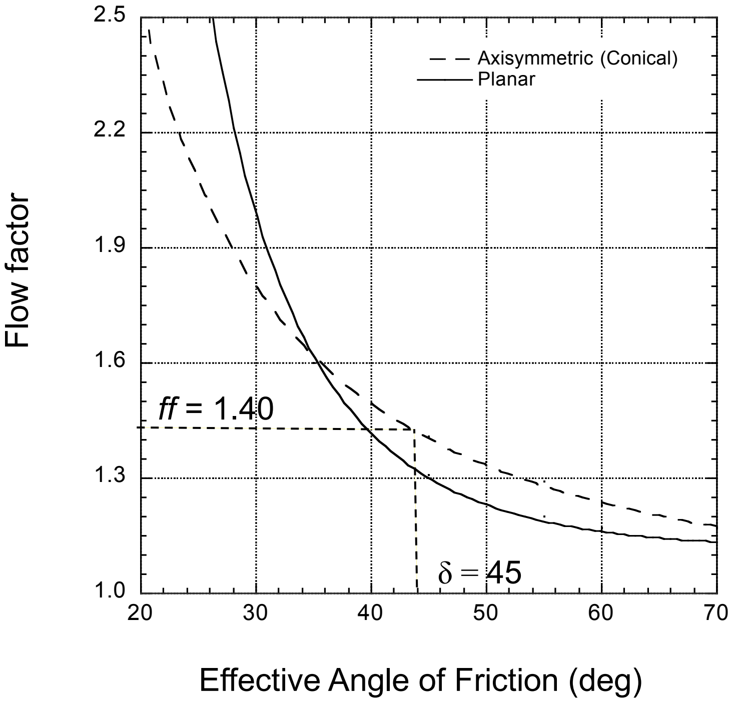

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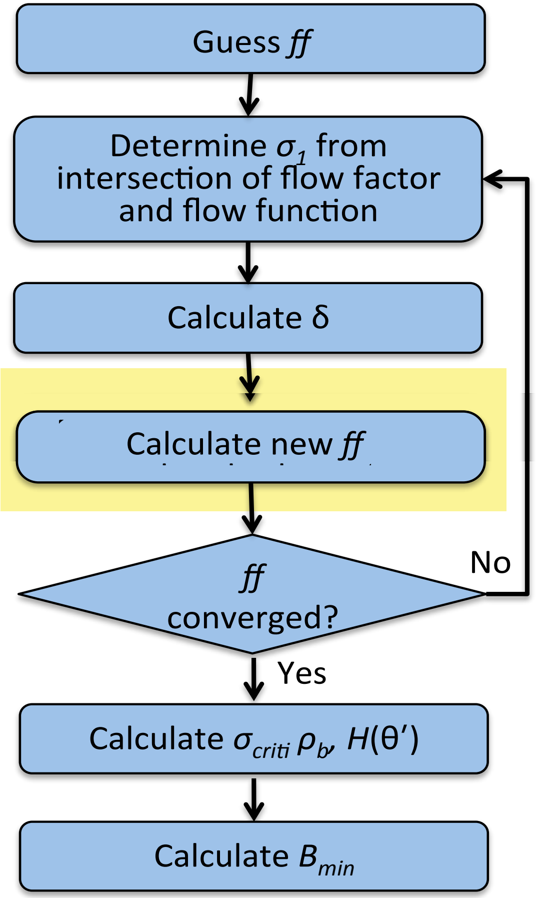

Guess *ff*

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No

Yes

Calculate σ_{crit}, ρ_b, $H(\theta')$

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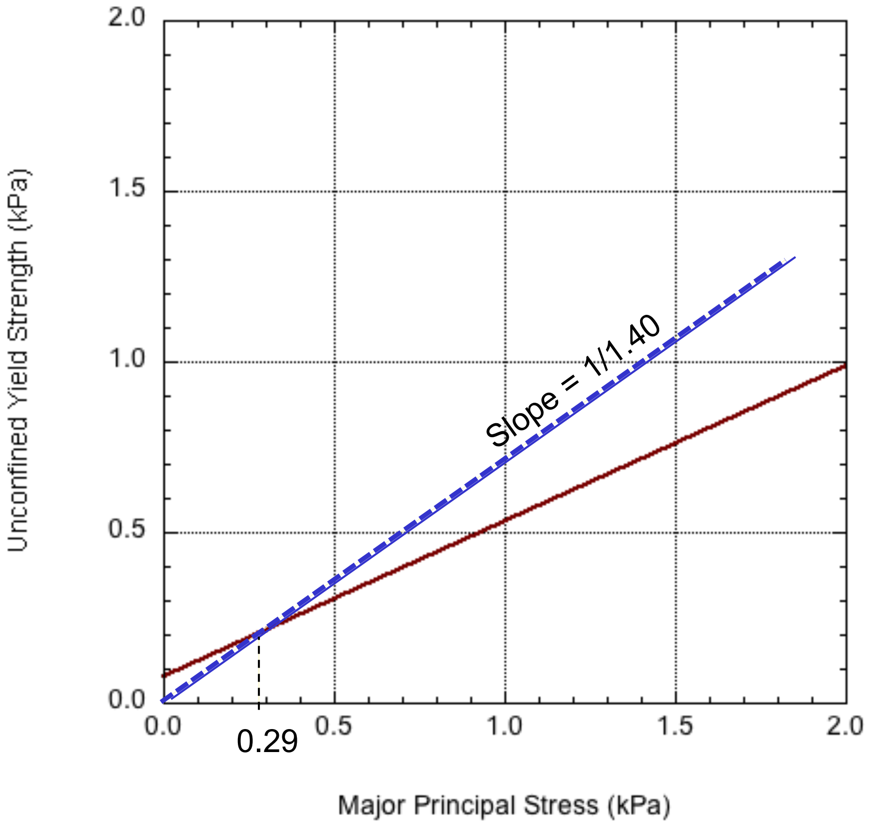

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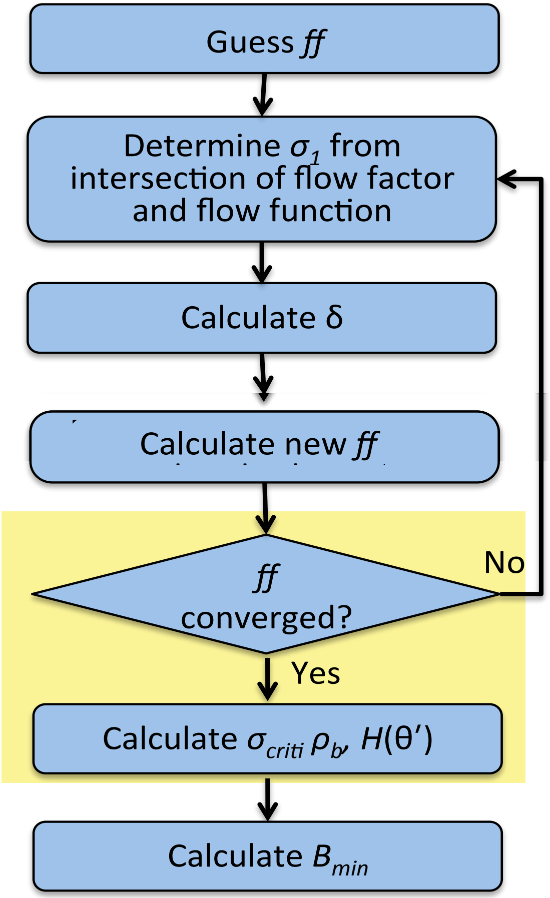

Guess *ff*

Determine σ_1 from intersection of flow factor and flow function

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Calculate σ_{crit}, ρ_b, $H(\theta')$

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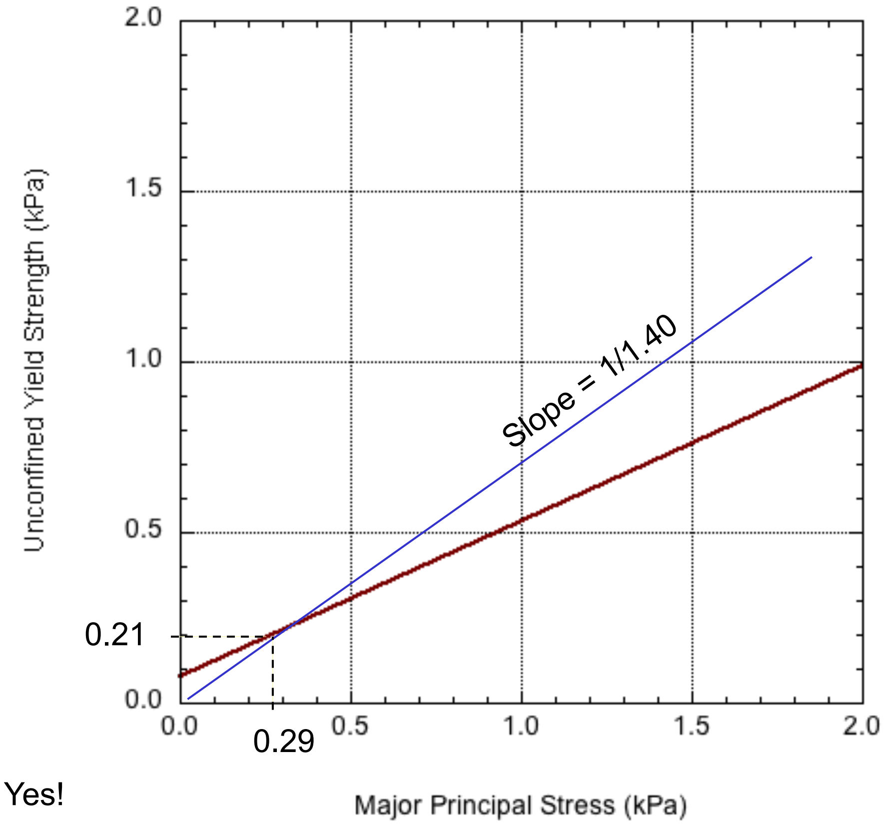
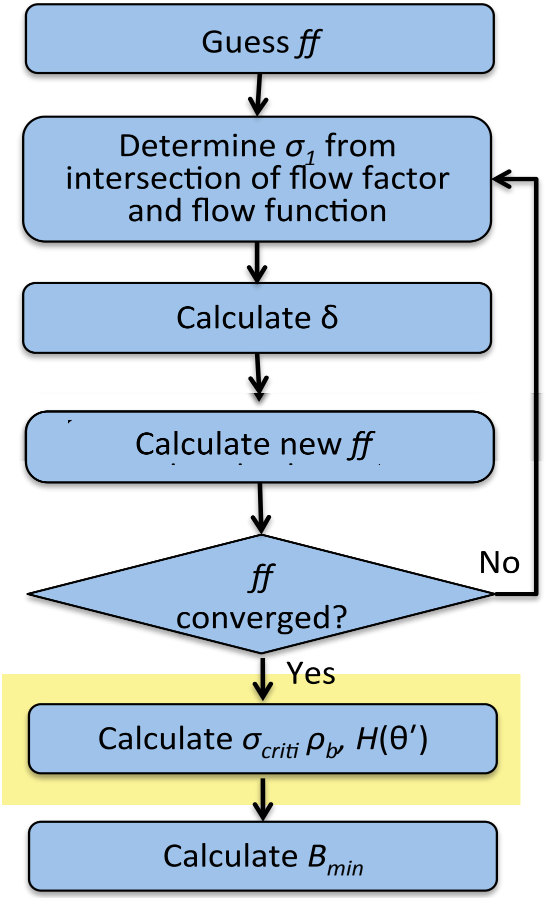


Yes!

Short-cut method



Guess *ff*

Determine σ_1 from intersection of flow factor and flow function

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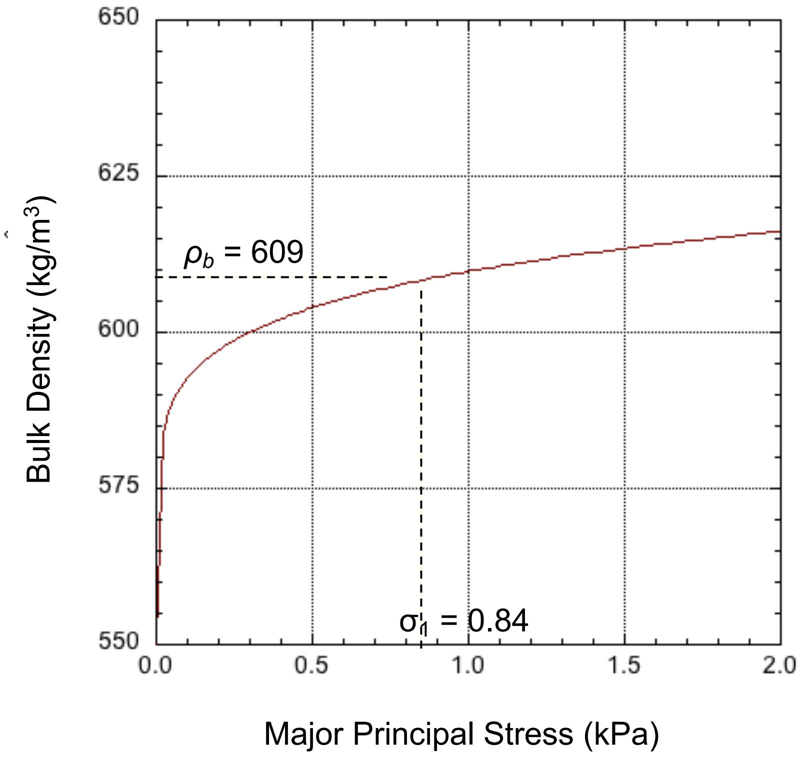
ff converged?

No

Yes

Calculate σ_{crit}, ρ_b, $H(\theta')$

Calculate B_{min}



$$B_{\min} = \frac{H(\theta')\sigma_{Critical}}{\rho_b g}$$

Use $H(20^{\circ}) = 2.3$

$$B_{\min} = \frac{(2.3)(0.21)(1000)}{(611)(9.8)}$$

$$= 0.081 \text{ m}$$

Bmin = 0.081 m
= 3.1 in.

TUNRA Analytical expressions

Mass flow hopper angles

Conical

$$\theta' = 90° - \frac{1}{2}\cos^{-1}\left(\frac{1-\sin\delta}{2\sin\delta}\right) - \beta$$

$$2\beta = \phi' + \sin^{-1}\left(\frac{\sin\phi'}{\sin\delta}\right)$$

Planar, slotted outlet

$$\theta' = \frac{\exp[3.75(1.01)^{(\delta-30°)/10}] - \phi'}{0.725(\tan\delta)^{1/5}}$$

Flow factors

i = 1: conical, *i* = 0: planar, slotted outlet

$$ff = \frac{Y(1+\sin\delta)H(\theta')}{2(X-1)(\sin\theta')}$$

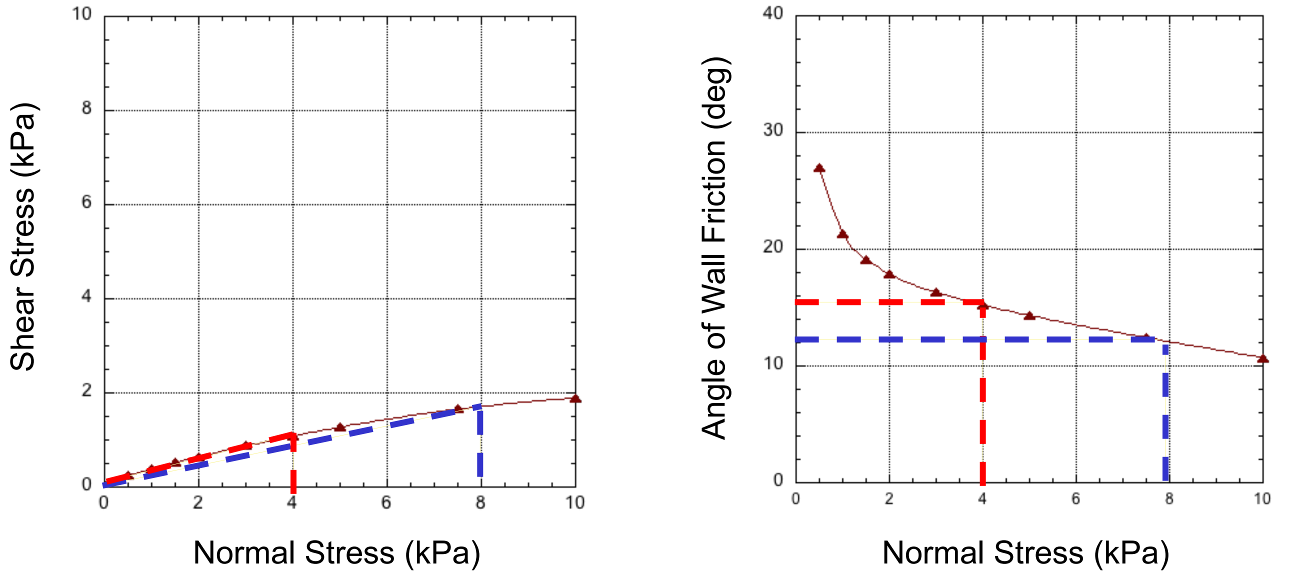
$$X = \frac{2^i\sin\delta}{1-\sin\delta}\left[\frac{\sin(2\beta+\theta')}{\sin\theta'} + 1\right]$$

$$Y = \frac{[2(1-\cos(\beta+\theta'))]^i\sin\theta'(\beta+\theta')^{1-i} + \sin\beta\sin^{1+i}(\beta+\theta')}{(1-\sin\delta)\sin^{2+i}(\beta+\theta')}$$

$$H(\theta') = \left(\frac{130° + \theta'}{65}\right)^i\left(\frac{200° + \theta'}{200°}\right)^{1-i}$$

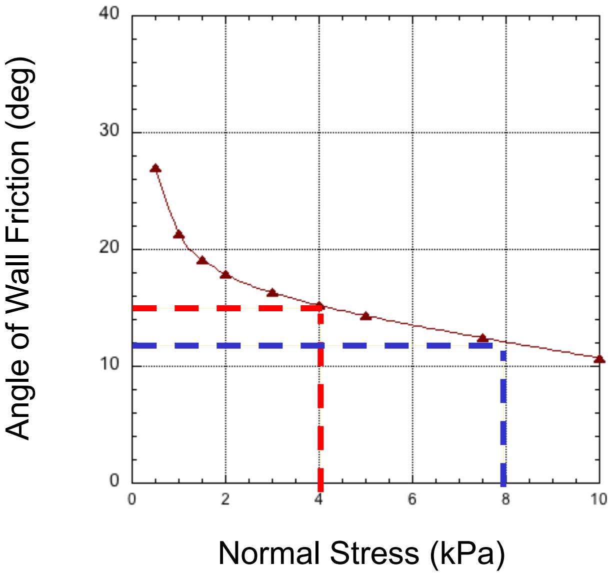
Mass flow hopper design

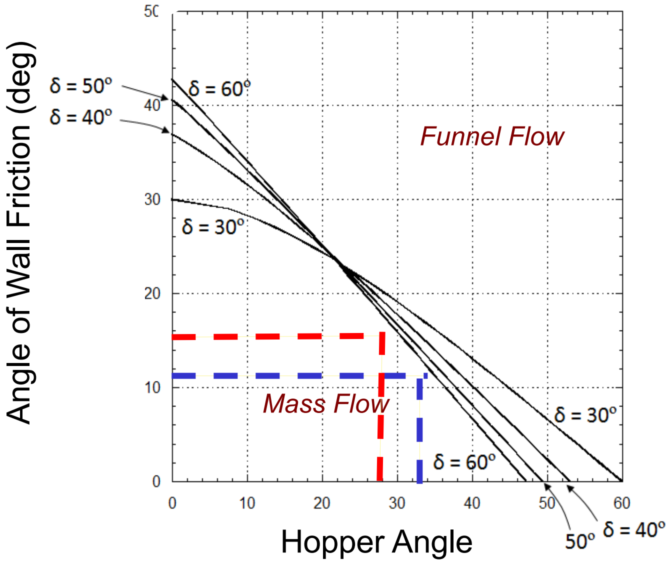
Recommended mass flow hopper angles for outlet dimensions > B_{min}



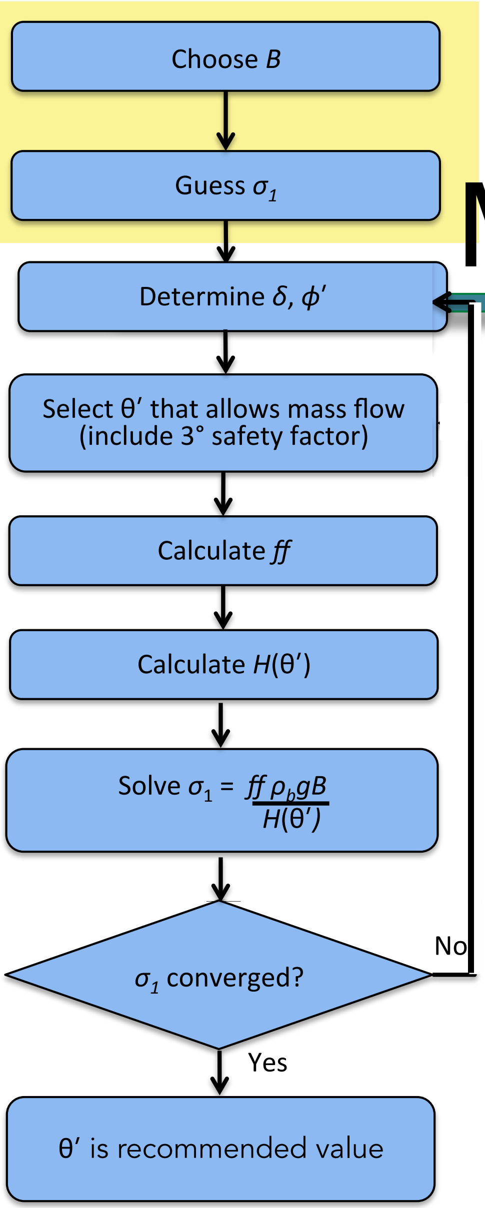
Mass flow hopper design

Recommended mass flow hopper angles for outlet dimensions > B_{min}

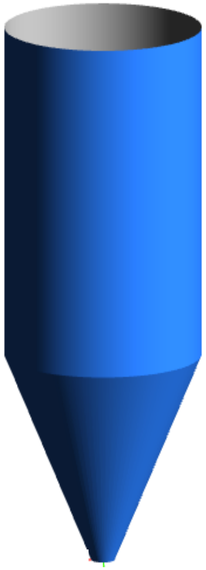




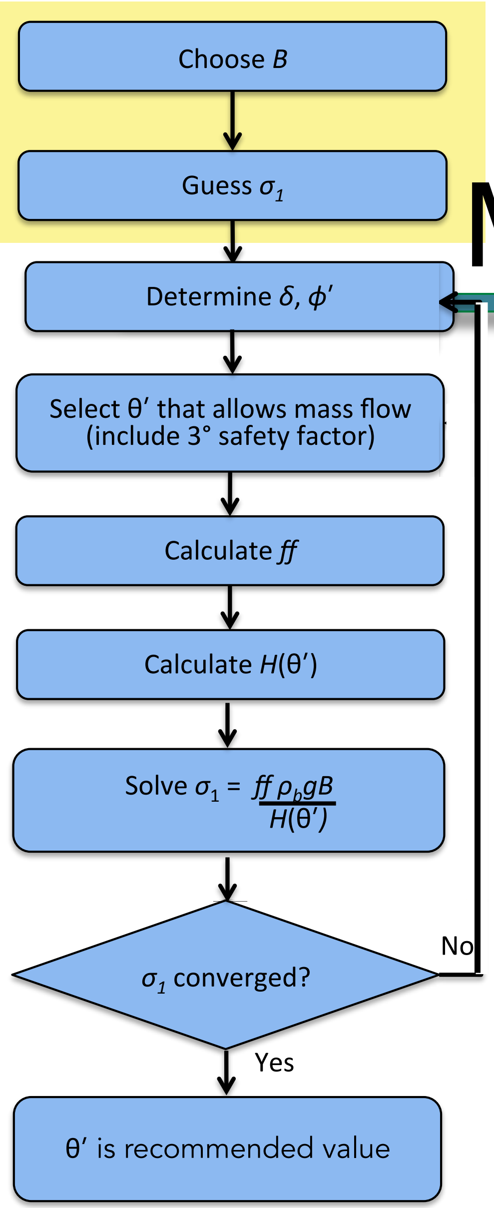
Mass flow hopper design



$B = 0.304$ m = 12 in. I.D.



Mass flow hopper design



Choose B

Guess σ_1

Determine δ, ϕ'

Select θ′ that allows mass flow (include 3° safety factor)

Calculate ff

Calculate $H(\theta')$

Solve $\sigma_1 = \dfrac{ff\,\rho_b g B}{H(\theta')}$

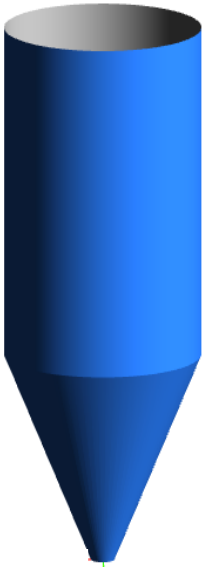
σ_1 converged?

No

Yes

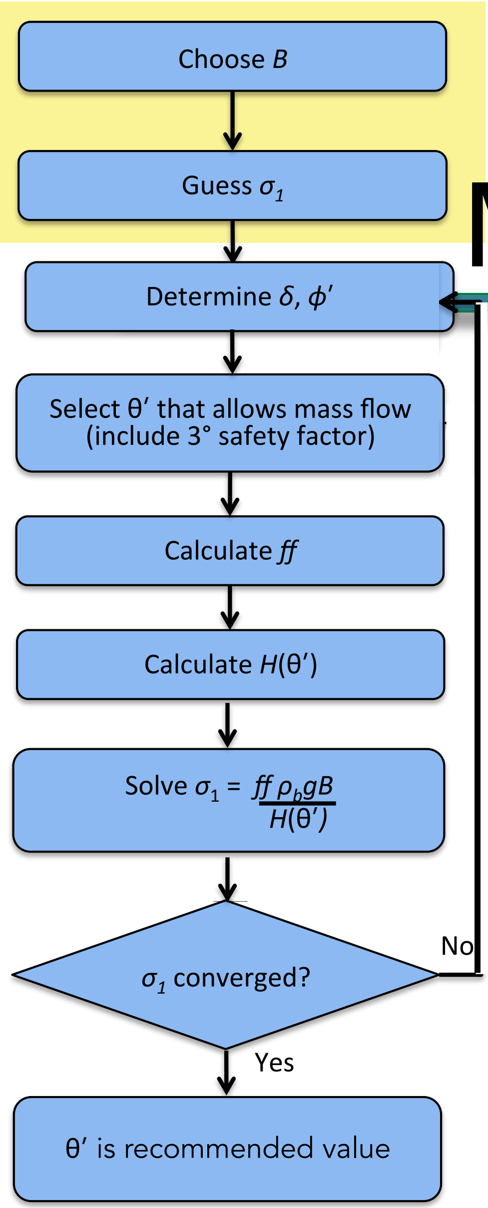
θ′ is recommended value

$$B = 0.304\ \text{m} = 12\ \text{in. I.D.}$$



$$ff = \frac{\sigma_1}{\bar{\sigma}_1}$$

Mass flow hopper design



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Select θ′ that allows mass flow (include 3° safety factor)

Calculate ff

Calculate $H(\theta')$

Solve $\sigma_1 = \frac{ff\, \rho_b g B}{H(\theta')}$

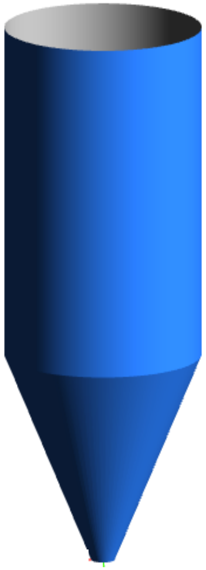
σ_1 converged?

No

Yes

θ′ is recommended value

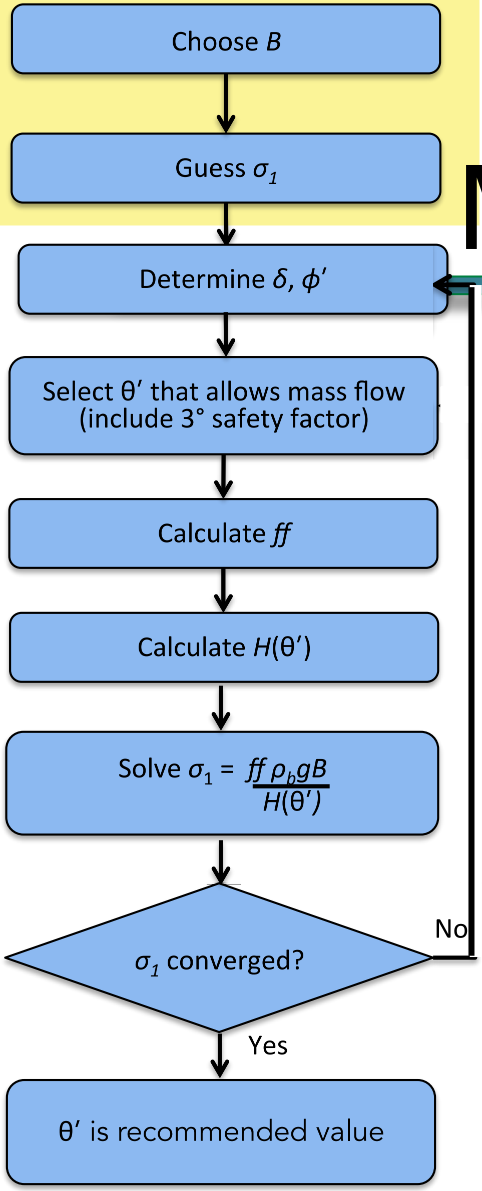
$$\boxed{B = 0.304 \text{ m} = 12 \text{ in. I.D.}}$$



$$ff = \frac{\sigma_1}{\bar{\sigma}_1}$$

$$\bar{\sigma}_1 = \frac{\rho_b g B}{H(\theta')}$$

Mass flow hopper design



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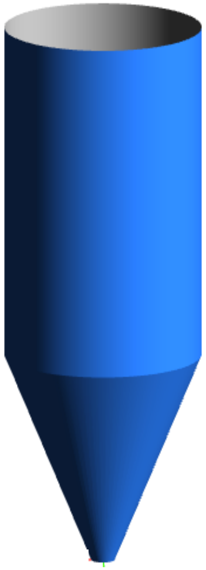
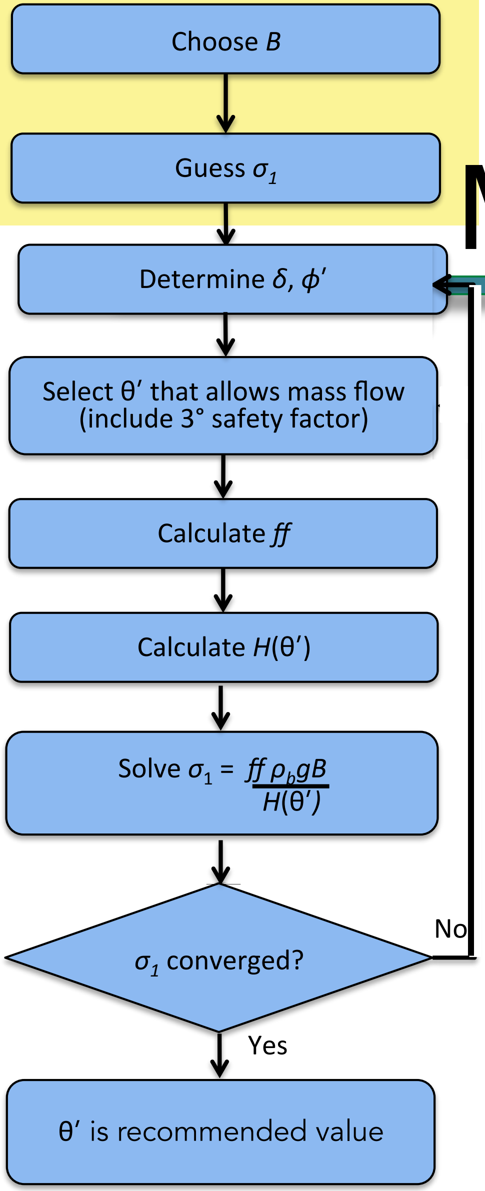


$$ff = \frac{\sigma_1}{\bar{\sigma}_1}$$

$$\bar{\sigma}_1 = \frac{\rho_b g B}{H(\theta')}$$

$$\sigma_1 = \frac{ff\,\rho_b(\sigma_1) g B}{H(\theta')}$$

Mass flow hopper design



Choose B

Guess σ_1

Determine δ, ϕ'

Select θ′ that allows mass flow (include 3° safety factor)

Calculate ff

Calculate $H(\theta')$

Solve $\sigma_1 = \dfrac{ff\,\rho_b gB}{H(\theta')}$

σ_1 converged?

No

Yes

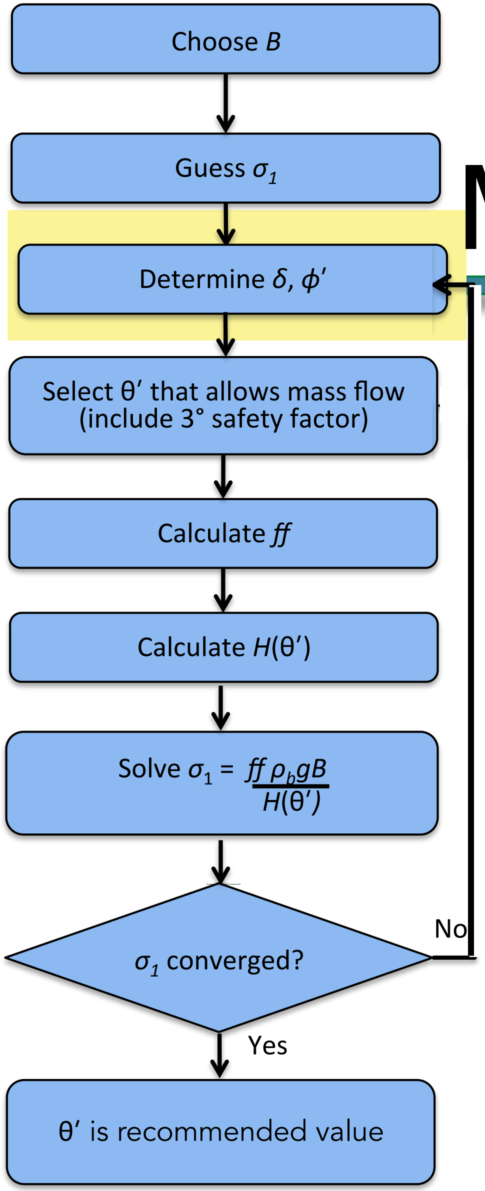
θ′ is recommended value

$$\boxed{B = 0.304 \text{ m} = 12 \text{ in. I.D.}}$$

$$\sigma_1 = \frac{ff\,\rho_b gB}{H(\theta')}$$

$$= \frac{(1.4)(610)(9.8)(0.305)}{(2.3)(1000)} = 1.2$$

Mass flow hopper design



Choose B

Guess σ_1

Determine δ, ϕ'

Select θ′ that allows mass flow (include 3° safety factor)

Calculate ff

Calculate $H(\theta')$

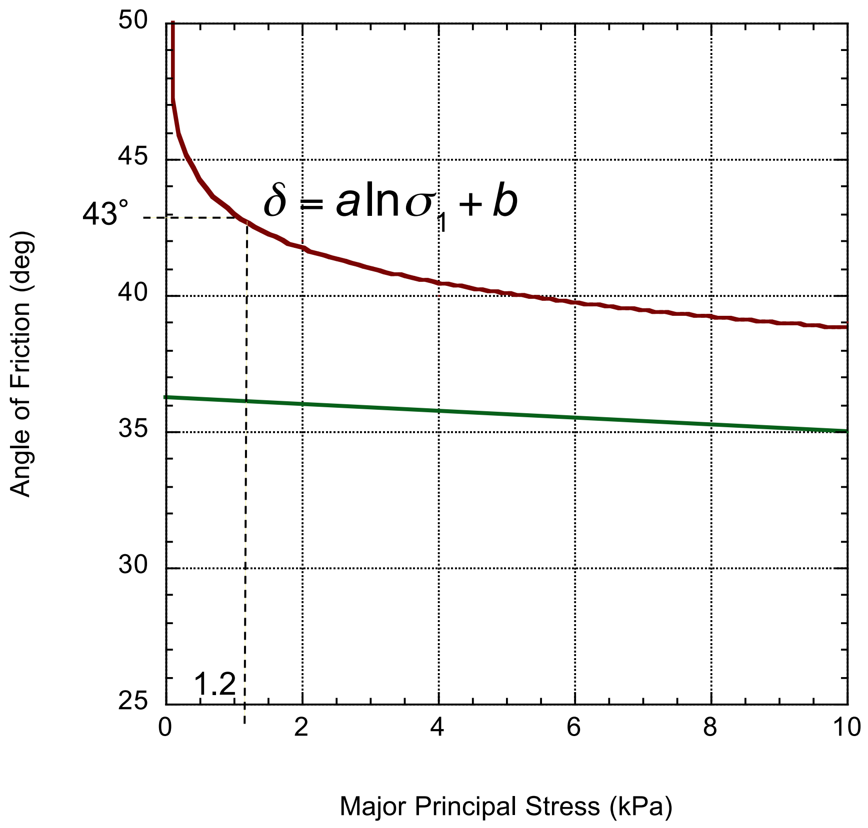
Solve $\sigma_1 = \dfrac{ff\,\rho_b g B}{H(\theta')}$

σ_1 converged?

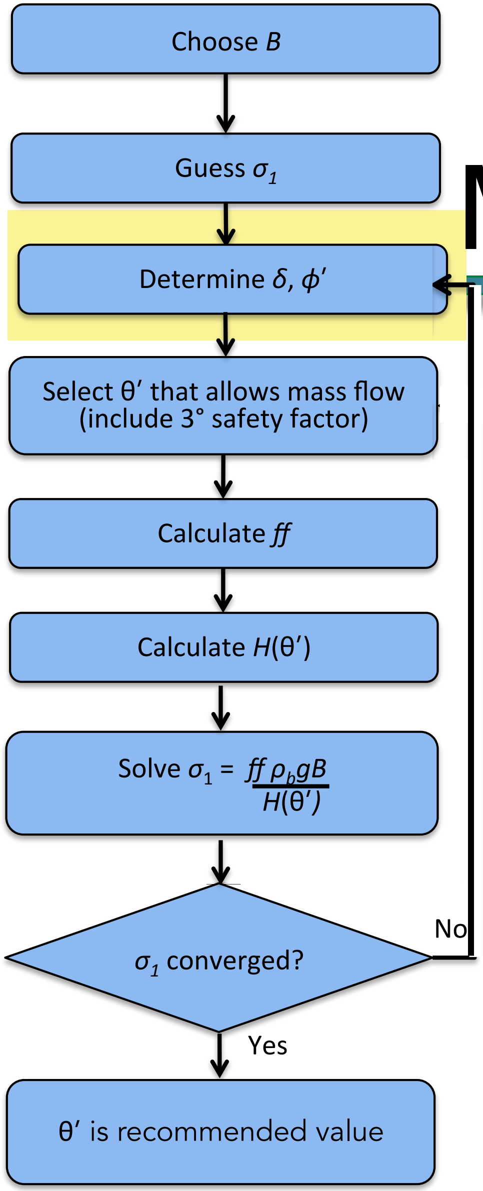
No

Yes

θ′ is recommended value



Mass flow hopper design



Choose B

Guess σ_1

Determine δ, ϕ'

Select θ′ that allows mass flow (include 3° safety factor)

Calculate ff

Calculate $H(\theta')$

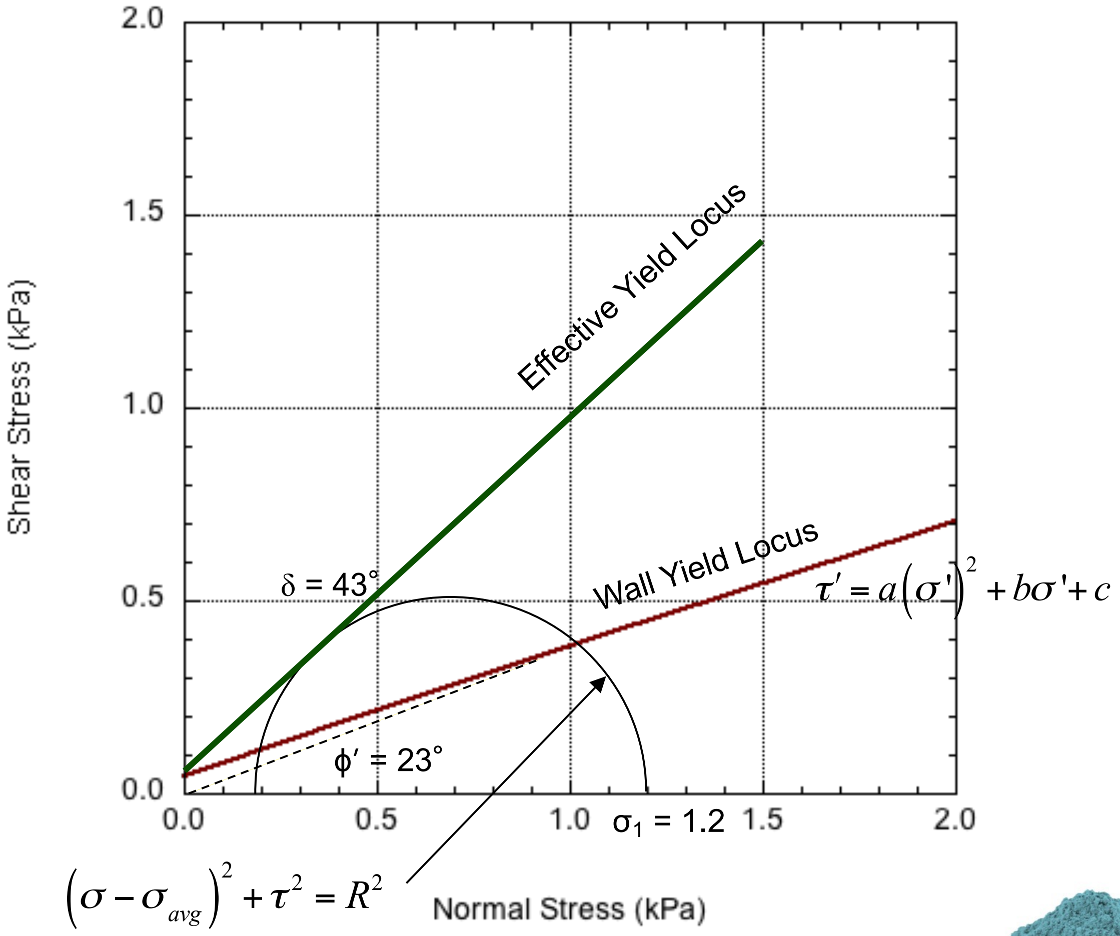
Solve $\sigma_1 = \dfrac{ff\,\rho_b g B}{H(\theta')}$

σ_1 converged?

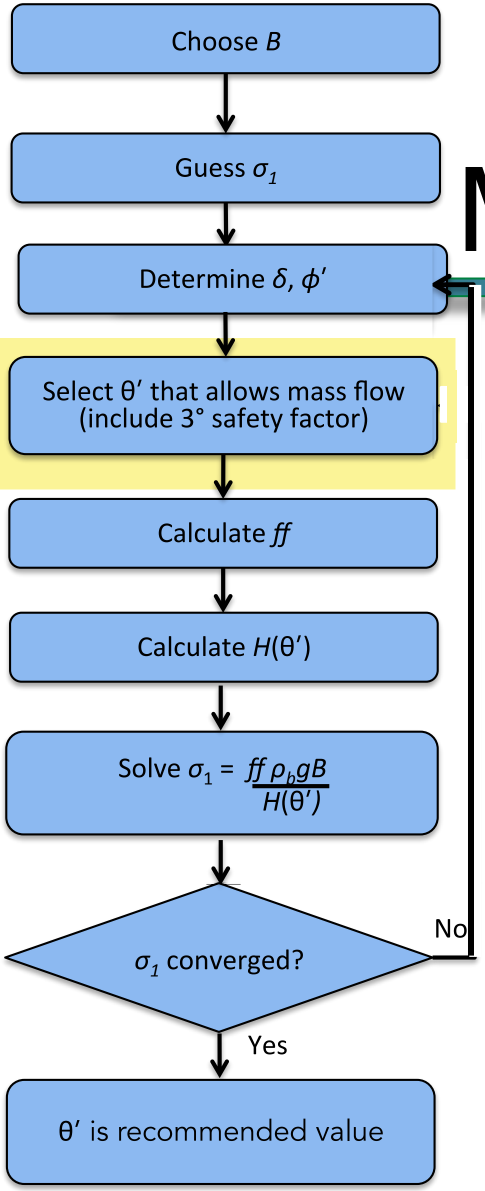
No

Yes

θ′ is recommended value



Mass flow hopper design



Choose *B*

Guess σ_1

Determine δ, ϕ'

Select θ′ that allows mass flow (include 3° safety factor)

Calculate *ff*

Calculate *H*(θ′)

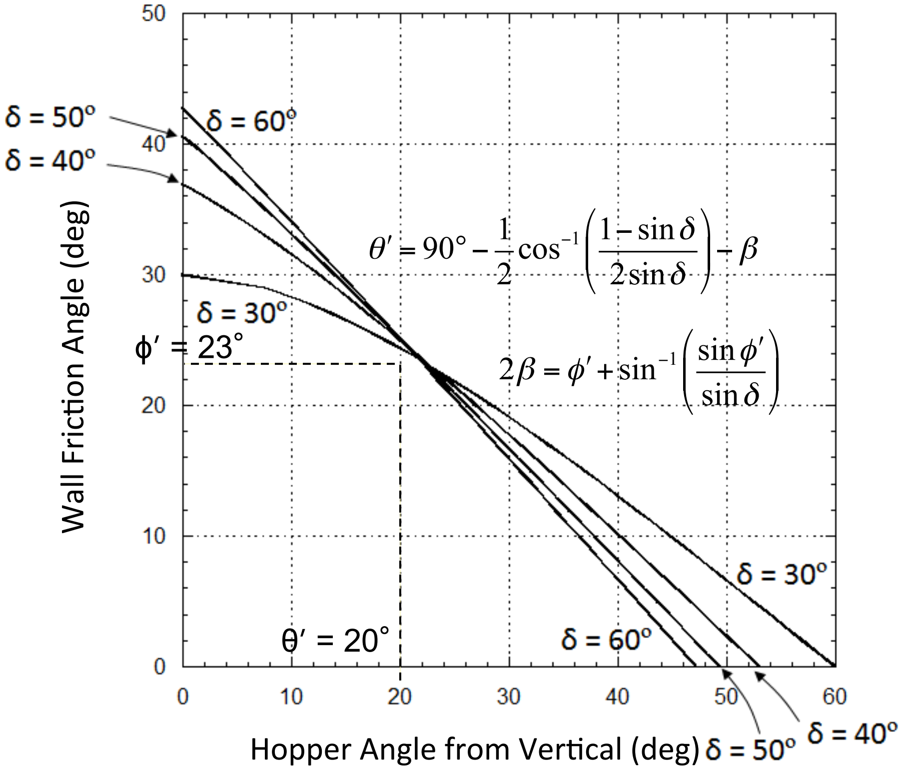
Solve $\sigma_1 = \dfrac{ff\,\rho_b g B}{H(\theta')}$

σ_1 converged?

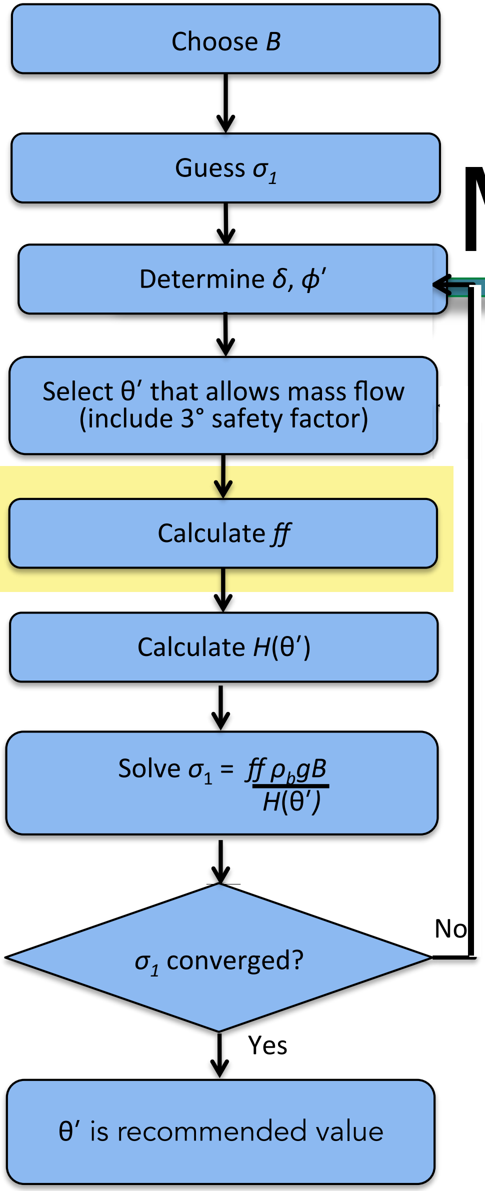
No

Yes

θ′ is recommended value



Mass flow hopper design



Choose B

Guess σ_1

Determine δ, ϕ'

Select θ′ that allows mass flow (include 3° safety factor)

Calculate ff

Calculate $H(\theta')$

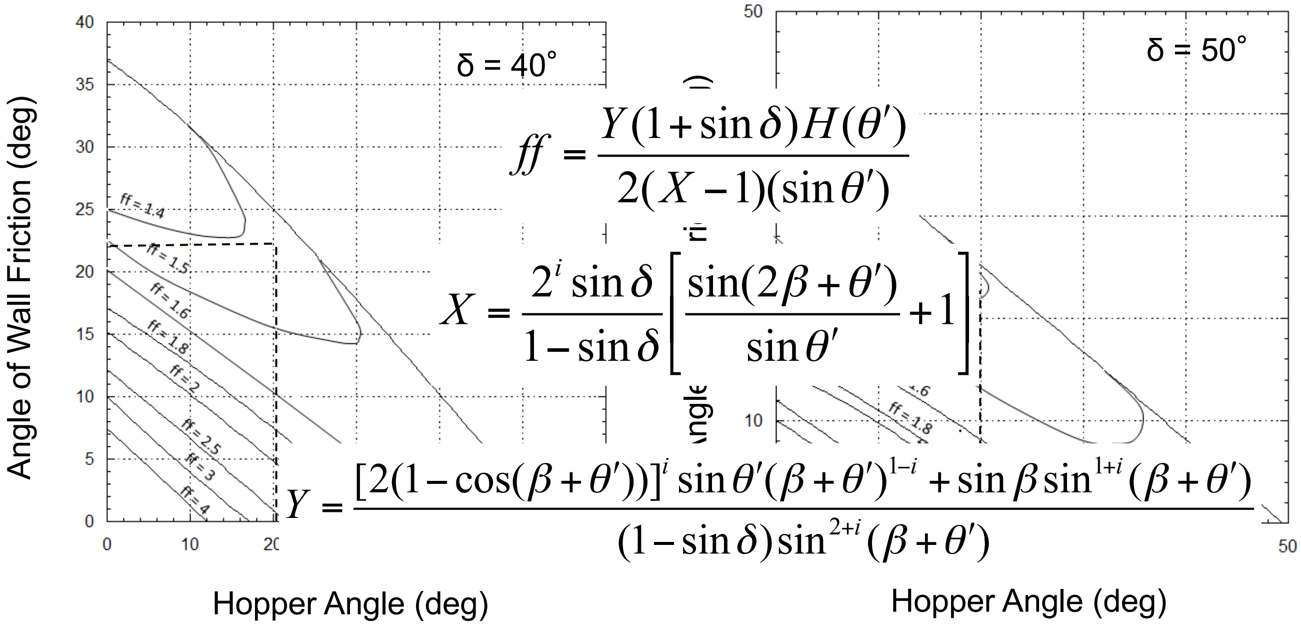
Solve $\sigma_1 = \dfrac{ff\,\rho_b g B}{H(\theta')}$

σ_1 converged?

No

Yes

θ′ is recommended value



$$ff = \frac{Y(1+\sin\delta)H(\theta')}{2(X-1)(\sin\theta')}$$

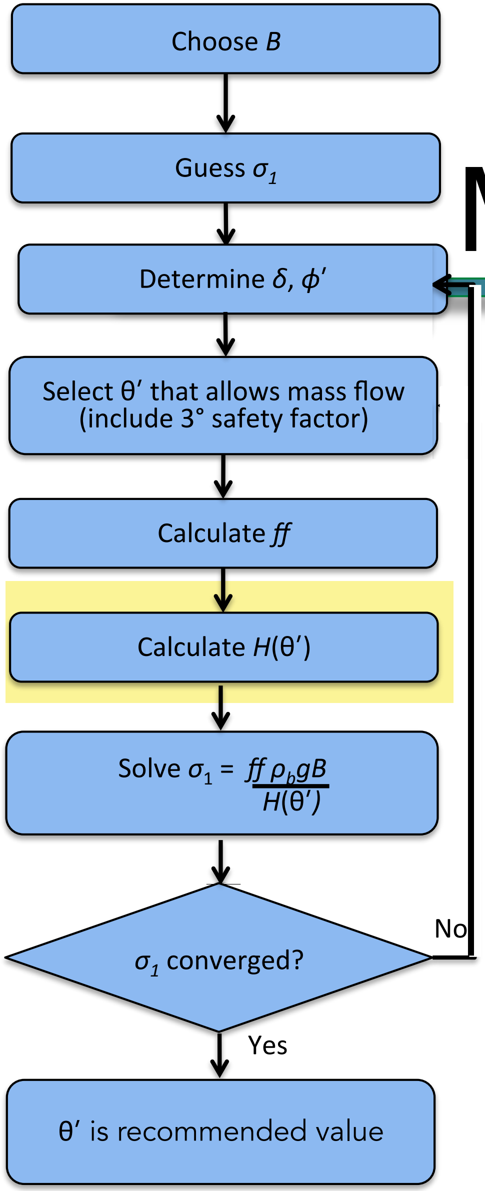
$$X = \frac{2^i \sin\delta}{1-\sin\delta}\left[\frac{\sin(2\beta+\theta')}{\sin\theta'}+1\right]$$

$$Y = \frac{[2(1-\cos(\beta+\theta'))]^i \sin\theta'(\beta+\theta')^{1-i} + \sin\beta \sin^{1+i}(\beta+\theta')}{(1-\sin\delta)\sin^{2+i}(\beta+\theta')}$$

@ δ = 43° , ϕ' = 23° , θ' = 22°

ff = 1.39

Mass flow hopper design



Choose B

Guess σ_1

Determine δ, ϕ'

Select θ′ that allows mass flow (include 3° safety factor)

Calculate ff

Calculate $H(\theta')$

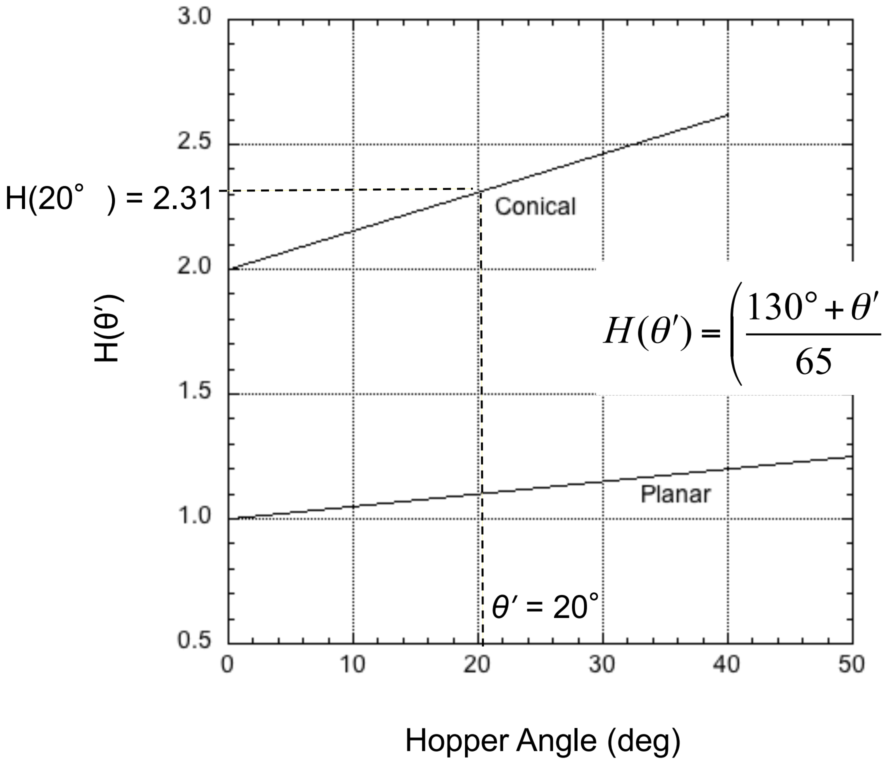
Solve $\sigma_1 = \frac{ff\,\rho_b g B}{H(\theta')}$

σ_1 converged?

No

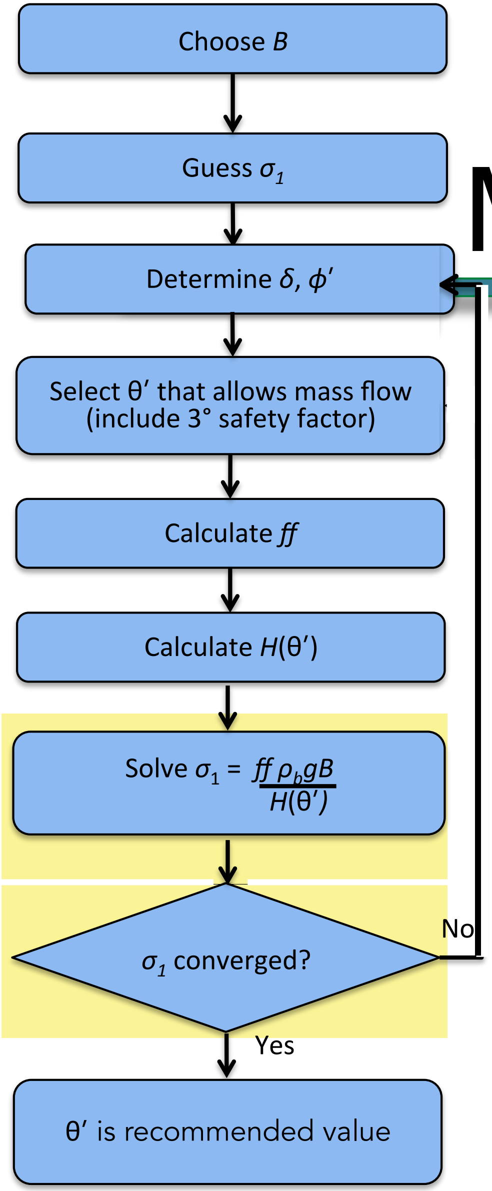
Yes

θ′ is recommended value



$$H(\theta') = \left(\frac{130° + \theta'}{65}\right)^{i}\left(\frac{200° + \theta'}{200°}\right)^{1-i}$$

Mass flow hopper design



Choose B

Guess σ_1

Determine δ, ϕ'

Select θ′ that allows mass flow (include 3° safety factor)

Calculate ff

Calculate $H(\theta')$

Solve $\sigma_1 = \frac{ff\,\rho_b g B}{H(\theta')}$

σ_1 converged?

No

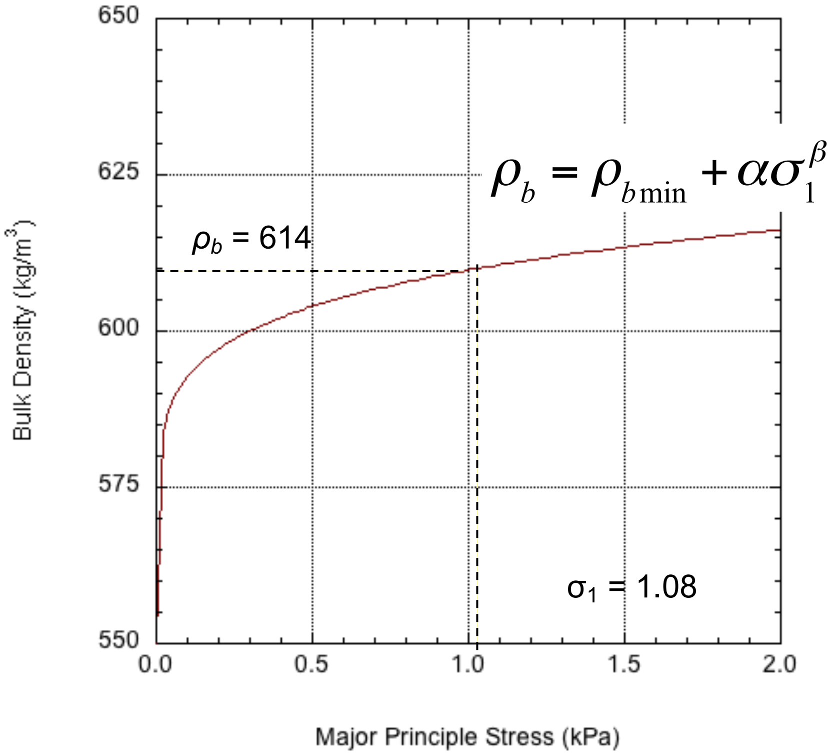
Yes

θ′ is recommended value

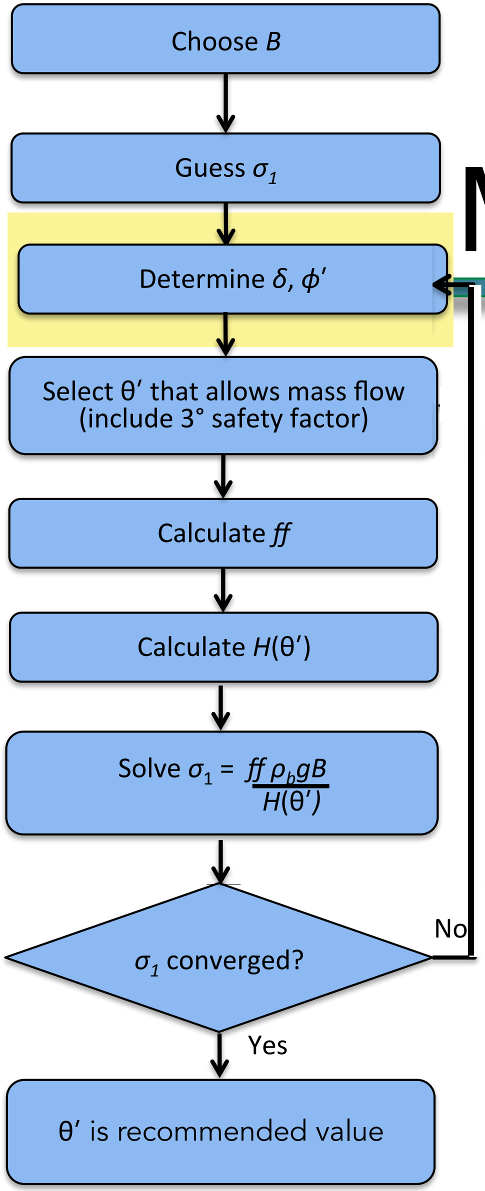
$$\sigma_1 = \frac{ff\,\rho_b(\sigma_1)gB}{H(\theta')}$$

$$\rho_b = \rho_{b\min} + \alpha\sigma_1^{\beta}$$

$$\sigma_1 = 1.06$$



Mass flow hopper design



Choose B

Guess σ_1

Determine δ, ϕ'

Select θ′ that allows mass flow (include 3° safety factor)

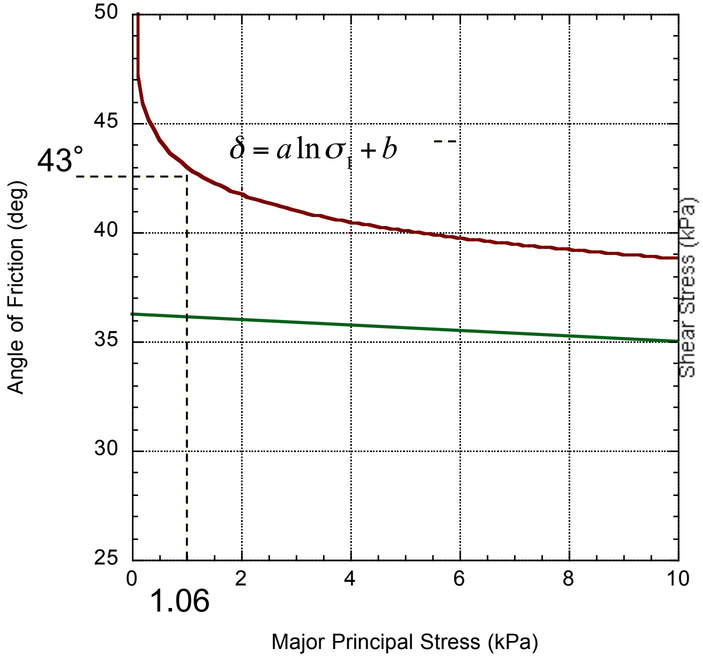
Calculate ff

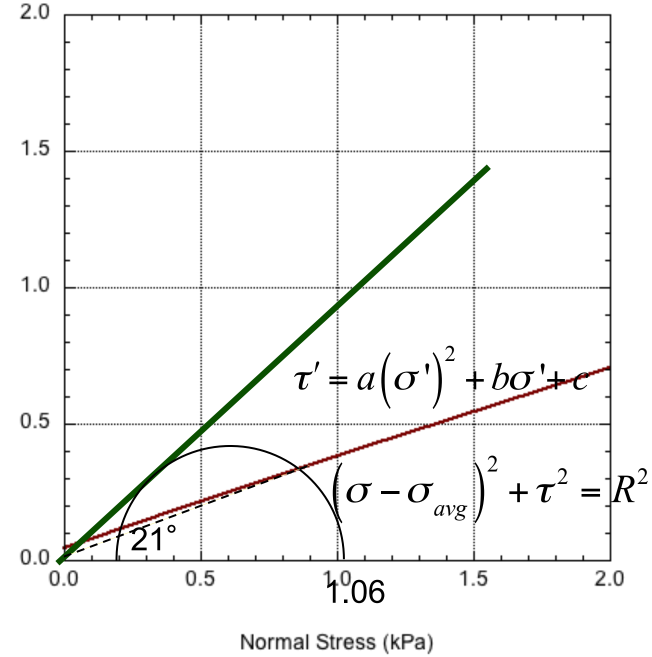
Calculate $H(\theta')$

Solve $\sigma_1 = \dfrac{ff\,\rho_b g B}{H(\theta')}$

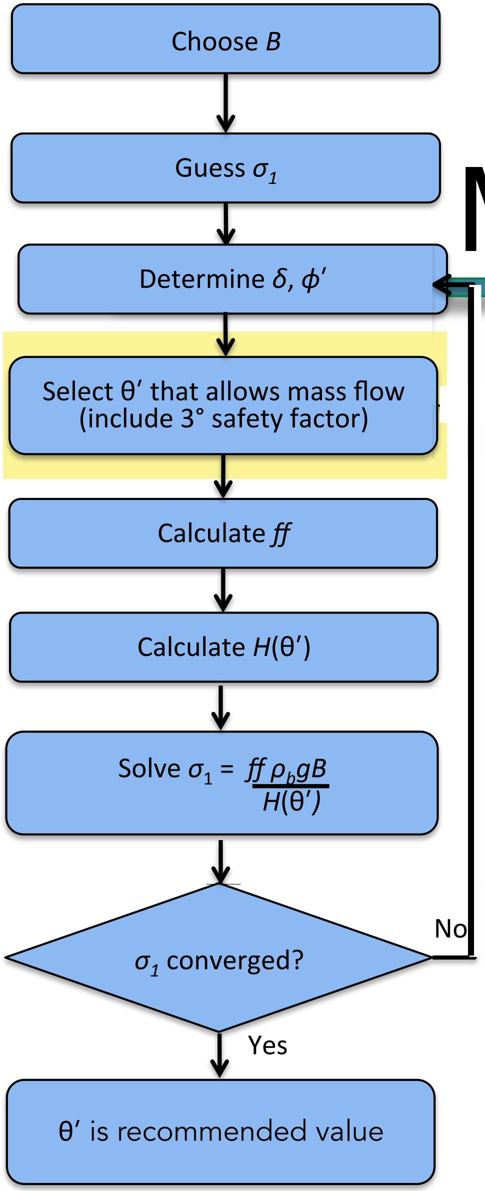
σ_1 converged?

θ′ is recommended value





Mass flow hopper design



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Guess σ_1

Determine δ, ϕ'

Select θ′ that allows mass flow (include 3° safety factor)

Calculate ff

Calculate $H(\theta')$

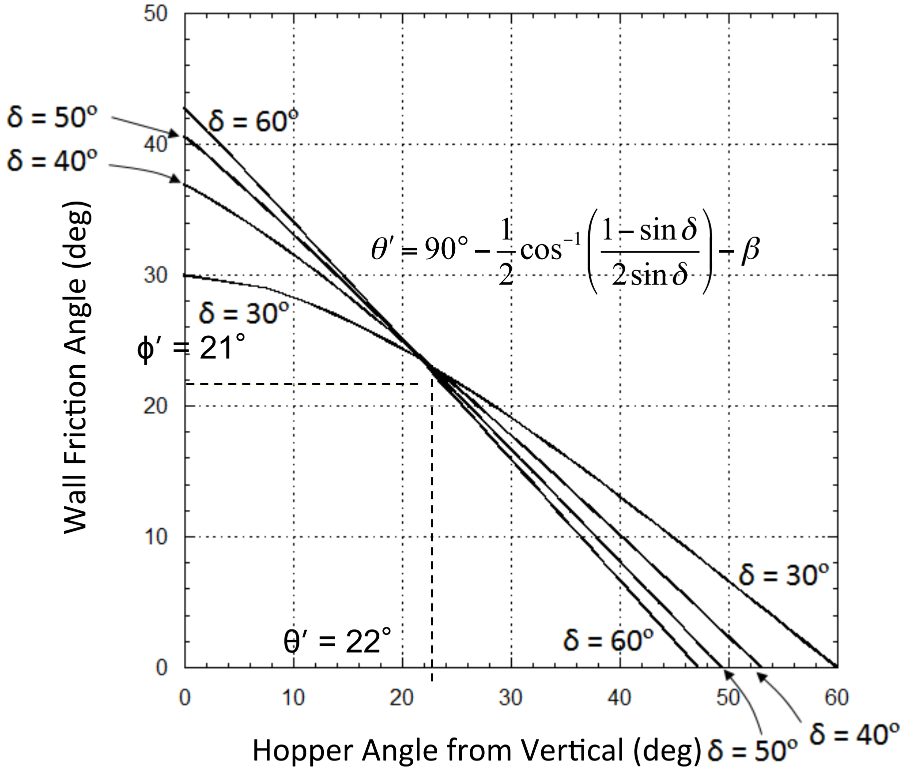
Solve $\sigma_1 = \frac{ff\,\rho_b g B}{H(\theta')}$

σ_1 converged?

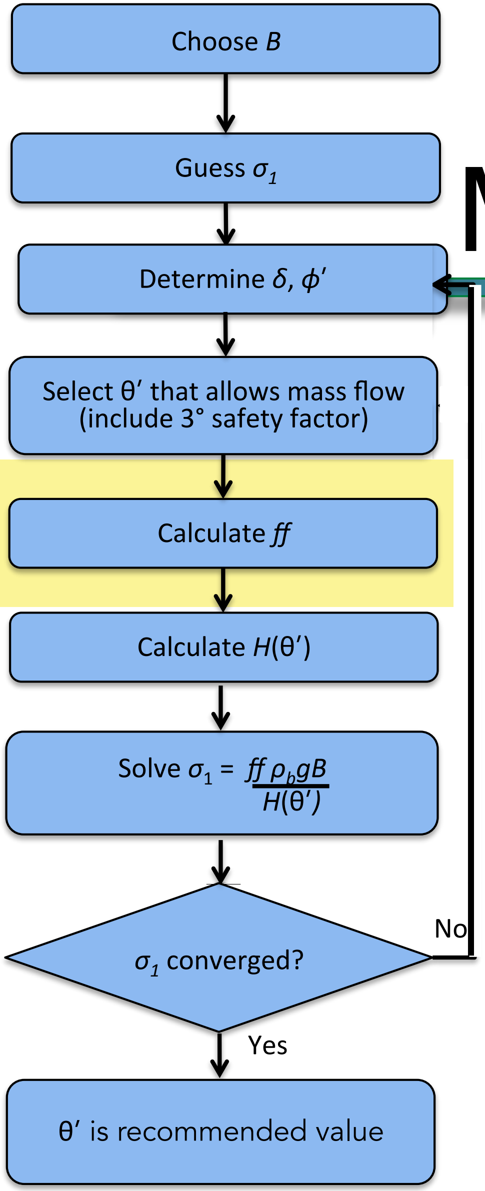
No

Yes

θ′ is recommended value



Mass flow hopper design



Choose B

Guess σ_1

Determine δ, ϕ'

Select θ′ that allows mass flow (include 3° safety factor)

Calculate ff

Calculate $H(\theta')$

Solve $\sigma_1 = \frac{ff\,\rho_b g B}{H(\theta')}$

σ_1 converged?

No

Yes

θ′ is recommended value

$$ff = \frac{Y(1+\sin\delta)H(\theta')}{2(X-1)(\sin\theta')}$$

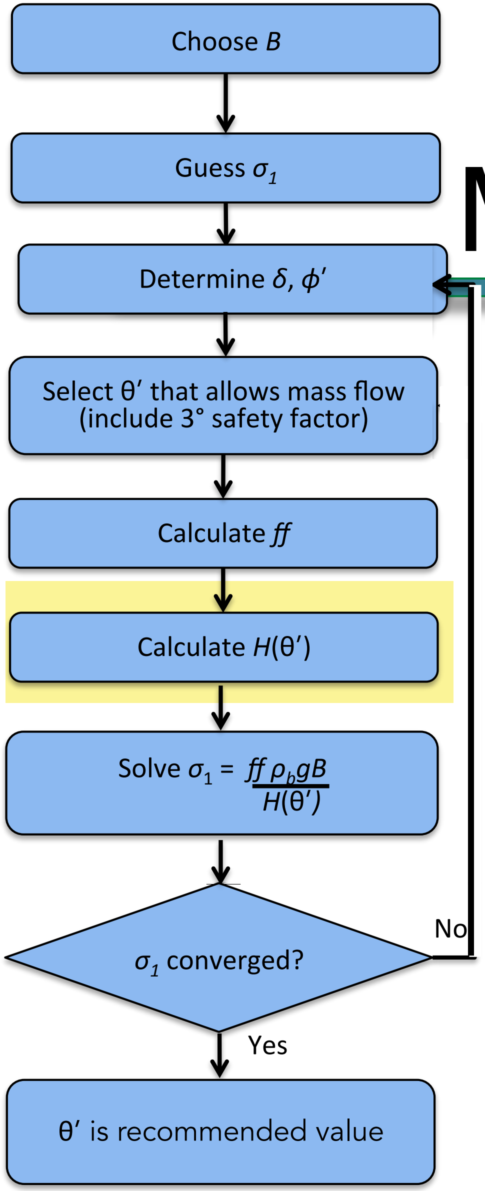
$$X = \frac{2^i \sin\delta}{1-\sin\delta}\left[\frac{\sin(2\beta+\theta')}{\sin\theta'}+1\right]$$

$$Y = \frac{[2(1-\cos(\beta+\theta'))]^i \sin\theta'(\beta+\theta')^{1-i} + \sin\beta\sin^{1+i}(\beta+\theta')}{(1-\sin\delta)\sin^{2+i}(\beta+\theta')}$$

@ δ = 43° , ϕ′ = 21° , θ′ = 22°

= 1.39

Mass flow hopper design



Choose B

Guess σ_1

Determine δ, ϕ'

Select θ′ that allows mass flow (include 3° safety factor)

Calculate *ff*

Calculate $H(\theta')$

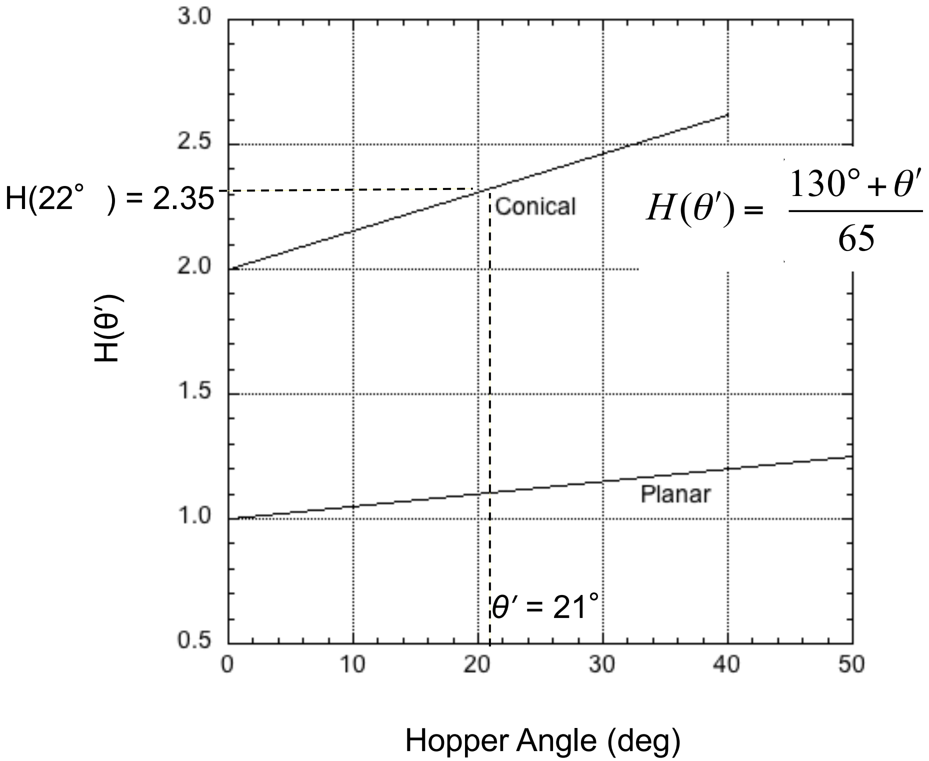
Solve $\sigma_1 = \frac{ff \rho_b g B}{H(\theta')}$

σ_1 converged?

No

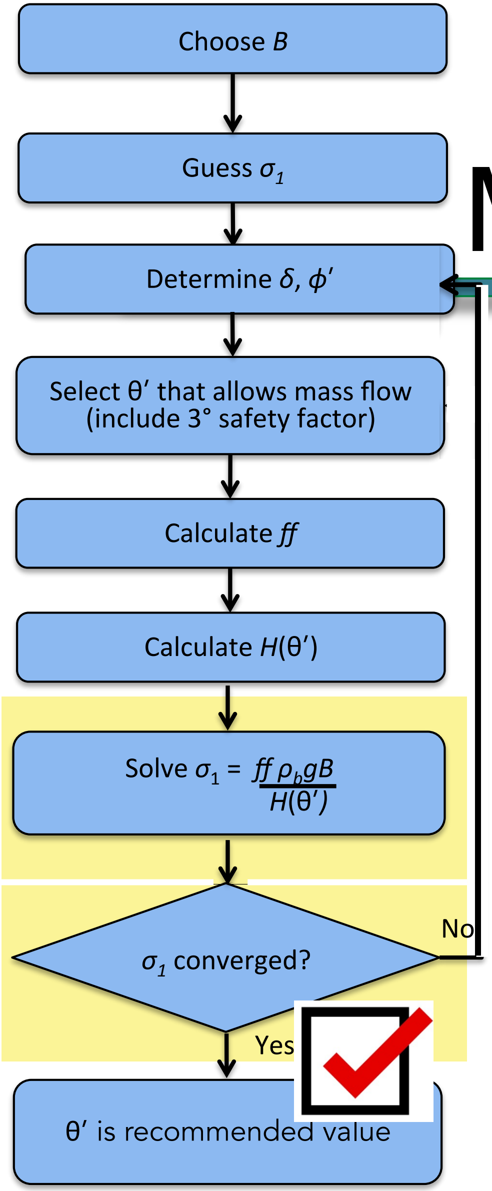
Yes

θ′ is recommended value



$$H(\theta') = \frac{130° + \theta'}{65}$$

Mass flow hopper design



Choose B

Guess σ_1

Determine δ, ϕ'

Select θ′ that allows mass flow (include 3° safety factor)

Calculate ff

Calculate $H(\theta')$

Solve $\sigma_1 = \dfrac{ff\,\rho_b g B}{H(\theta')}$

σ_1 converged?

No

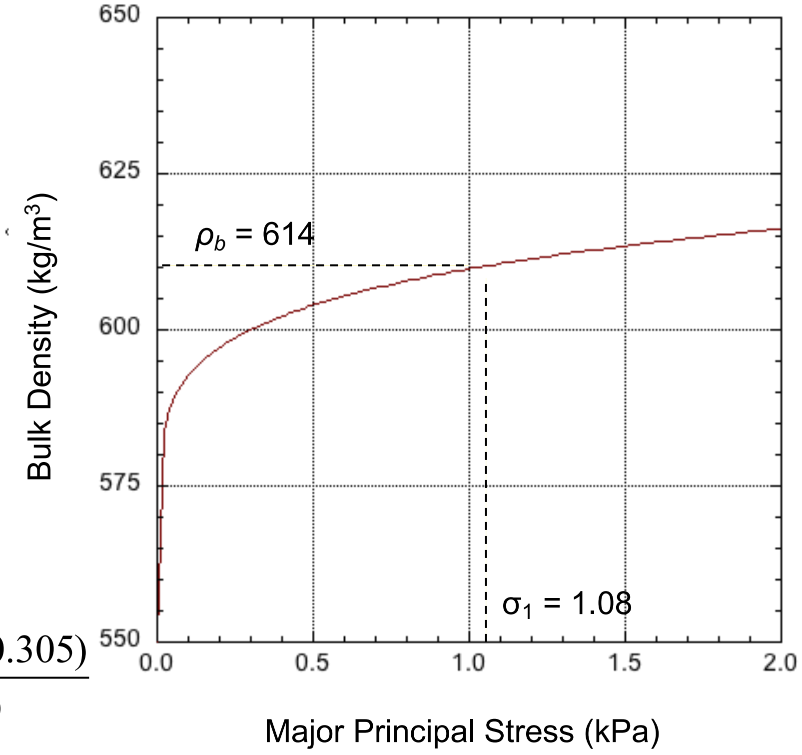
Yes

θ′ is recommended value

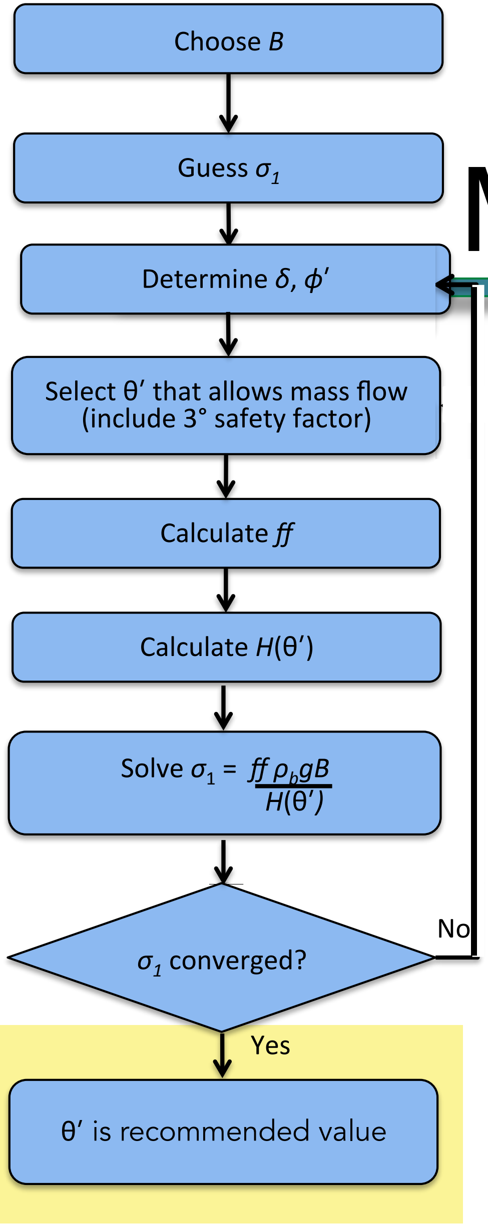
$$\sigma_1 = \frac{ff\,\rho_b(\sigma_1)gB}{H(\theta')}$$

$$\rho_b = \rho_{b\min} + \alpha\sigma_1^{\beta}$$

$$1.08 = \frac{(1.08)(614)(9.8)(0.305)}{(2.32)(1000)}$$



Mass flow hopper design



Choose B

Guess σ_1

Determine δ, ϕ'

Select θ′ that allows mass flow (include 3° safety factor)

Calculate ff

Calculate $H(\theta')$

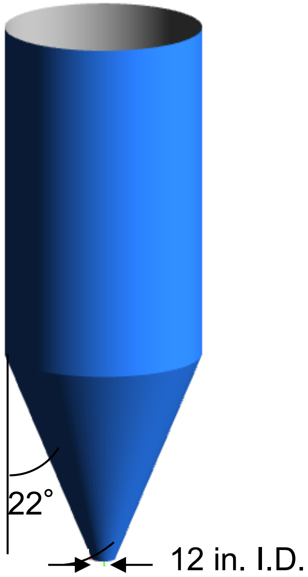
Solve $\sigma_1 = \dfrac{ff\,\rho_b g B}{H(\theta')}$

σ_1 converged?

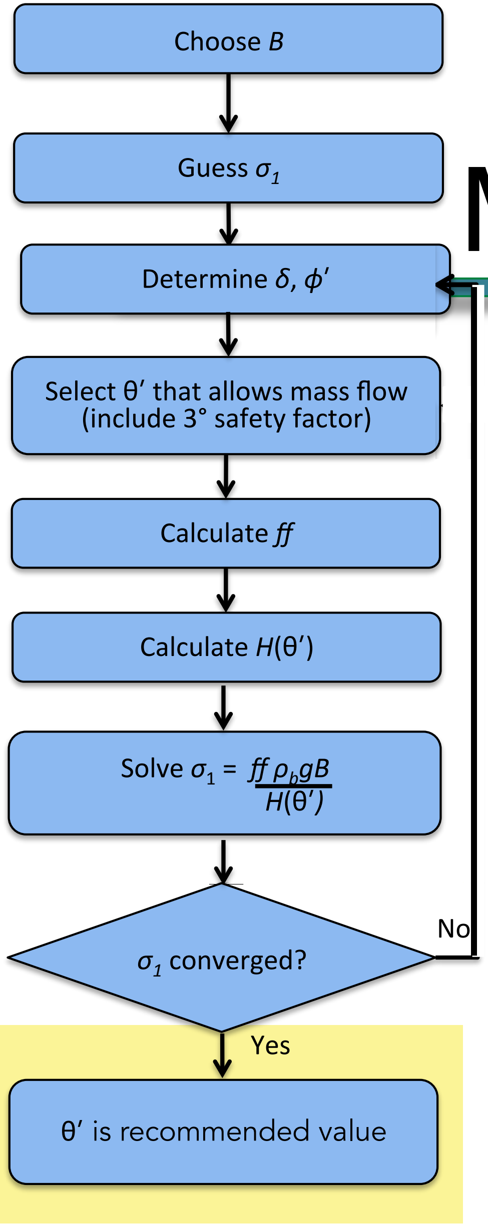
No

Yes

θ′ is recommended value



Mass flow hopper design



Choose *B*

Guess σ_1

Determine δ, ϕ'

Select θ′ that allows mass flow (include 3° safety factor)

Calculate *ff*

Calculate $H(\theta')$

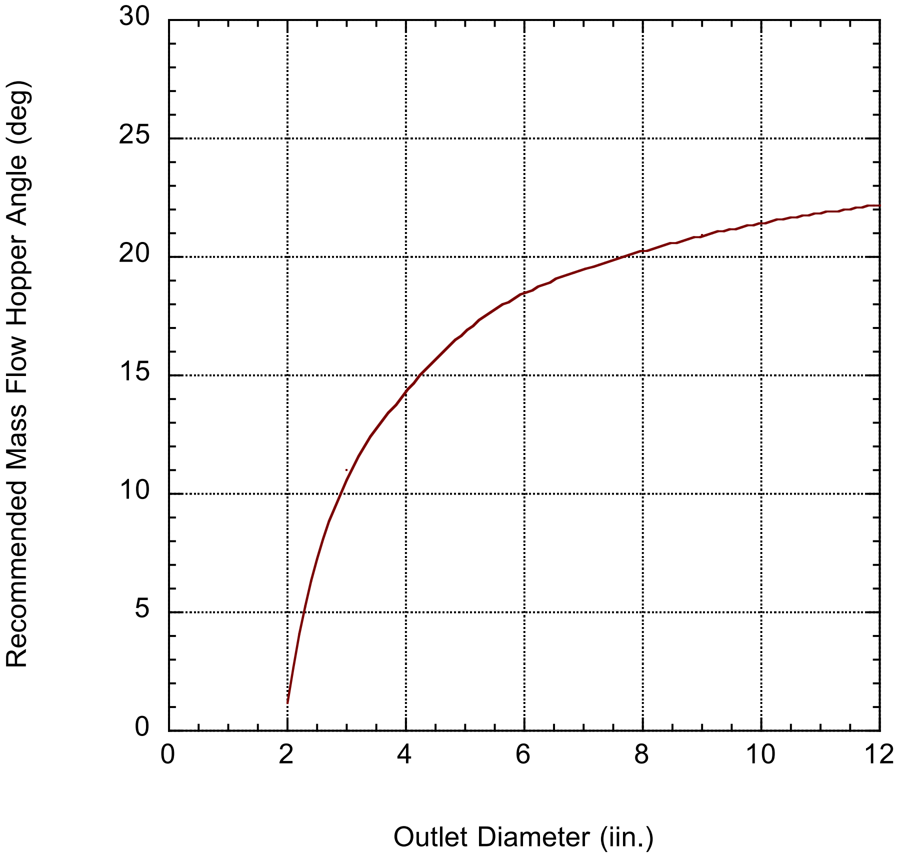
Solve $\sigma_1 = \frac{ff\,\rho_b g B}{H(\theta')}$

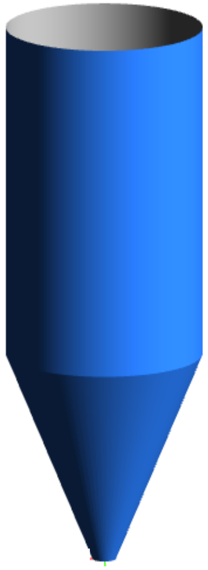
σ_1 converged?

No

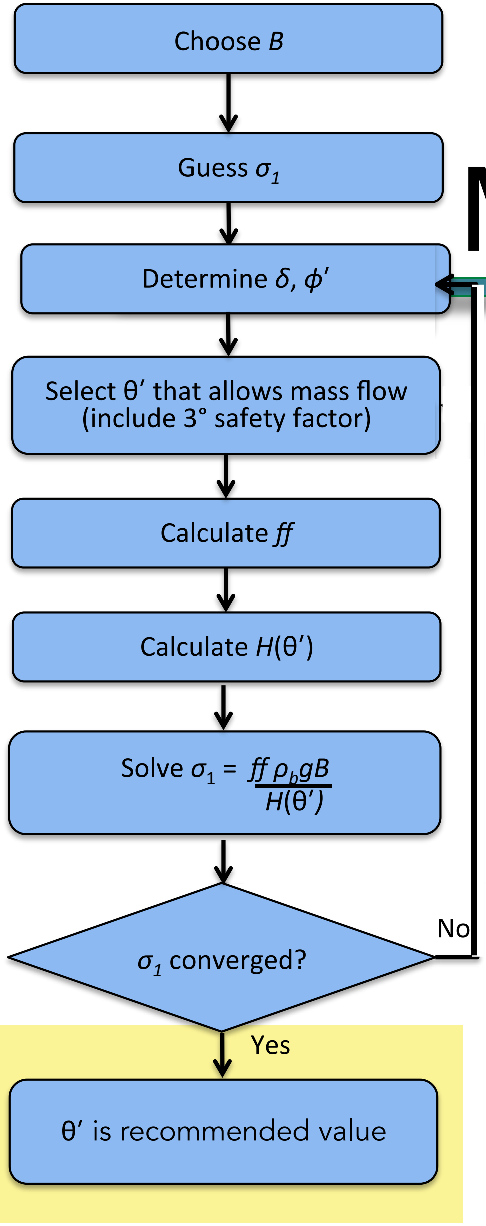
Yes

θ′ is recommended value





Mass flow hopper design



Choose B

Guess σ_1

Determine δ, ϕ'

Select θ′ that allows mass flow (include 3° safety factor)

Calculate ff

Calculate $H(\theta')$

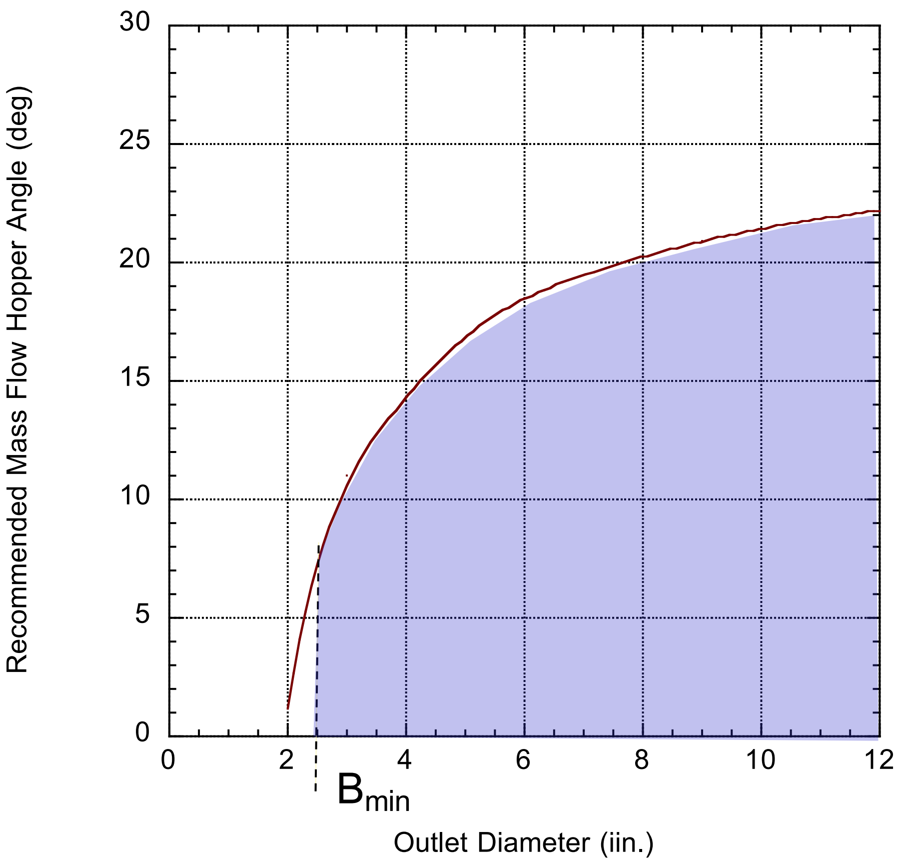
Solve $\sigma_1 = \dfrac{ff\,\rho_b g B}{H(\theta')}$

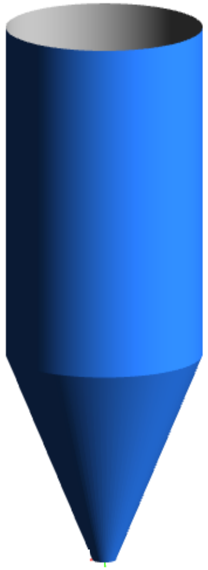
σ_1 converged?

No

Yes

θ′ is recommended value





Flow and Storage of Powders

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