

Flow and Storage of Powders

Greg Mehos, Ph.D., P.E.

gmehos@uri.edu

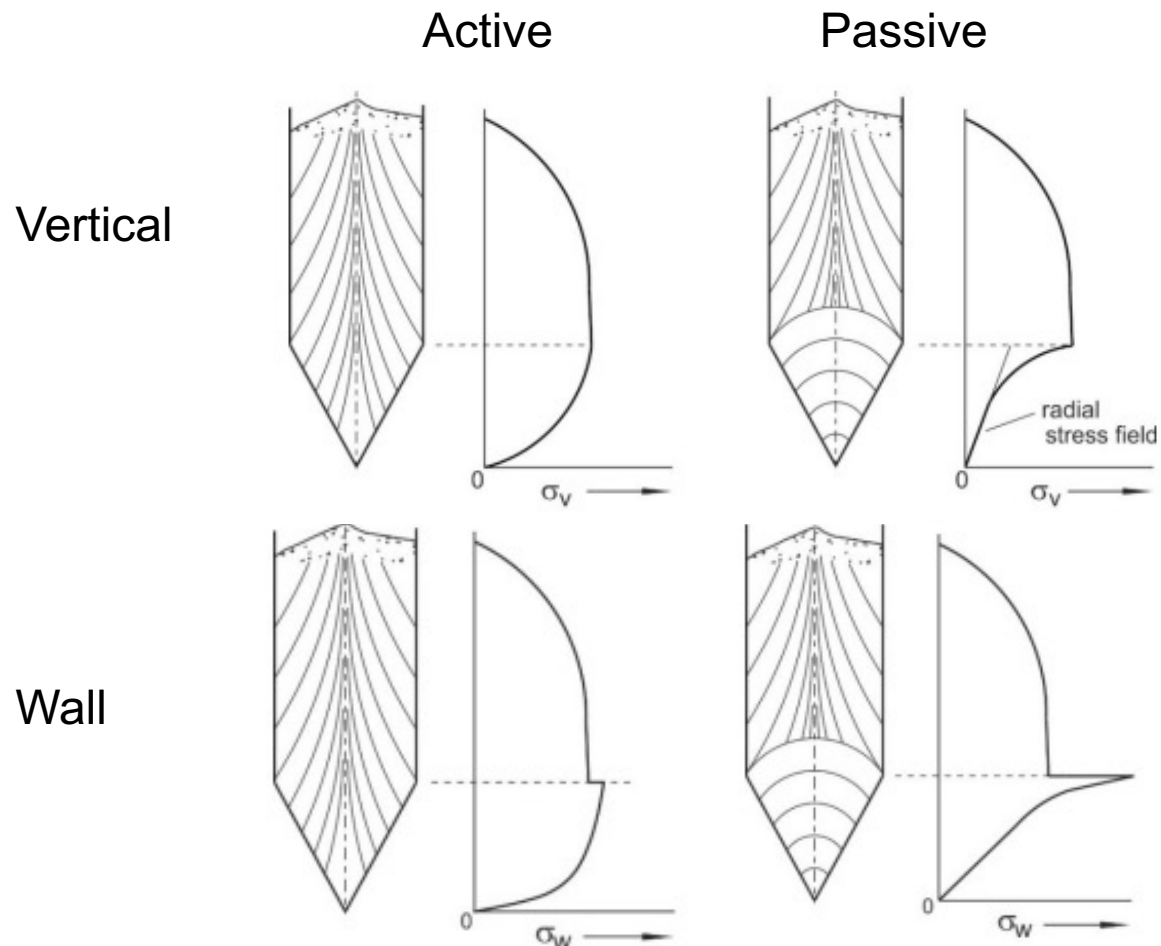
greg@mehos.net

www.mehos.net

978-799-7311

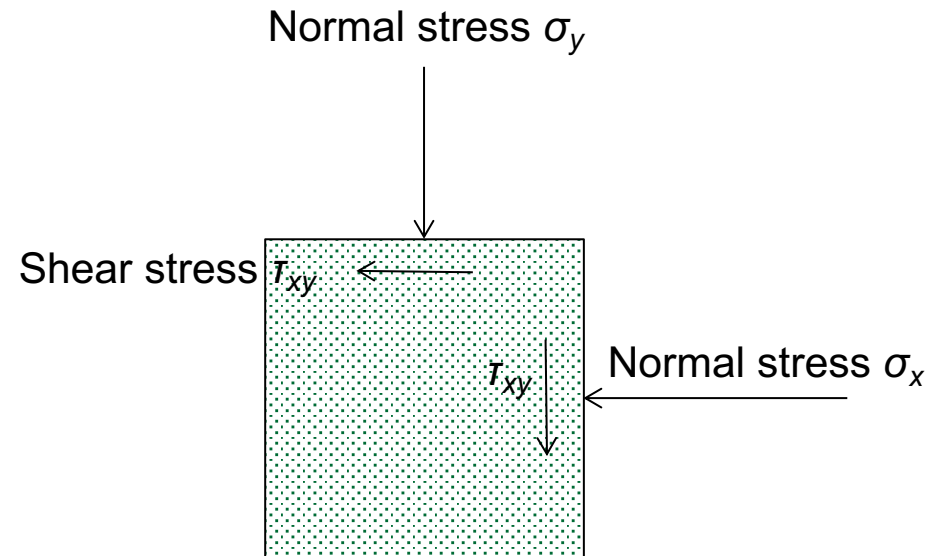


Solids stresses

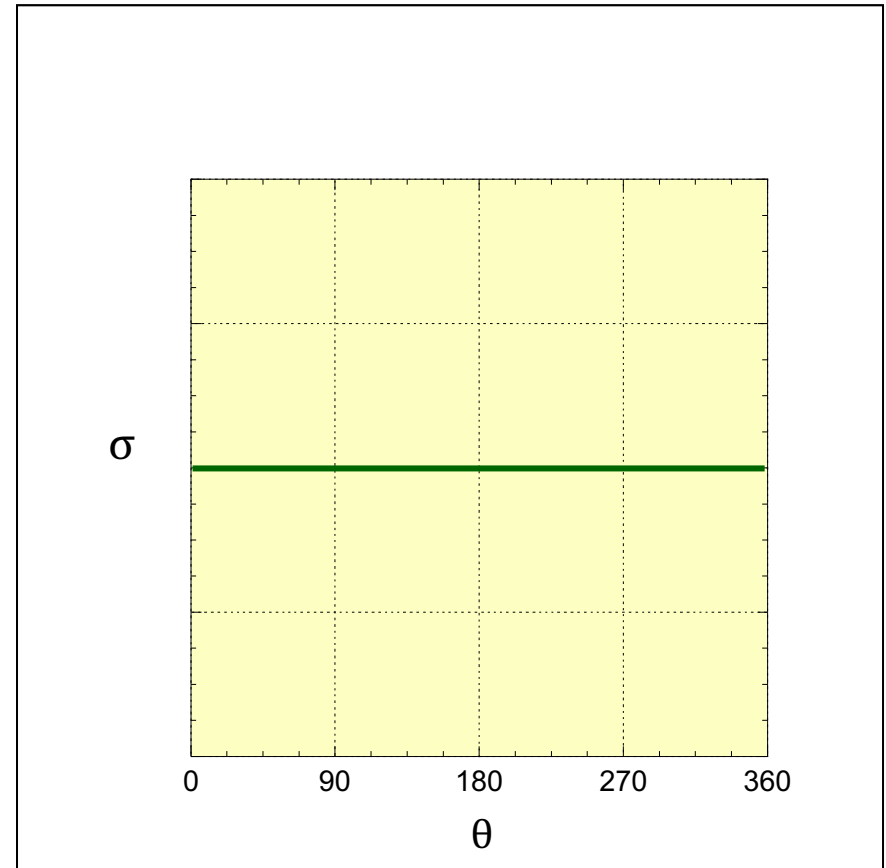
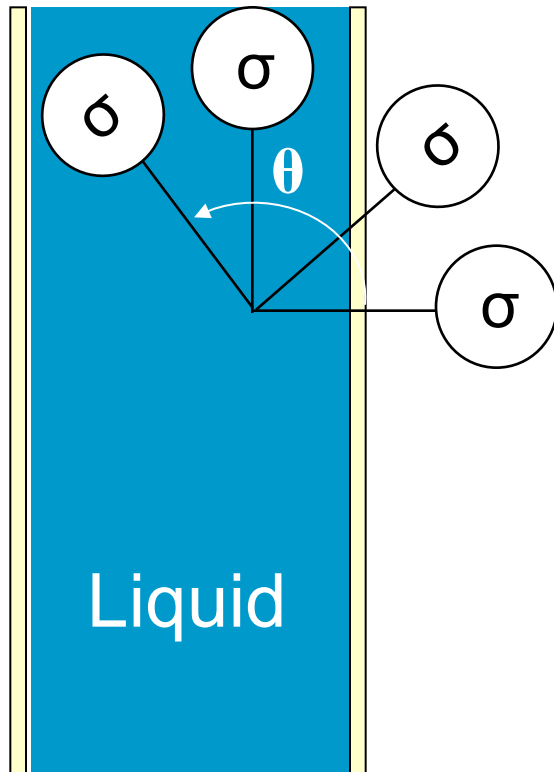


Solids stresses

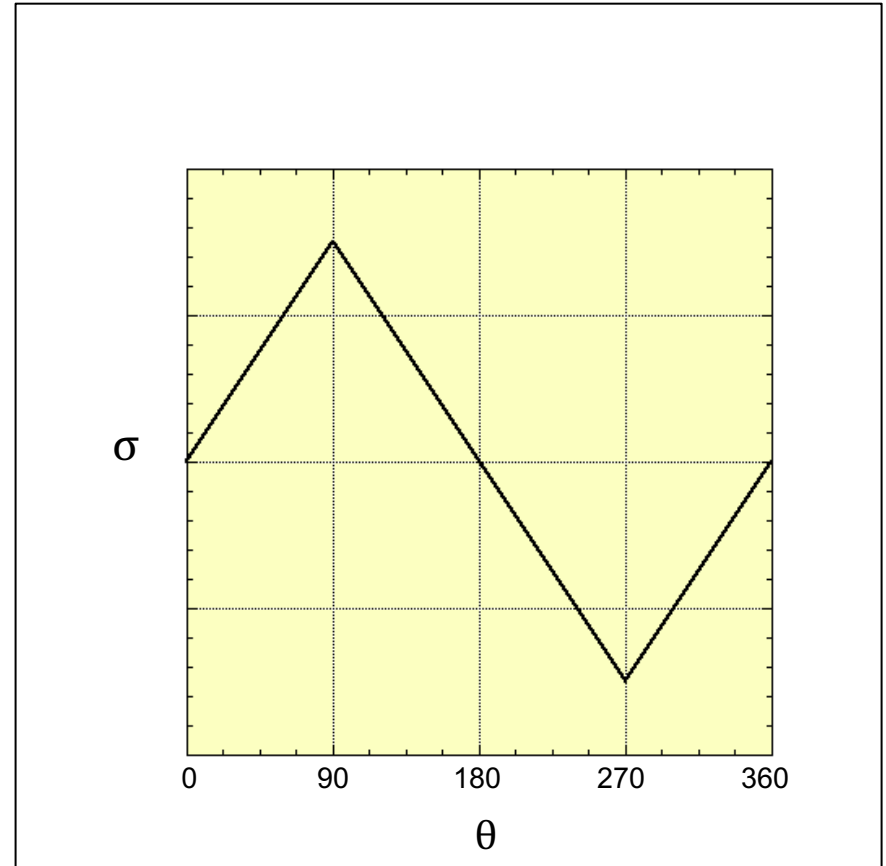
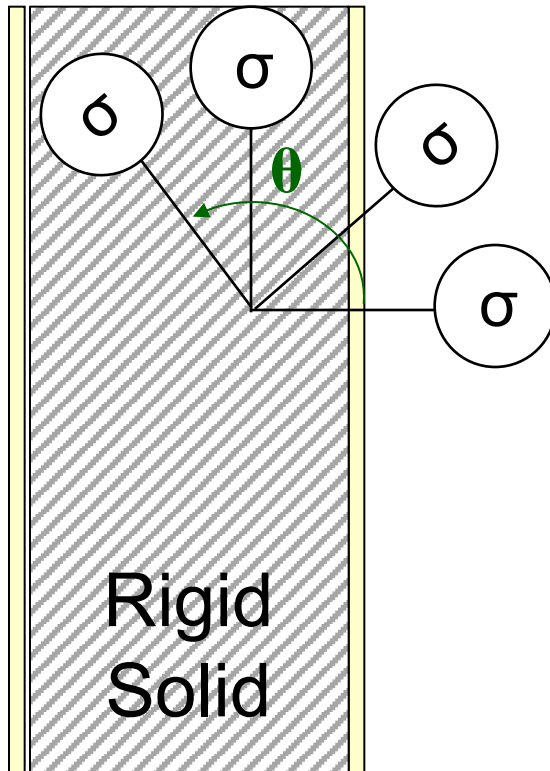
- Normal stresses
act perpendicular to plane
- Shear stresses
act parallel to plane
(plane can be hopper wall,
or slice of bulk material)



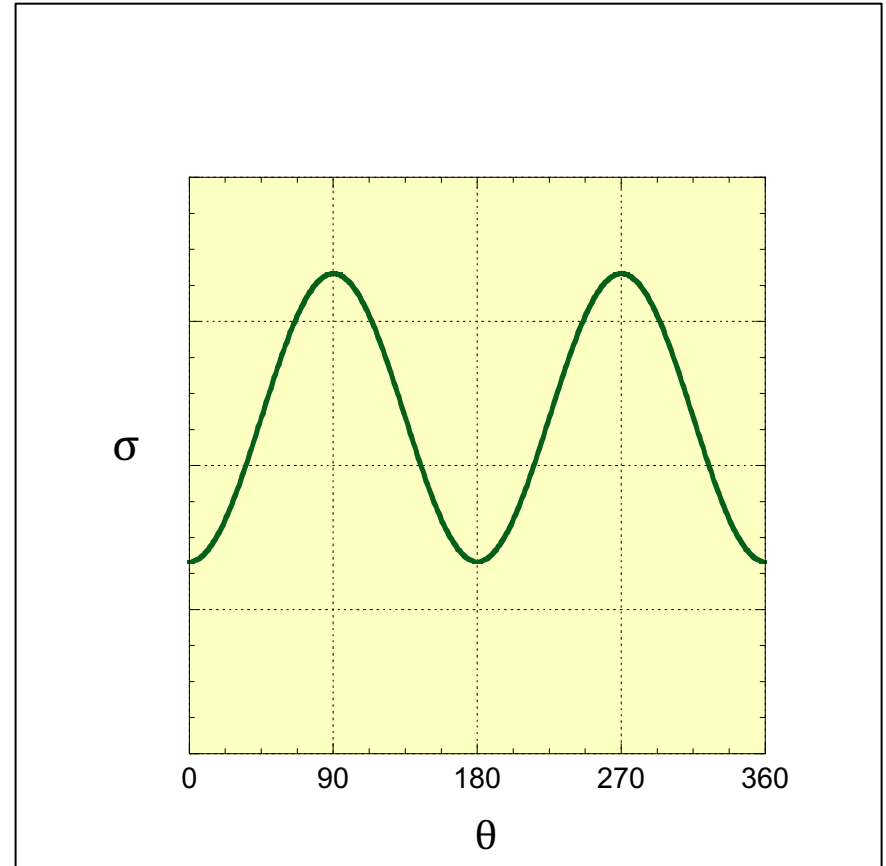
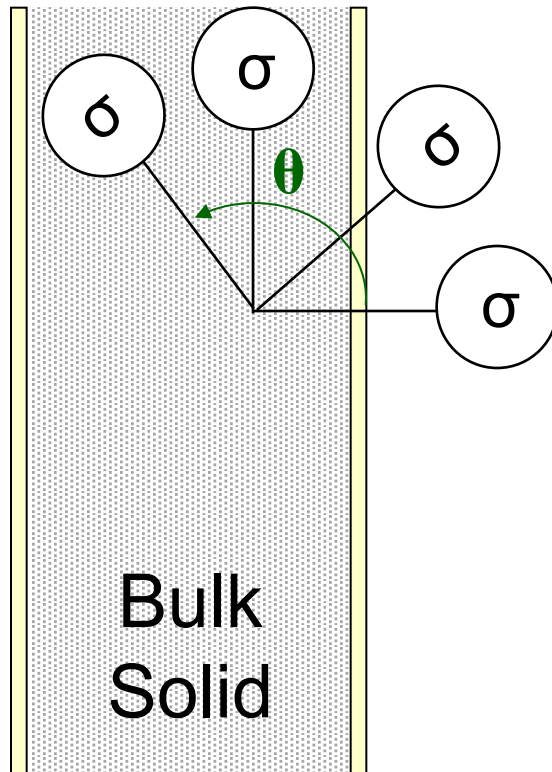
Stresses



Stresses



Stresses



Warning!

- Lots of math ahead!
- The first step is simple! It's usually just a force balance.
- The derived equations are easy to use.

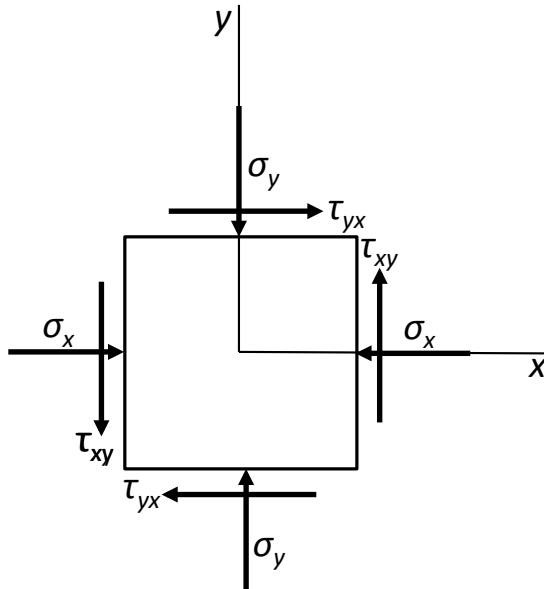


Warning!

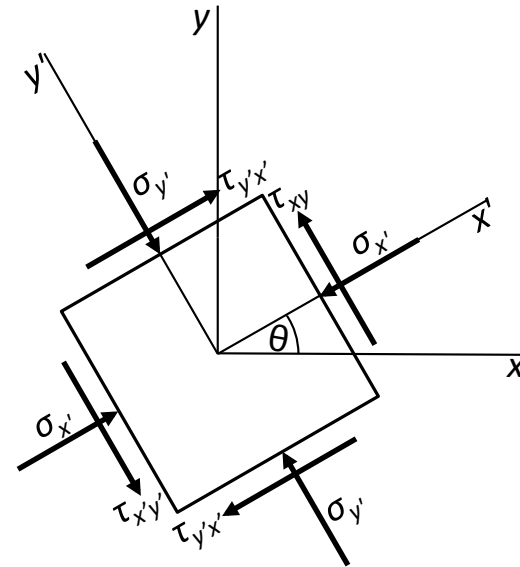
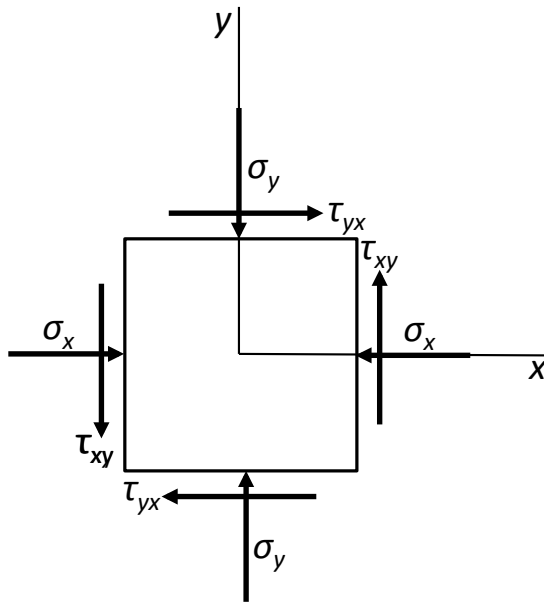
- Lots of math ahead!
- The first step is simple! It's usually just a force balance.
- The derived equations are easy to use.
- Understand the first step.
- Know how to apply the final step.
- Be thankful that someone smarter than us came up with all the other steps!



Analysis of stress

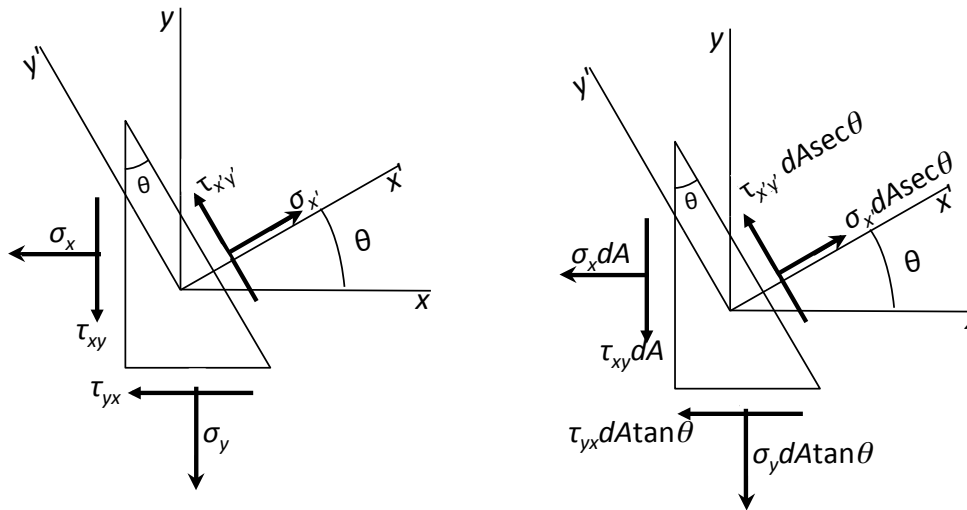


Analysis of stress



Analysis of stress

Equilibrium force balance in x' direction:

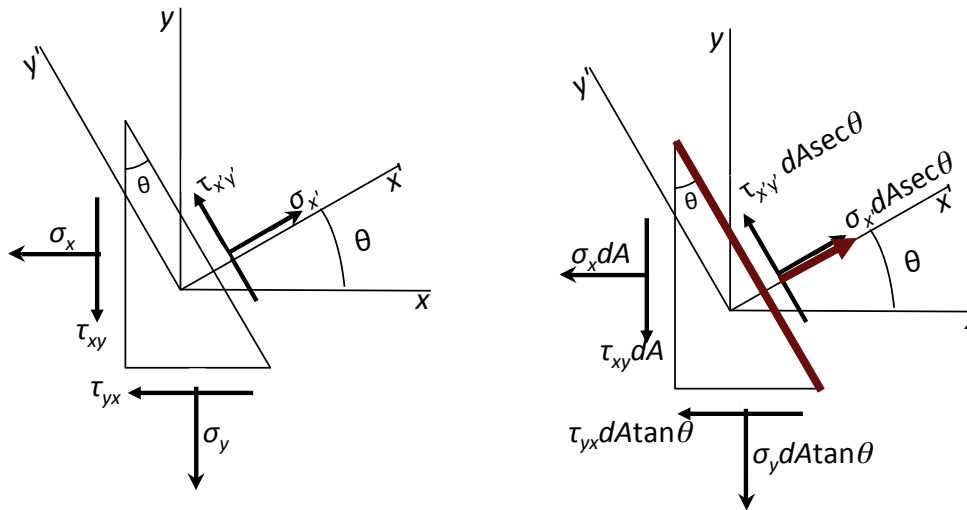


$$\sigma_{x'} \sec \theta = \sigma_x \cos \theta + \tau_{xy} \sin \theta + \sigma_y \tan \theta + \tau_{yx} \tan \theta \cos \theta$$



Analysis of stress

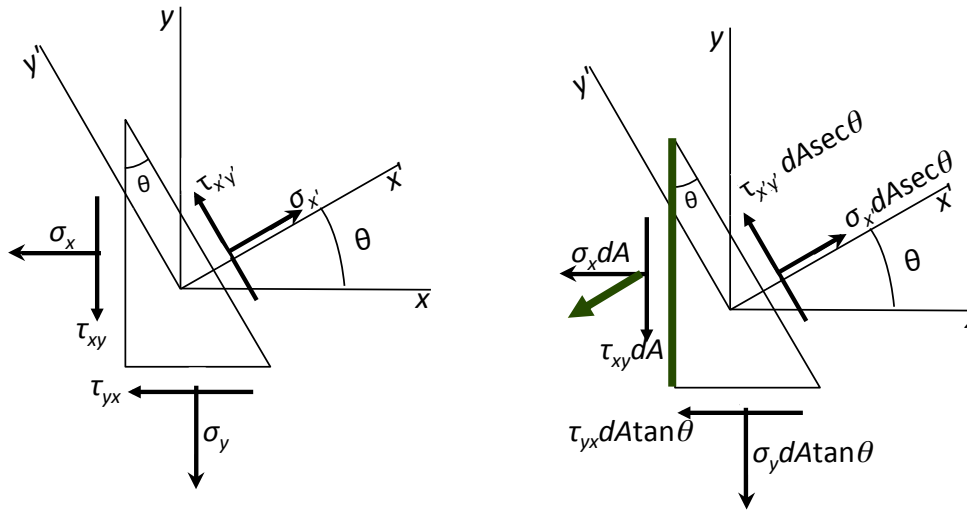
Equilibrium force balance in x' direction:



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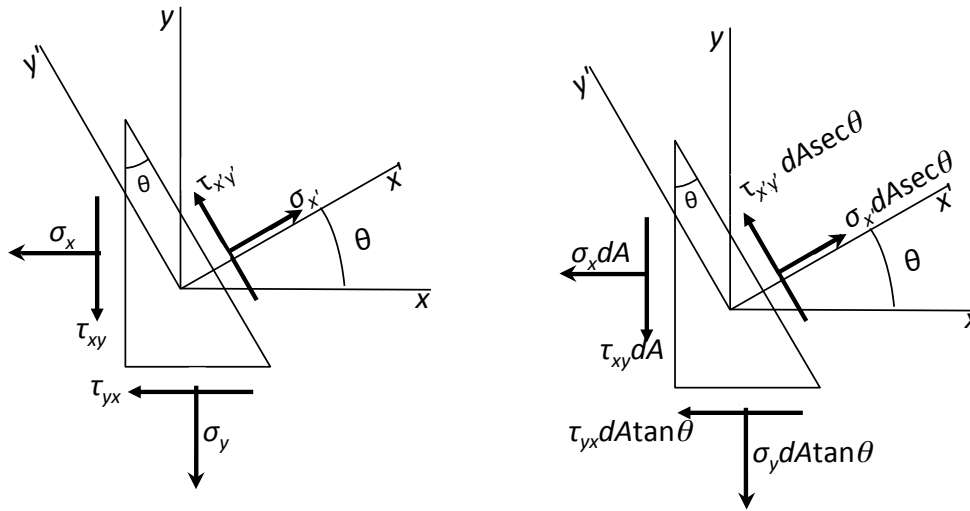
Analysis of stress

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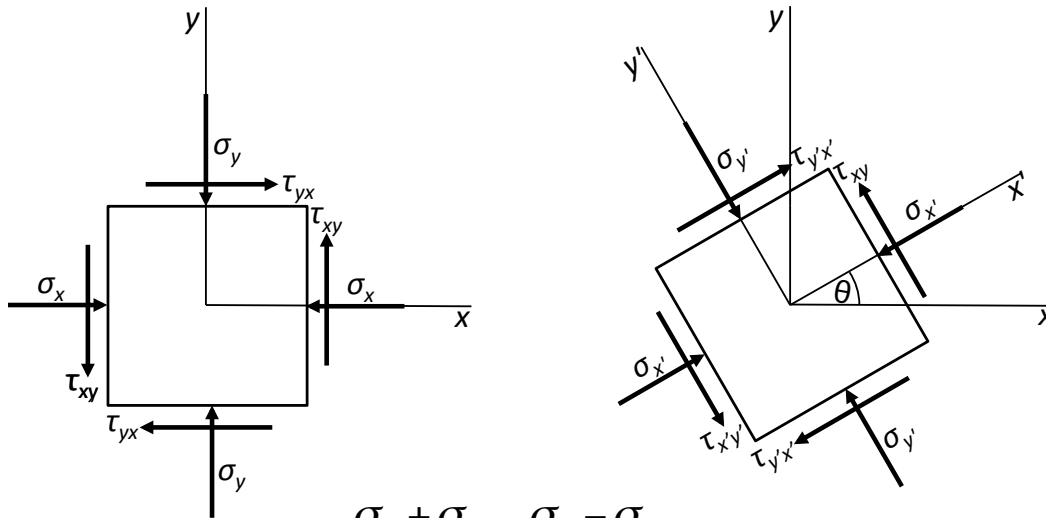
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Analysis of stress

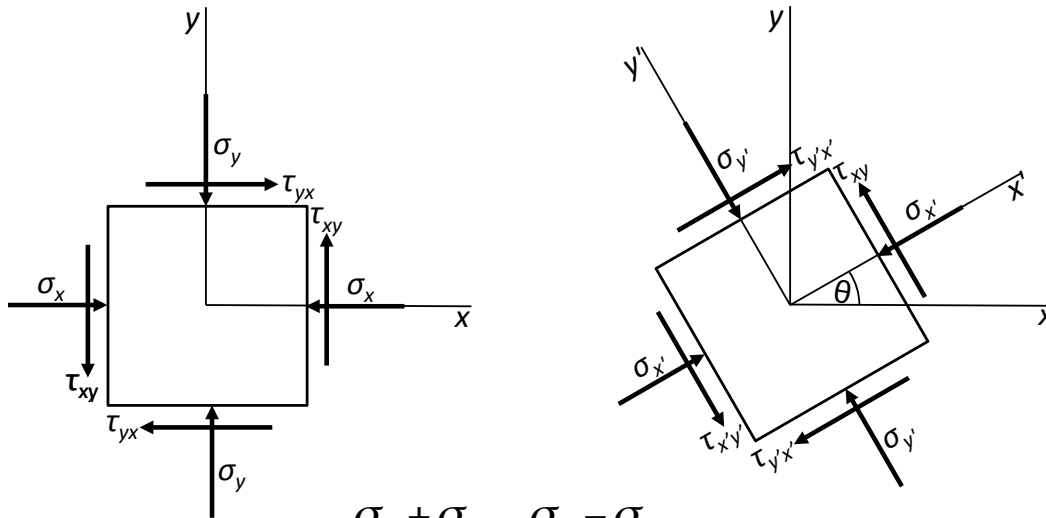


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$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta$$

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Analysis of stress



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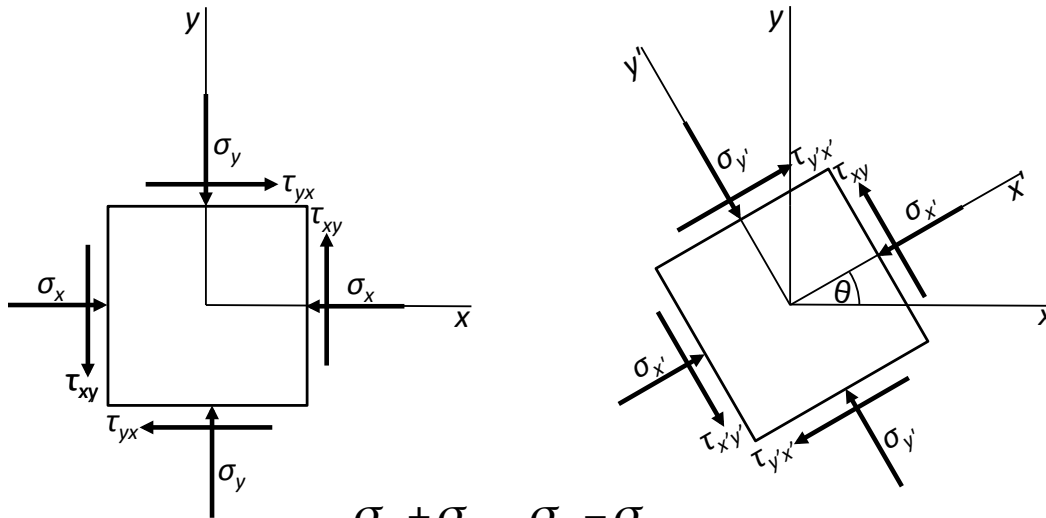
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$$(\sigma_x - \sigma_{avg})^2 + \tau_{xy}^2 = R^2$$

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} \quad R^2 = \left(\frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau_{xy}^2$$

Mohr's circle



$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

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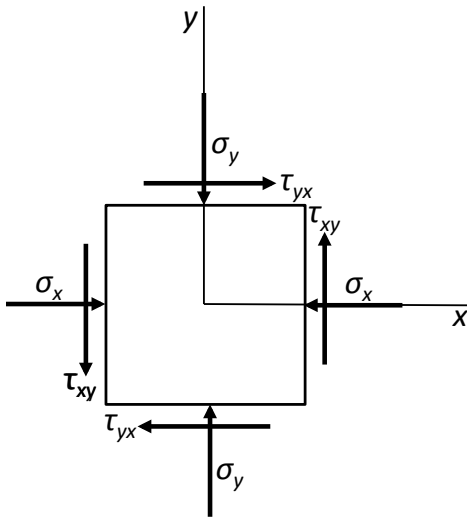
Equation of a Circle!

$$(\sigma_x - \sigma_{avg})^2 + \tau_{xy}^2 = R^2$$

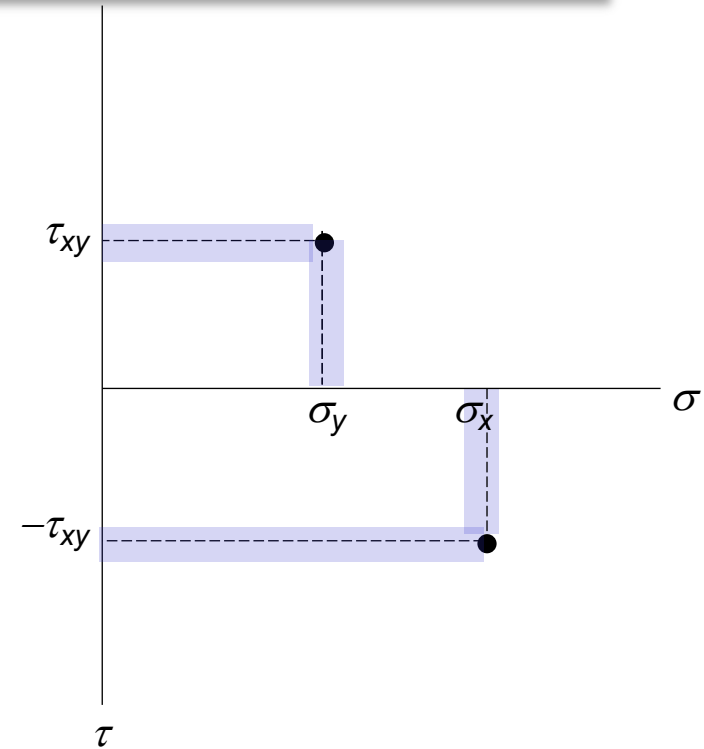
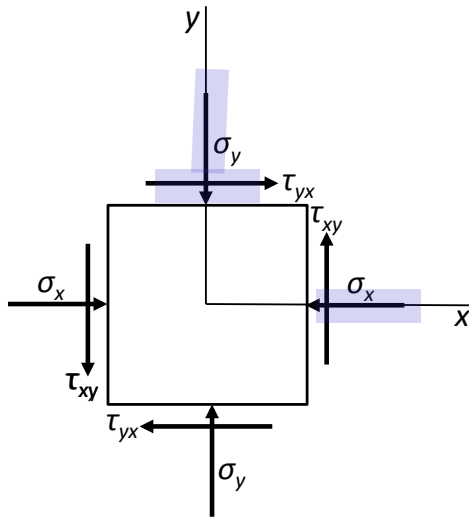
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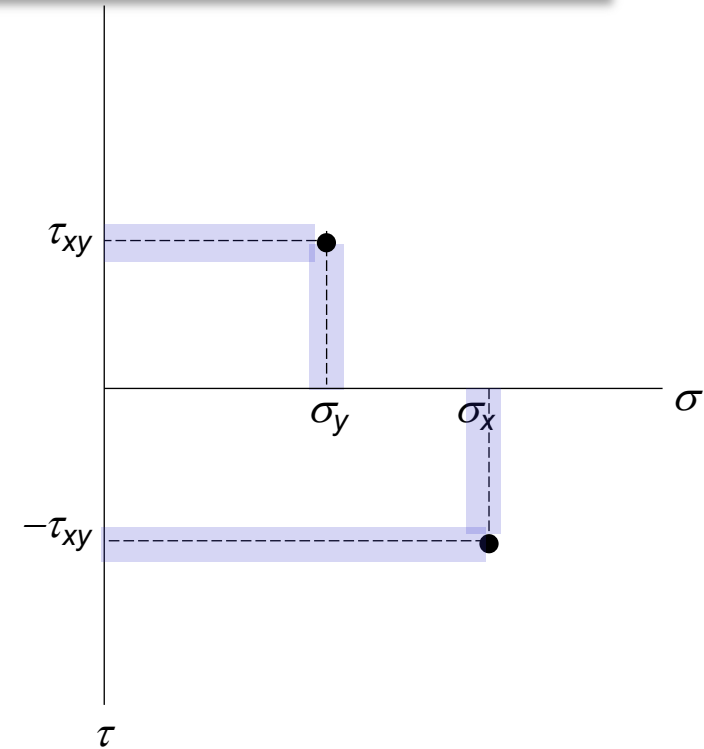
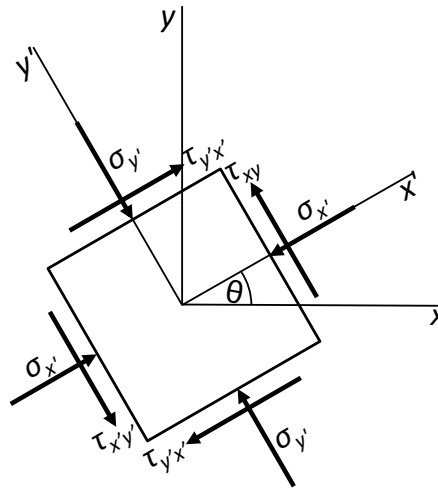
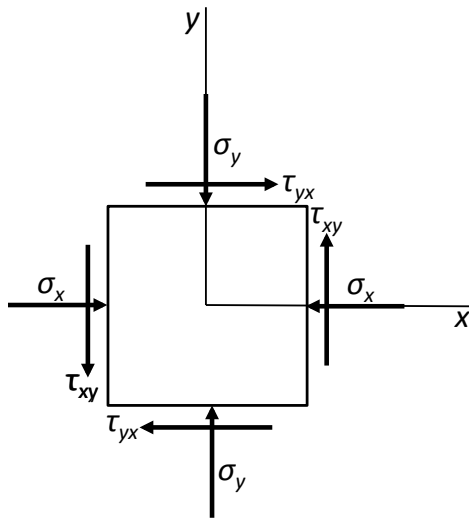
Mohr's circle



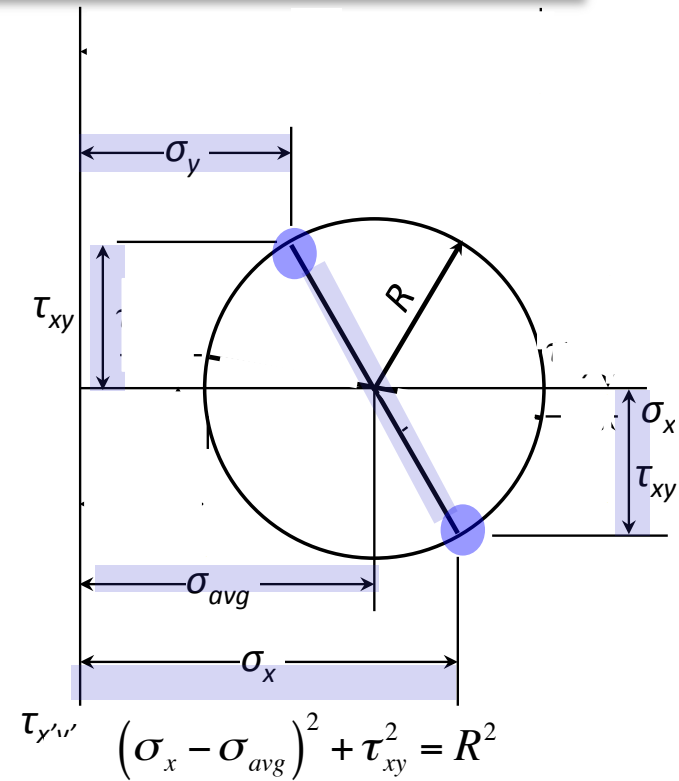
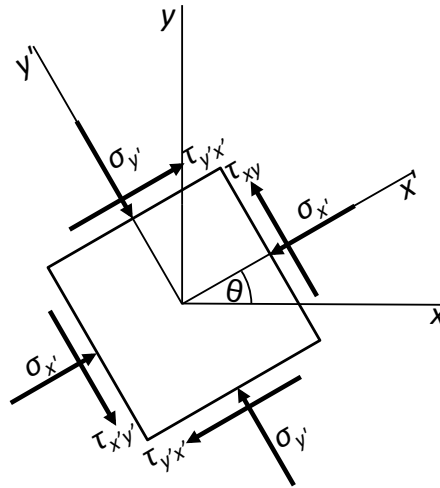
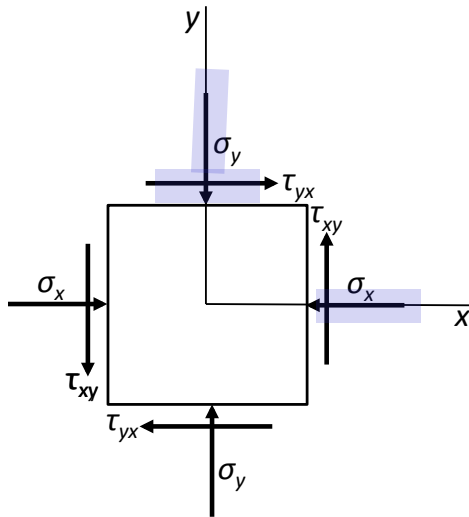
Mohr's circle



Mohr's circle

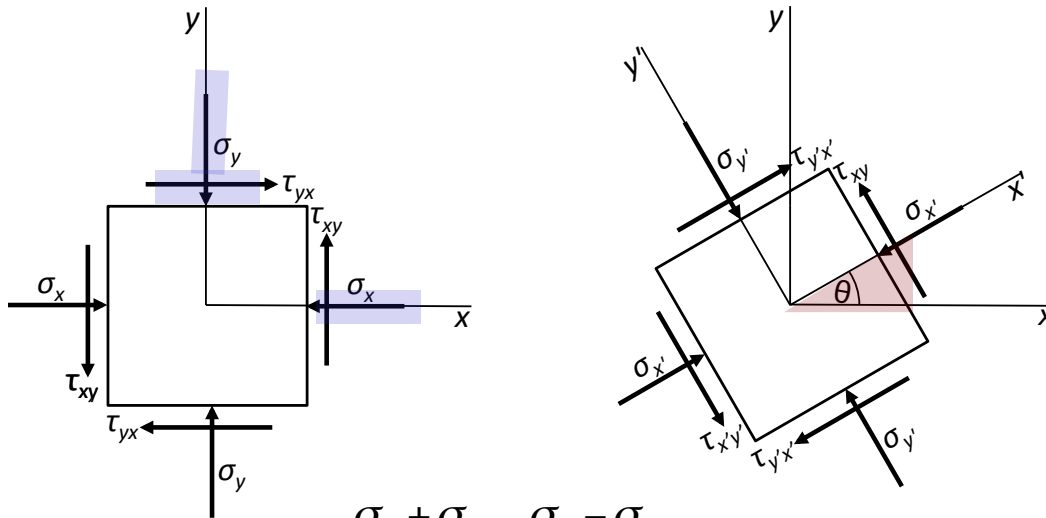


Mohr's circle



$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} \quad R^2 = \left(\frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau_{xy}^2$$

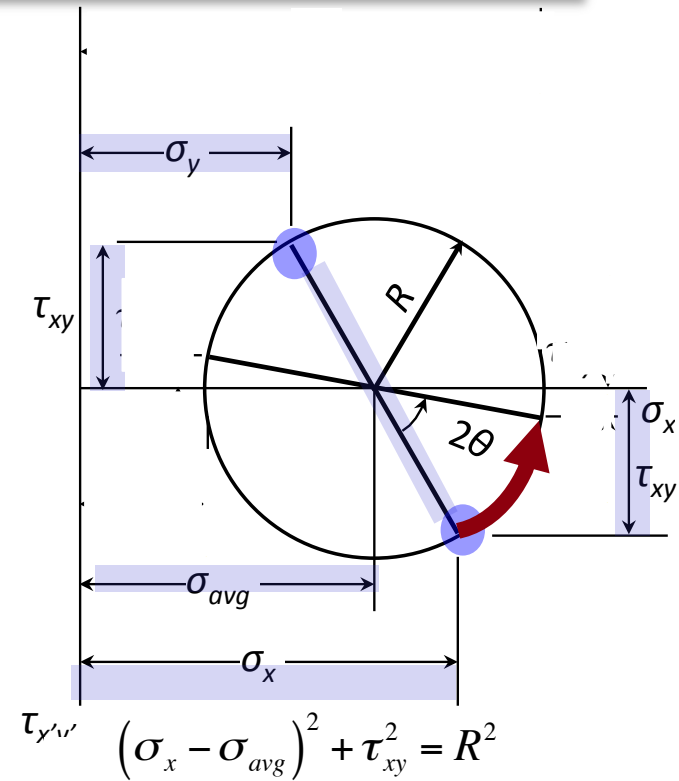
Mohr's circle



$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

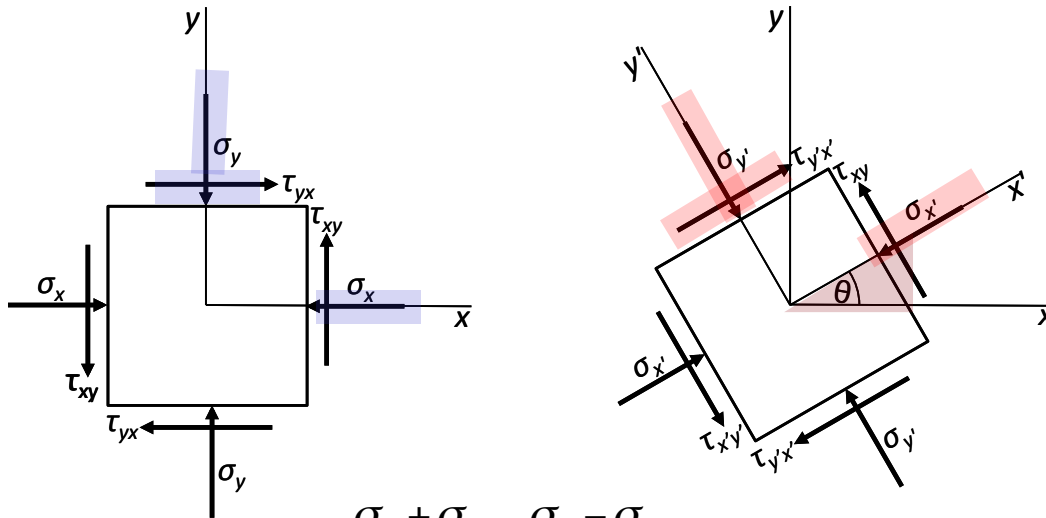
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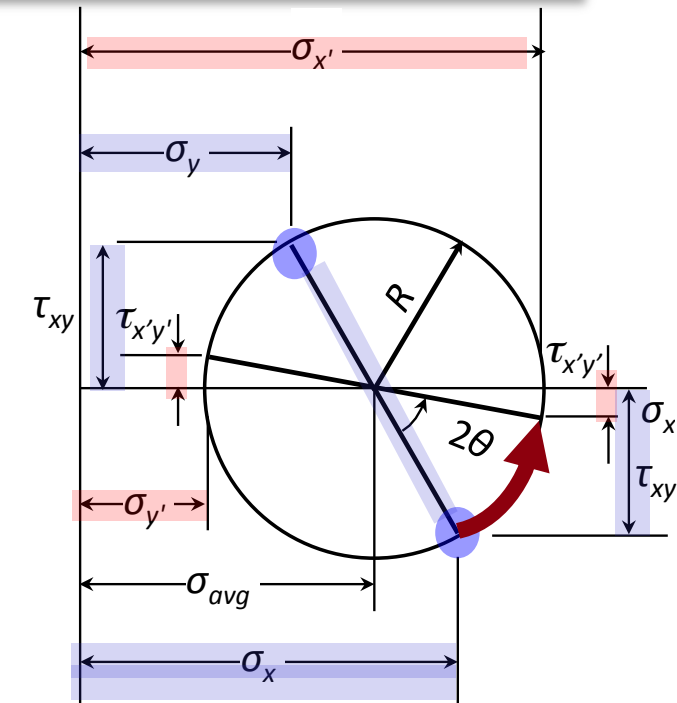
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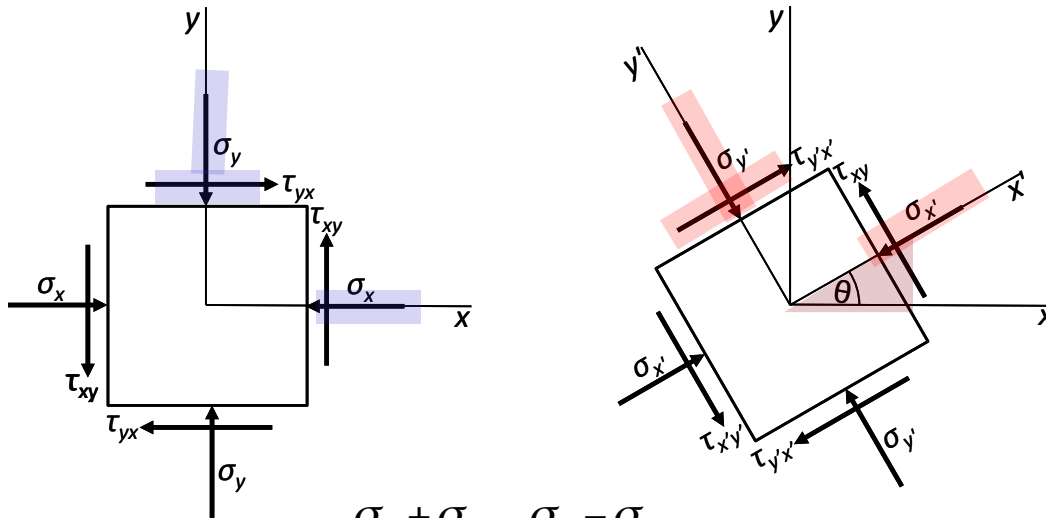


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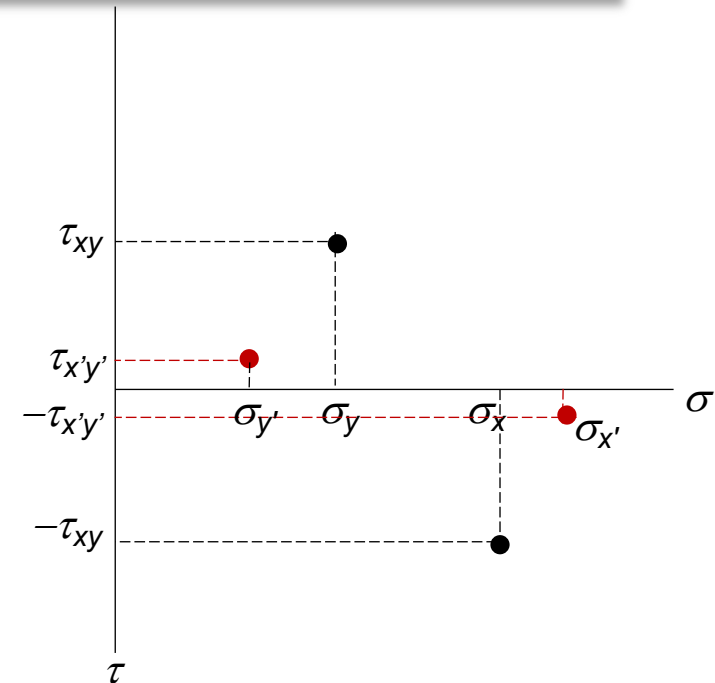
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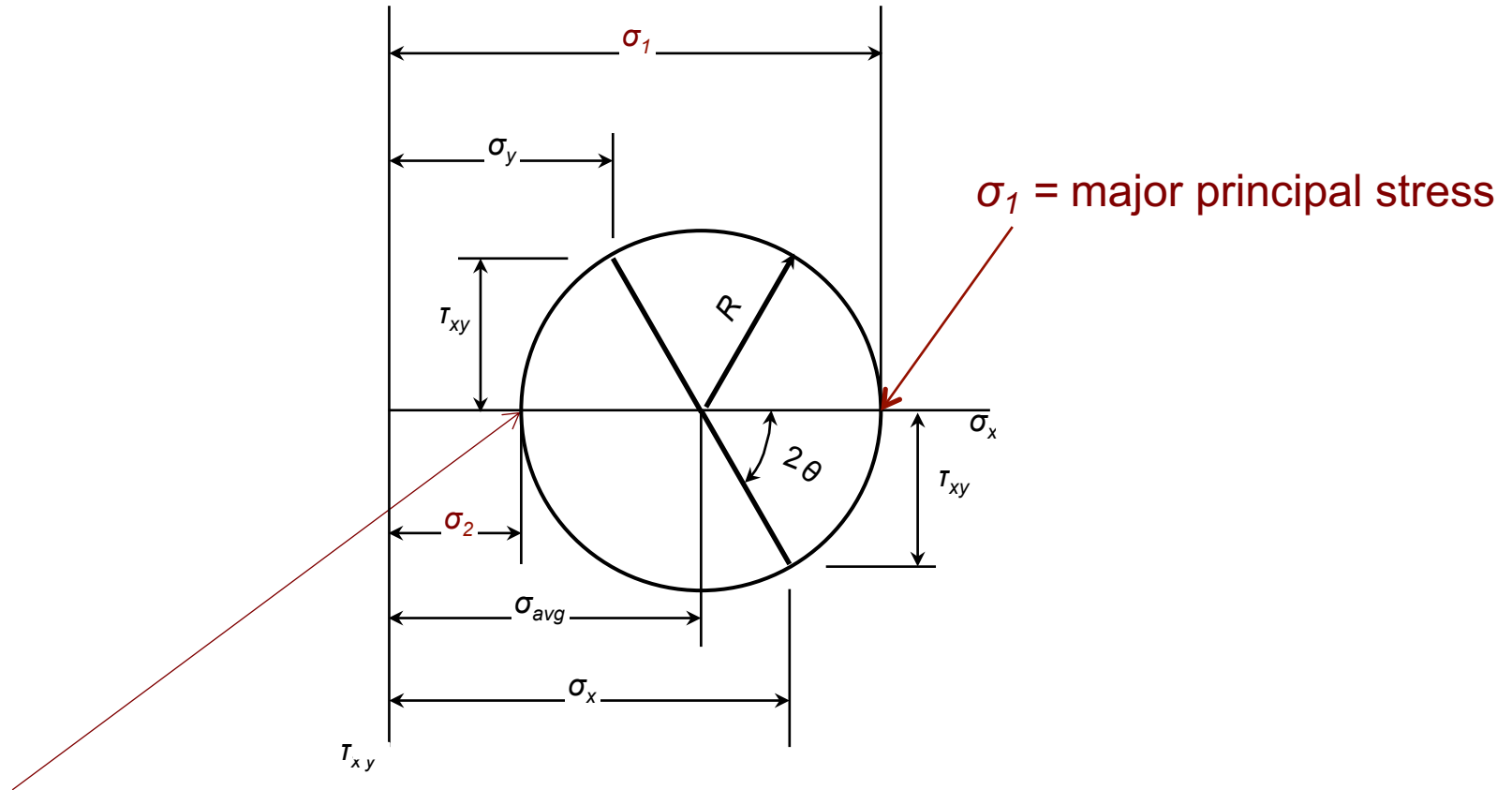
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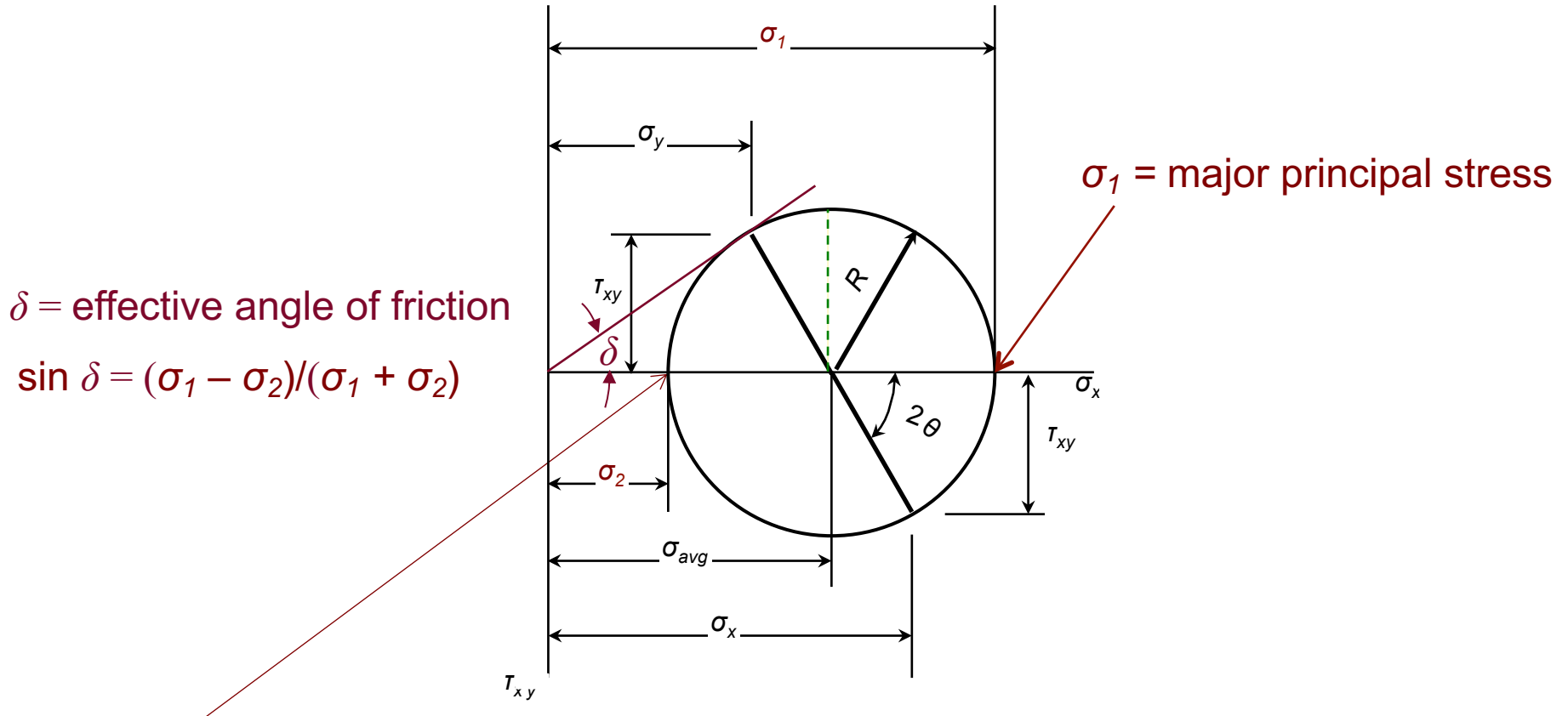
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Principal stresses



$\sigma_2 = \text{minor principal stress}$

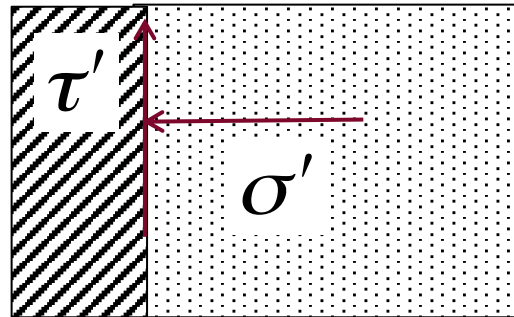
Effective angle of friction



$\sigma_2 = \text{minor principal stress}$

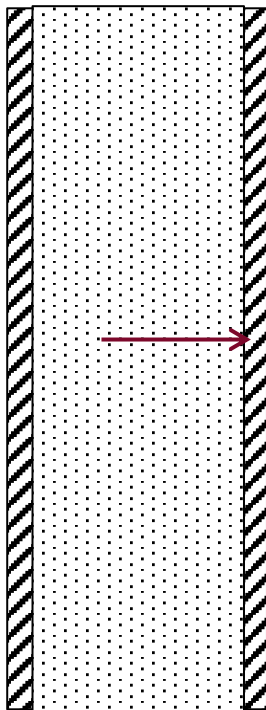


Wall stresses



$$\frac{\tau'}{\sigma'} = \mu = \tan \phi'$$

Wall stresses



$$\sigma' = k\sigma_v$$

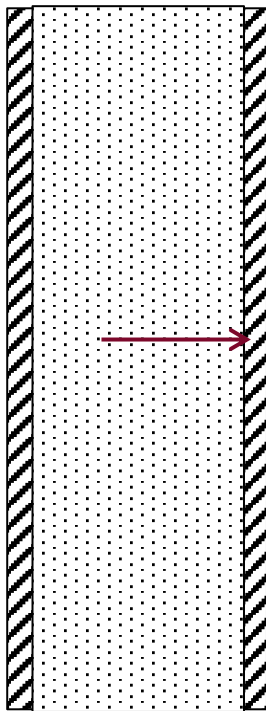
$$\sigma_v(z) = \frac{\rho_b g R_H}{k \tan \phi'} \left[1 - \exp\left(\frac{-k \tan \phi' z}{R_H}\right) \right]$$

$$k = \frac{\sigma_h}{\sigma_v}$$

$$\sigma_h(z) = \frac{\rho_b g R_H}{\tan \phi'} \left[1 - \exp\left(\frac{-k \tan \phi' z}{R_H}\right) \right]$$



Wall stresses



$$\sigma' = k\sigma_v$$

$$\sigma_v(z) = \frac{\rho_b g R_H}{k \tan \phi'} \left[1 - \exp\left(\frac{-k \tan \phi' z}{R_H}\right) \right]$$

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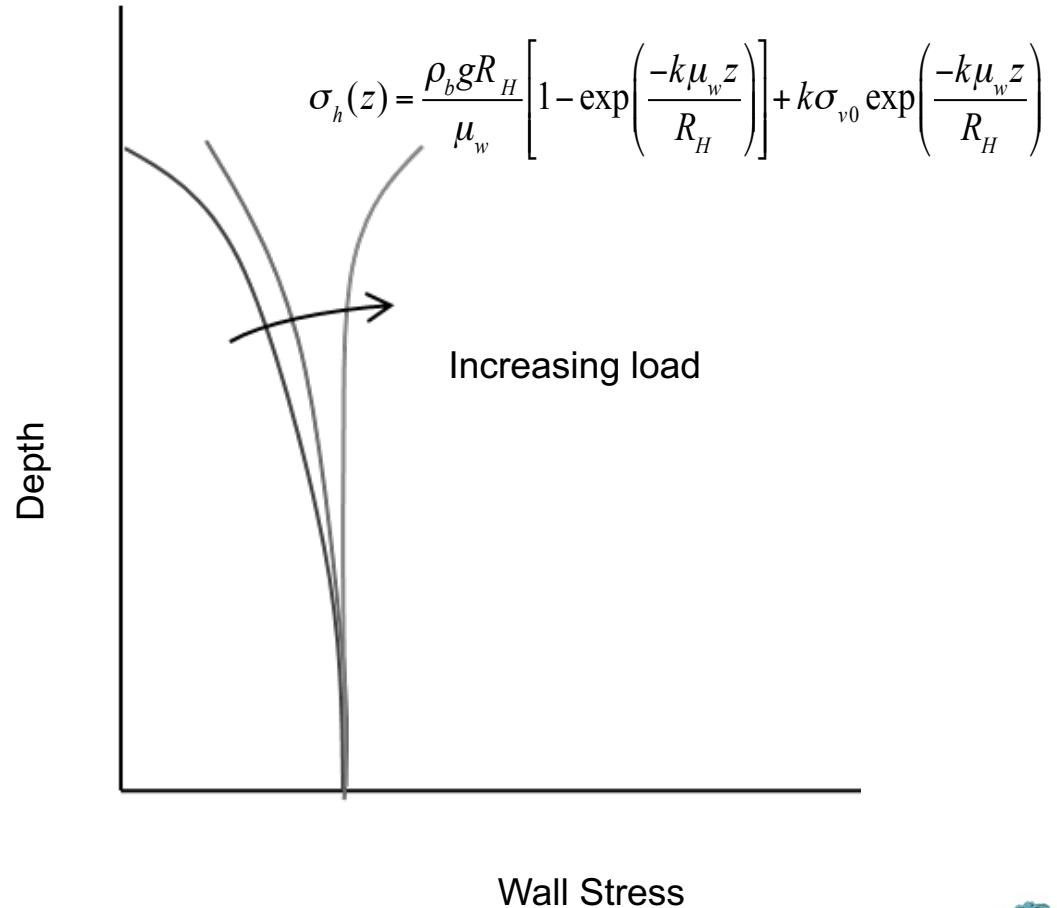
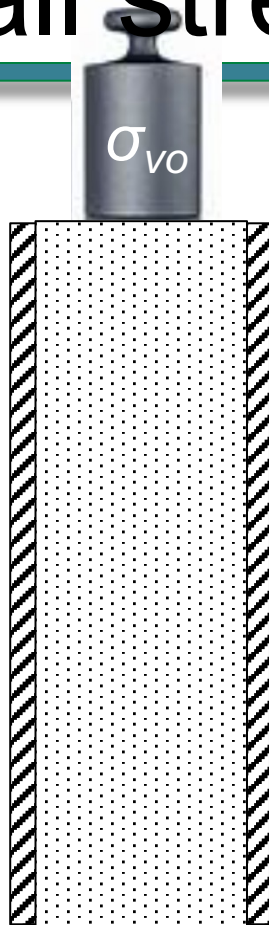
$$k = 0.4 - 0.6$$

Max. vertical load calculations

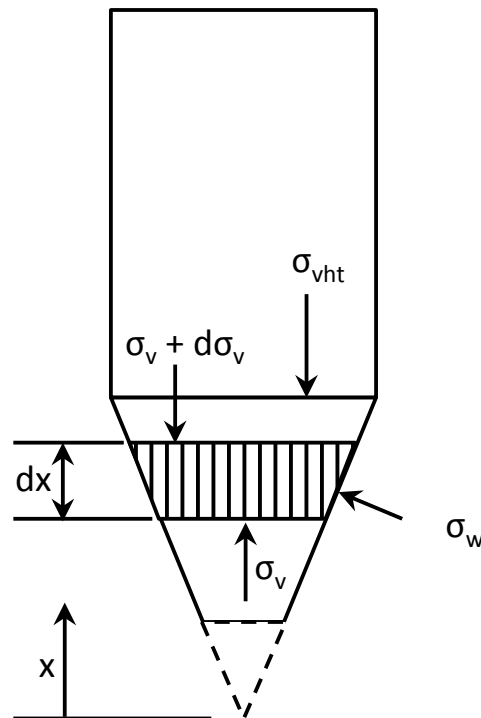
Max. wall stress calculations



Wall stresses



Wall stresses



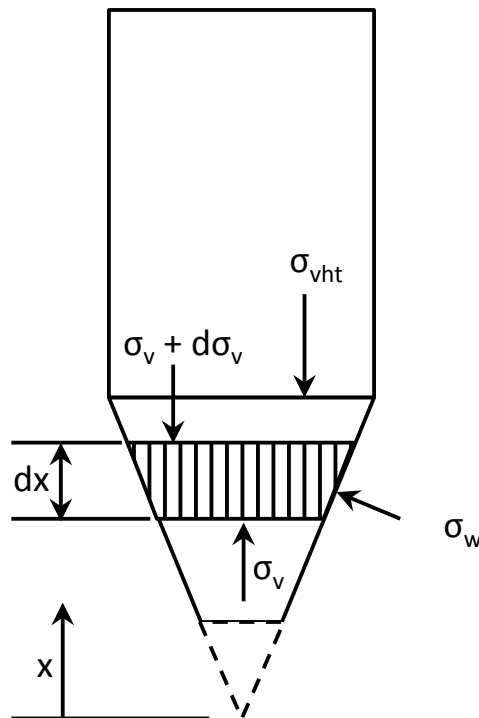
$$\frac{d\sigma_v}{dz} - n \frac{\sigma_v}{z} = -g\rho_b$$

$$n = (m+1) \left[k \left(1 + \frac{\tan \phi'}{\tan \theta} \right) - 1 \right]$$

$$\sigma_v = \frac{\rho_b g h}{n-1} \left[\frac{x}{h} - \left(\frac{x}{h} \right)^n \right] + \sigma_{vht} \left(\frac{x}{h} \right)^n$$

$$\sigma_w = k\sigma_v$$

Wall stresses



$$\sigma_v = \frac{\rho_b g h}{n-1} \left[\frac{x}{h} - \left(\frac{x}{h} \right)^n \right] + \sigma_{vht} \left(\frac{x}{h} \right)^n$$

Initial fill

$$k = 1 - \frac{0.2}{1 + \frac{\tan \theta'}{\tan \phi'}}$$

Discharge

$$k = \frac{1 + \sin \delta \cos \varepsilon}{1 - \sin \delta \cos(2\theta' + \varepsilon)}$$

$$\varepsilon = \phi' + \sin^{-1} \left(\frac{\sin \phi'}{\sin \delta} \right)$$

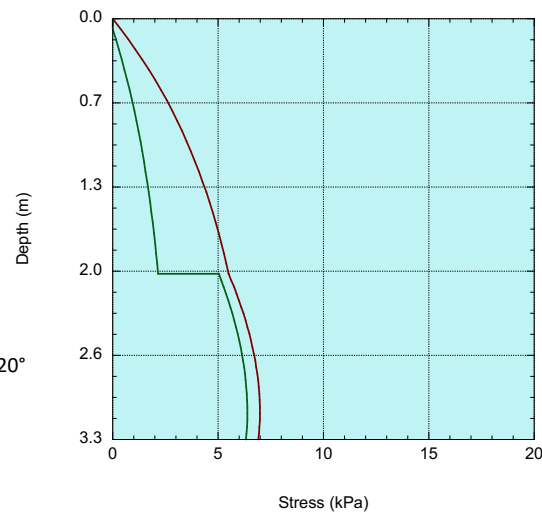
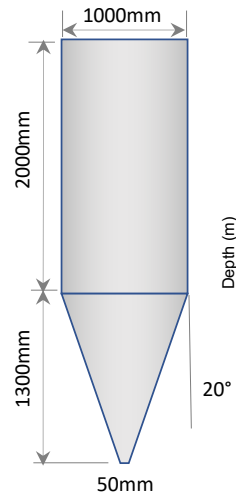
$$k = \left(\frac{1}{1 + \frac{\tan \phi'}{\tan \theta'}} \right) \left\{ 1 + 2 \left[1 + \left(\frac{\sin \delta}{1 + \sin \delta} \right) \left(\frac{\cos \varepsilon \sin(\varepsilon - \theta')}{\sin \theta'} \right) \right] \right\}$$

$$\varepsilon = \theta' + \frac{1}{2} \left[\phi' + \sin^{-1} \left(\frac{\sin \phi'}{\sin \delta} \right) \right]$$

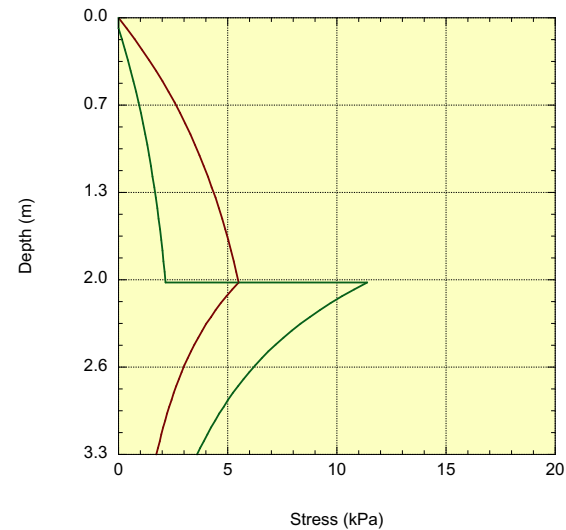


Solids stresses

Initial fill



After discharge



Fundamental flow properties

- Cohesive strength
- Internal friction
- Compressibility
- Wall friction
- Permeability

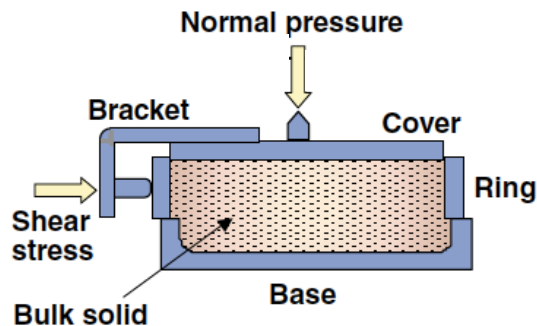


Test equipment

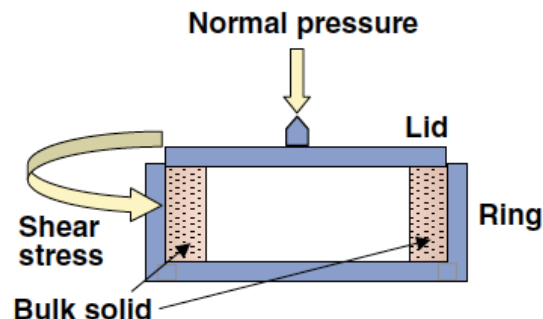
- Shear cell testers
 - Cohesive strength
 - Internal friction
 - Compressibility
 - Wall friction
- Permeability testers



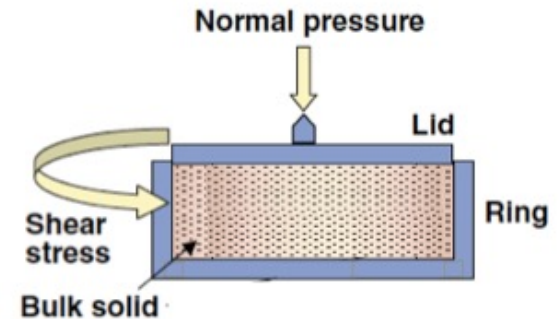
Shear cell testers



Direct



Annular



Torsional

- Cohesive strength
- Internal friction
- Compressibility
- Wall friction (use caution)

Shear cell testers



Jenike &
Johanson



Dietmar
Schulze



Brookfield
Engineering



Anton
Paar



Freeman
Technology

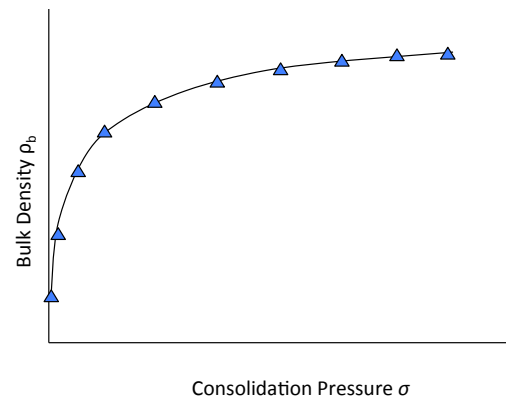
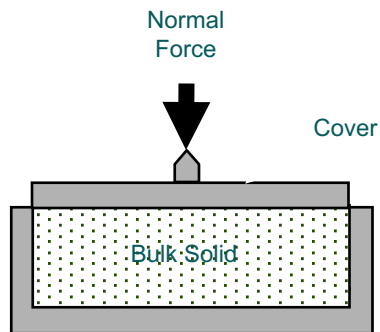


E&G
Associates

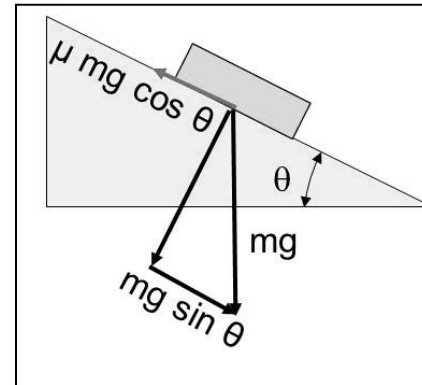
Compressibility



Compressibility



Wall friction

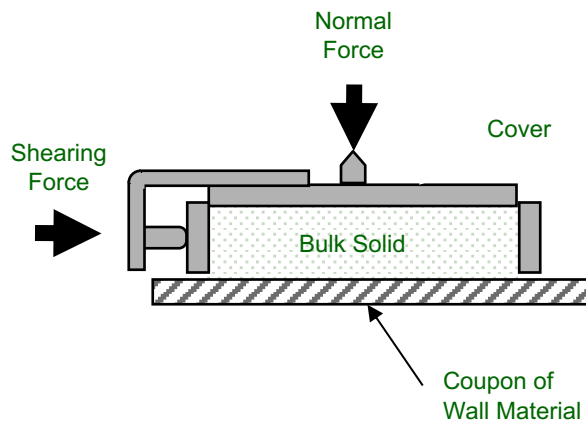


$$mg \sin \theta = \mu mg \cos \theta$$

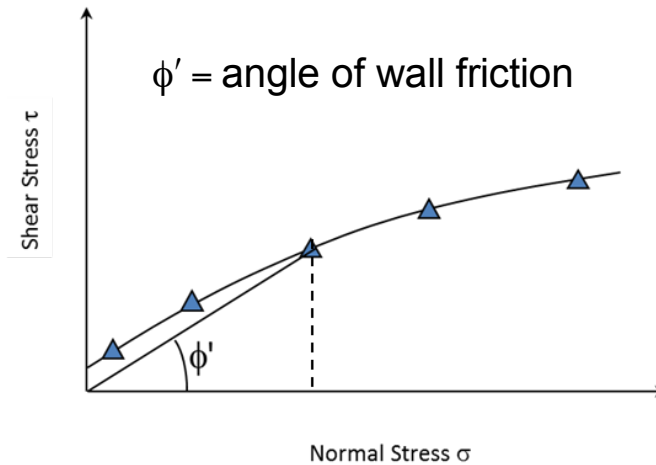
$$\mu = \tan \theta$$



Wall friction



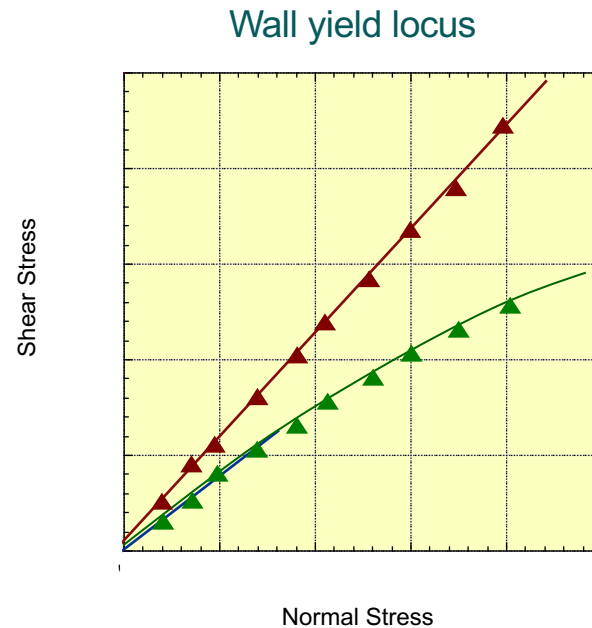
Wall Yield Locus



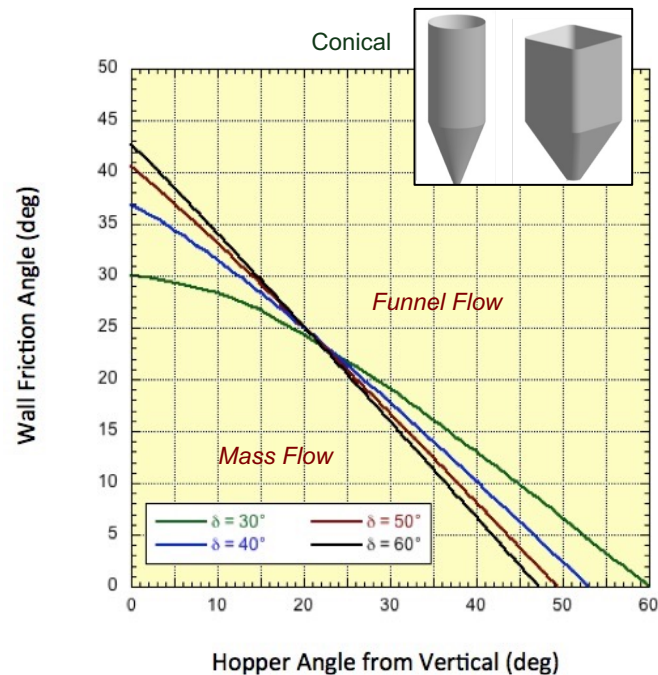
$$\tan \phi' = \frac{\text{Shear Stress}}{\text{Normal Stress}} = \mu$$



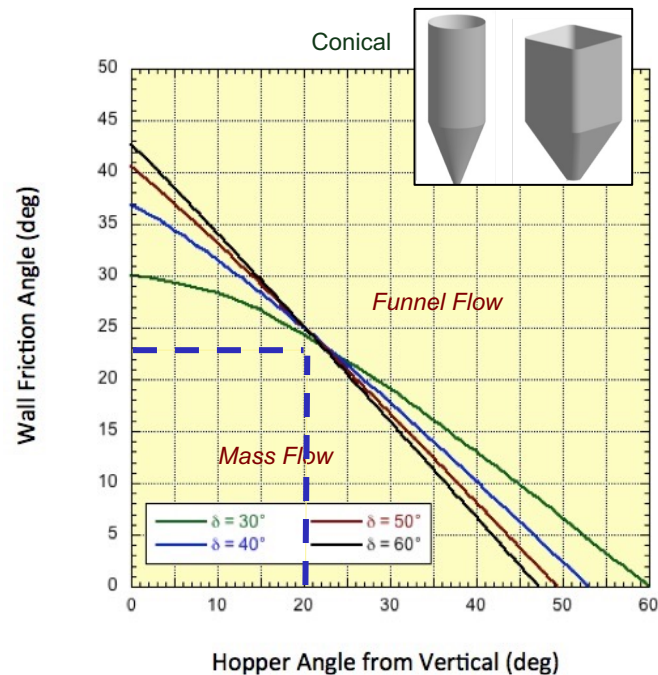
Wall friction



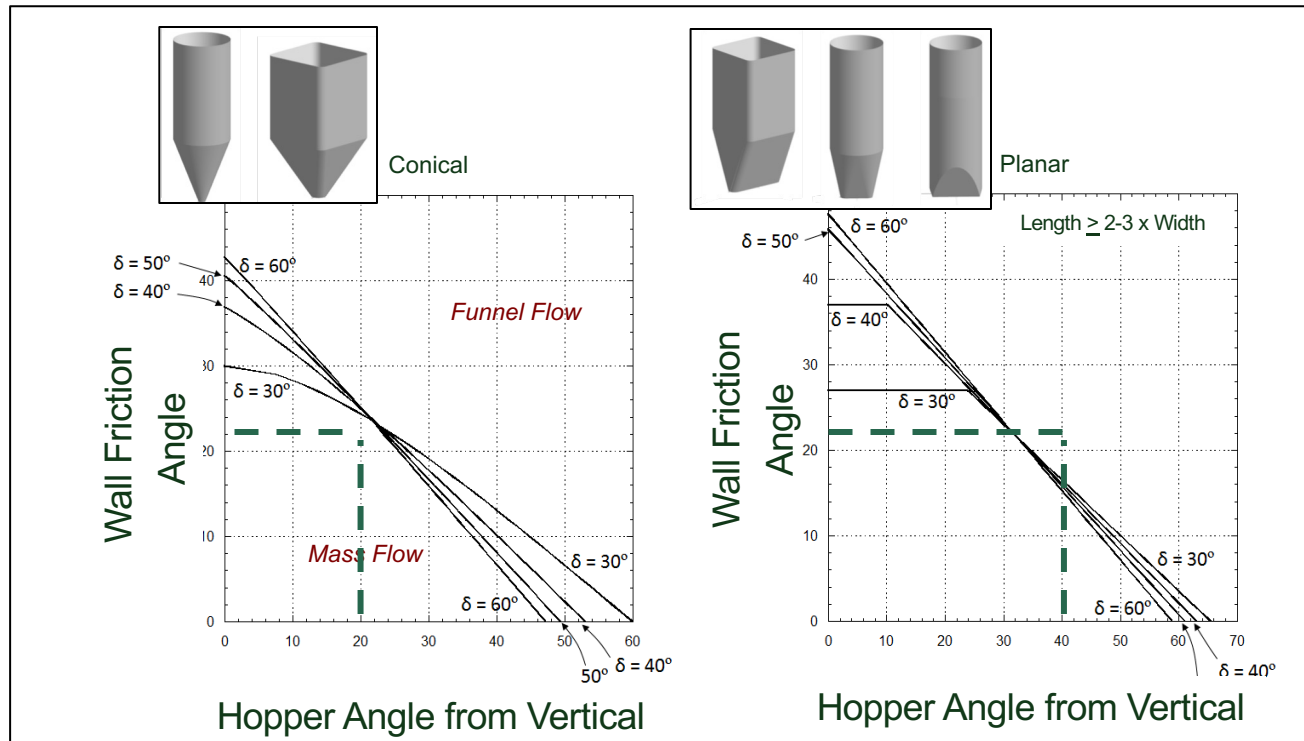
Mass flow hopper angle



Mass flow hopper angle



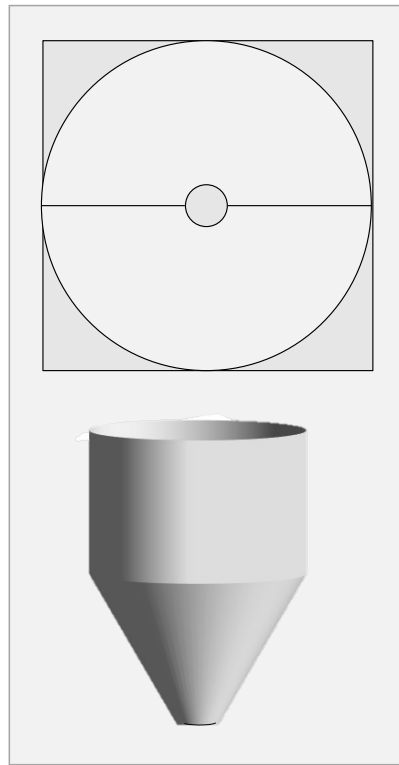
Mass flow hopper angles



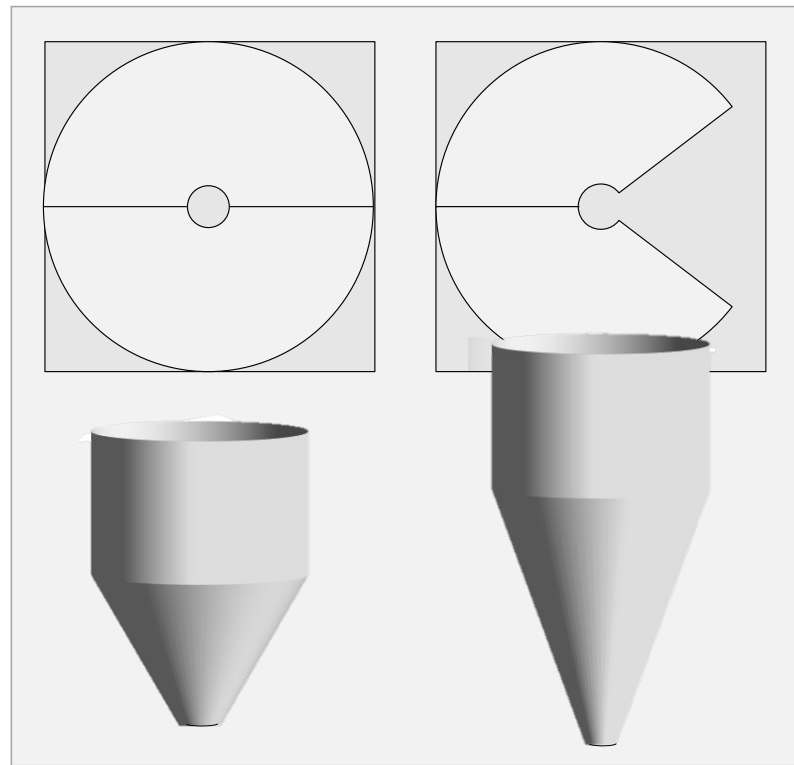
From Jenike Bulletin 123



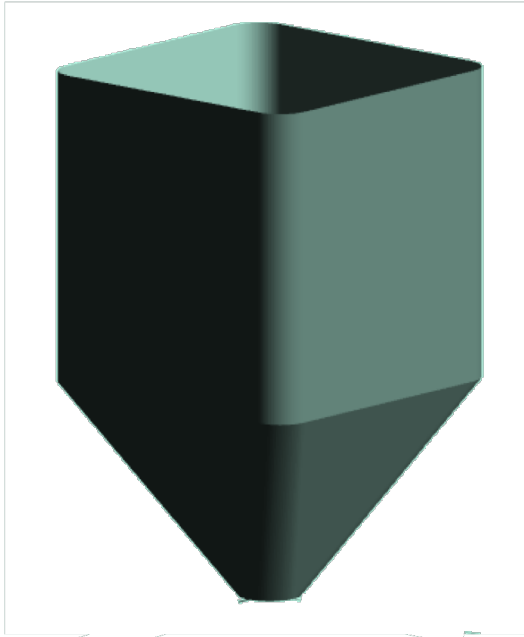
Mass flow hopper angle



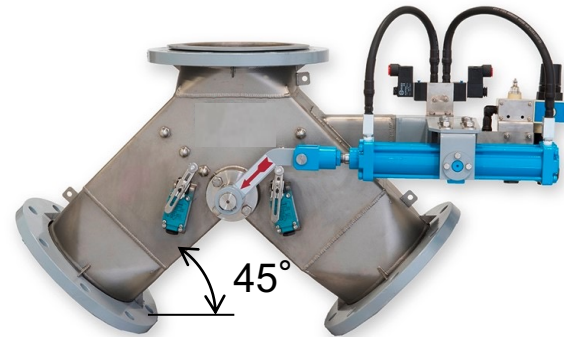
Mass flow hopper angle



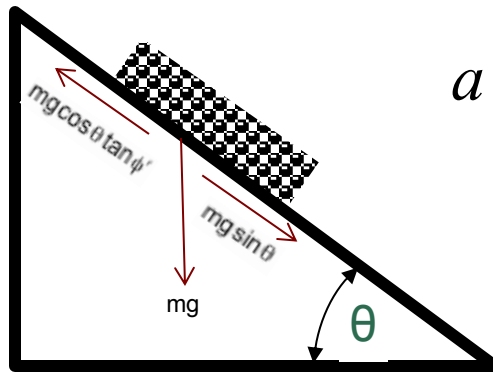
Valley angles



Transfer chutes and diverter valves



Transfer chutes

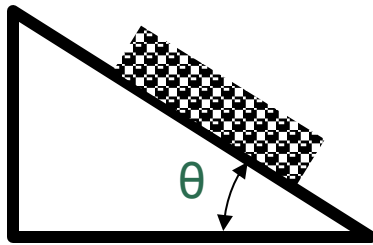


$$\begin{aligned} a &= g(\sin \theta - \cos \theta \tan \phi') \\ &= g \cos \theta (\tan \theta - \tan \phi') \end{aligned}$$

For dependable flow, $\theta' = \phi' + 5^\circ$

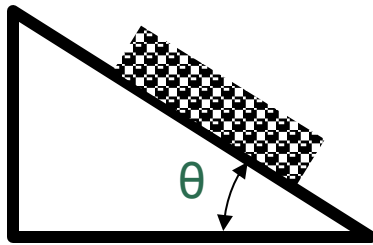


Transfer chutes



$$a = g \cos \theta (\tan \theta - \tan \phi')$$

Transfer chutes

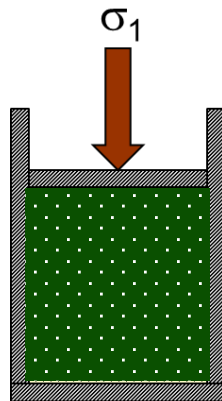


$$a = g \cos \theta (\tan \theta - \tan \phi')$$

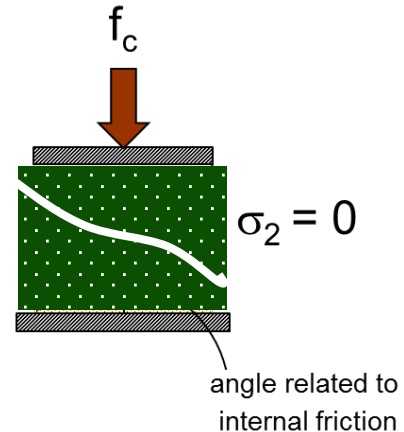
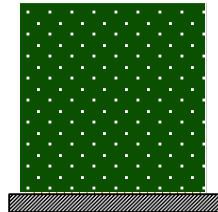
Cohesive strength



Cohesive strength

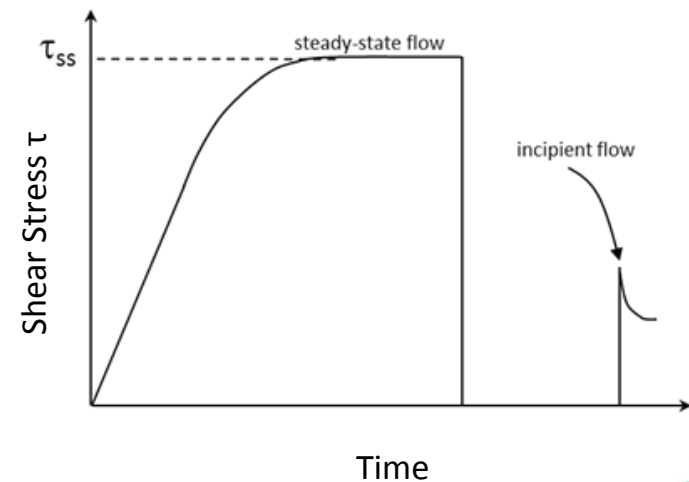
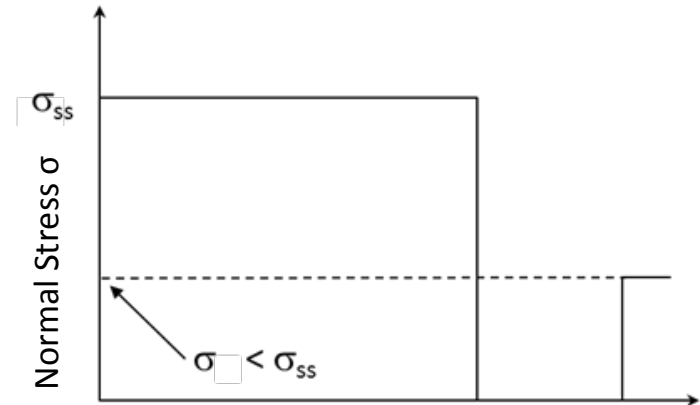
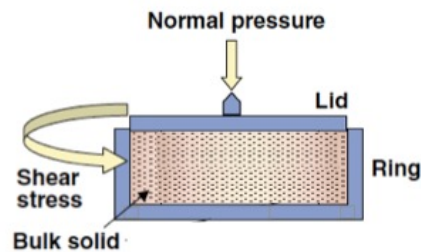
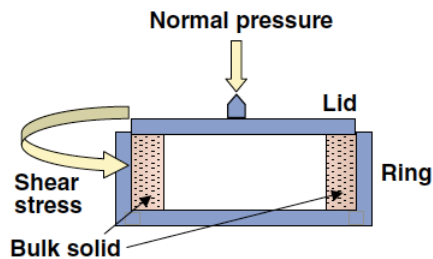
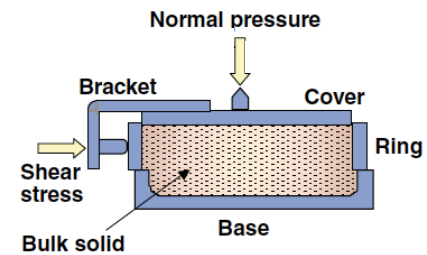


Consolidation

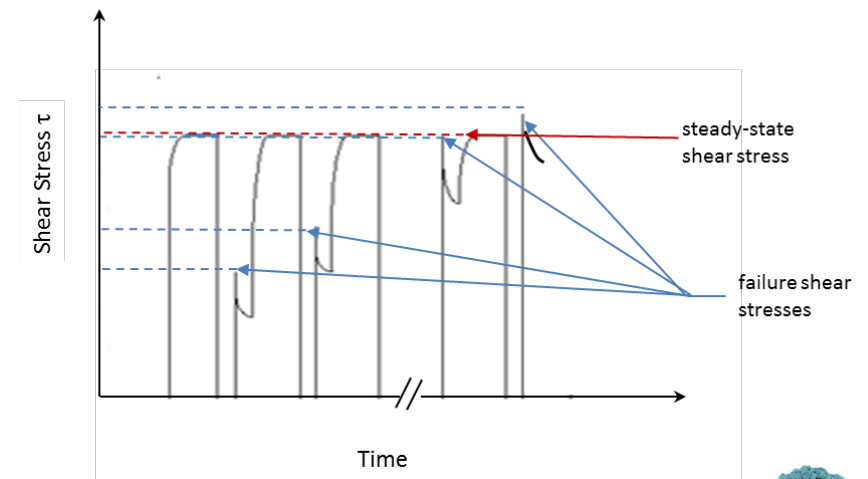
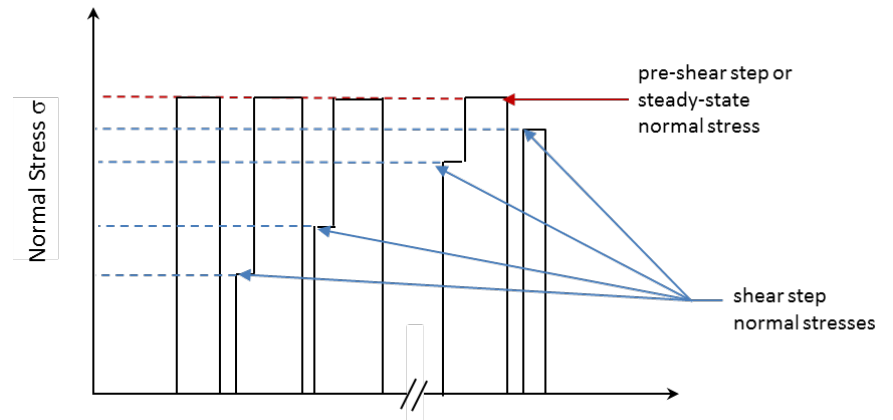
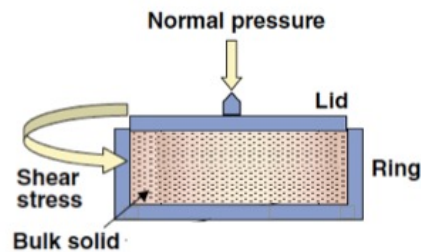
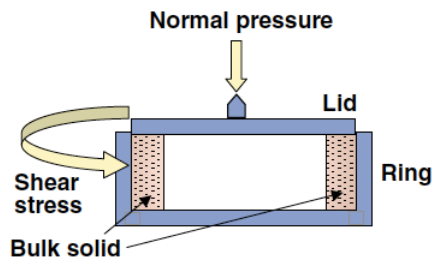
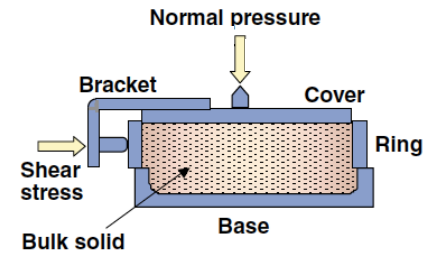


Failure

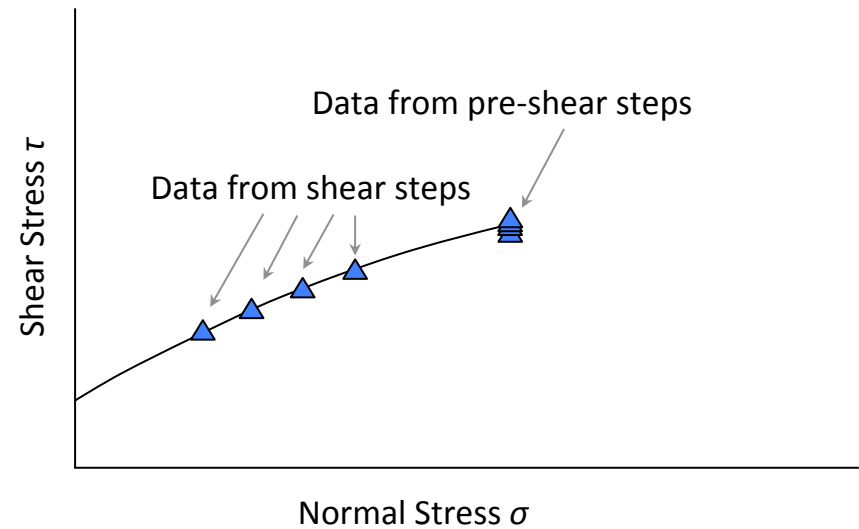
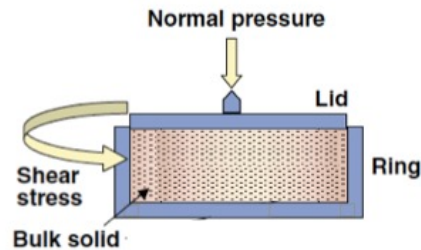
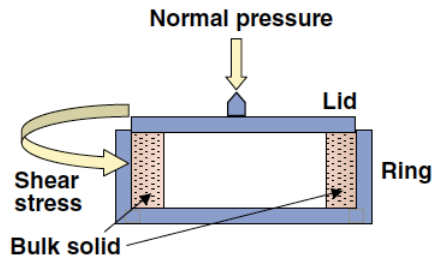
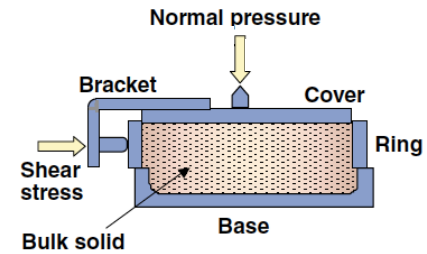
Cohesive strength



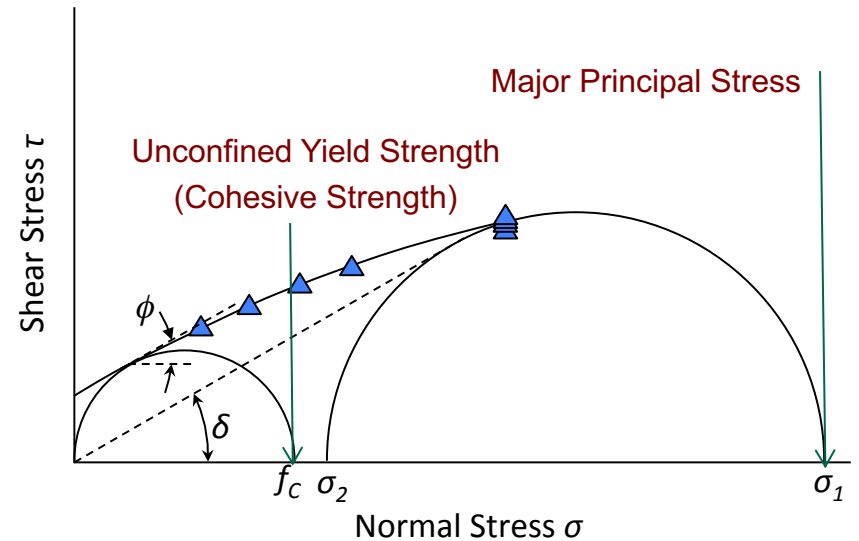
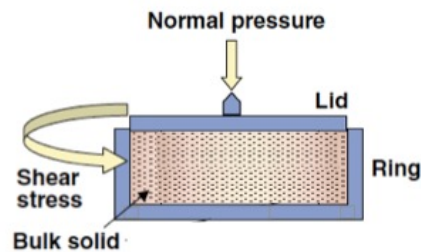
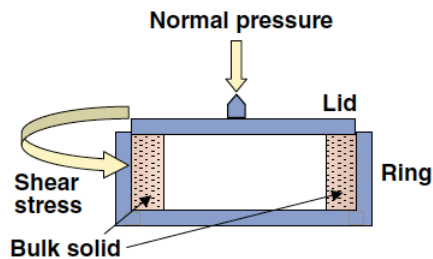
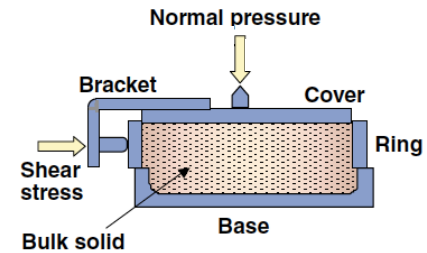
Cohesive strength



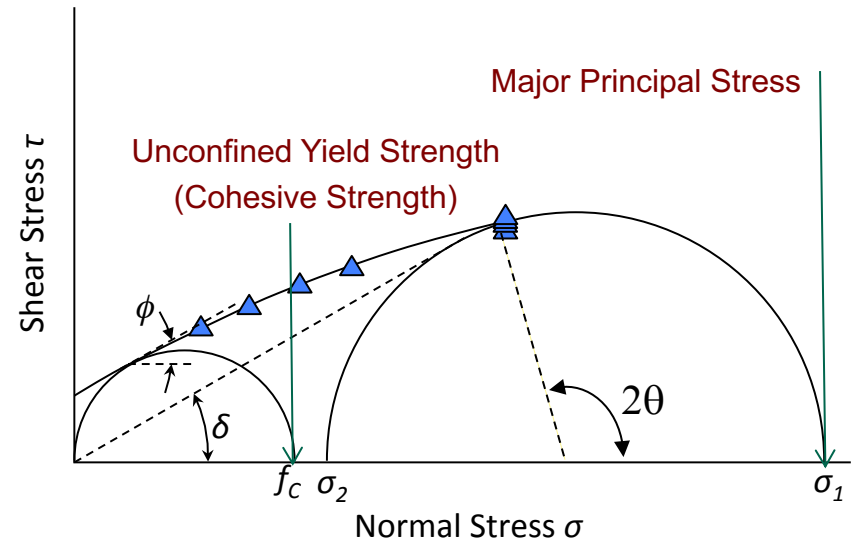
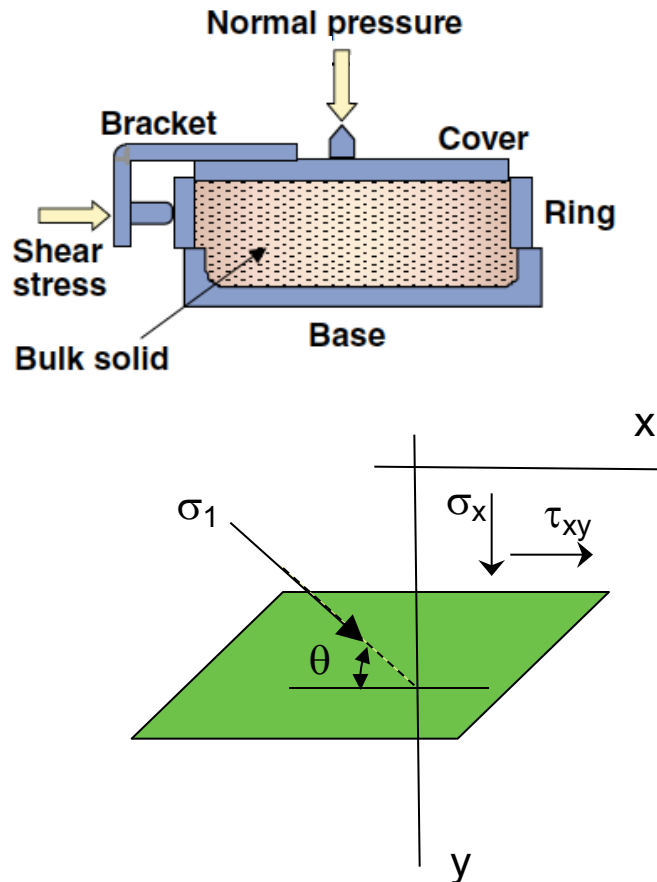
Yield locus



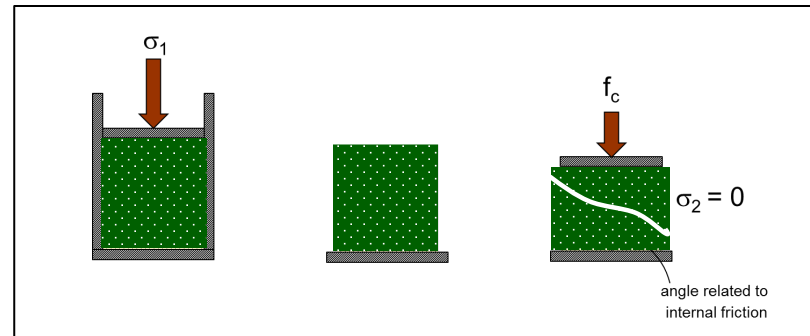
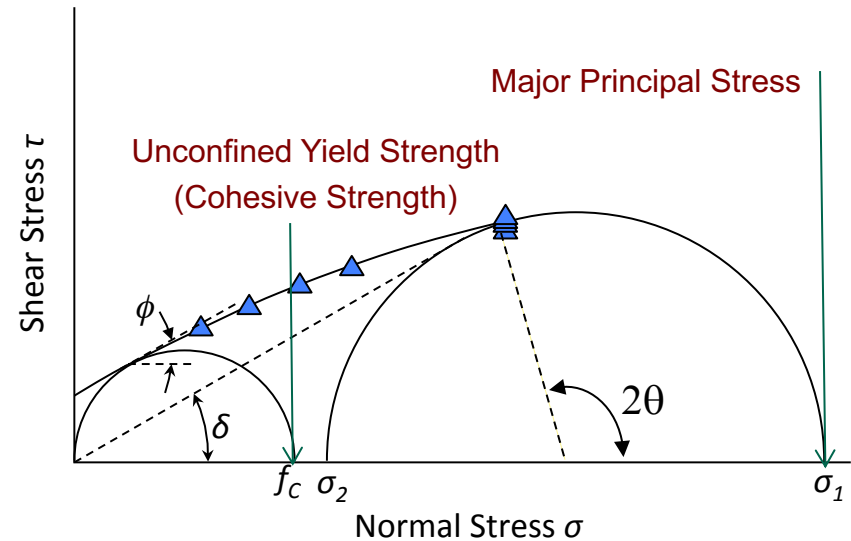
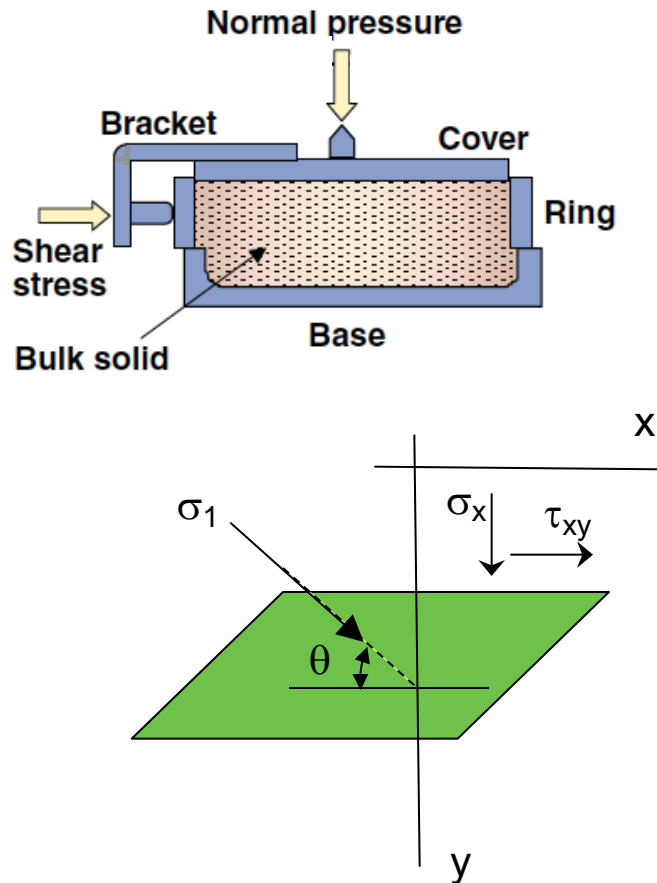
Yield locus



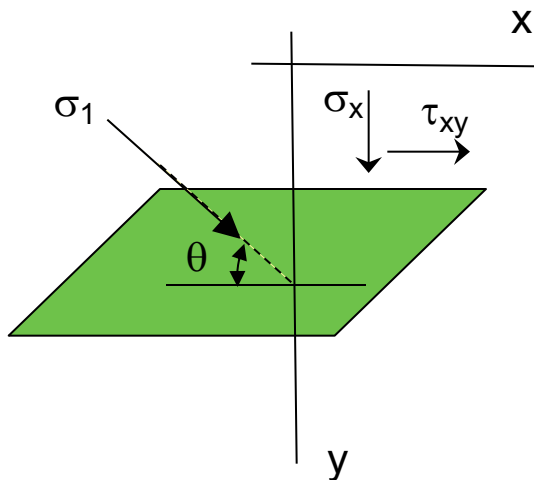
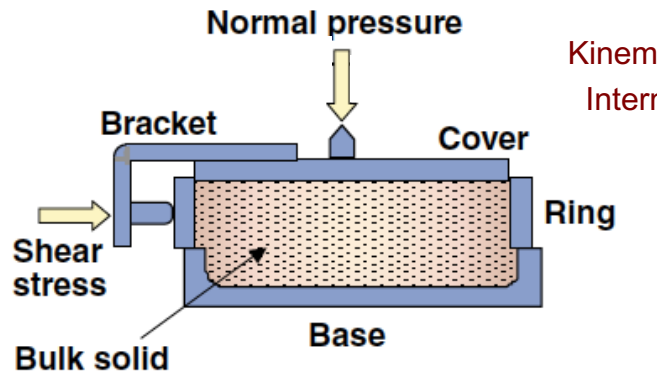
Yield locus



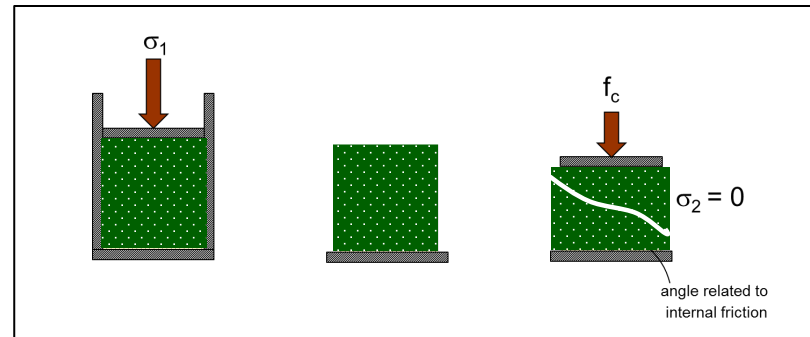
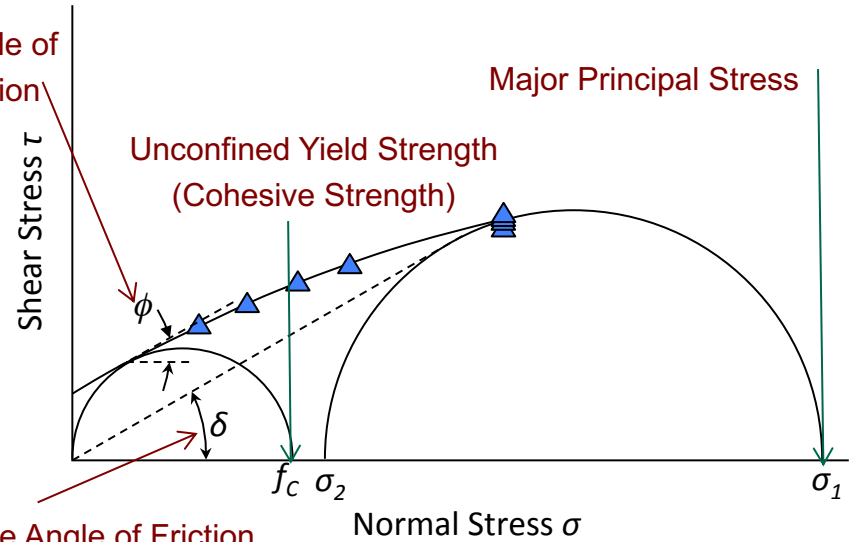
Yield locus



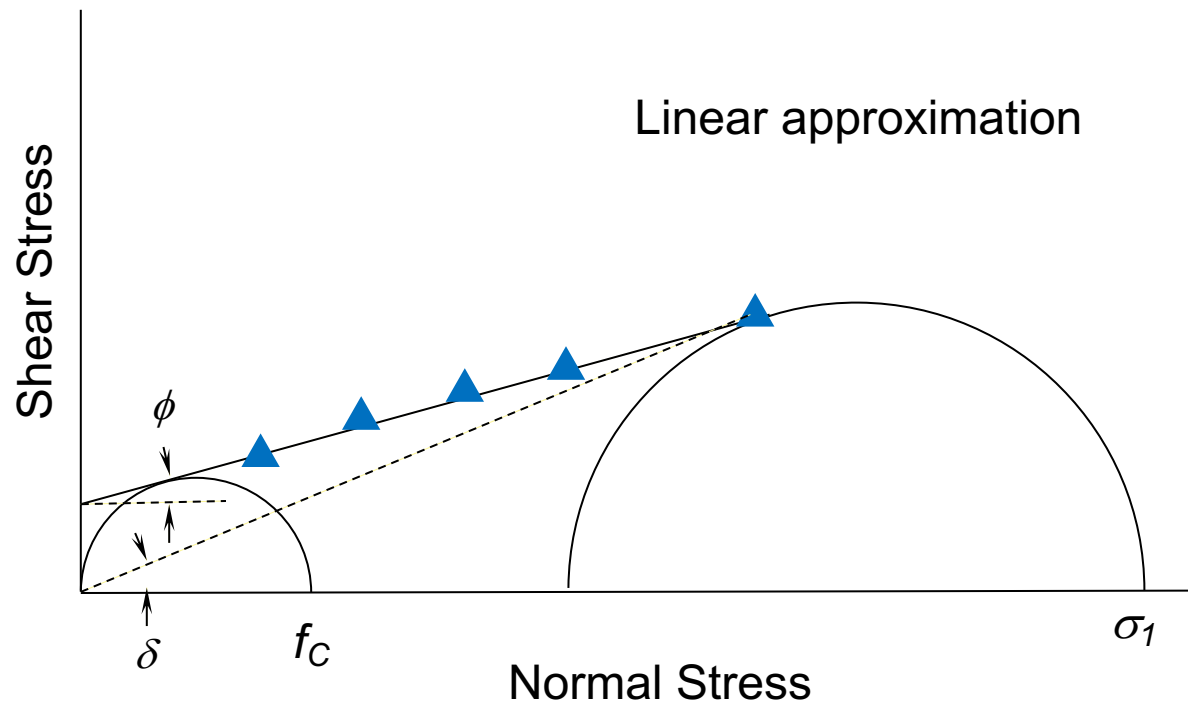
Yield locus



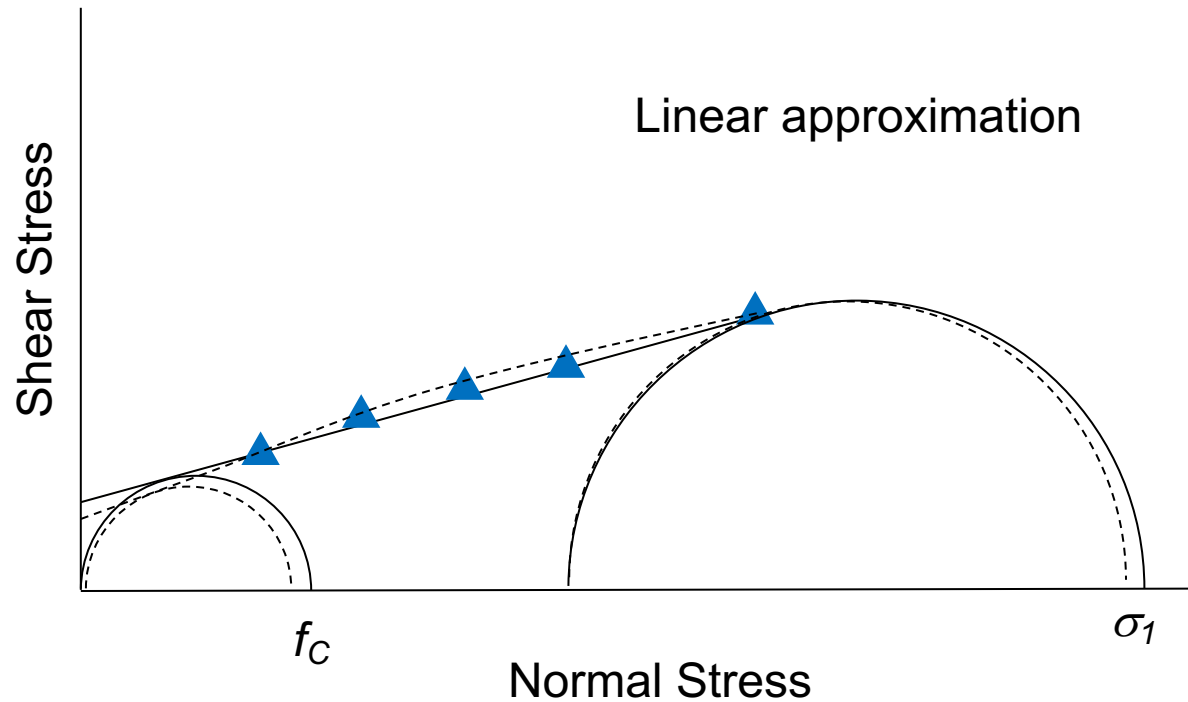
Kinematic Angle of Internal Friction



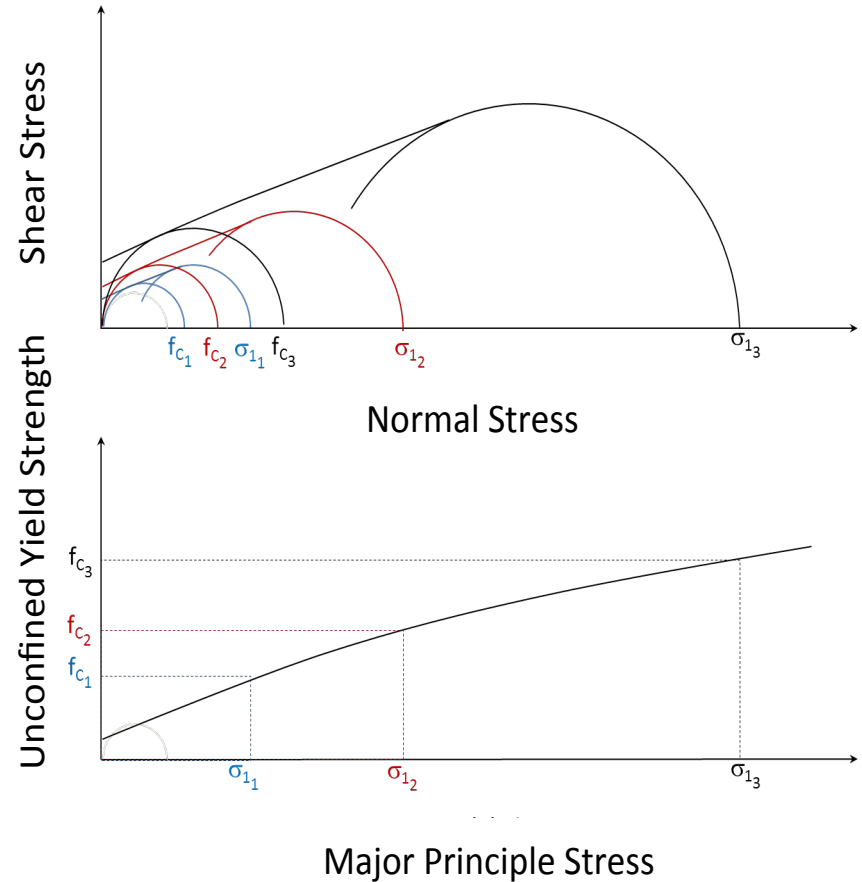
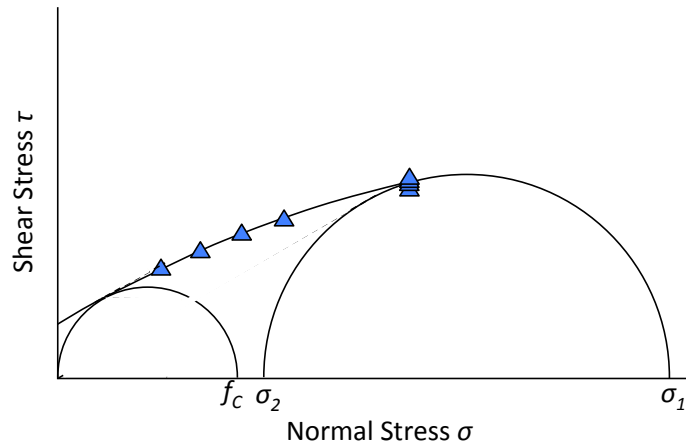
Yield locus



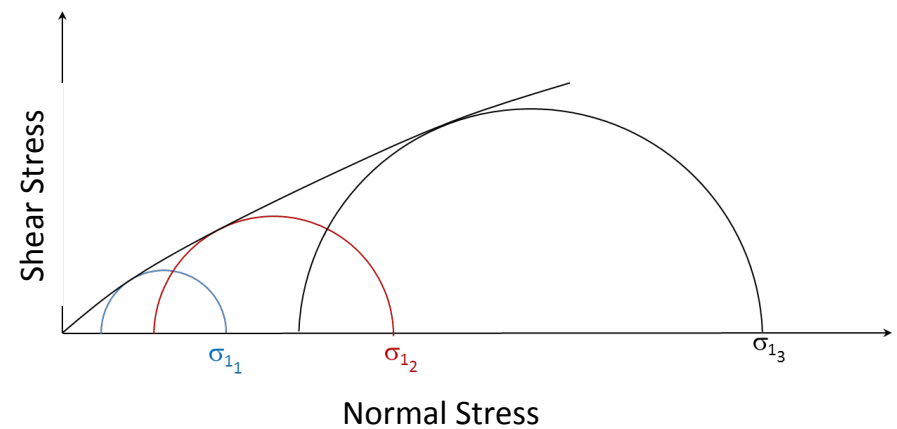
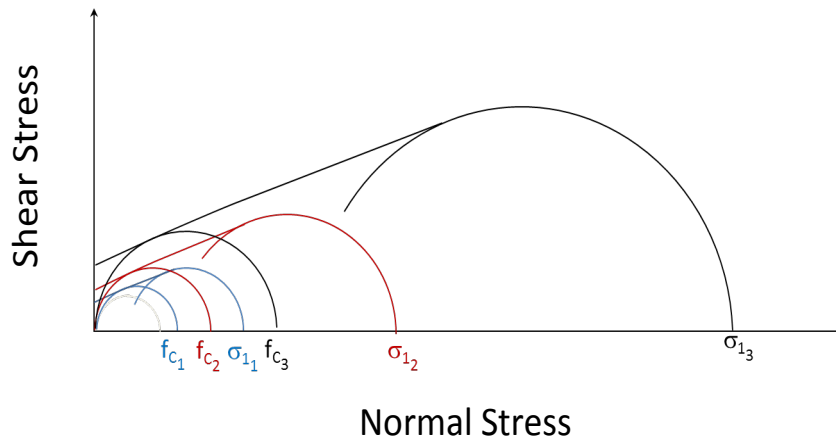
Yield locus



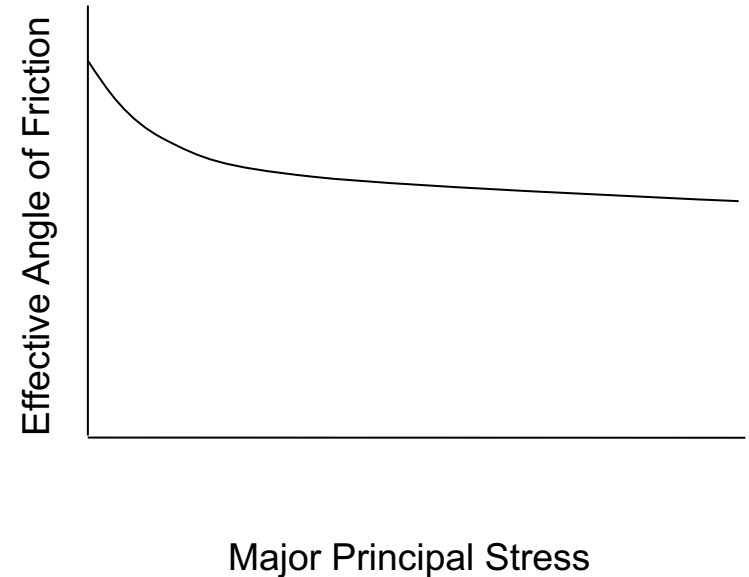
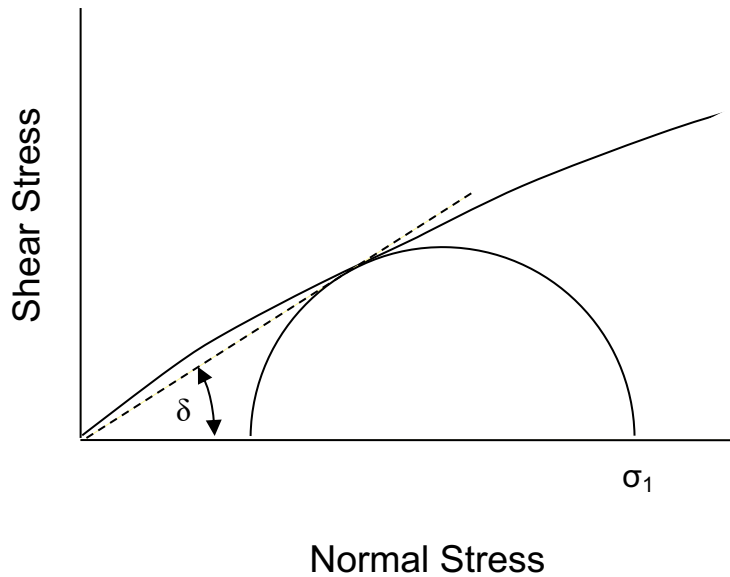
Flow function



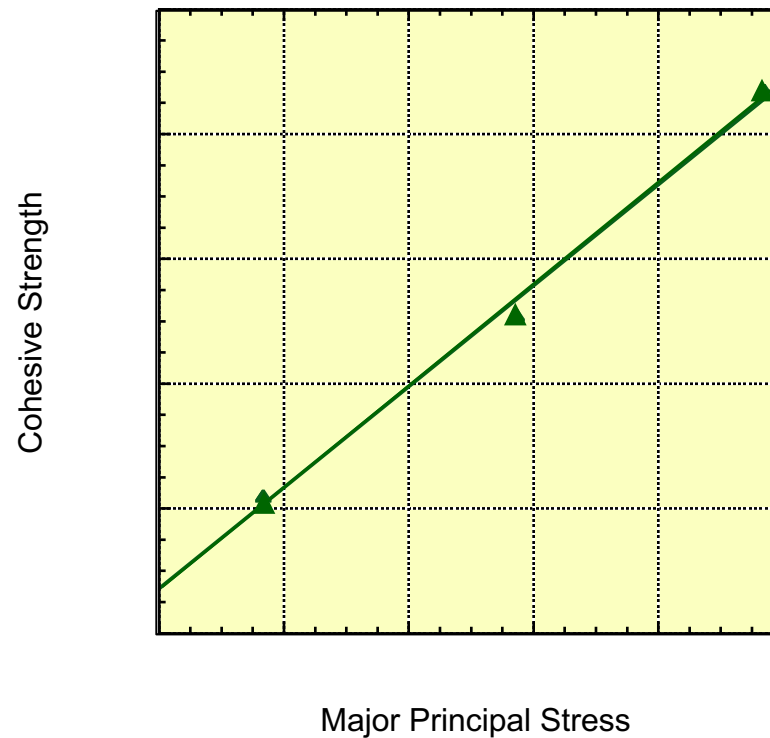
Effective yield locus



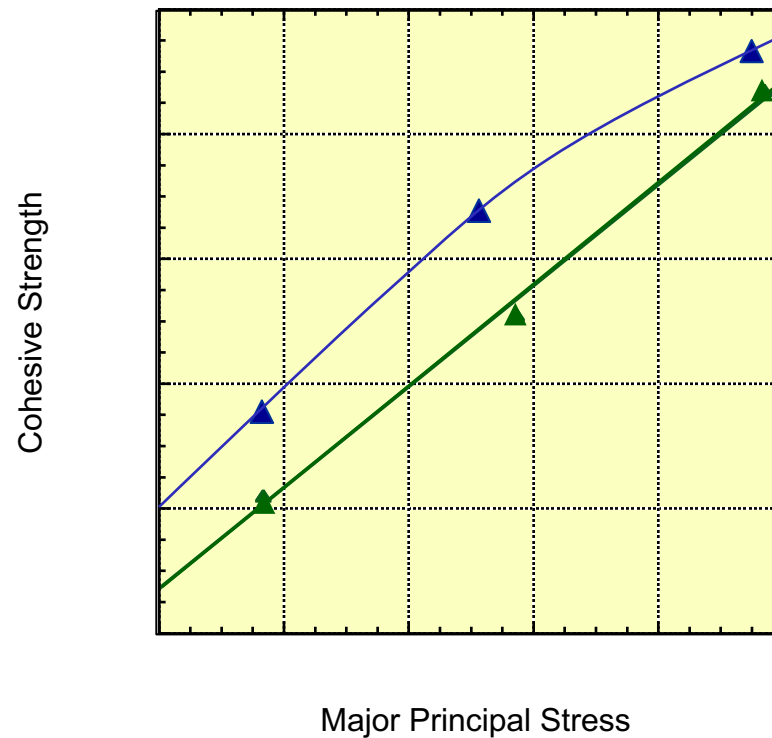
Effective yield locus



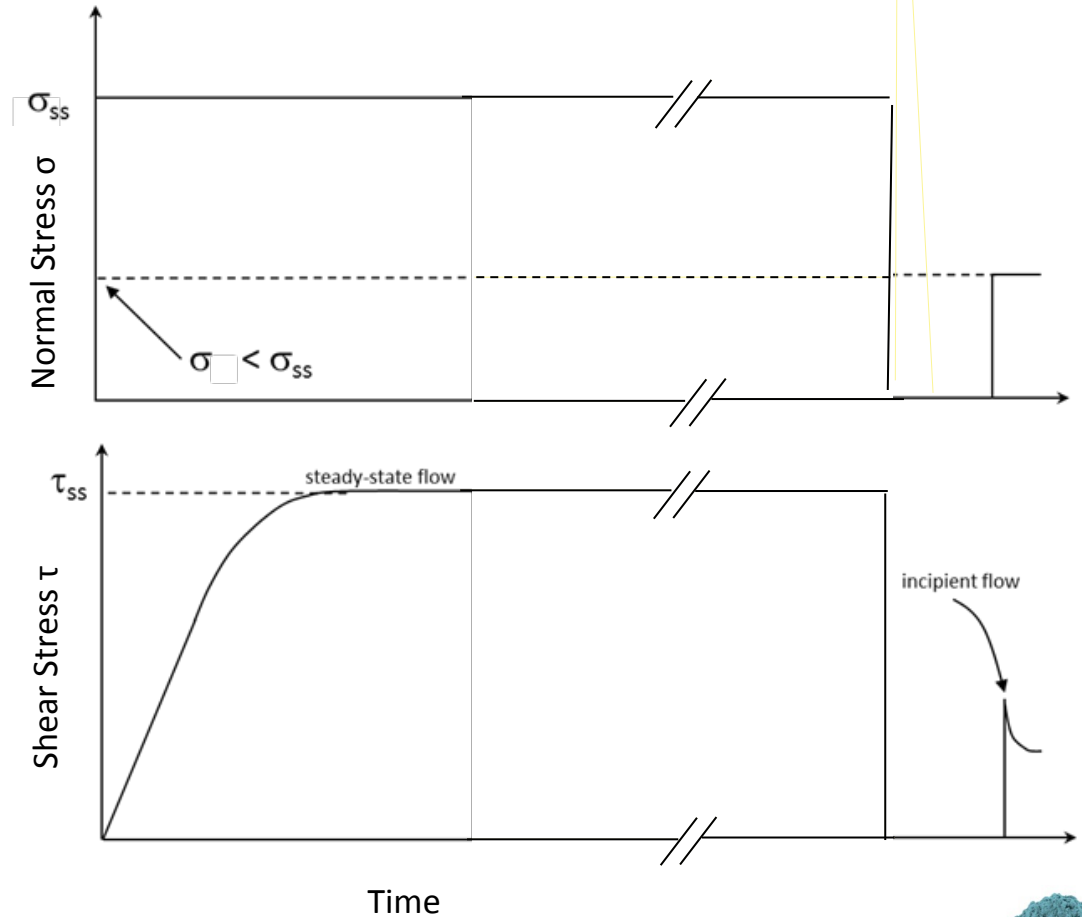
Flow function



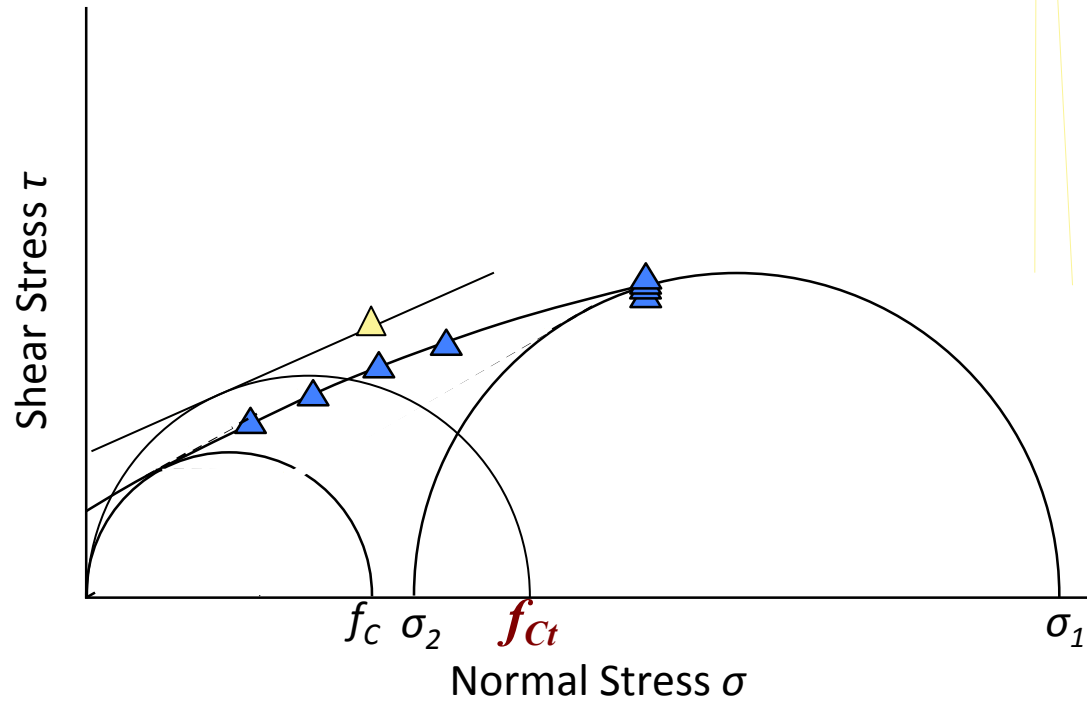
Flow function



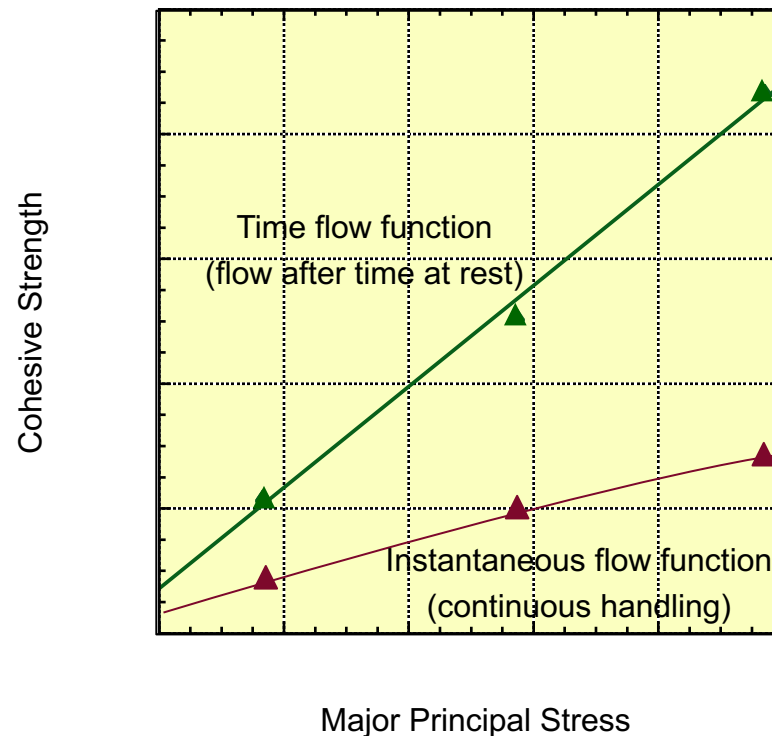
Time tests



Time tests



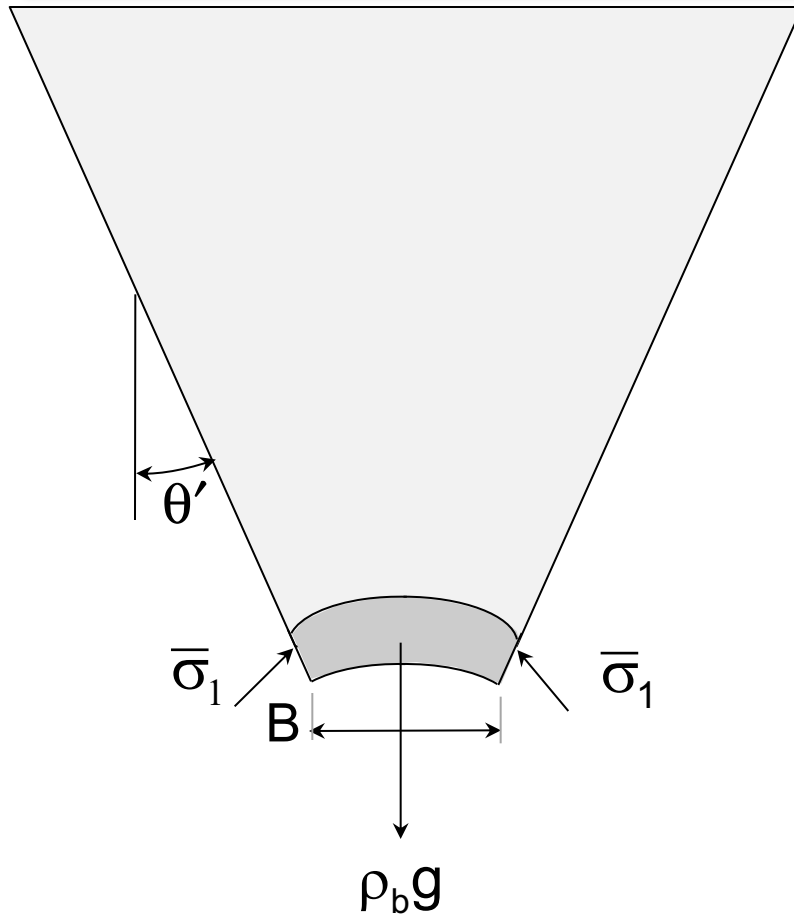
Time flow function



Arch stress



Arch stress

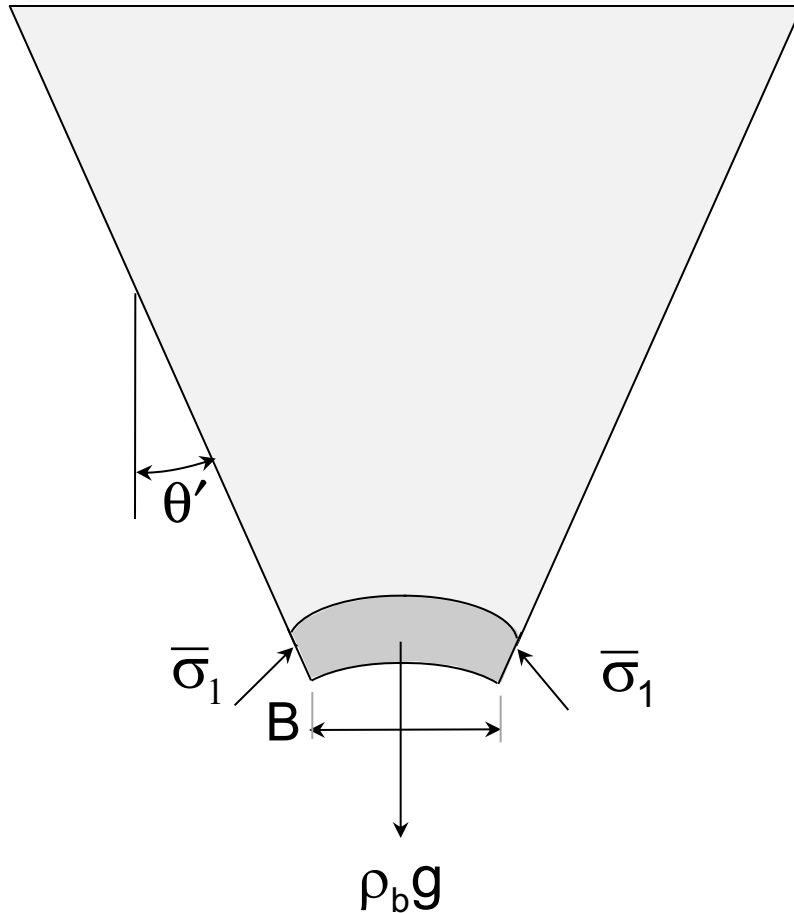


Stress on abutments of arch:

$$\bar{\sigma}_1 \approx \frac{\rho_b g B}{2}$$



Arch stress



Stress on abutments of arch:

$$\bar{\sigma}_1 = \frac{\rho_b g B}{H(\theta')}$$

Conical:

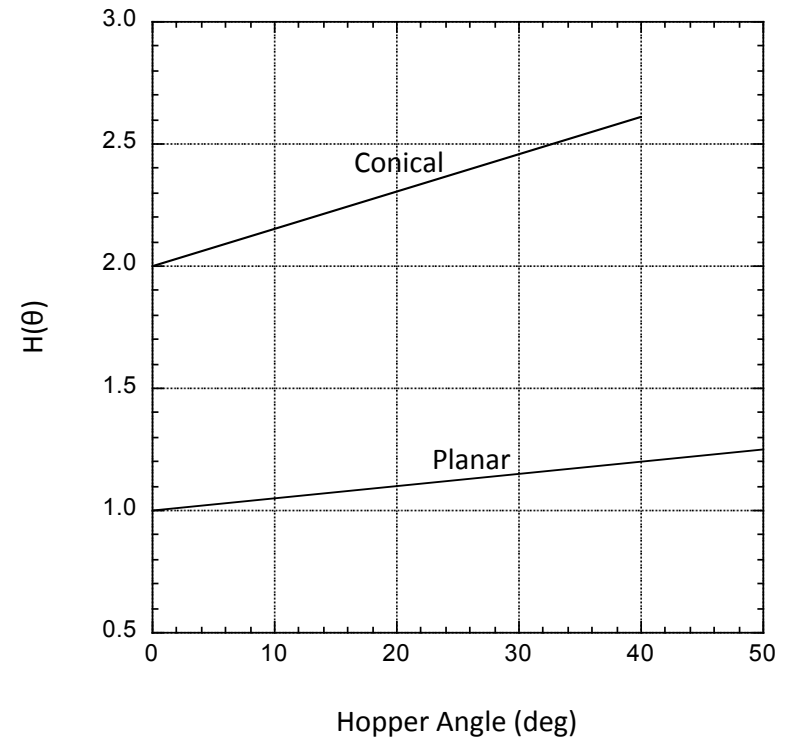
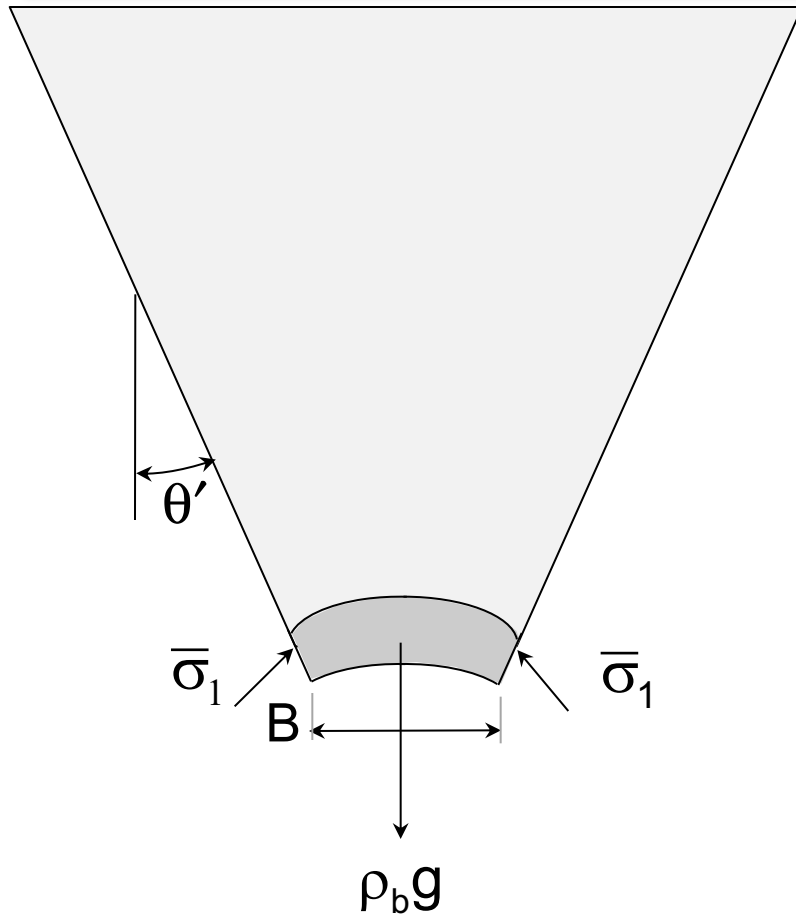
$$H(\theta') \approx 2$$

Slot:

$$H(\theta') \approx 1$$



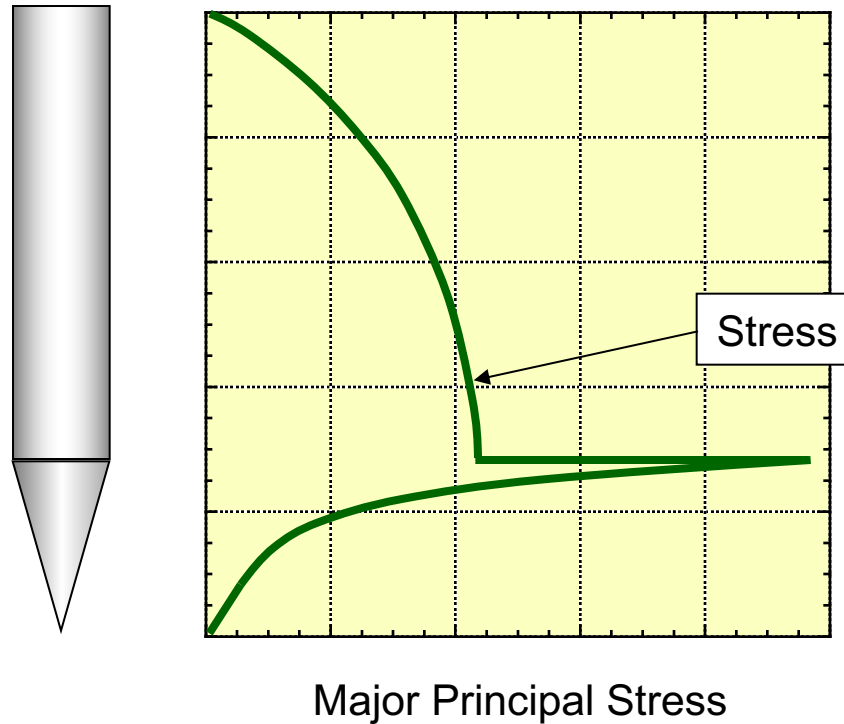
Arch stress



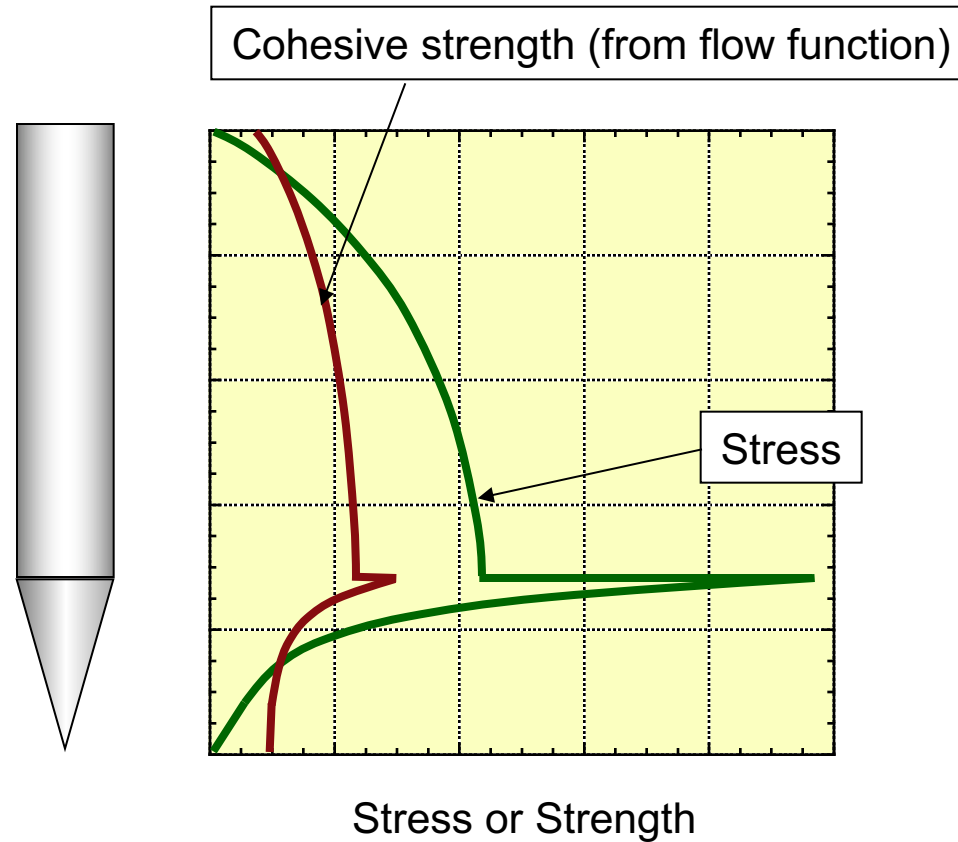
$$\bar{\sigma}_1 = \frac{\rho_b g B}{H(\theta')}$$



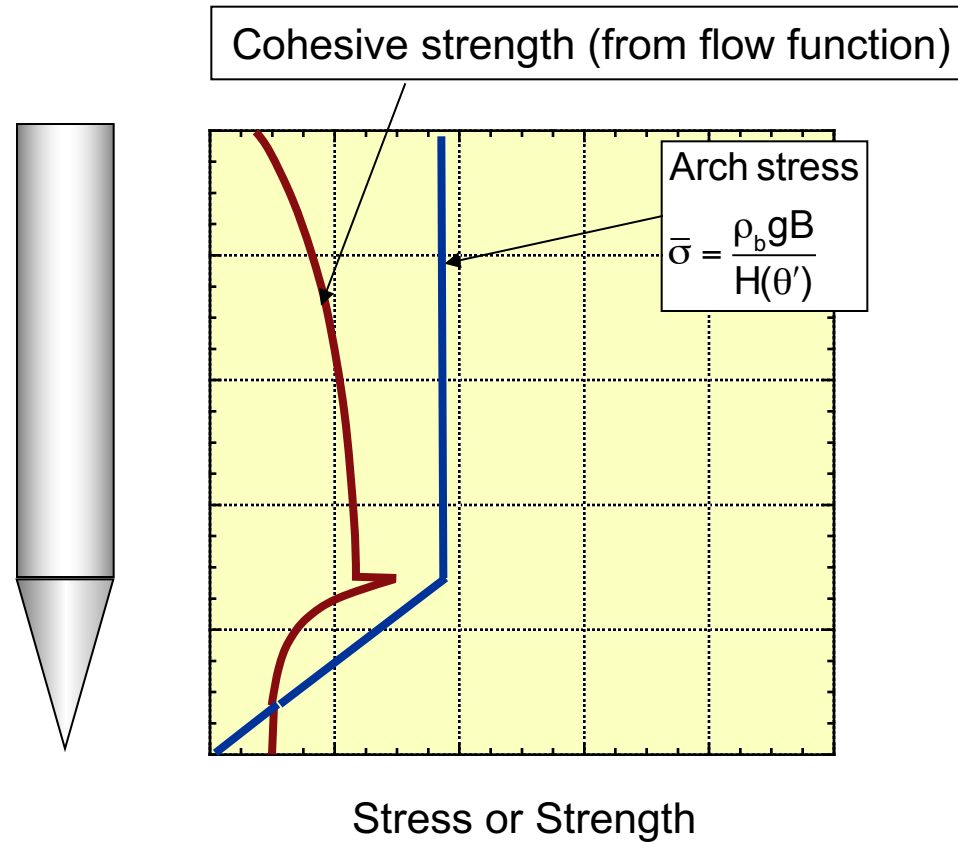
Pressure distribution



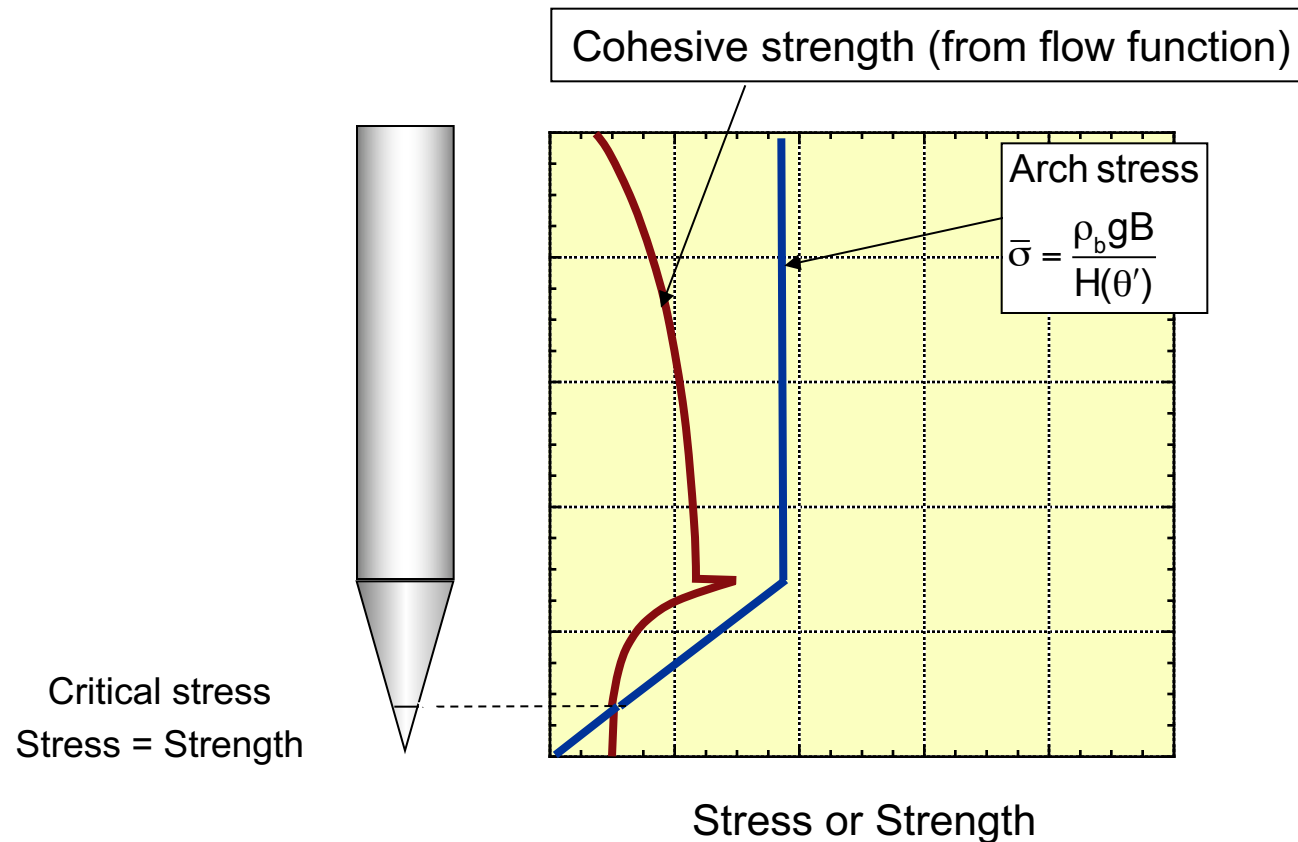
Cohesive strength



Arch stress



Critical arching diameter



Critical arching diameter

$$B_{\min} = \frac{H(\theta')\sigma_{\text{Critical}}}{\rho_b g}$$

Critical stress
Stress = Strength



σ_{Critical}

Cohesive strength (from flow function)

Arch stress

$$\bar{\sigma} = \frac{\rho_b g B}{H(\theta')}$$

Stress or Strength



Flow factor

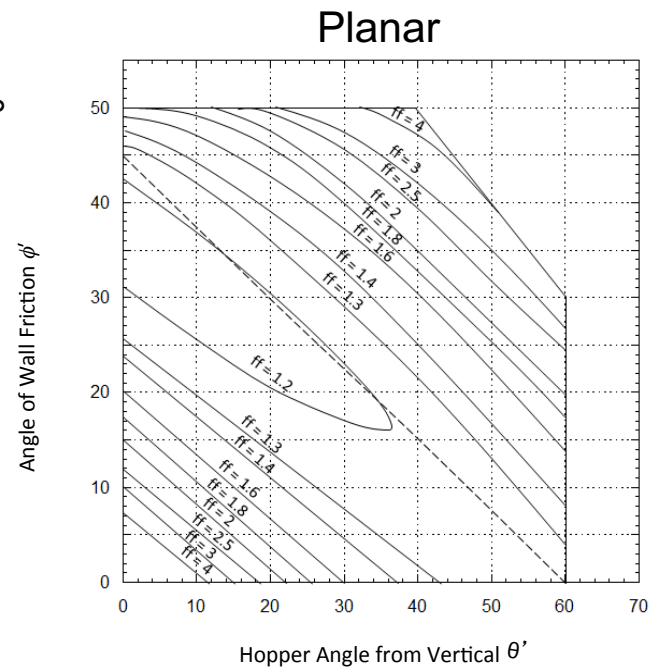
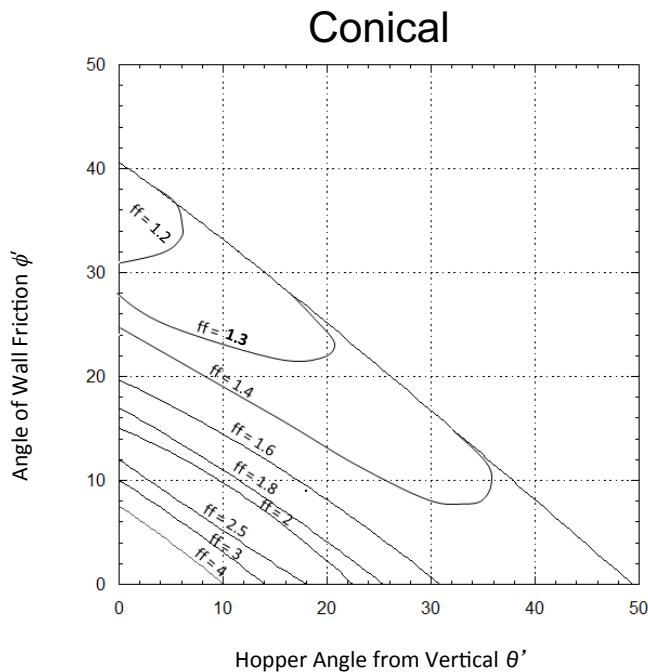
- Flow function (FF)
 - Relationship between major principal stress and cohesive strength
- Flow factor (ff)
 - Ratio of major principal stress and arch supporting stress

$$ff = \frac{\sigma_1}{\bar{\sigma}_1} = \frac{\text{major principal stress}}{\text{arch stress}}$$



Flow factor

$$ff = \frac{\sigma_1}{\bar{\sigma}_1} = \frac{\text{major principal stress}}{\text{arch stress}}$$



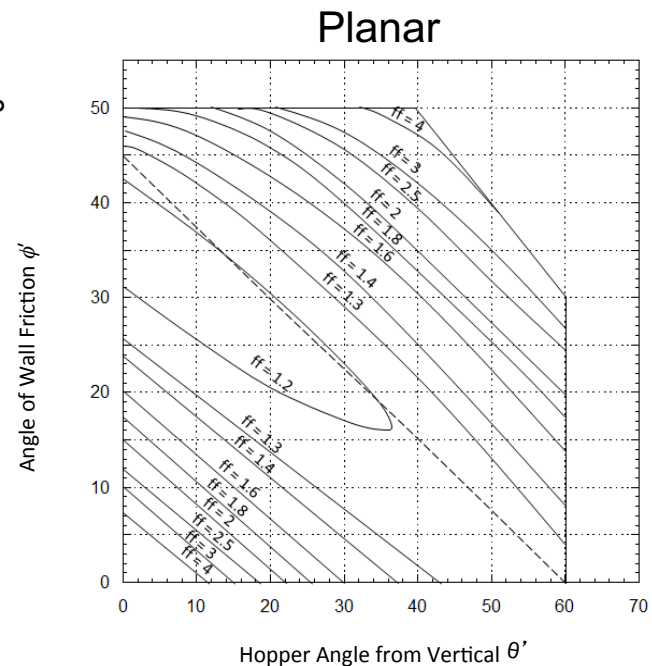
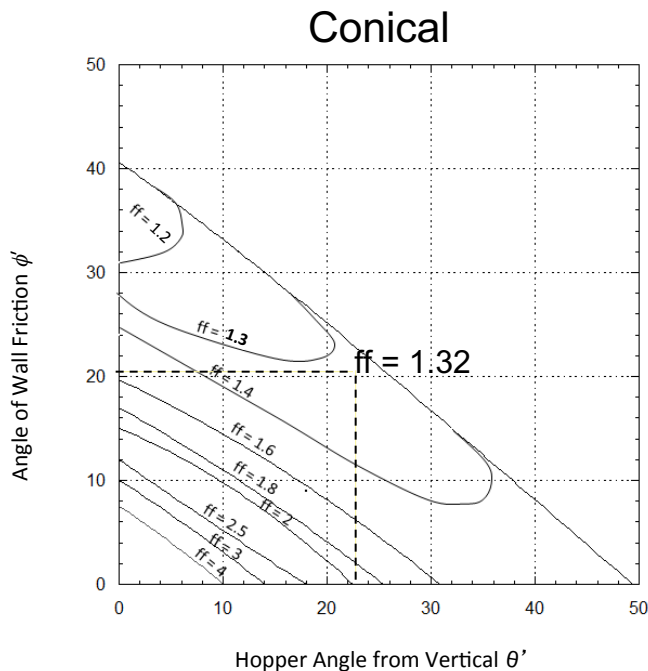
https://digital.library.unt.edu/ark:/67531/metadc1067072/m2/1/high_res_d/5240257.pdf

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Flow factor

$$ff = \frac{\sigma_1}{\bar{\sigma}_1} = \frac{\text{major principle stress}}{\text{arch stress}}$$

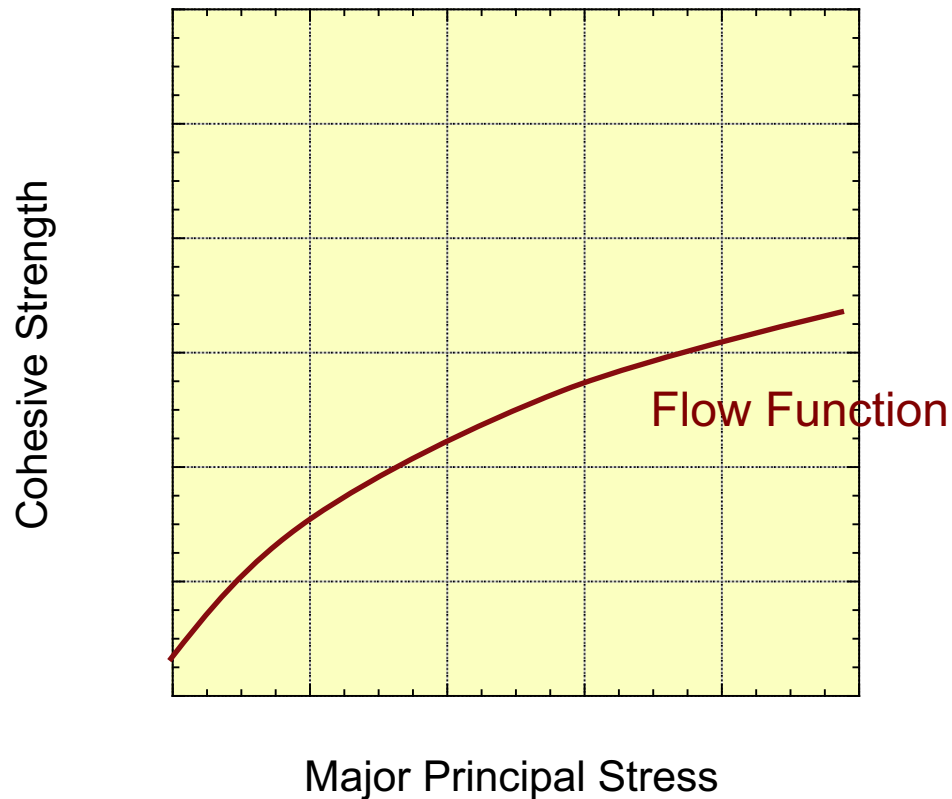


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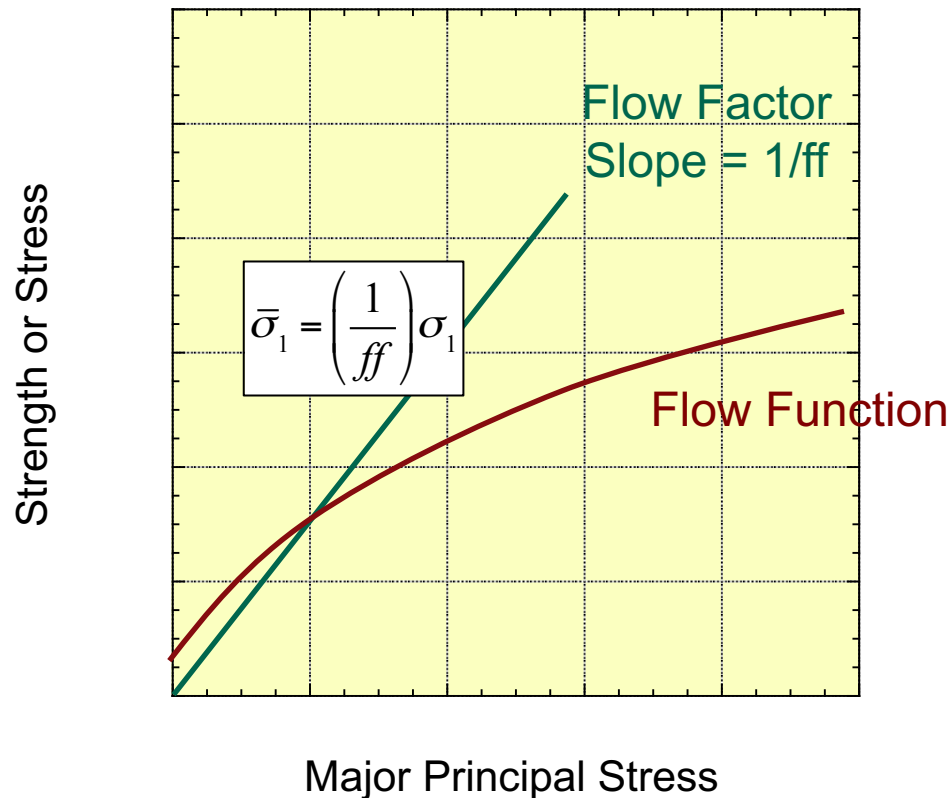
From Jenike Bulletin 123



Flow function



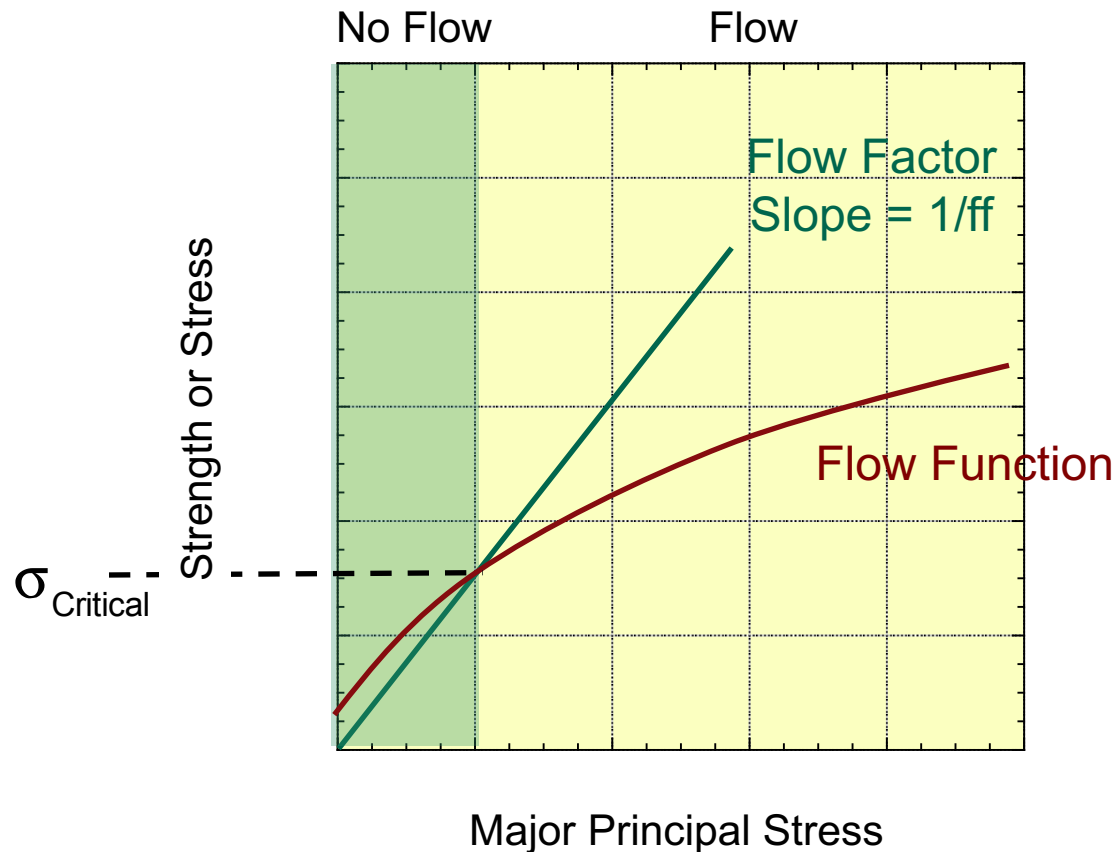
Flow factor



$$ff = \frac{\sigma_1}{\bar{\sigma}_1}$$
$$\bar{\sigma}_1 = \left(\frac{1}{ff} \right) \sigma_1$$



Critical stress



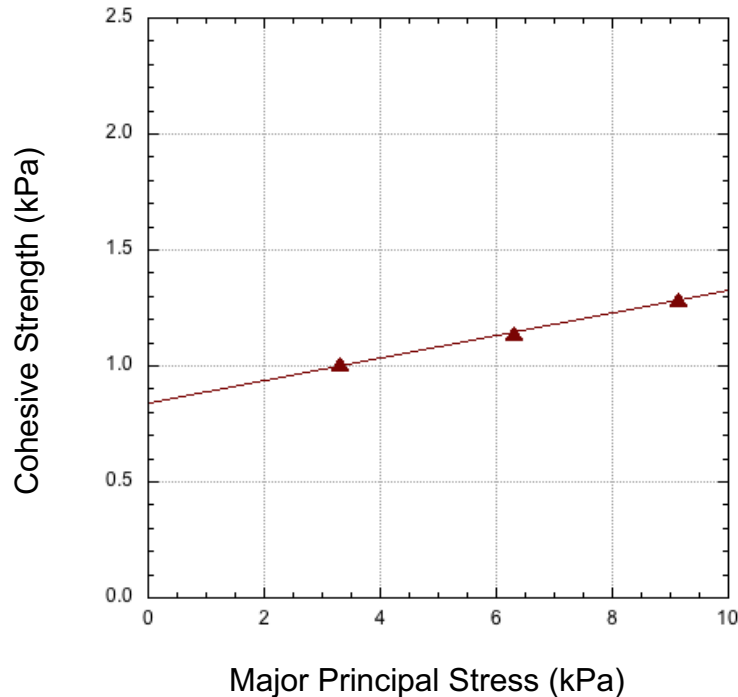
$$B_{min} = \frac{H(\theta')\sigma_{Critical}}{\rho_b g}$$



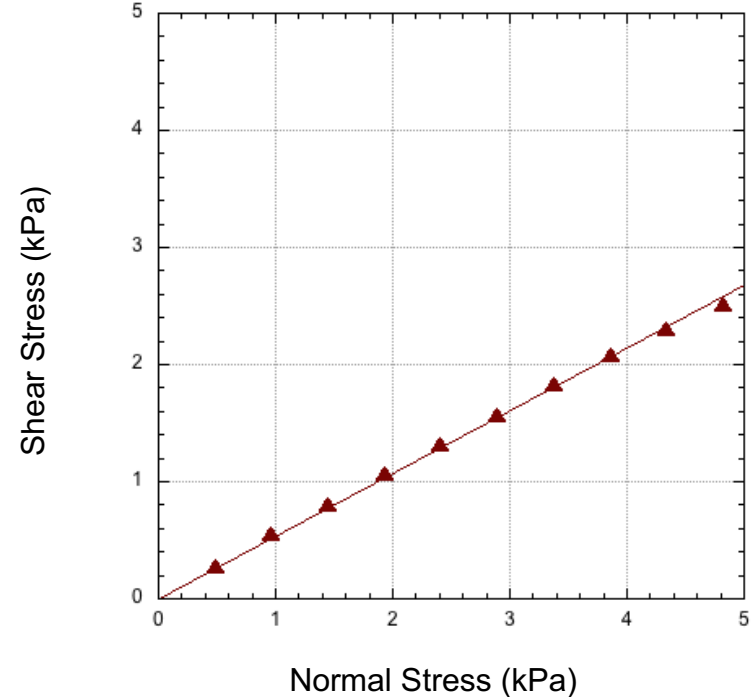
Example

Design a hopper that reliably handles 67/29/4 blend of acetaminophen/Avicel/hydroxypropyl cellulose

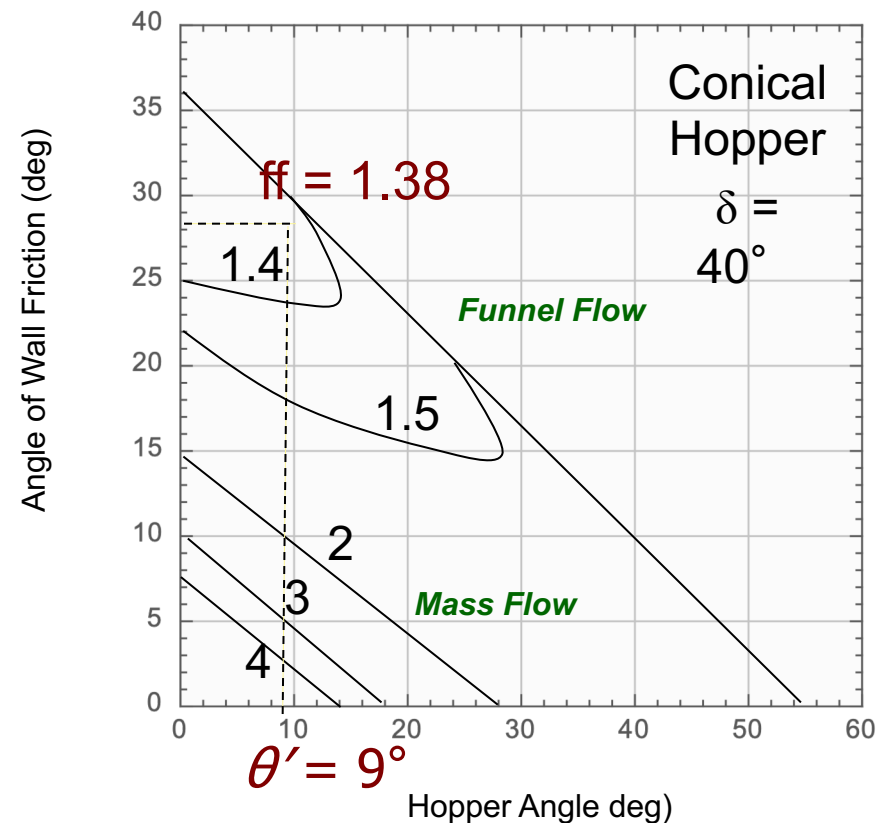
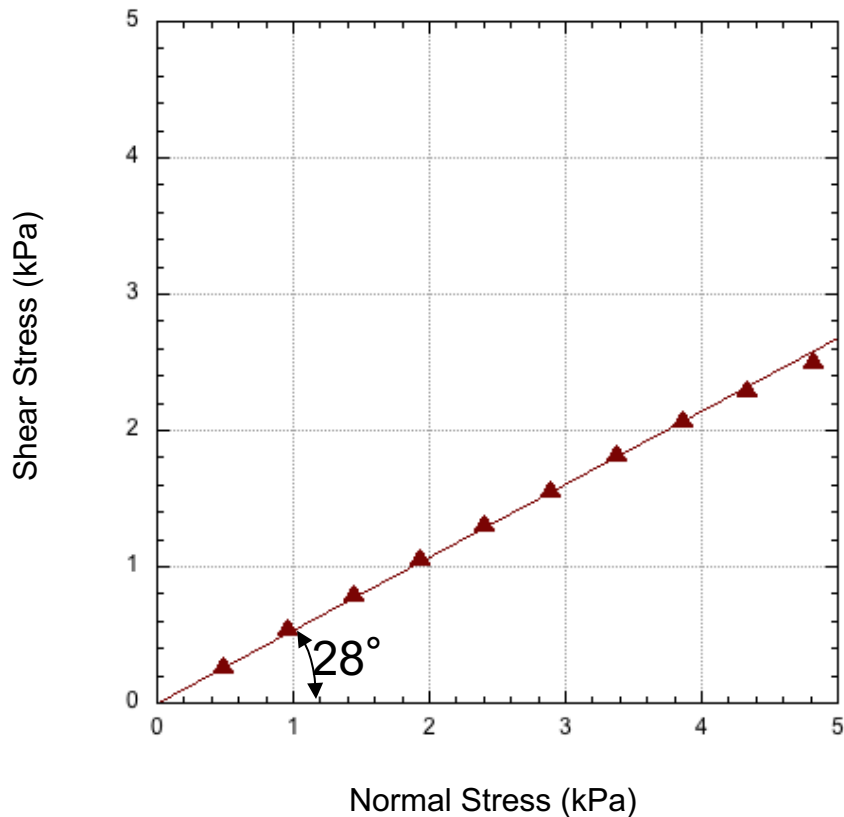
Flow Function



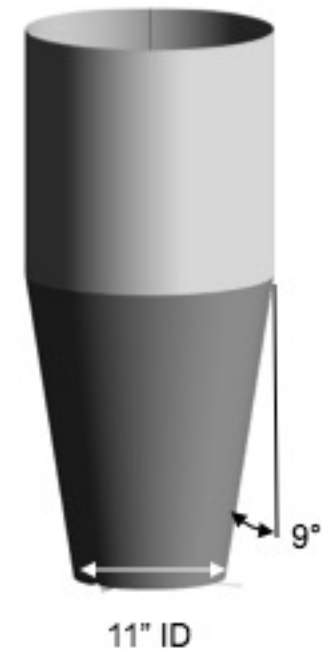
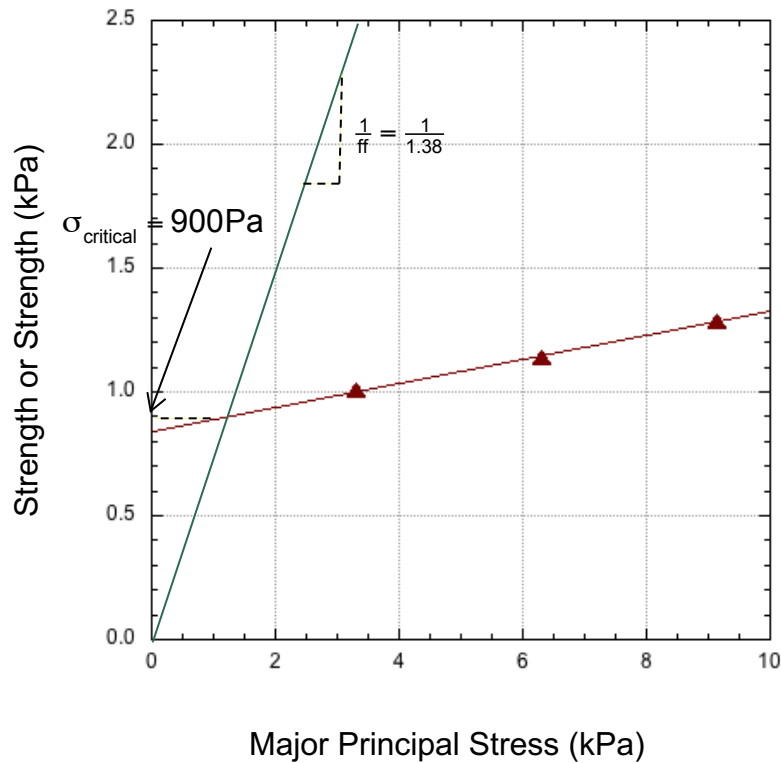
Wall Friction



Example



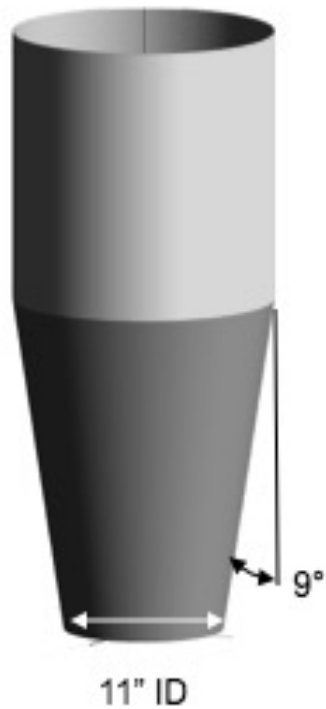
Example



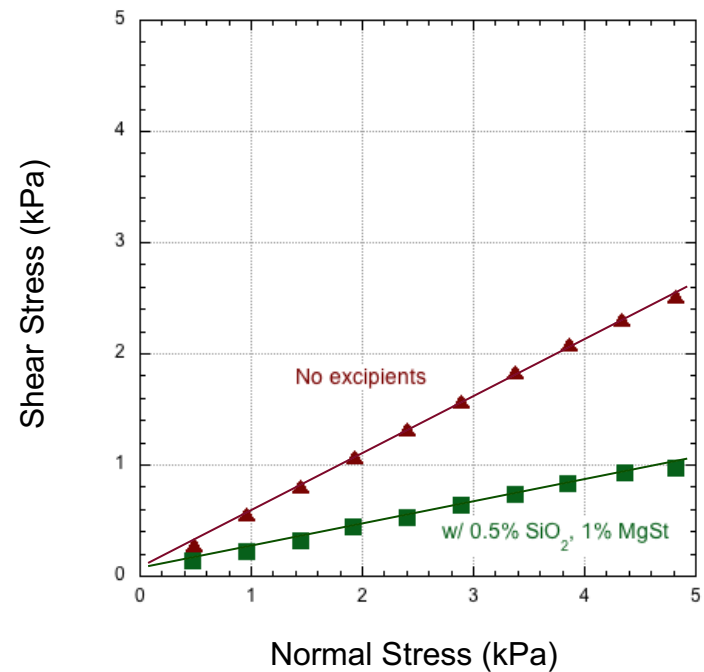
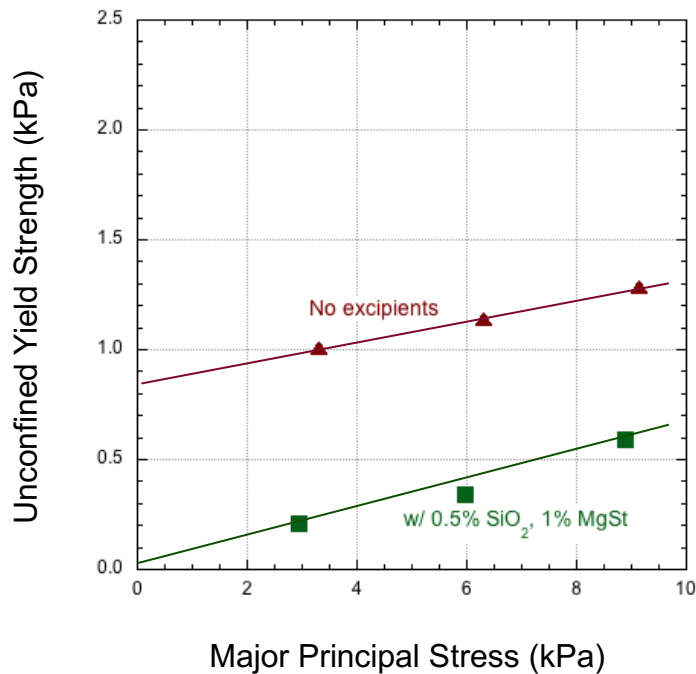
$$B_{min} = \frac{H(\theta')\sigma_{Critical}}{\rho_b g} = \frac{(2.14)(900)}{(690)(9.8)} = 0.28 \text{ m} = 11 \text{ in.}$$



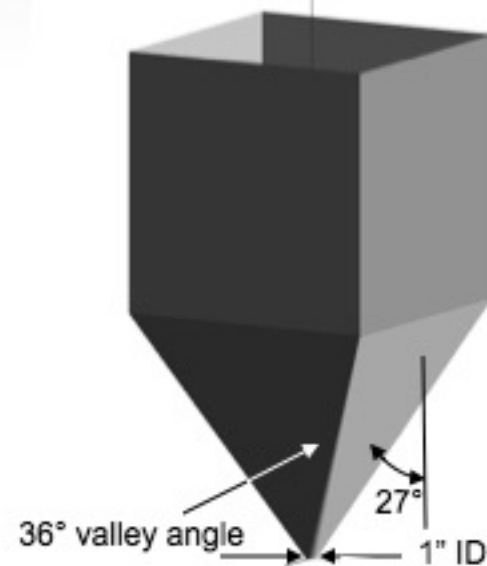
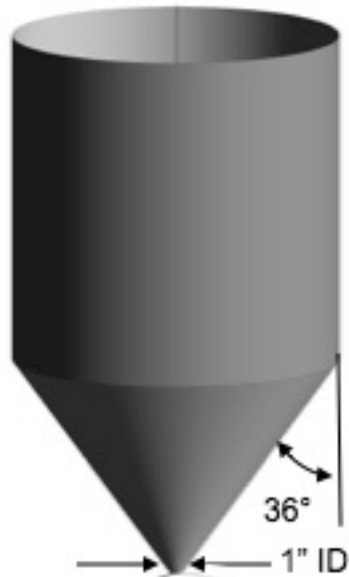
Example



Example



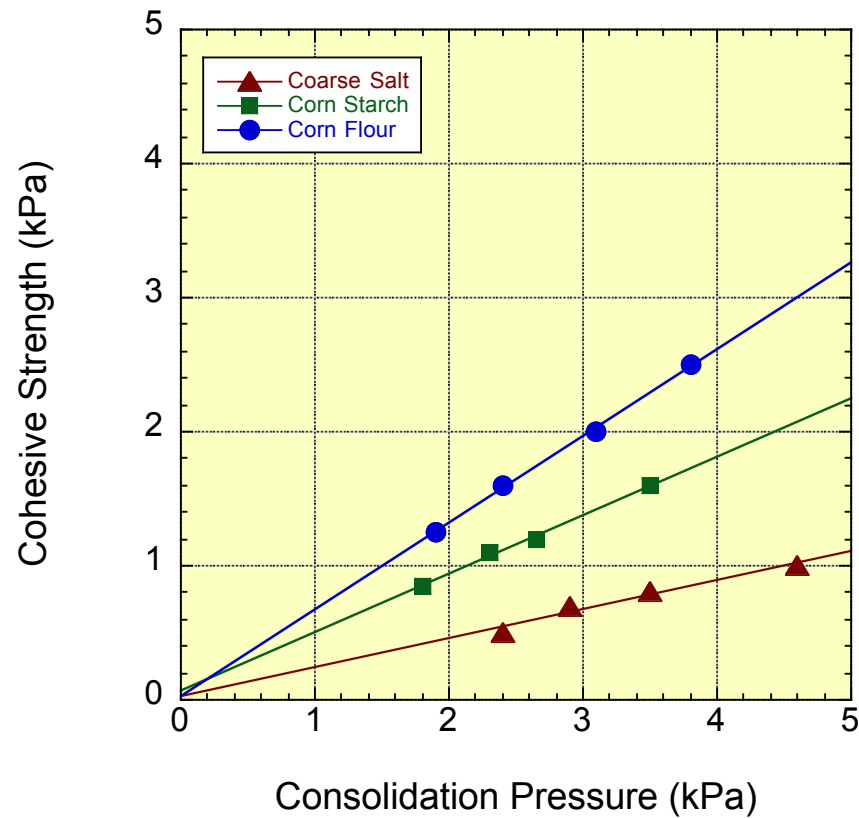
Example



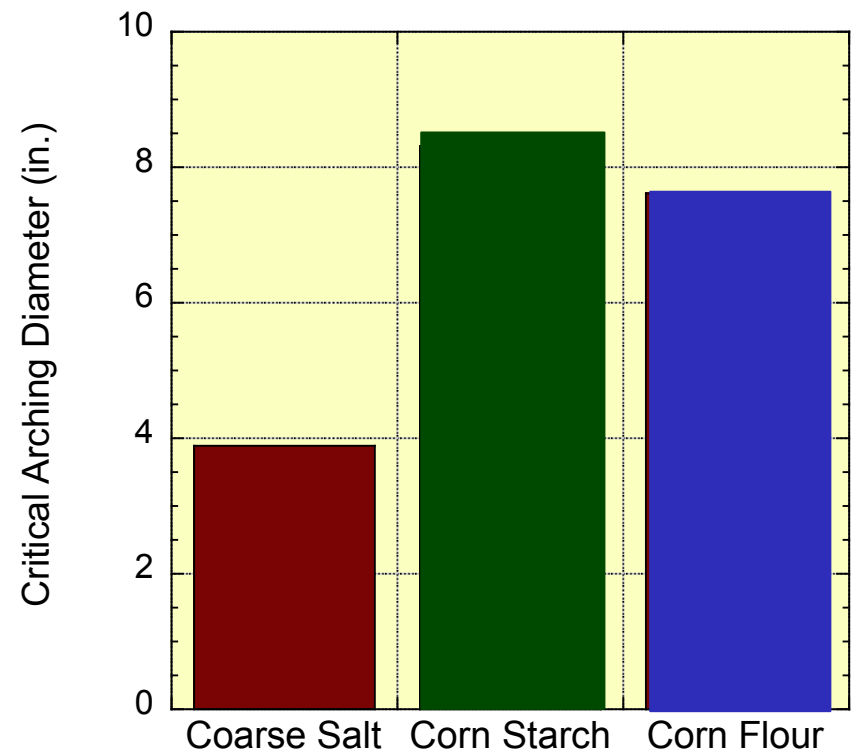
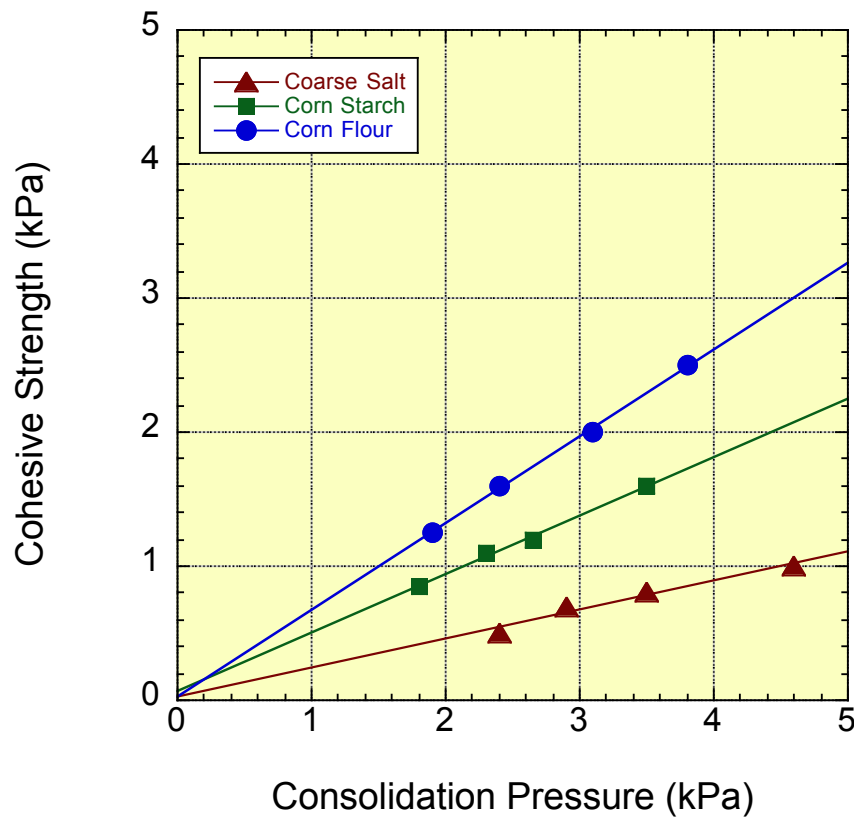
Note ♪: need to confirm outlet size will allow required discharge rate



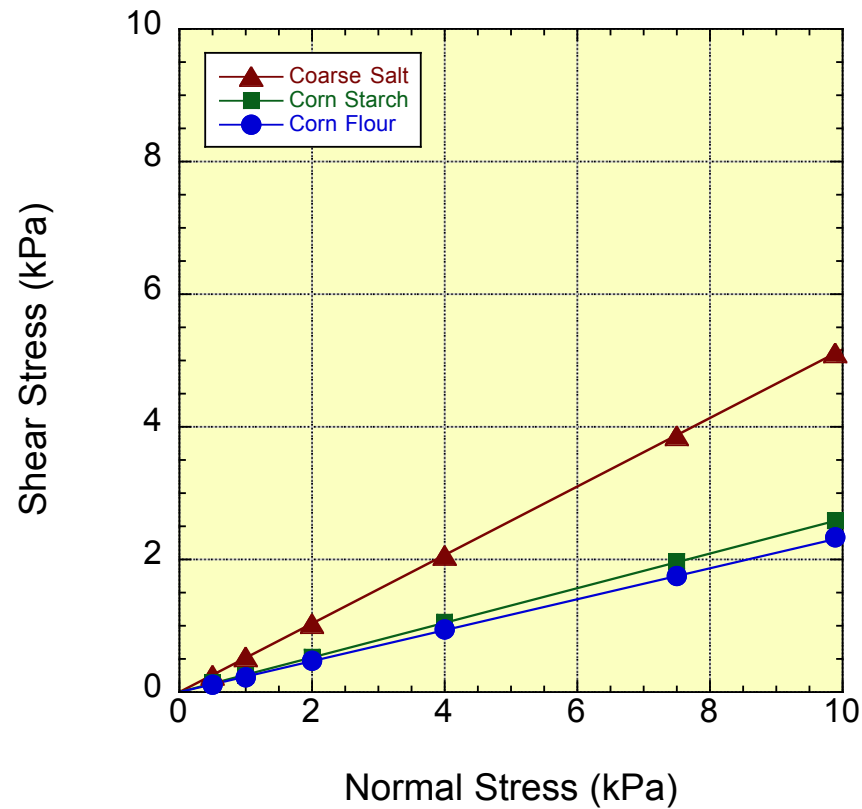
Example



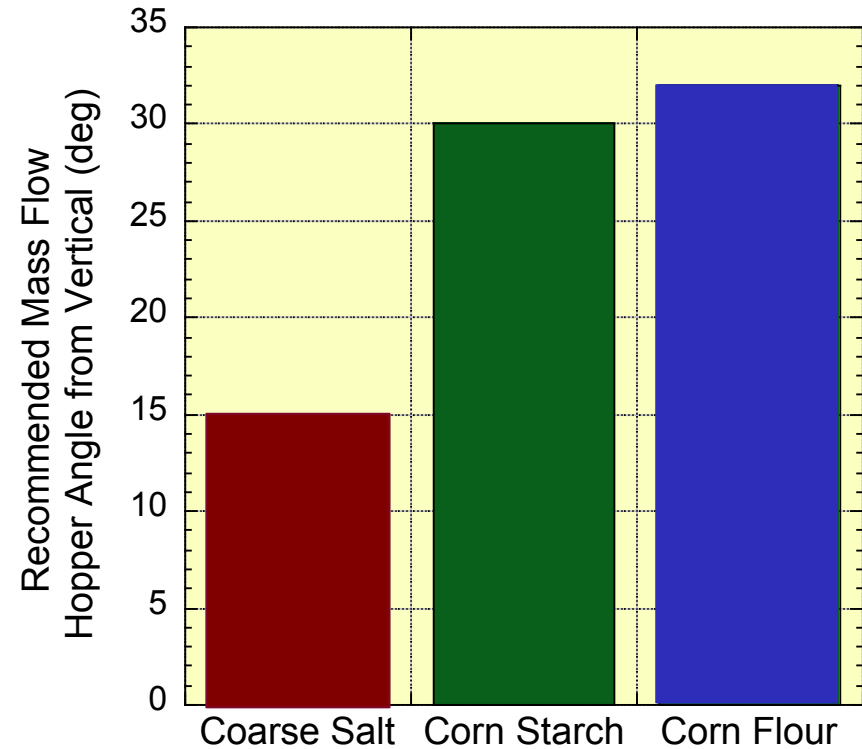
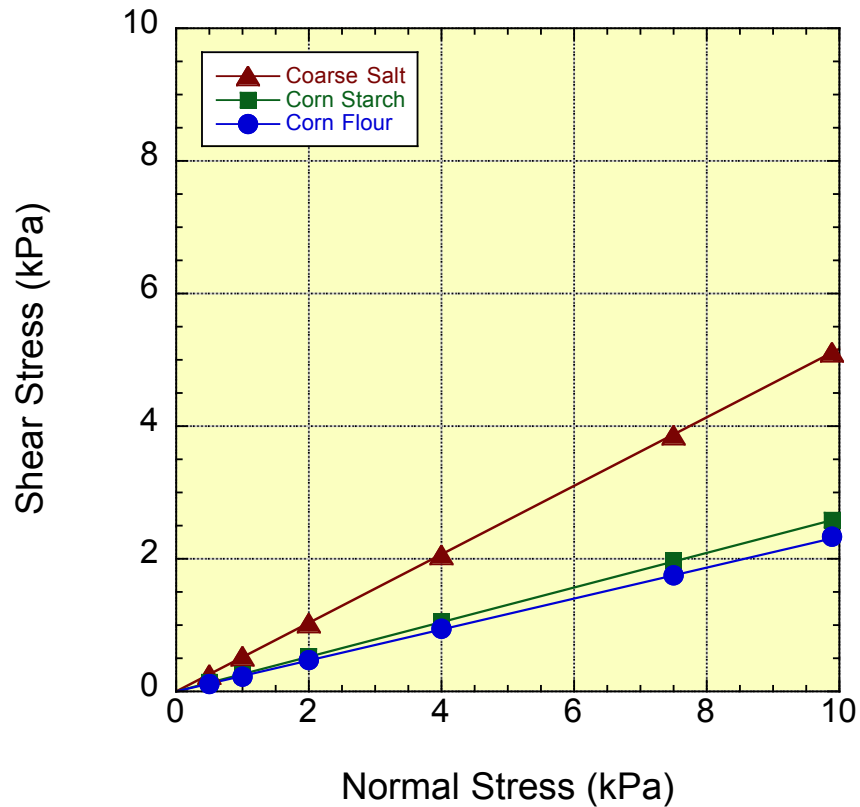
Example



Example



Example



Complications

- Bulk density, cohesive strength, wall friction, and internal friction depend on solids stress
- Choice of regression model affects outcome
- Stresses at outlet are important
- Major principal stress is not equal to wall stress
- Mass flow hopper angle depends on outlet size
- Feeder selection is important
- There is always more than one answer



2002 Albuterol recall

- Schering-Plough
 - 59 million asthma inhalers recalled
 - Unknown number contained no AI (albuterol)
 - \$500,000,000 fine



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- WSJ September 9, 2003:

“The pharmaceutical industry has a little secret: Even as it invents futuristic new drugs, its manufacturing techniques lag far behind those of potato-chip and laundry-soap makers.”



Flow and Storage of Powders

Greg Mehos, Ph.D., P.E.

gmehos@uri.edu

greg@mehos.net

www.mehos.net

978-799-7311

