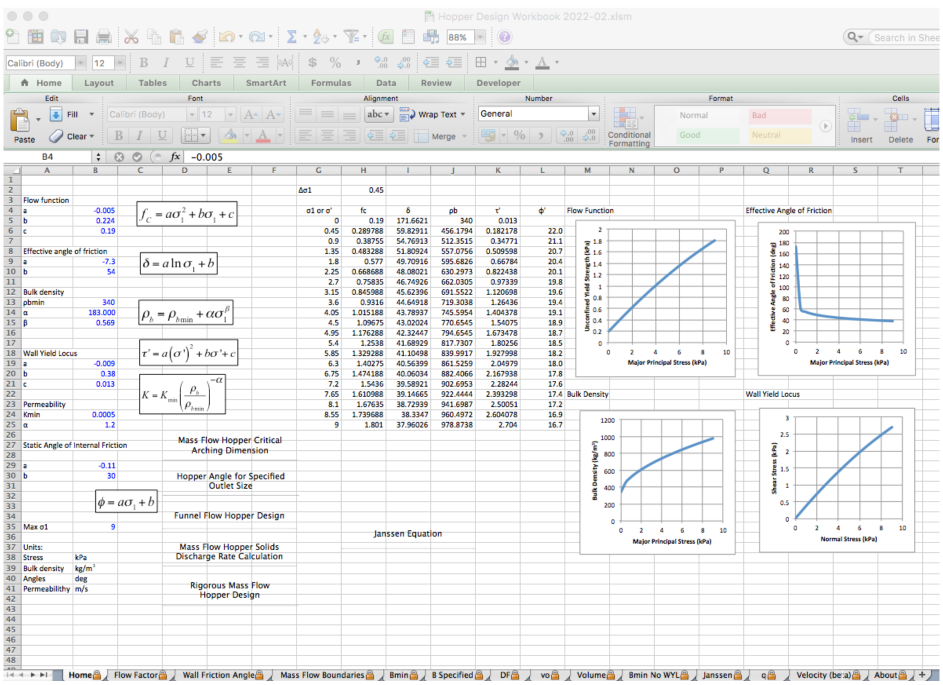
Note: My companion regression workbook is useful as it calculates the regression constants that are used in the hopper design workbook. A couple of pointers:

- Do not be fooled by R^2 values. For example, if you have three data points for the flow function to regress and you choose a quadratic model, the correlation factor will be equal to 1, but you'll likely calculate a very small arching dimension as the critical stress must be determined by extrapolation.
- Note the unique functions for the effective angle of friction and the bulk density.
- I like to use a quadratic to fit the wall friction test results.
- The value of *c* (the intercept) should never be negative. If you get this result, it's best to use the "Linear through Origin" or "Quadratic with Fixed Intercept" workbooks, using zero for the intercept.
- Read my textbook for advice for choosing a model for the flow function. The choice of regression model and greatly affect the calculations.

<u>Powder flow properties input</u>

Clicking the *Home* tab will display the following worksheet:



Powder flow property correlations are entered here. The flow function and wall yield locus are nonlinear. (Linear correlations of course can be used by setting the coefficient *a* equal to zero.) The effective angle of friction is linear with respect to the logarithm of the major principle stress. If regression yields a positive slope, the effective angle of friction can be assumed constant and equal to the average of the test results by setting *b* equal to the average value of σ_1 and *a* equal to zero. The bulk density is nonlinear. Note the form of its empirical relation. A constant value of the static angle of friction is used, while *K* uses a power-law relationship. Note that the results of the analyses can be very dependent on your choice of regression model.

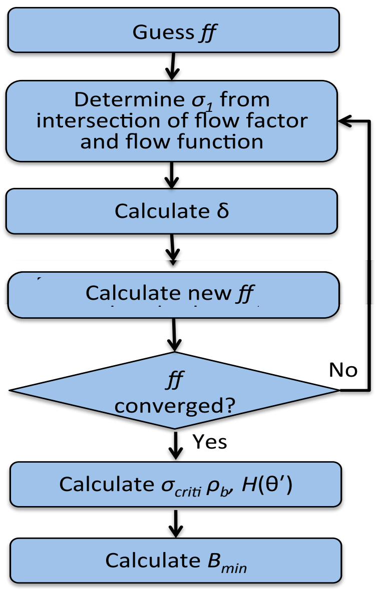
The units for stress and strength are kilopascals. Units for bulk density are kilograms per cubic meter. Angles are in degrees. Permeability is in meters per second.

The user should confirm the regression inputs and compare the plots and tabulated values of the strength, effective angle of friction, bulk density, and wall friction with those given by test results. Enter a value for the maximum major principal stress so that the plots will cover the range of stresses for which you have data.

Critical hopper outlet dimensions can be sensitive to the choice of the model used in the regression of the cohesive strength data to define the flow function. Use engineering judgment – determination of the critical stress requires extrapolation of the flow function data.

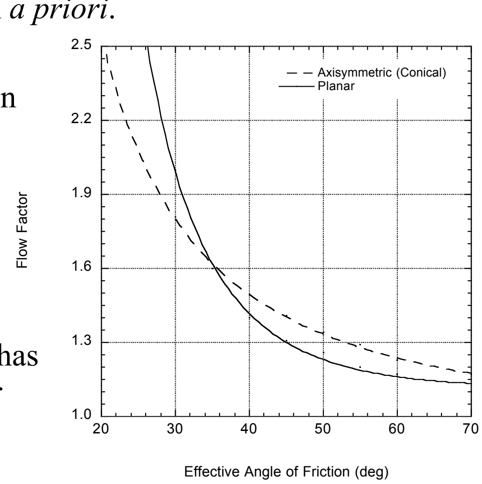
<u>Mass flow hopper critical arching dimension</u>

Refer to the flow chart shown below:



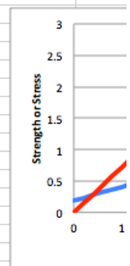
The critical arching diameter requires calculation of the critical stress, which is found by determining the intersection of the flow function (relationship between the powder's unconfined yield strength and major principal stress) and flow factor (relationship between the stress on the abutments of an arch at a hopper outlet and the major principal stress). The flow factor depends on the angle of wall friction, the geometry of the hopper, and the hopper angle, which may not be known *a priori*.

The adjacent figure plots values of the flow factor as a function of the effective angle of friction and is based on a figure provided by Jerry Johanson in Kulwiec (<u>Materials Handling Handbook</u>, John Wiley and Sons, Hoboken, NJ, 1985). The flow factor determined from Johanson's relation is approximately the same that would be calculated for wall friction angles greater than about 12° and hopper angles in the neighborhood of the mass flow boundary. If a powder has exceptionally low wall friction, the rigorous method for determining B_{min} should be followed.



From the *Home* tab, click the *Mass Flow Hopper Critical Arching Dimension* button. This will open the following worksheet:

	A	B	C	D	E	F	G	H
1								
2		$f_C = a\sigma_1^2 + b\sigma_1 + c$		$\rho_b = \rho_{bmin} + \alpha\sigma_1^{\beta}$			$\delta = a\ln\sigma + b$	
3								
4		kPa		kg/m3 vs. kPa				
5	a	-0.005		pbmin	340		Effective angle of frict	
6	b	0.224		α	183		a	
7	c	0.19		β	0.569		b	
8								
9	i	1	'= 1 for round, 0 for slot					
10	ff	1.4						
11	θ'	19.5		Confirm Δ = 0				
12				Δ	-1.0712E-05			
13	P-factor	1						
14								
15								
16				ff	1.22			
17		Conical		ff * P-factor	1.22			
18	H	2.30						
19	Bmin (m)	0.14			0.14			
20	Bmin (mm)	140.56			140.56			
21	Bmin (in.)	5.53			5.53			
22								
23								
24	Successive substitution							
25	b-1/ff	-0.49028571		σ1crit	0.318076687			
26	σ1	0.386009581		σcrit	0.260743314			
27	δ	60.94881955		pbo	435.3655914			
28	ff	1.22990884		δ	62.36163239			
29	b-1/ff	-0.58906839						
30	σ1	0.321664956						
31	δ	62.27998693						
32	ff	1.220447059						
33	b-1/ff	-0.59537188						
34	σ1	0.31827754						
35	δ	62.35727002						
36	ff	1.219914362						
37	b-1/ff	-0.59572967						
38	σ1	0.318087399						
39	δ	62.36163239						
40	ff	1.219884346						
41	b-1/ff	-0.59574984						



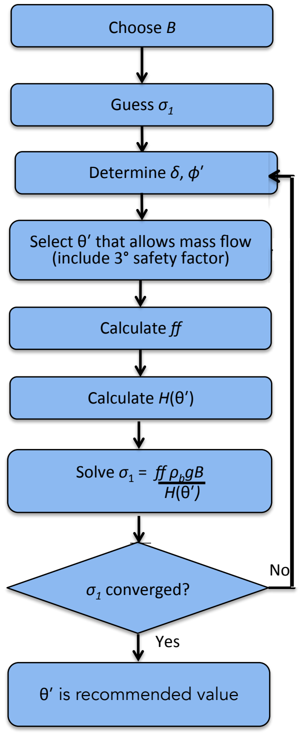
An initial guess of the flow factor (default value equals 1.4) is input along with the shape factor *i* (enter 1 for a round outlet, 0 for a slotted outlet) and the hopper angle. (19.5° is suitable for design purposes; this value can be overridden.) A P-factor can be entered to determine the effect of vibration, impact, *etc.* on critical arching dimensions.

<u>Calculation of recommended mas flow hopper angle for specified outlet size.</u>

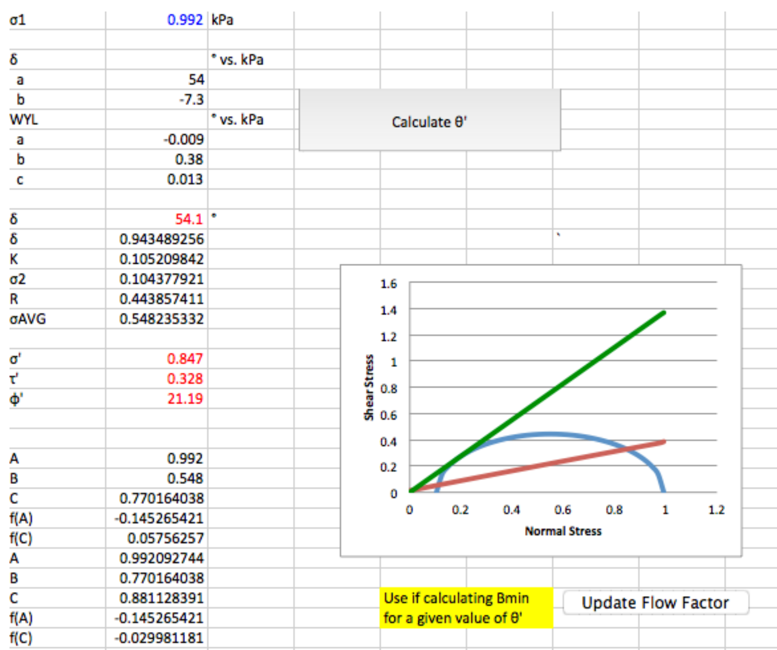
Refer to flow chart on the following page.

1. If on the *Home* worksheet, click the *Hopper Angle for Specified Outlet Size* button. This will bring you to the *B Specified* worksheet.

A	B	C	D	E	F	G	H	I
pbmin	340	$\rho_b = \rho_{bmin} + \alpha\sigma_1^{\beta}$						
α	183							
β	0.569				Calculate δ and ϕ'			
B	0.3048							
i	1	'= 1 for round, 0 for slot						
σ1	0	Confirm Δ = 0	Δ	0.0024				
ff	1.4							
H	2.2							



2. Enter an initial estimate of the flow factor. (The flow factor used to calculate the minimum arching dimension can be used.) A successive substitution algorithm is used to calculate the major consolidation stress σ_1. An initial guess is required; 0 can be used. Convergence should be confirmed by verifying that Δ is 0.
3. Click *Calculate δ and ϕ′* to open the *Wall Friction Angle* worksheet.



4. Click the *Calculate θ′* to open the *Mass Flow Angle* worksheet.

i	1	= 1 for round, 0 for slot				
Conical				Planar		
ϕ'	21.1927628	°		ϕ'	21.1927628	°
δ	54.0579524	°		δ	54.0579524	°
ϕ'	0.36988349			ϕ'	0.36988349	
δ	0.94348926			δ	0.94348926	
β	0.41637738			β	0.41637738	
θC	24.519576			θP	32.8	°
Safety Factor	3					
				ϕ' - δ	-32.86519	
θC	21.5	°		θP	32.8	°
Recommended Mass Flow Hopper Angle			Conical	21.519576		
	Update Flow Factor					

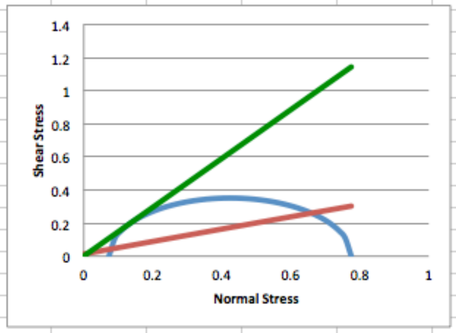
5. Click the *Update Flow Factor* button to open the *Flow Factor* worksheet.

pbmin	340	$\rho_b = \rho_{bmin} + \alpha\sigma_1^{\beta}$				
α	183					
β	0.569				Calculate δ and ϕ'	
B	0.3048					
i	1	'= 1 for round, 0 for slot				
σ1	0	Confirm Δ = 0	Δ	0.0017		
ff	1.21462533					
H	2.3310704					
Successive Substitution						

6. Click *Update* σ_1 button to open the *B Specified* worksheet.

σ1	0.775	kPa
δ		° vs. kPa
a	54	
b	-7.3	
WYL		° vs. kPa
a	-0.009	
b	0.38	
c	0.013	
δ	55.9	°
δ	0.97488934	
K	0.094307542	
σ2	0.073125086	
R	0.351132251	
σAVG	0.424257338	
σ'	0.660	
τ'	0.260	
ϕ'	21.49	
A	0.775	
B	0.424	
C	0.599823463	
f(A)	-0.091347194	
f(C)	0.035971565	
A	0.775389589	
B	0.599823463	
C	0.687606526	
f(A)	-0.091347194	
f(C)	-0.018977976	
A	0.687606526	

Calculate θ'



Use if calculating Bmin for a given value of θ'

Update Flow Factor

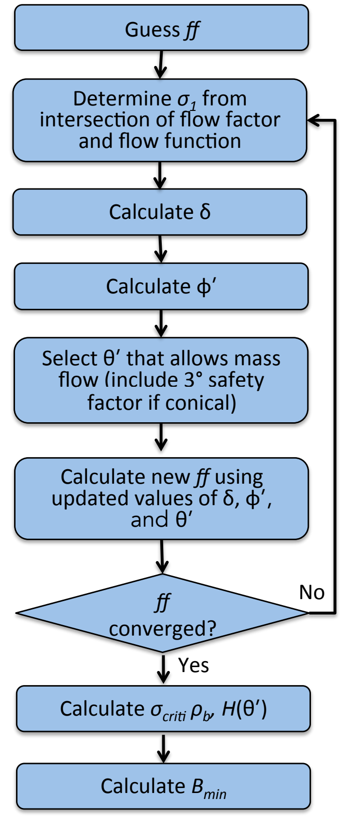
7. Note the value of recommended mass flow hopper angle. Repeat steps 3-7 until convergence has been reached.

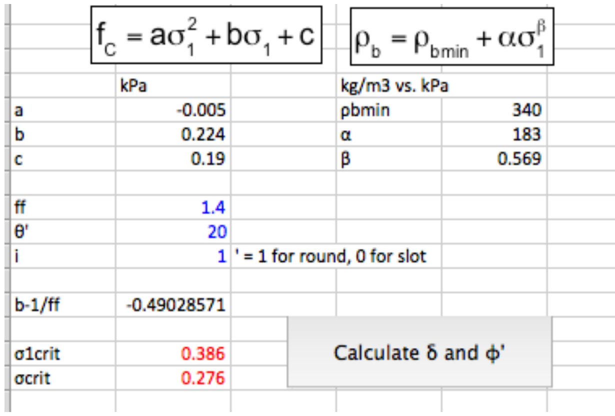
Rigorous mass flow hopper design

Refer to flow sheet to the right:

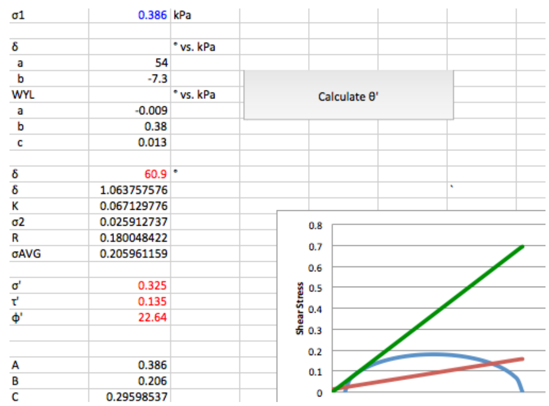
1. If on the *Home* worksheet, click the *Rigorous Mass Flow Hopper Design* button. This will bring you to the *Bmin* worksheet.
2. Any cells that are colored blue can be overridden. Enter an initial estimate for the flow function *ff* (1.2 – 1.4 is usually a good choice) and the geometry factor *i* (1 for a round outlet; 0 for a slot).
3. Optionally enter a value of the hopper angle θ.
4. Updated values of the major principle stress σ_1 at the hopper outlet and other calculation results will appear in red.
5. Click *Calculate* δ *and* ϕ' to open the *Wall Friction Angle* tab.

   The spreadsheet will calculate values of the effective angle of friction and wall friction angle.





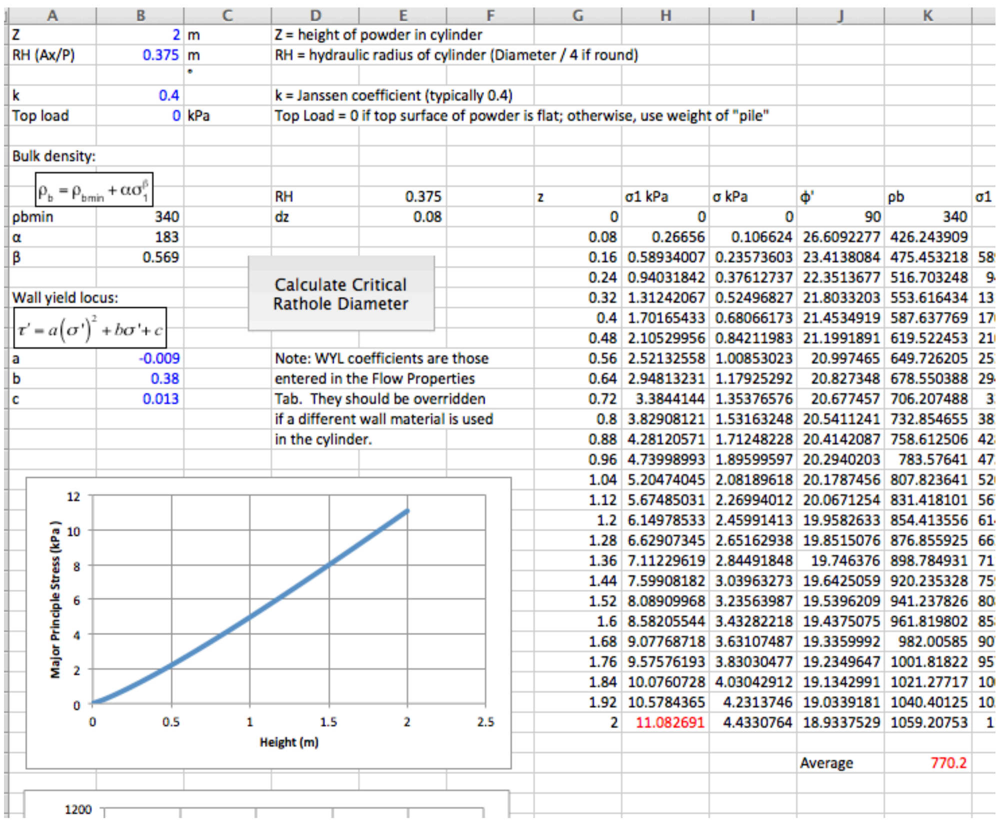
6. Click the *Calculate θ′* button. This will bring you to the *Mass Flow Boundaries* tab.



7. Note the value of the recommended mass flow hopper angle and click the *Update Flow Factor* button. This will bring you to the *Flow Factor* worksheet.
8. The spreadsheet will update the value of the flow factor based on new values of δ, ϕ' and θ'. Click on the *Update Flow Factor* tab. This will return you to the *Bmin* worksheet.
9. Note the value of the flow factor. If it had not appreciably changed, the solution converged. Note the values of the minimum outlet diameter or width and the recommended mass flow hopper angle. If the flow factor changed appreciably, then click the *Calculate δ and ϕ′* button and repeat Steps 5 through 8 until the flow factor calculation has converged.

<u>Janssen equation</u>

1. Click the *Janssen* button will bring you to the *Janssen* worksheet.
2. Enter the cylinder height and hydraulic radius (Cross-sectional area divided by the perimeter).
3. The default value of the Janssen coefficient is 0.4, but it may be overridden.
4. You may replace wall yield locus regression parameters if cylinder section is different from cone.
5. You may also enter an external load, *e.g.*, the weight of the surcharge.

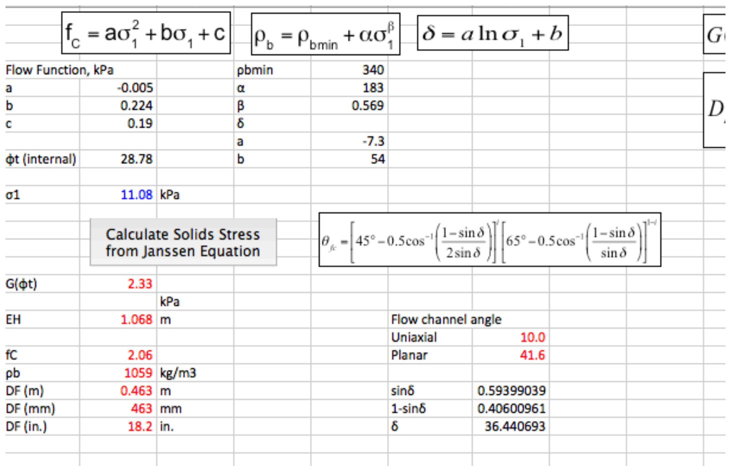


A	B	C	D–F
Z	2	m	Z = height of powder in cylinder
RH (Ax/P)	0.375	m	RH = hydraulic radius of cylinder (Diameter / 4 if round)
k	0.4		k = Janssen coefficient (typically 0.4)
Top load	0	kPa	Top Load = 0 if top surface of powder is flat; otherwise, use weight of "pile"
Bulk density:			
$\rho_b = \rho_{bmin} + \alpha\sigma_1^{\beta}$			RH 0.375
pbmin	340		dz 0.08
α	183		
β	0.569		Calculate Critical Rathole Diameter
Wall yield locus:			
$\tau' = a(\sigma')^2 + b\sigma' + c$			
a	-0.009		Note: WYL coefficients are those
b	0.38		entered in the Flow Properties
c	0.013		Tab. They should be overridden
			if a different wall material is used
			in the cylinder.

z	σ1 kPa	σ kPa	ϕ'	ρb	σ1
0	0	0	90	340	
0.08	0.26656	0.106624	26.6092277	426.243909	
0.16	0.58934007	0.23573603	23.4138084	475.453218	58
0.24	0.94031842	0.37612737	22.3513677	516.703248	9
0.32	1.31242067	0.52496827	21.8033203	553.616434	13
0.4	1.70165433	0.68066173	21.4534919	587.637769	17
0.48	2.10529956	0.84211983	21.1991891	619.522453	21
0.56	2.52132558	1.00853023	20.997465	649.726205	25
0.64	2.94813231	1.17925292	20.827348	678.550388	29
0.72	3.3844144	1.35376576	20.677457	706.207488	3
0.8	3.82908121	1.53163248	20.5411241	732.854655	38
0.88	4.28120571	1.71248228	20.4142087	758.612506	42
0.96	4.73998993	1.89599597	20.2940203	783.57641	47
1.04	5.20474045	2.08189618	20.1787456	807.823641	52
1.12	5.67485031	2.26994012	20.0671254	831.418101	56
1.2	6.14978533	2.45991413	19.9582633	854.413556	61
1.28	6.62907345	2.65162938	19.8515076	876.855925	66
1.36	7.11229619	2.84491848	19.746376	898.784931	71
1.44	7.59908182	3.03963273	19.6425059	920.235328	75
1.52	8.08909968	3.23563987	19.5396209	941.237826	80
1.6	8.58205544	3.43282218	19.4375075	961.819802	85
1.68	9.07768718	3.63107487	19.3359992	982.00585	90
1.76	9.57576193	3.83030477	19.2349647	1001.81822	95
1.84	10.0760728	4.03042912	19.1342991	1021.27717	10
1.92	10.5784365	4.2313746	19.0339181	1040.40125	10
2	11.082691	4.4330764	18.9337529	1059.20753	1
			Average	770.2	

Calculation of critical outlet dimensions for funnel flow bins

1. On the *Home* worksheet, click the *Funnel Flow Hopper Design* button. This will bring you to the *DF* worksheet.



2. Click the *Calculate Solids Stress from Janssen Equation* button.
3. Enter the cylinder height, hydraulic radius, Janssen coefficient, and if necessary the additional solids loading.
4. Click the *Calculate Critical Rathole Diameter* button.
5. Read the calculated value for *DF,* the critical ratholing diameter. *DF* is the diameter of a conical hopper or the diagonal of a hopper with flat walls and a slotted outlet that will prevent the formation of a stable rathole.
6. To determine the diameter of a planar funnel flow hopper required to prevent a stable rathole from developing, open the *Bmin* worksheet and enter $ff = 1.7$, $\theta' = 30°$, and $i = 0$.
7. To determine the diameter of a conical funnel flow hopper required to prevent a stable rathole from developing, enter $ff = 1.3$, $\theta' = 7°$, and $i = 1$.

Note that the critical rathole diameter D_F is equal to the diameter of a slotted outlet or the diameter of the round outlet that must be exceeded to ensure that a rathole will collapse.

The material of construction in the cylinder is not necessarily the same as that of the hopper section. The appropriate wall yield locus coefficients can be entered in the *Janssen Equation* sheet.

When designing a conical funnel flow hopper, the critical arching diameter is not typically specified since the critical rathole diameter will be significantly larger than the arching diameter.

The spreadsheet also provides the flow channel angle, *i.e.*, the angle referenced from vertical that marks the boundary between flowing material and stagnant material.

Calculation of steady discharge rate from mass flow bins

Click on the v_o tab or from the *Home* sheet, click *Mass Flow Hopper Discharge Rate Calculation* to open the *vo* worksheet.

A	B	C	D	E	F	G	H	I
σ1	9	kPa	ϕ'	21				
			δ	54				
B	0.5	m	ff	1.41		Cohesive Powder		
θ'	20							
i	1	= 1 for round, 0 for slot						
					Flow function			
			Bulk density		a	-0.005		
θ'	0.34906585		α	183	b	0.224		
RH	9		β	0.569	c	0.19		
			α		K			
tanθ'	0.36397023		fc		Kmin	0.00045		
B	0.5		a		α	1.2		
pbmax	979							
pbmin	340							
Kmin	4.50E-04							
pbo	340							
A	0.29711856							
B	1450.4							
C	-1.000							
Geometery	Conical							
vo	0.0007	m/s				vo	0.00050	m/s
Mdot	166	kg/hr				Mdot	122	kg/hr

1. Enter the maximum solids stress in the cylinder.
2. Enter a value of *B*, which is the diameter of a round outlet or the width of a slotted outlet.
3. Enter the hopper angle θ'. (For funnel flow hoppers, use the flow channel angle.)
4. Enter *i* equal to 1 for a round outlet, 0 for a slotted outlet, and enter the appropriate outlet dimensions.

Kerry Johanson recommends using the maximum solids stress in the cylinder to calculate the bulk density at the location of the hopper where the interstitial gas pressure is a minimum.

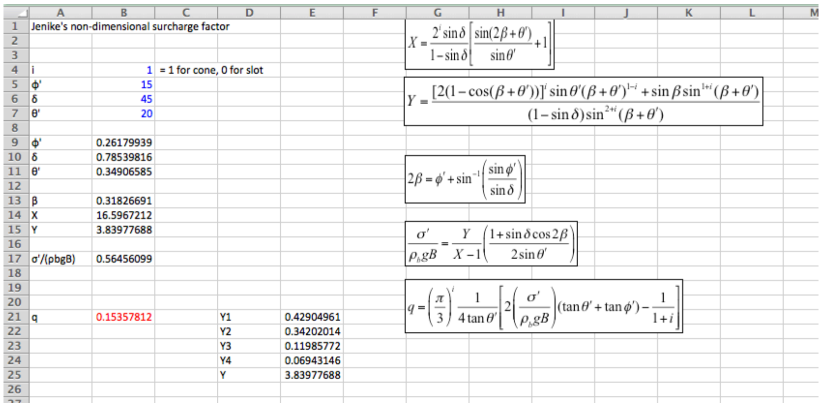
I have found that the predicted discharge rate better matches observations by reducing the calculated value by 80 percent.

Note: the discharge rate equation for no cohesion can be found in the 9th edition of Perry's. I've modified that equation following Jerry Johanson's method to account for the cohesive strength of the powder.

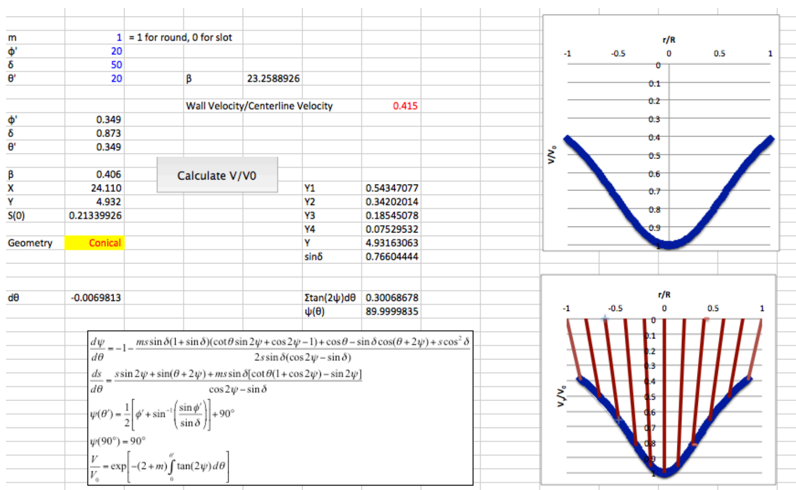
To account for cohesive strength, enter the flow factor, effective angle of friction, and wall friction angle, and click *Cohesive Powder.* If necessary, change the initial estimate of the solids stress at the outlet. This generally gives a larger discharge rate, so when designing a hopper, I do not bother using this feature.

Other useful tabs

The *q* spreadsheet calculates Jenike's non-dimension surcharge factors which can be used to calculate the vertical solids stress at a hopper outlet. The wall friction angle, effective angle of friction, hopper angle, and *i* are entered.



The *Velocities* spreadsheet calculates solids velocity profiles in converging hoppers. After the wall friction angle, effective angle of friction, hopper angle, and *m* are entered, the *Calculate V/V0* button is clicked to calculate the radial and vertical solids velocity profiles.



The *Volumes* spreadsheet calculates volumes, hopper angles. You can determine both the actual volume and working volume (if you have a tall enough cylinder).

Note: use the Regression Workbook to regress the flow property test data. If a negative intercept is found, fit the equation through the origin. For the flow function, consider using linear regression of the low-stress data points to determine the intercept, and then fit the data to a quadratic fixing the value of the intercept equal to that obtained from the linear regression.

Disclaimer: I make no warranty and the user should always check the outputs of the workbook with hand calculations based on Jenike Bulletin 123 and other references.