
TECHNICAL
DIFFICULTIES
PLEASE
STAND
BY

Happiness Equals Reality Minus Expectations

Outline

- Definition of flowability
- Fundamental properties
- Shear cell testing

Flowability

- Angle of repose
- Carr ratio
- Hausner ratio
- Carr index

Flowability

- Angle of repose
- Carr ratio
- Hausner ratio
- Carr index
- FFC
- Flodex
- Flow energy

Flowability

- Angle of repose
- Carr ratio
- Hausner ratio
- Carr index
- FFC
- Flodex
- Flow energy

None will tell you whether or not your powder will flow!

Good flowability

Poor flowability

Obstructions to flow

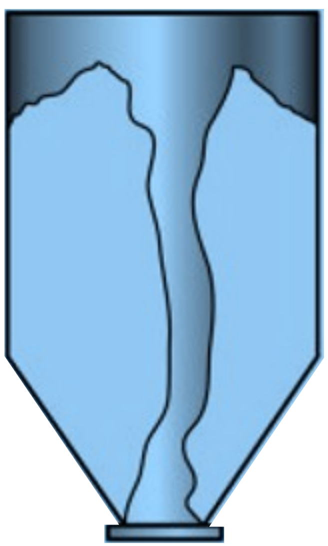
Arching/Bridging

Bridging

GM&A

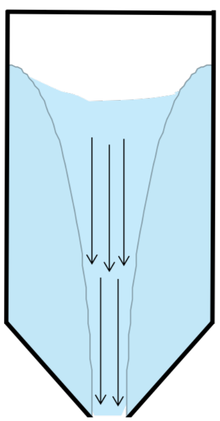
Obstructions to flow

Arching/Bridging

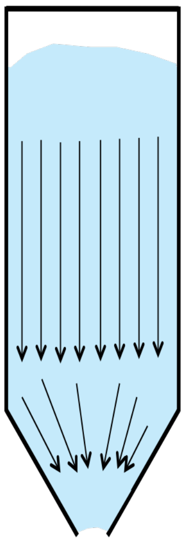


Ratholing

Flow patterns

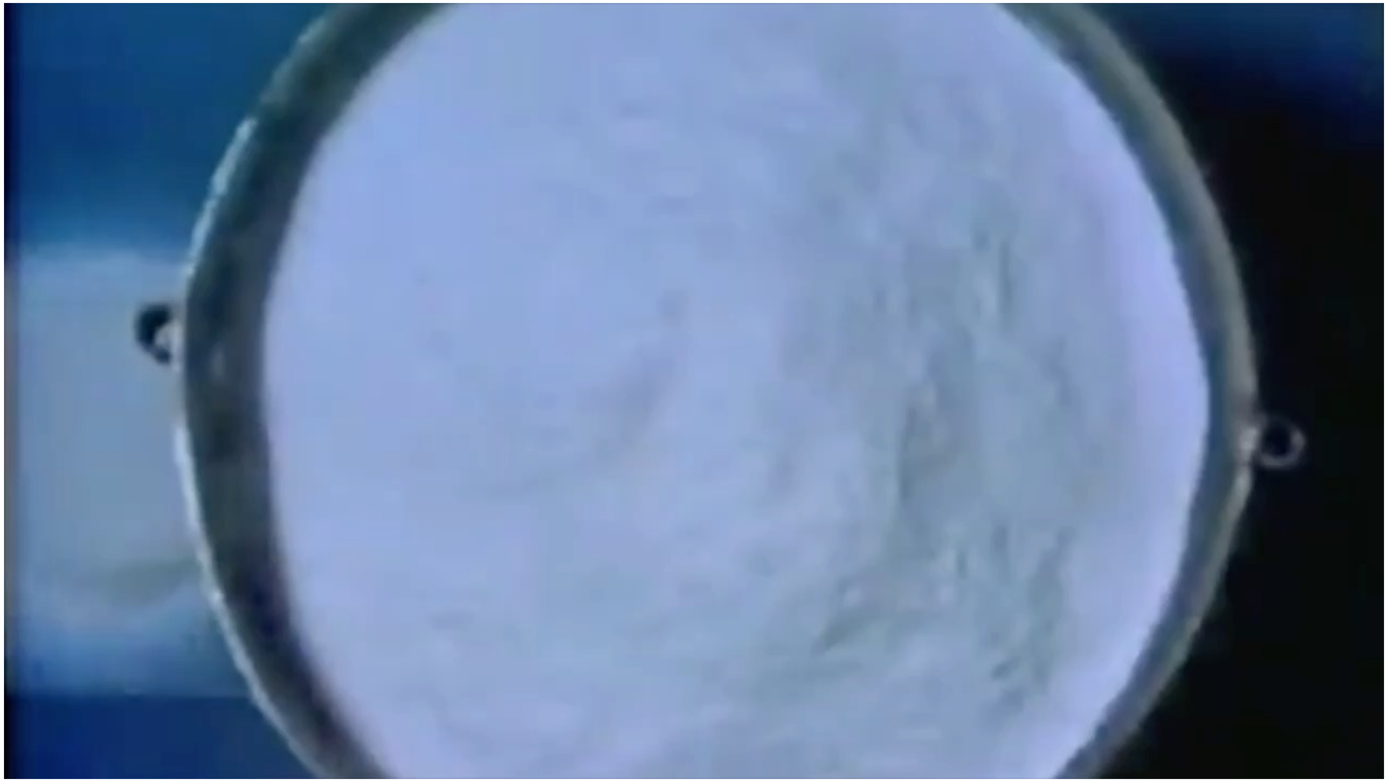


Funnel flow



Mass flow

Funnel flow



Mass flow

Andrew Jenike

Fundamental flow properties

- Cohesive strength
- Internal friction
- Bulk density/compressibility
- Wall friction

Fundamental flow properties

- Cohesive strength
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Allows calculation of hopper outlet dimensions required to prevent flow obstructions

Fundamental flow properties

- Cohesive strength
- Internal friction
- Bulk density/compressibility
- Wall friction

Allows calculation of hopper outlet dimensions required to prevent flow obstructions

Used to determine hopper angles that ensure mass flow

Shear cell testers

Jenike
Direct
Tester

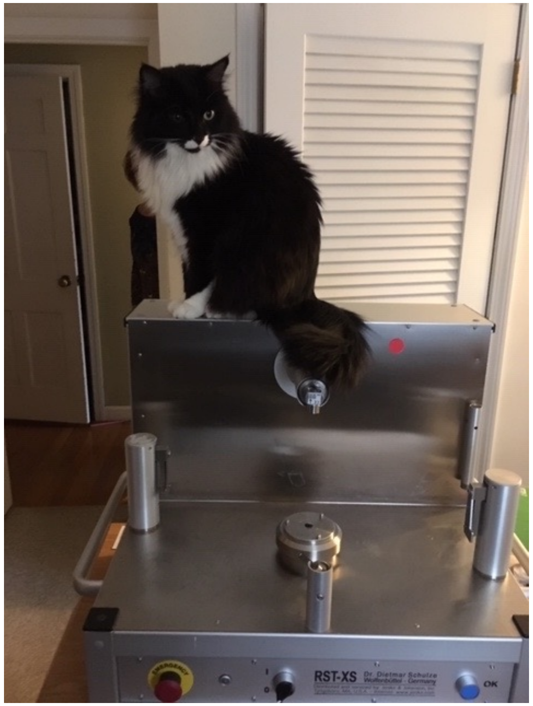
Schulze
RST-XS.s

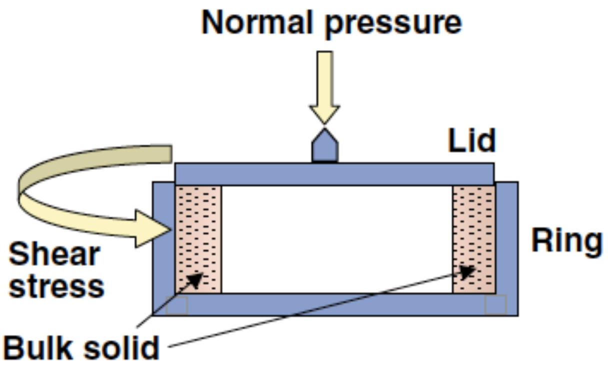
Brookfield
Engineering
PFT

Micromeritics
Freeman
Technology FT4

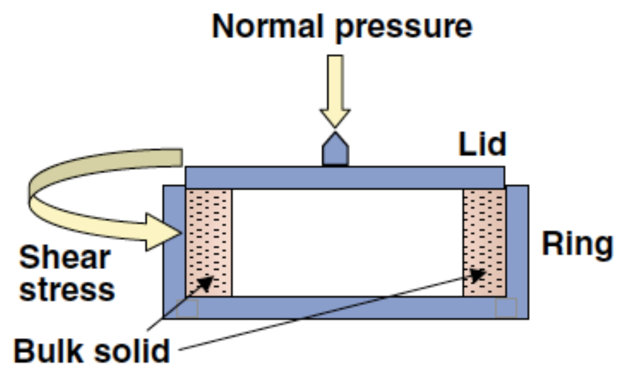
Cohesive strength

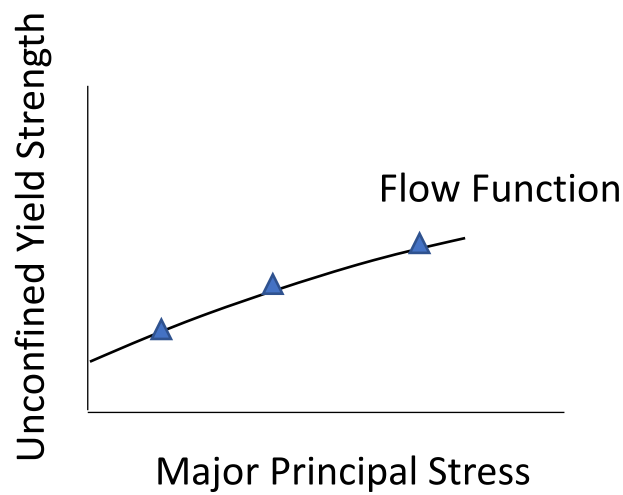
Cohesive strength



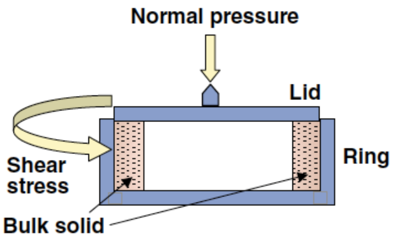


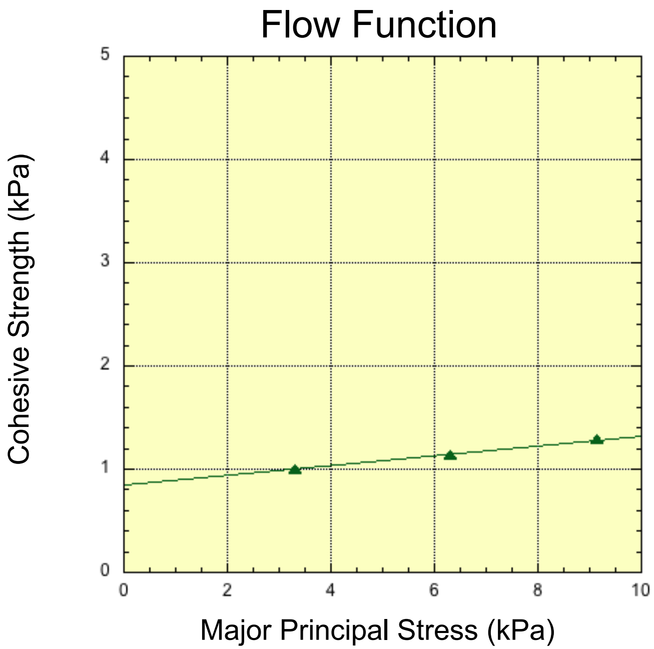
Cohesive strength





Cohesive strength





67/29/4 blend of acetaminophen/Avicel/hydroxypropyl cellulose

FFC

$$FFC = \frac{\sigma_1}{f_C}$$

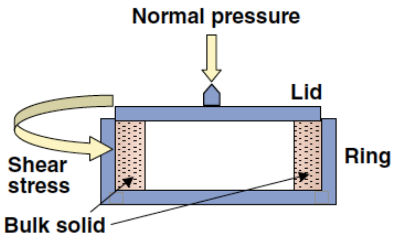
Major principal stress

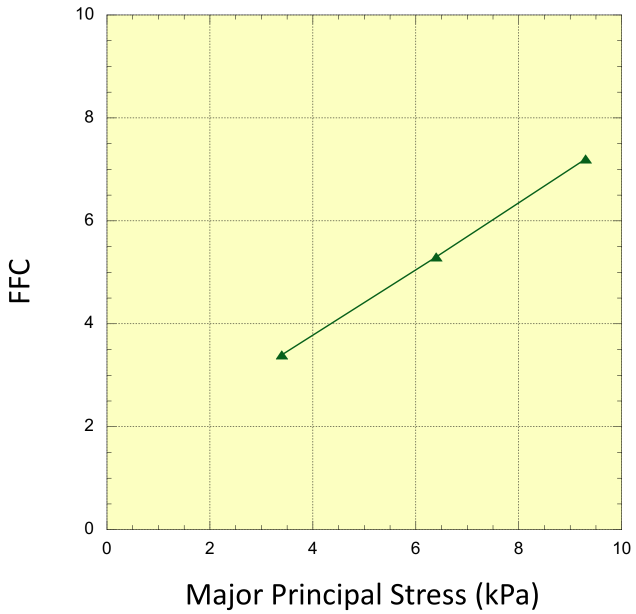
Unconfined yield strength

Flowability Coefficient
~~FF~~
~~Flow Function~~
~~Flow Factor~~

< 2	Non-flowing
2 – 4	Cohesive
4 – 10	Easy flowing
> 10	Free flowing

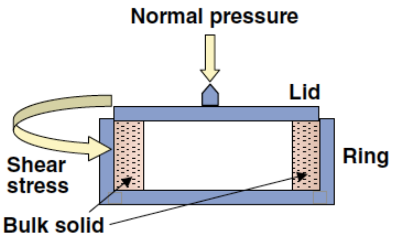
FFC

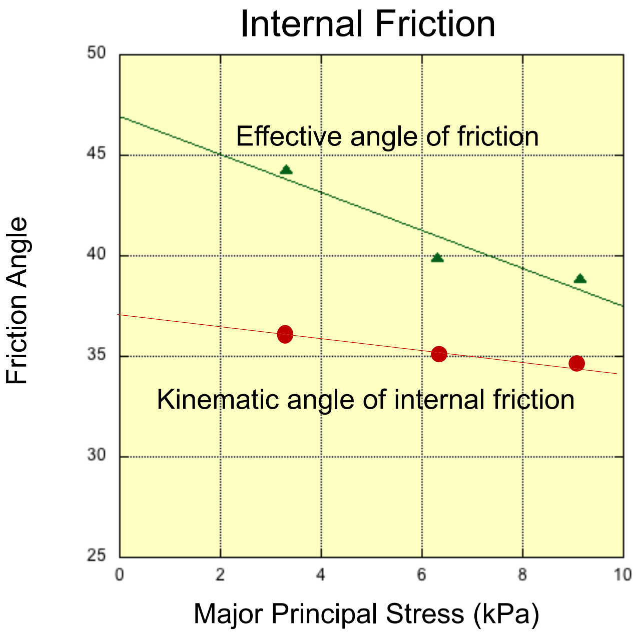




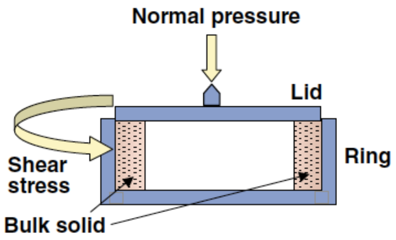
< 2	Non-flowing
2 – 4	Cohesive
4 – 10	Easy flowing
> 10	Free flowing

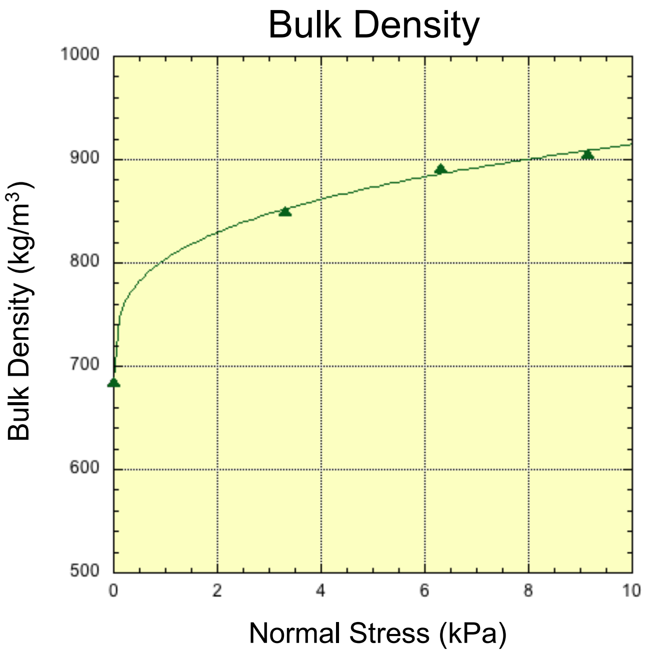
Internal friction



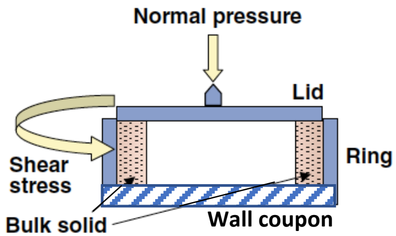


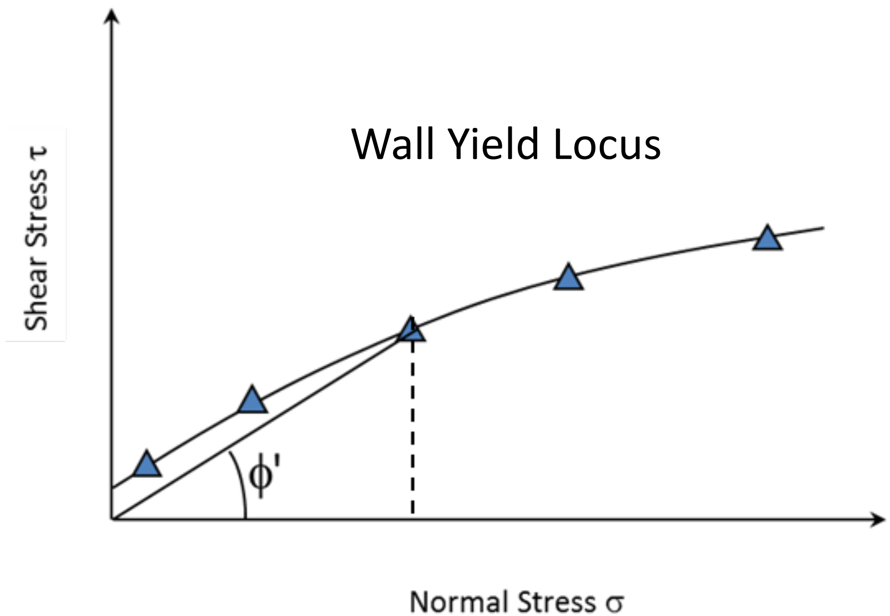
Compressibility





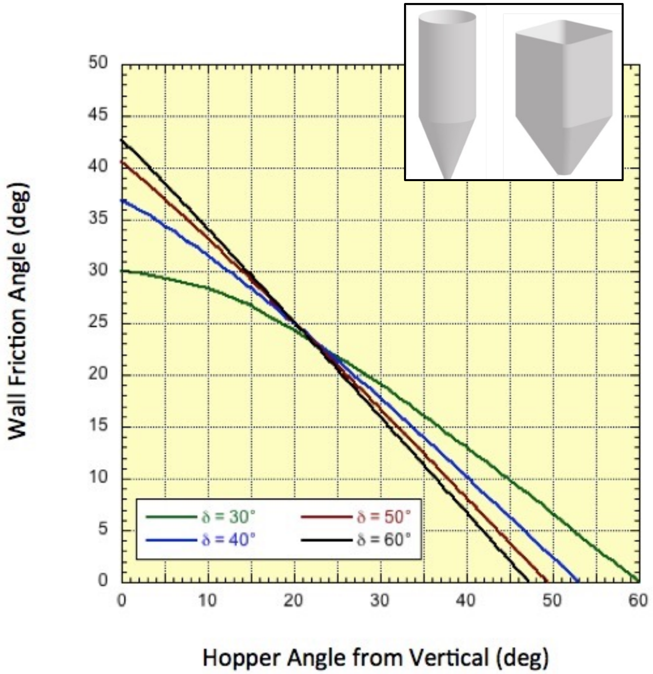
Wall friction



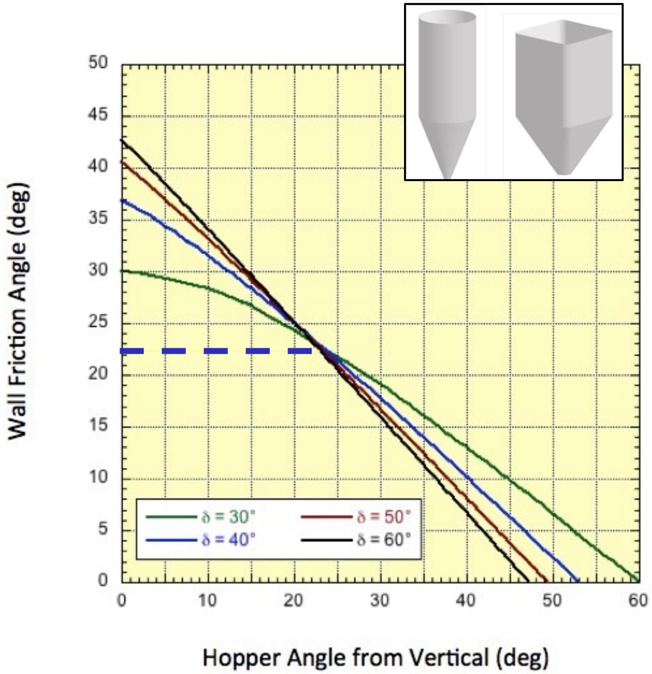


$$\tan\phi' = \frac{\text{Shear Stress}}{\text{Normal Stress}} = \mu$$

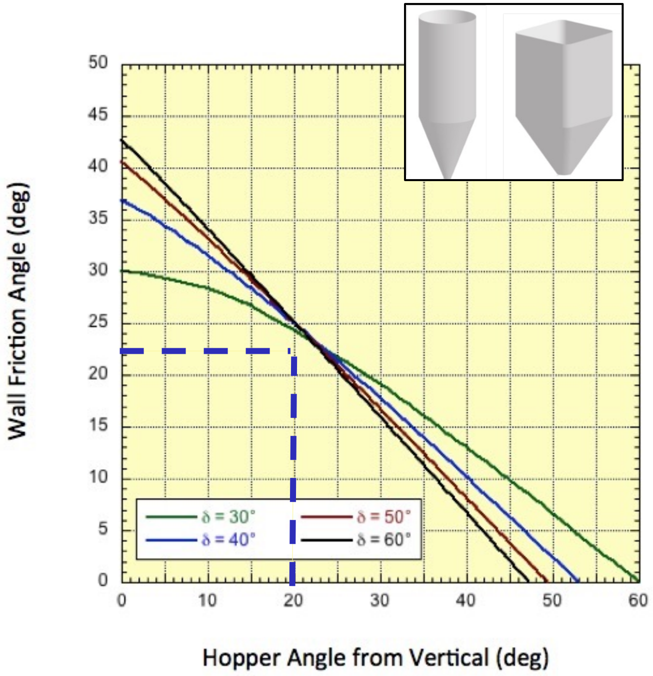
Mass flow hopper angle



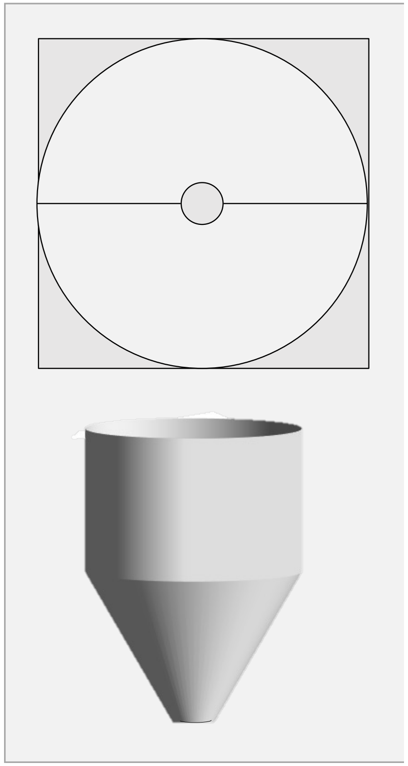
Mass flow hopper angle



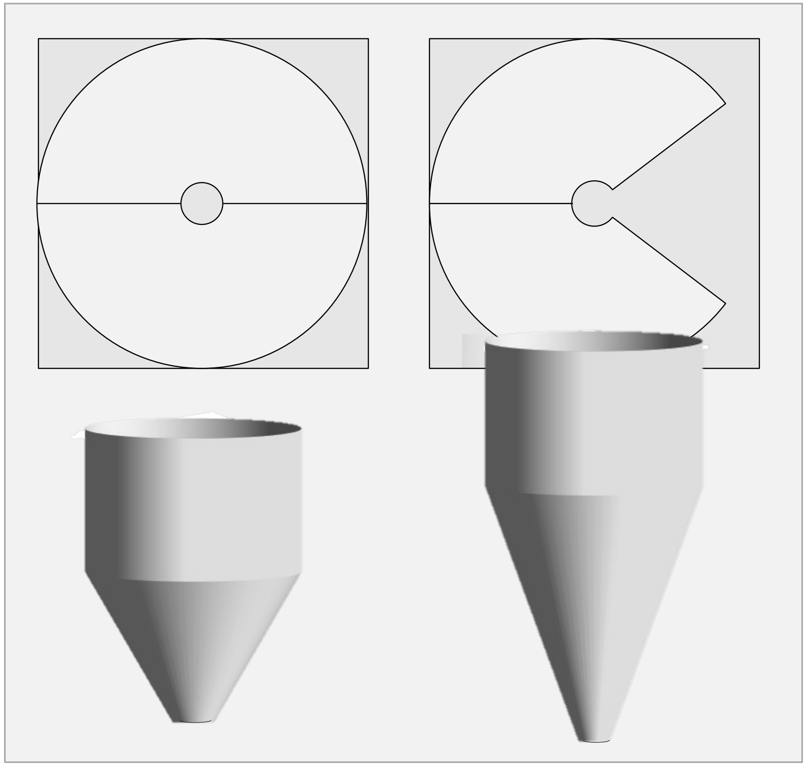
Mass flow hopper angle



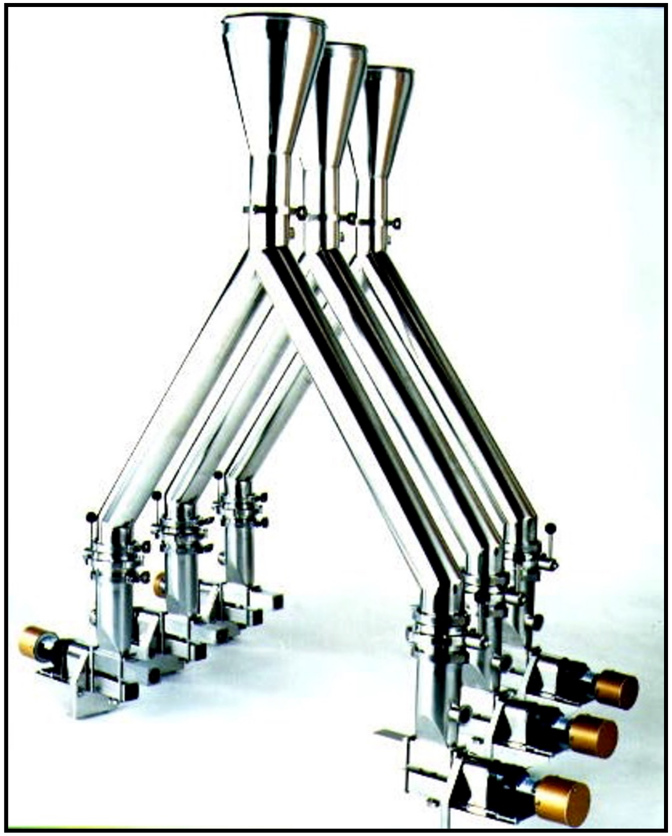
Mass flow hopper angle



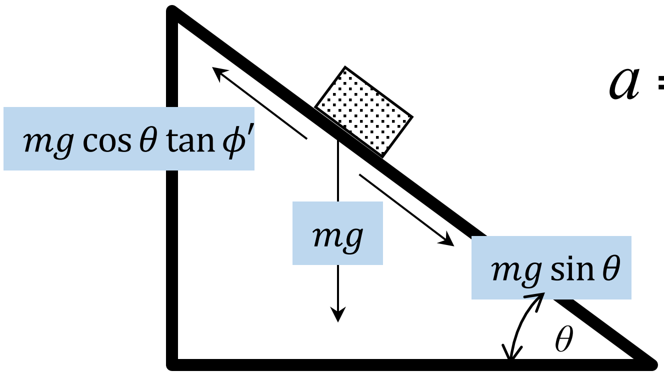
Mass flow hopper angle



Transfer chutes



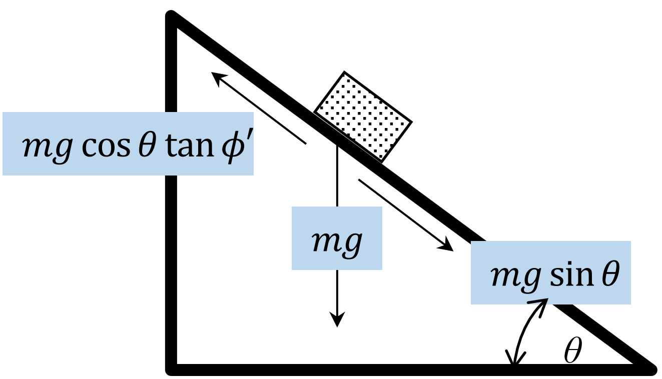
Transfer chutes



$$a = g(\sin\theta - \cos\theta\tan\phi')$$
$$= g\cos\theta(\tan\theta - \tan\phi')$$

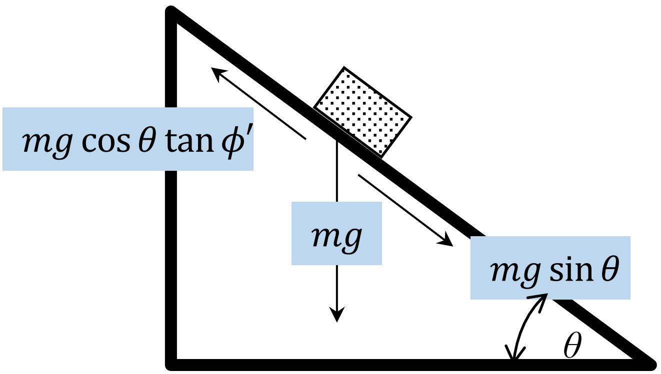
For flow, $\theta > \phi'$

Transfer chutes and diverter valves



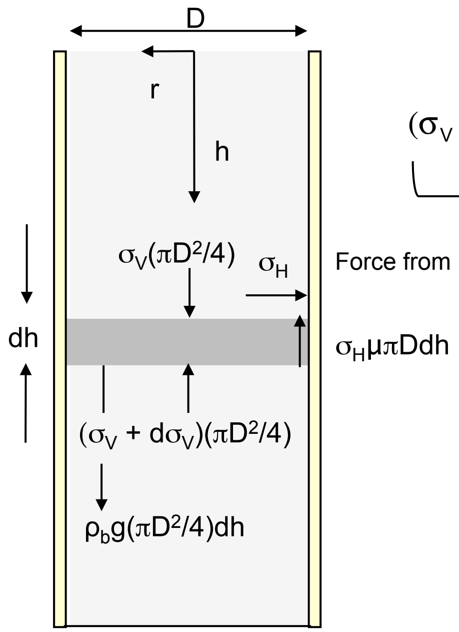
$$a = g\cos\theta(\tan\theta - \tan\phi')$$

Transfer chutes and diverter valves



$$a = g\cos\theta(\tan\theta - \tan\phi')$$

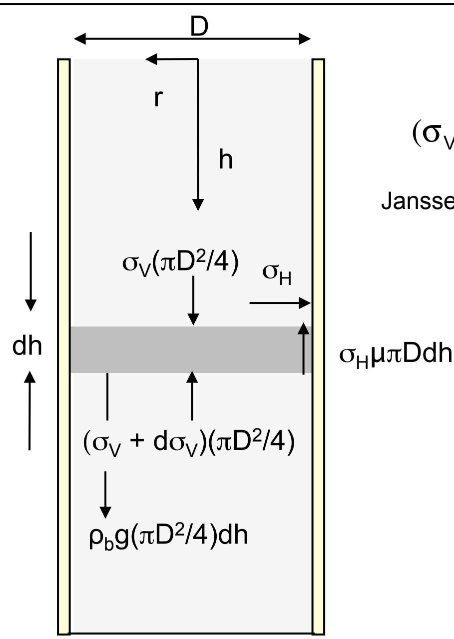
Pressure distribution



Cylinder

$$\underbrace{(\sigma_V + d\sigma_V)\frac{\pi D^2}{4}}_{\text{Force from powder below}} + \underbrace{\sigma_H \mu \pi D\,dh}_{\text{Force due to wall friction}} = \underbrace{\sigma_V \frac{\pi D^2}{4}}_{\text{Force from powder above}} + \underbrace{\rho_b g \frac{\pi D^2}{4} dh}_{\text{Force due to gravity}}$$

Pressure distribution



Cylinder

$$(\sigma_V + d\sigma_V)\frac{\pi D^2}{4} + \sigma_H \mu \pi D\, dh = \sigma_V \frac{\pi D^2}{4} + \rho_b g \frac{\pi D^2}{4} dh$$

Janssen coefficient

$$k = \frac{\sigma_H}{\sigma_V} \cong 0.4 - 0.6$$

$$\frac{D}{4}\frac{d\sigma_V}{dh} + k\mu\sigma_V = \frac{D}{4}\rho_b g$$

1st order linear differential equation

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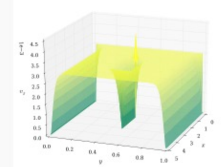
Integrating factor

From Wikipedia, the free encyclopedia
(Redirected from Integration factor)

In mathematics, an **integrating factor** is a function that is chosen to facilitate the solving of a given equation involving differentials. It is commonly used to solve ordinary differential equations, but is also used within multivariable calculus when multiplying through by an integrating factor allows an inexact differential to be made into an exact differential (which can then be integrated to give a scalar field). This is especially useful in thermodynamics where temperature becomes the integrating factor that makes entropy an exact differential.

Contents [hide]

Differential equations



Navier–Stokes differential equations used to simulate airflow around an obstruction.

Scope	[show]
Classification	
Types	[show]
Relation to processes	[show]
Solution	
General topics	[show]
Solution methods	[show]
People	[show]

v·t·e

Use in solving first order linear ordinary differential equations [edit]

Integrating factors are useful for solving ordinary differential equations that can be expressed in the form

$$y' + P(x)y = Q(x)$$

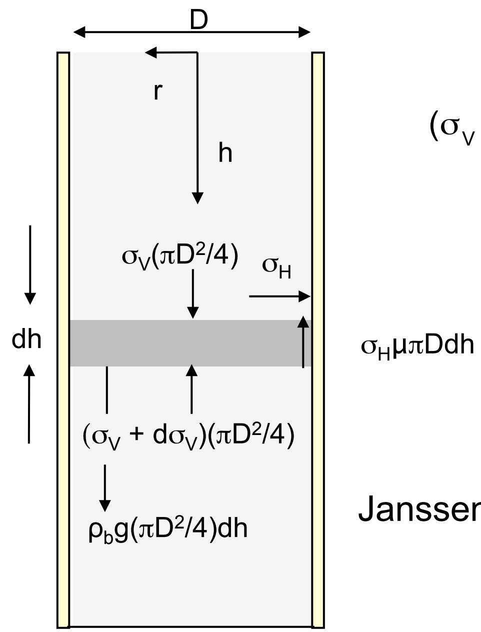
The basic idea is to find some function $M(x)$, called the "integrating factor," which we can multiply through our DE in order to bring the left-hand side under a common derivative. For the canonical first-order, linear differential equation shown above, our integrating factor is chosen to be

$$M(x) = e^{\int_{s_0}^{x} P(s)ds}$$

In order to derive this, let $M(x)$ be the integrating factor of a first order, linear differential equation such that multiplication by $M(x)$ transforms a partial derivative into a total derivative, then:

$$(1) \qquad \underbrace{M(x)(y' + P(x)y)}_{\text{partial derivative}}$$

Pressure distribution



Cylinder

$$(\sigma_V + d\sigma_V)\frac{\pi D^2}{4} + \sigma_H \mu \pi D\, dh = \sigma_V \frac{\pi D^2}{4} + \rho_b g \frac{\pi D^2}{4} dh$$

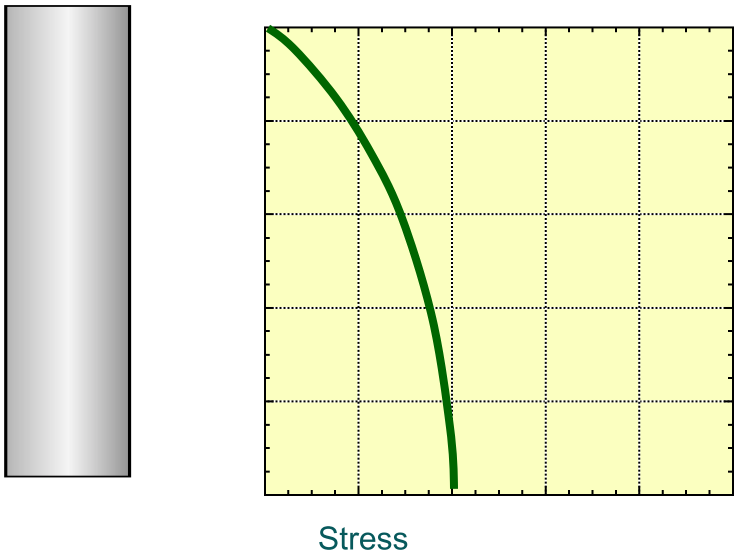
$$k = \frac{\sigma_H}{\sigma_V} \cong 0.4$$

$$\frac{D}{4}\frac{d\sigma_V}{dh} + k\mu\sigma_V \quad \frac{D}{4}\rho_b g$$

Janssen Equation:

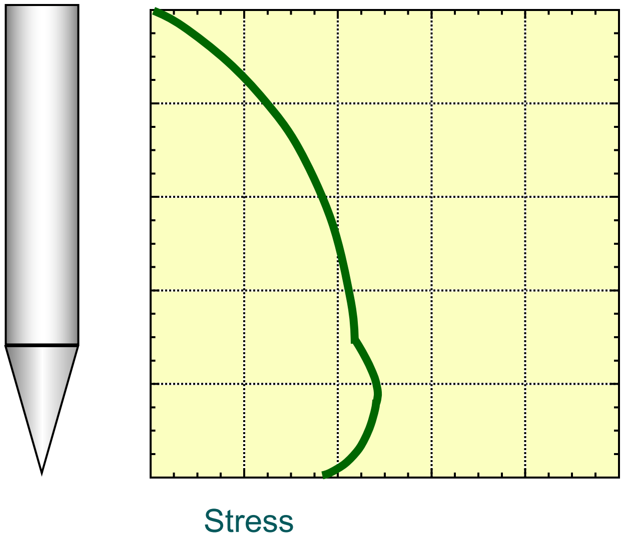
$$\sigma_v = \frac{\rho_b g D}{4k\mu}\left[1 - \exp\left(\frac{-4k\mu h}{D}\right)\right]$$

Pressure distribution



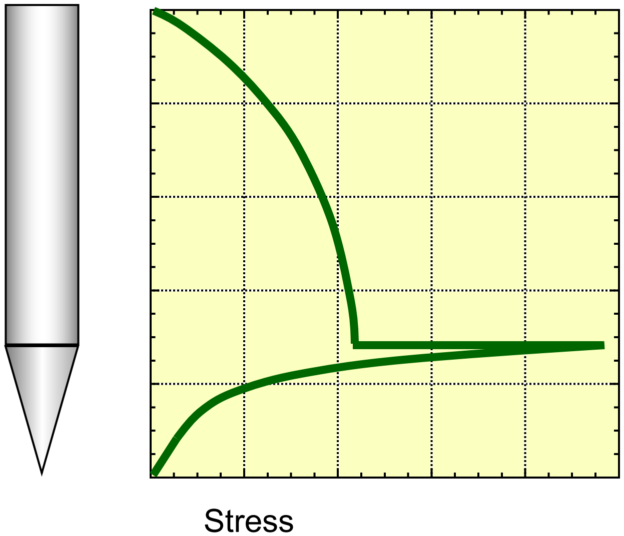
Pressure distribution

Initial fill



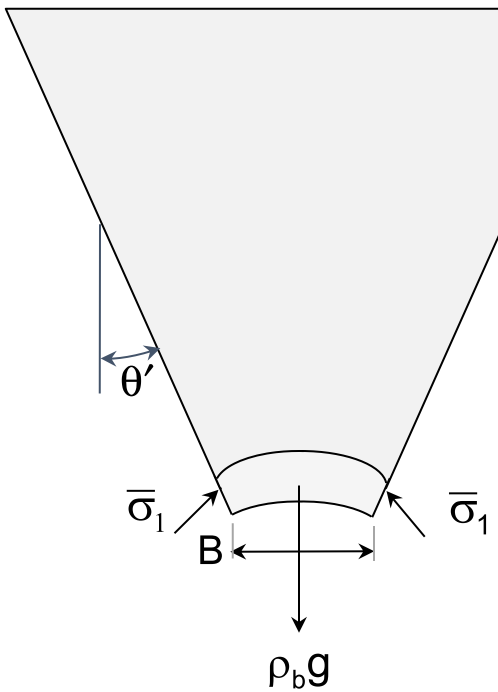
Pressure distribution

After discharge (mass flow)



Arch stress

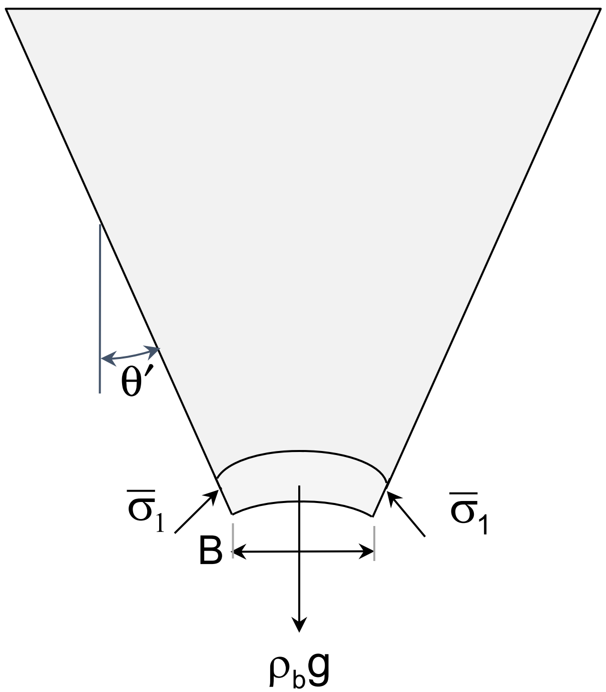
Arch stress



Stress on abutments of arch:

$$\boxed{\overline{\sigma}_1 \approx \frac{\rho_b g B}{2}}$$

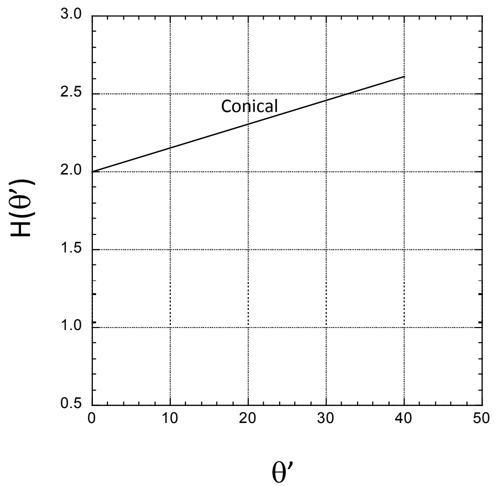
Arch stress



Stress on abutments of arch:

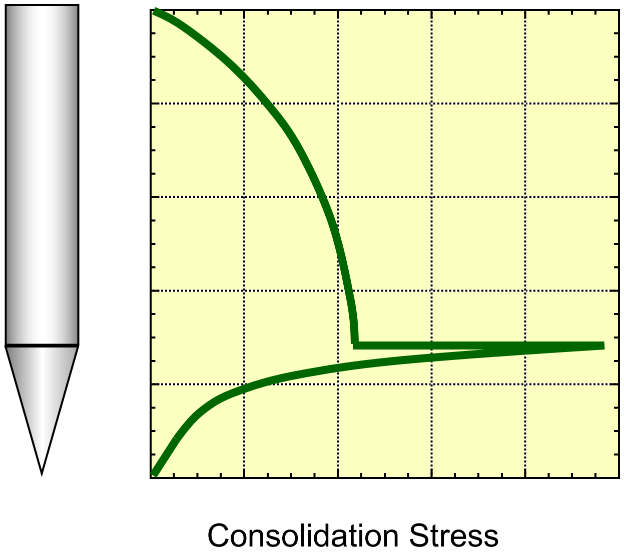
$$\boxed{\overline{\sigma}_1 = \frac{\rho_b g B}{H(\theta')}}$$

:

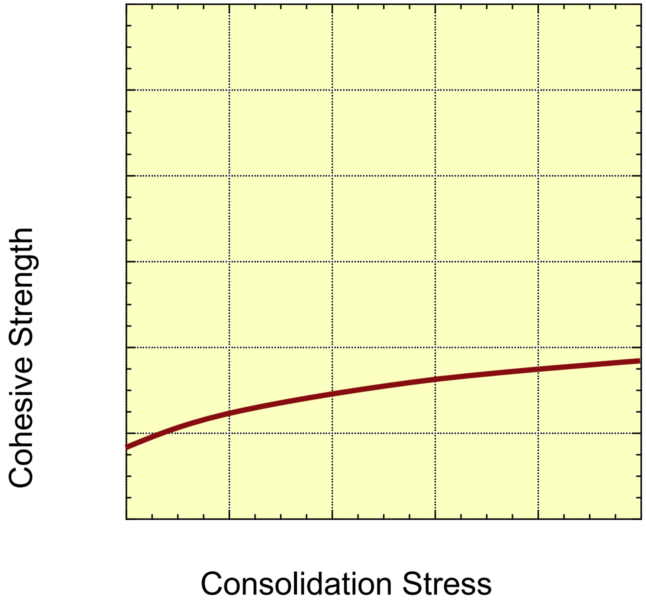


Pressure distribution

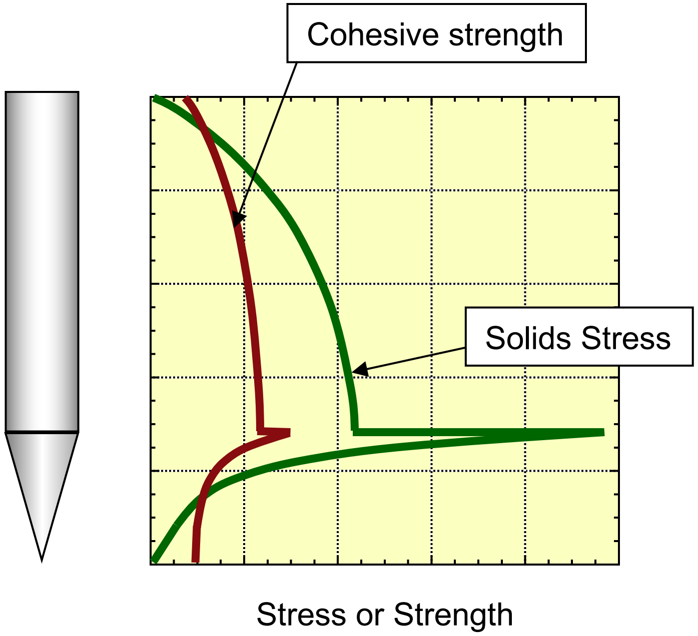
After discharge (mass flow)



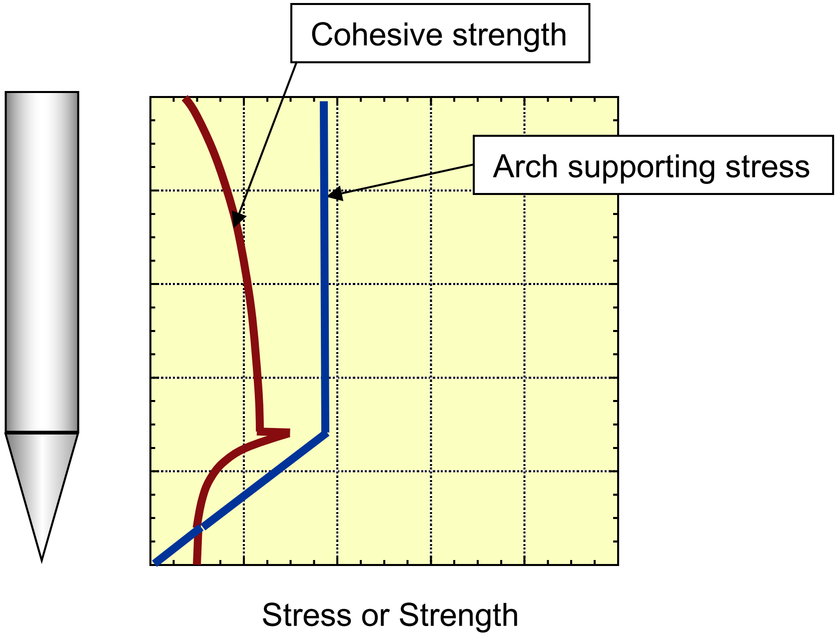
Cohesive strength



Cohesive strength

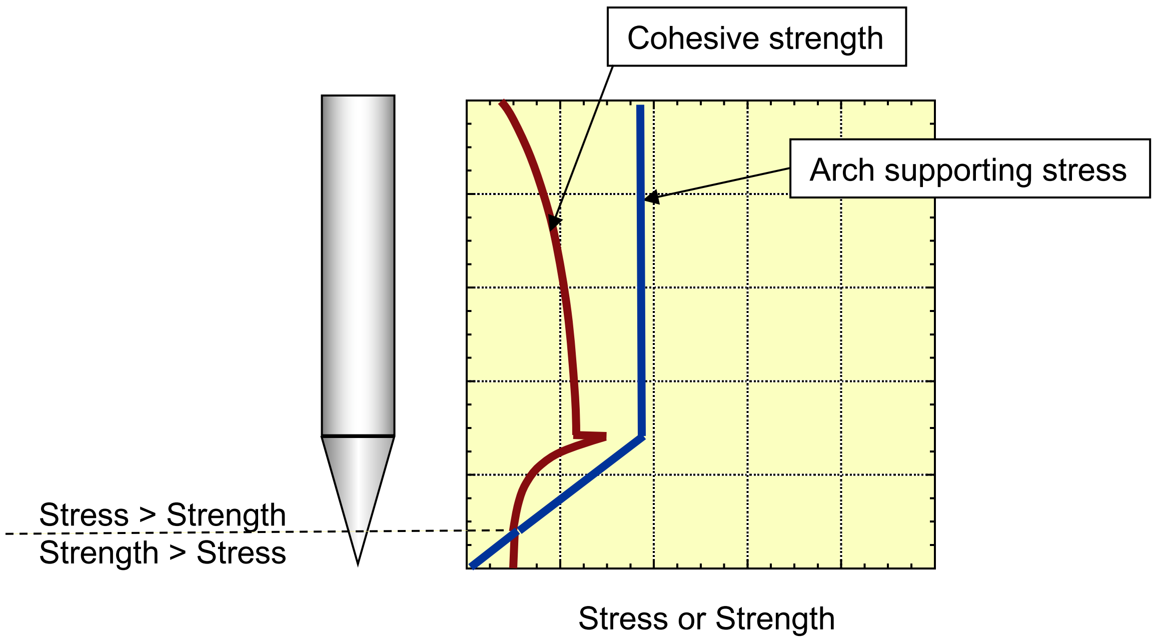


Arch stress

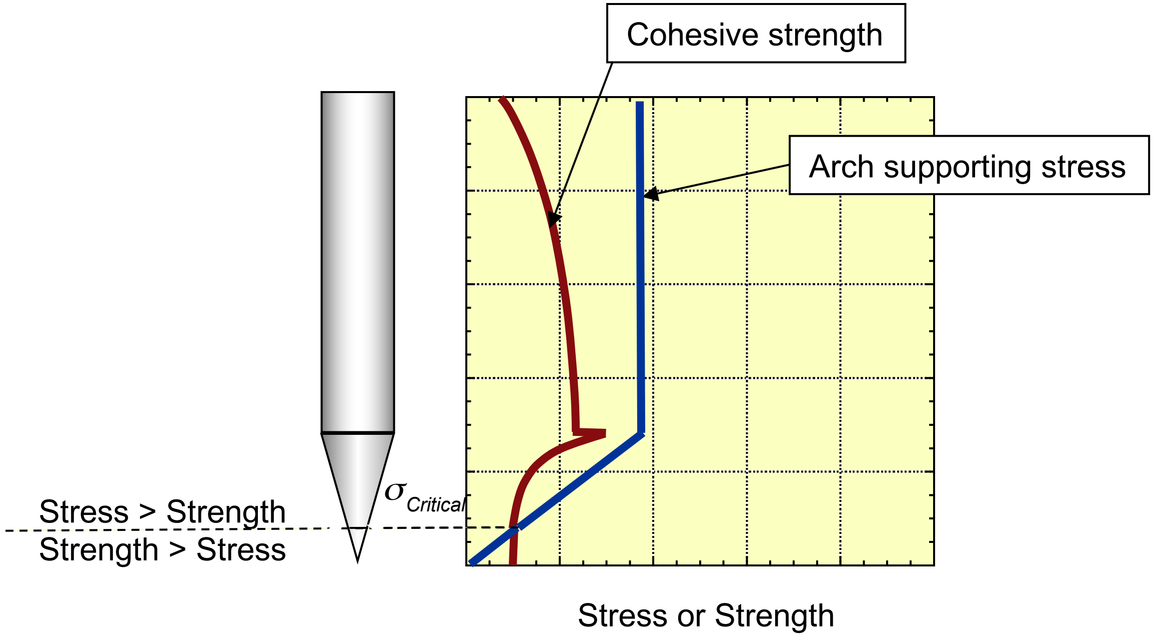


Cohesive strength
Arch supporting stress
Stress or Strength

Arch stress


Cohesive strength
Arch supporting stress
Stress > Strength
Strength > Stress
Stress or Strength

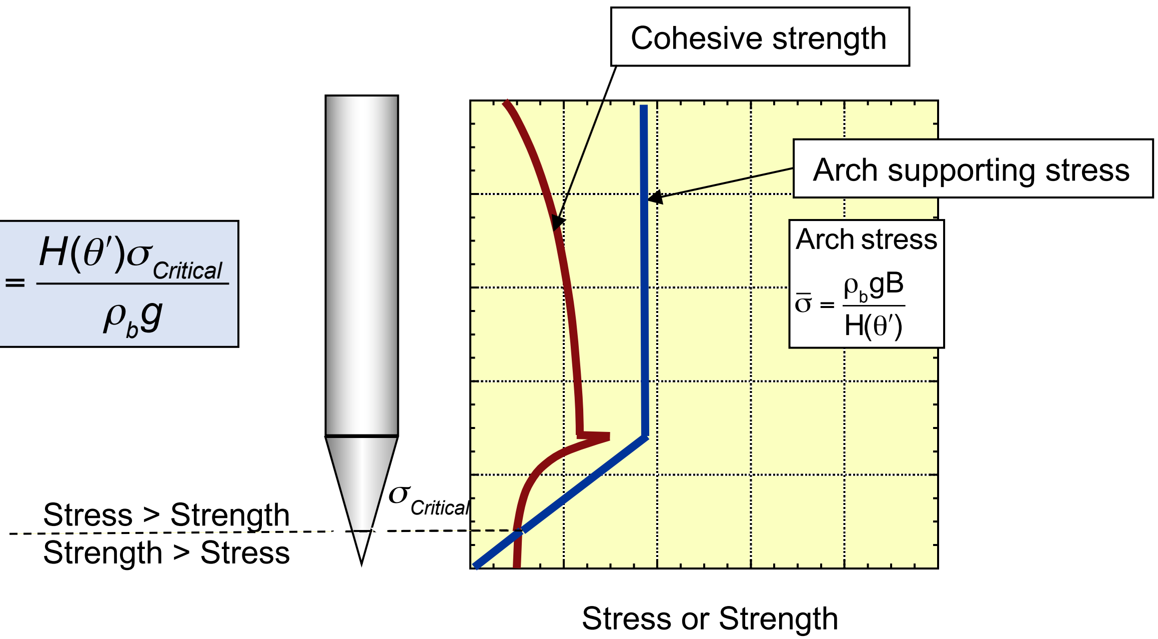
Arch stress



Cohesive strength
Arch supporting stress
$\sigma_{Critical}$
Stress > Strength
Strength > Stress
Stress or Strength

Critical arching dimension

$$B_{min} = \frac{H(\theta')\sigma_{Critical}}{\rho_b g}$$



Flow factor

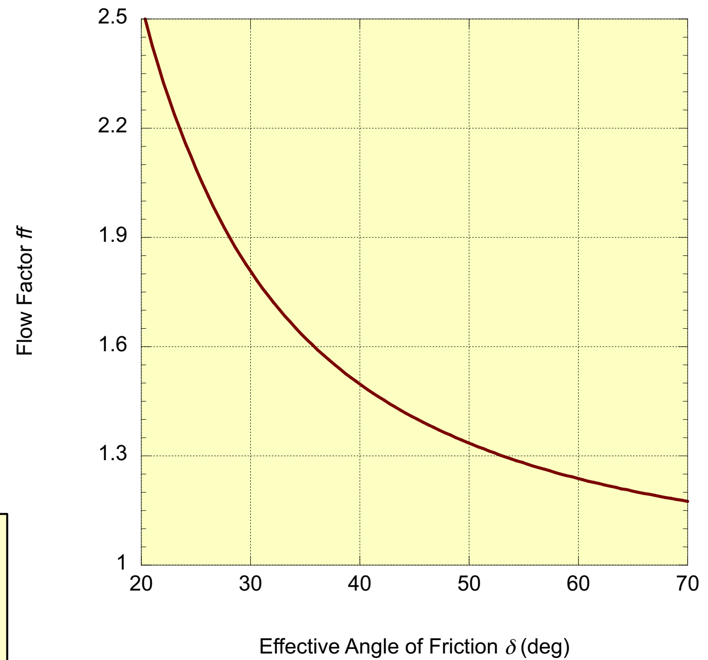
- Flow function (*FF*)

  Relationship between major principal stress and cohesive strength

- Flow factor (*ff*)

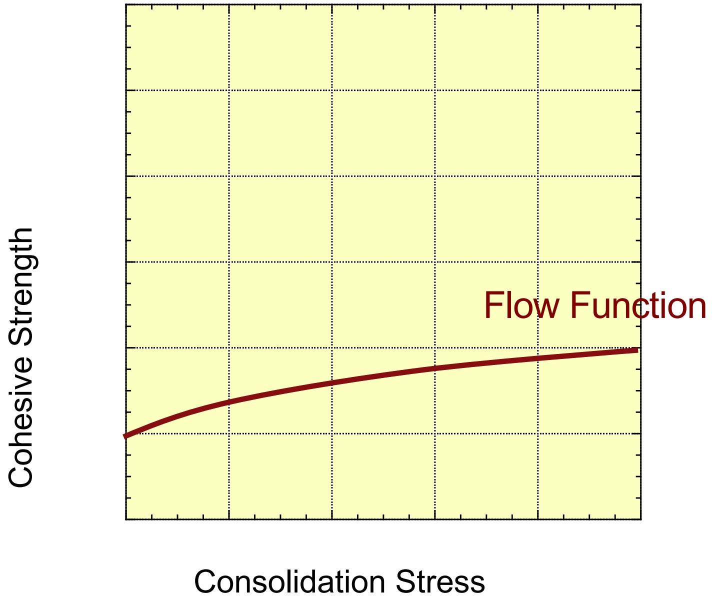
  Ratio of major principal stress and arch supporting stress

$$\text{ff} = \frac{\sigma_1}{\bar{\sigma}_1} = \frac{\text{major principle stress}}{\text{arch stress}}$$

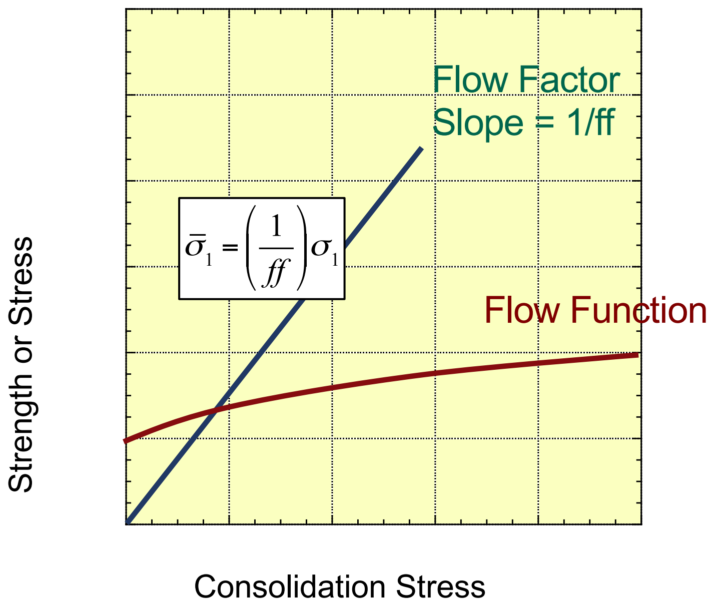


$$ff = 1.118 + \frac{0.285}{\left(\tan\delta\right)^2}$$

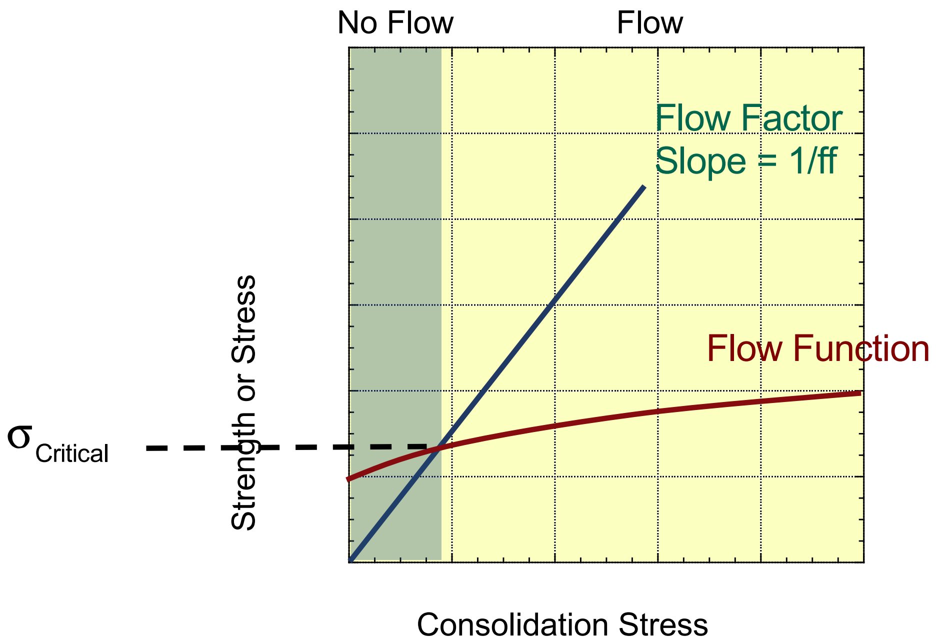
Flow function



Flow factor

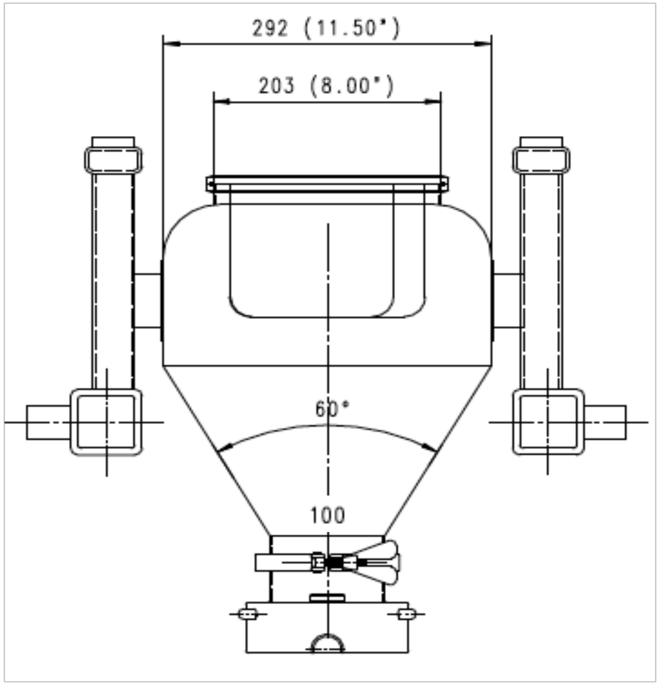

Flow Factor
Slope = 1/ff
$\bar{\sigma}_1 = \left(\frac{1}{ff}\right)\sigma_1$
Flow Function
Strength or Stress
Consolidation Stress

Critical arching diameter

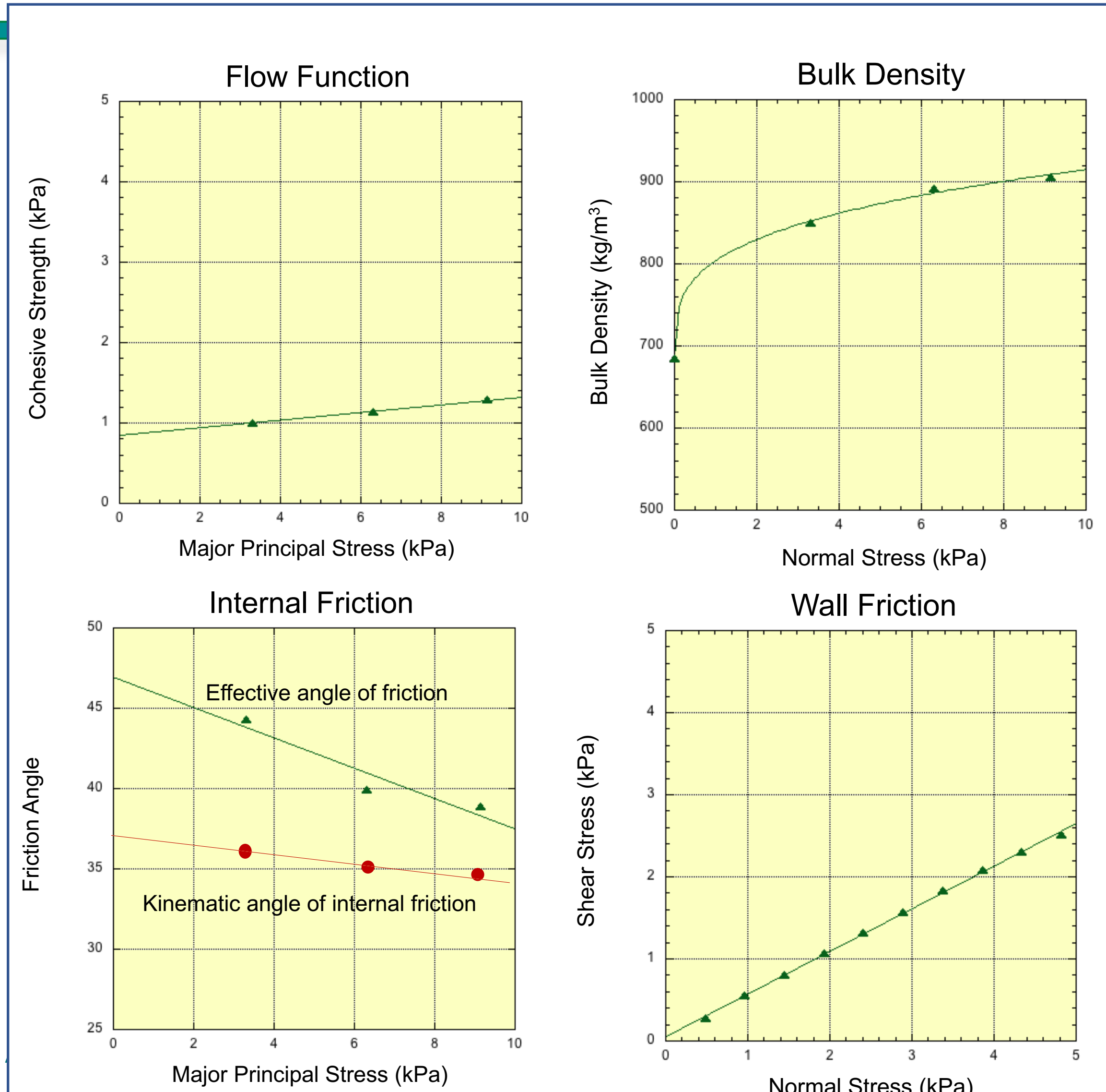


$$B_{\min} = \frac{H(\theta')\sigma_{Critical}}{\rho_b g}$$

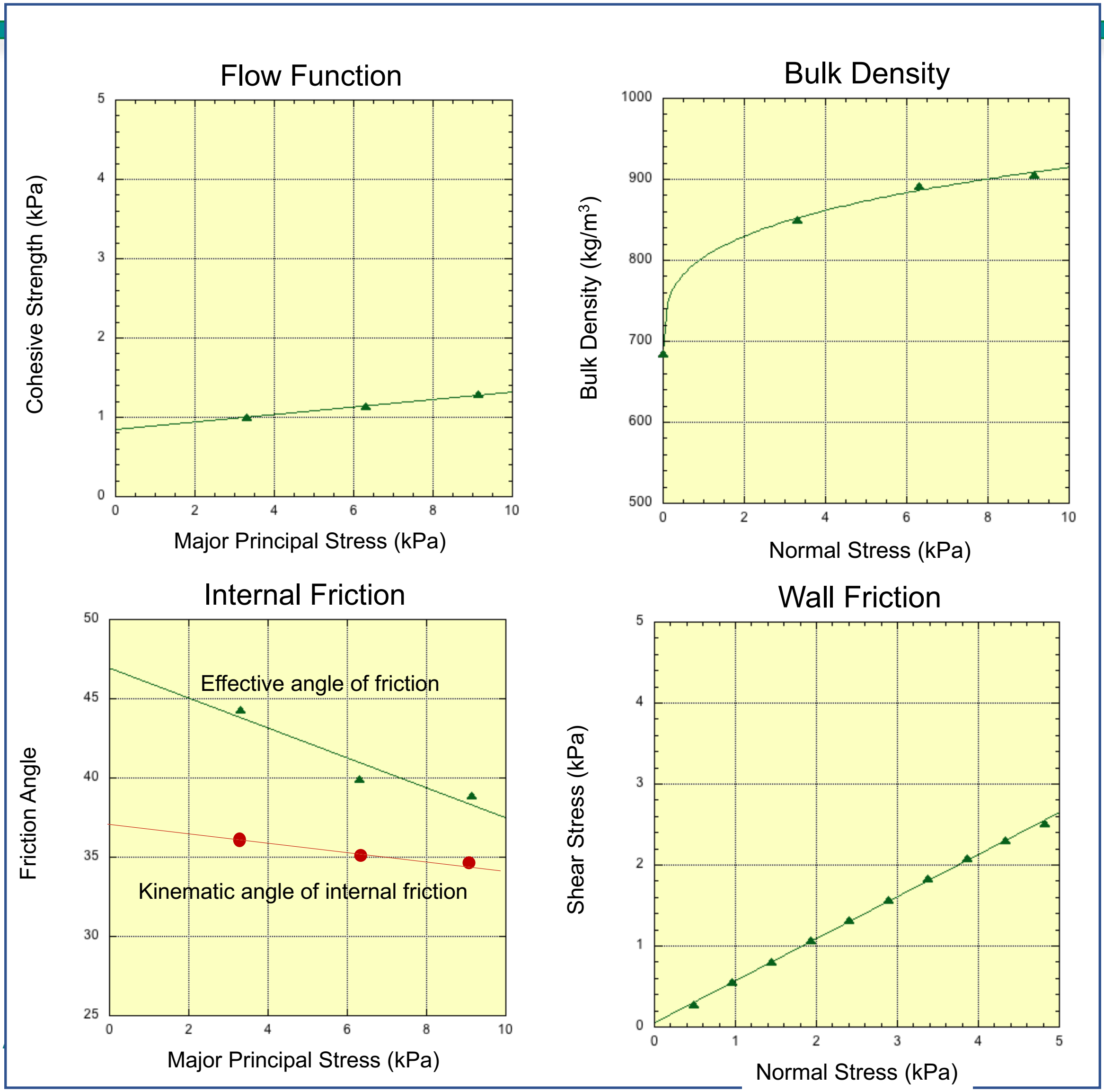
Example: existing hopper



Flow properties

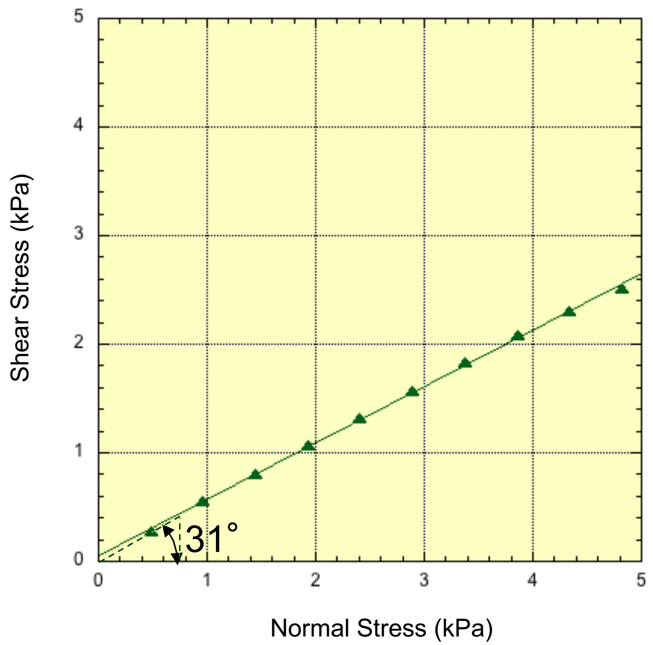


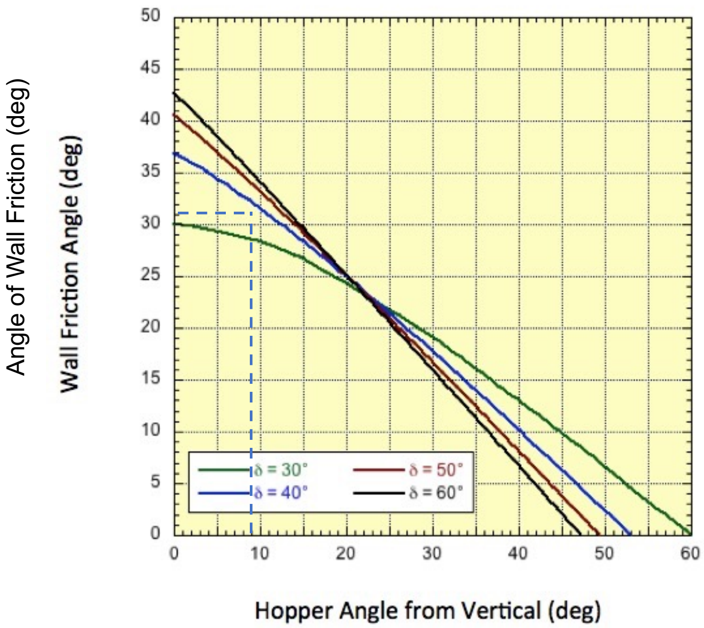
Flow properties



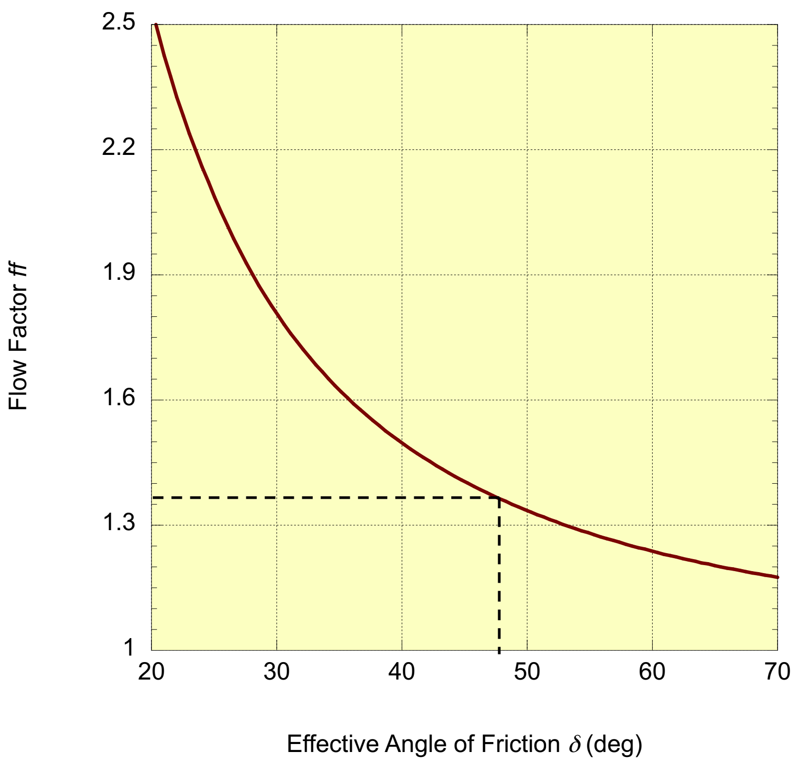
FFC = 7
“Easy-flowing”

Mass flow hopper angle

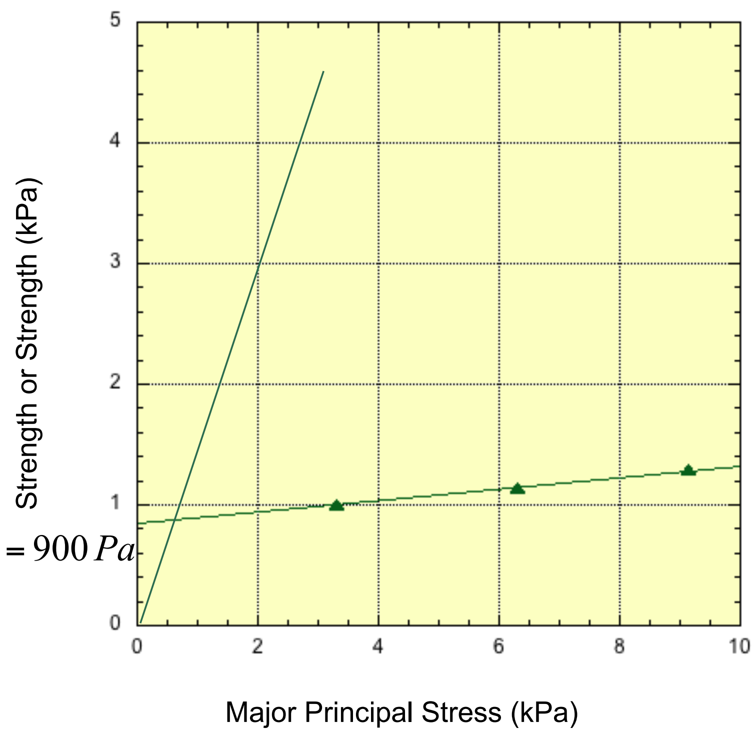




Outlet diameter

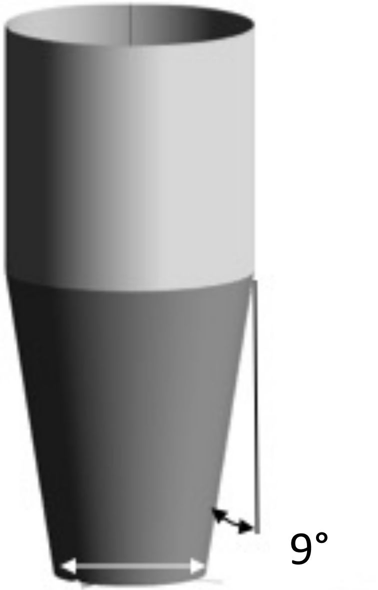


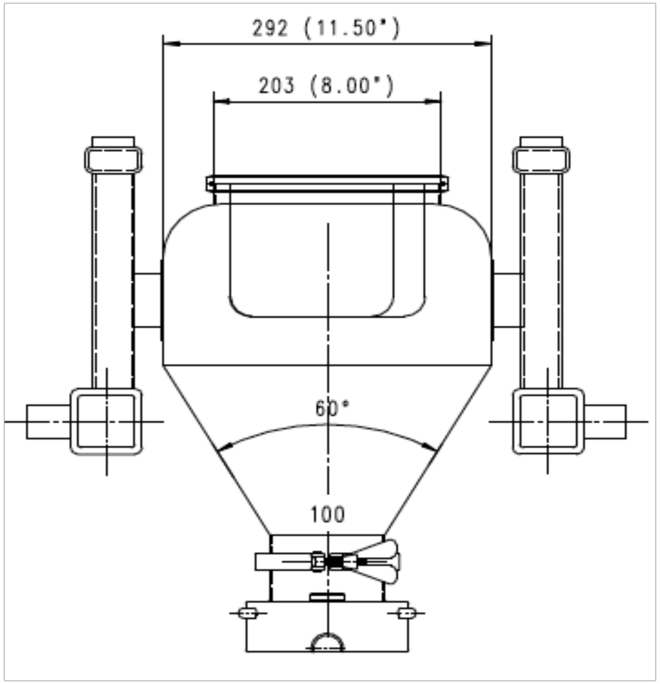
$$\sigma_{critical} = 900\,Pa$$



Recommended mass flow hopper

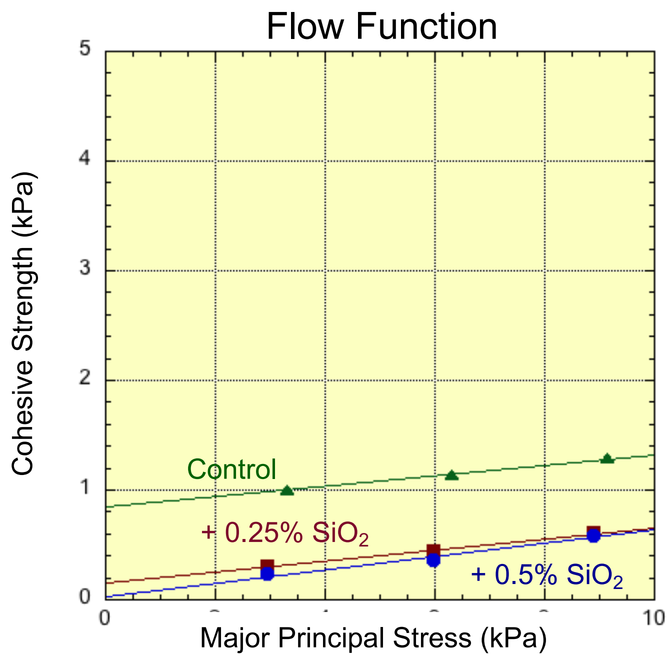
$$B_{\min} = \frac{H(\theta')\sigma_{Critical}}{\rho_b g} = \frac{(2.15)(900)(1000)}{(810)(9.8)} = 240 \text{ mm}$$

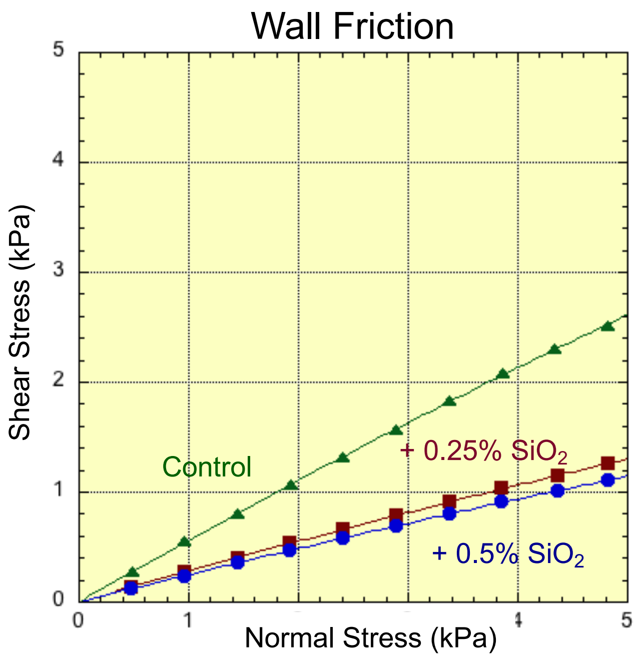




But this blend was "easy flowing!"

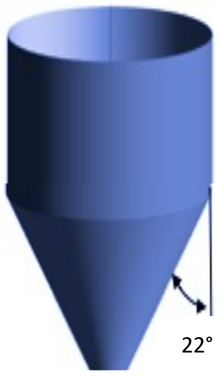
Add fumed silica (Cab-O-Sil M-5P)

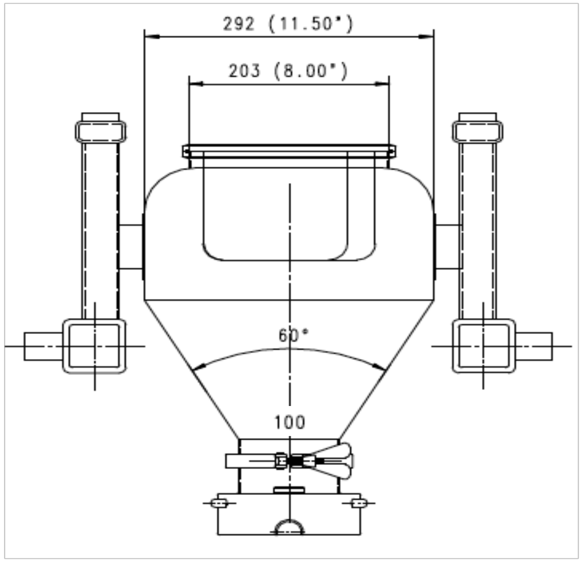




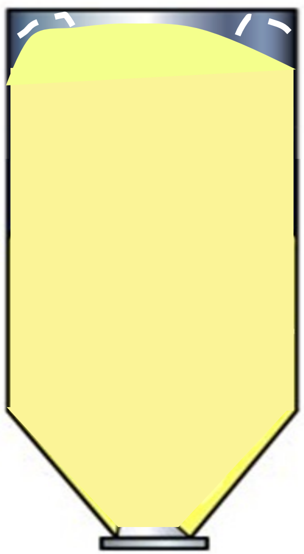
Add fumed silica (Cab-O-Sil M-5P)

FFC = 15 – "Free-flowing"
Critical arching dimension: 100mm
Rec'd mass flow hopper angle: 22°
Funnel flow

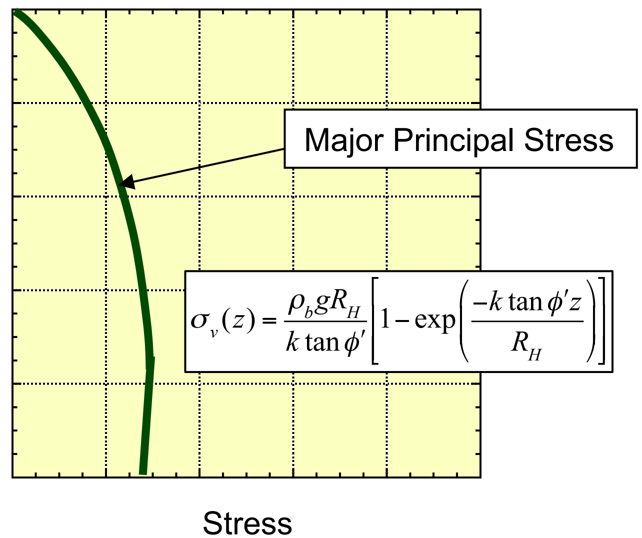




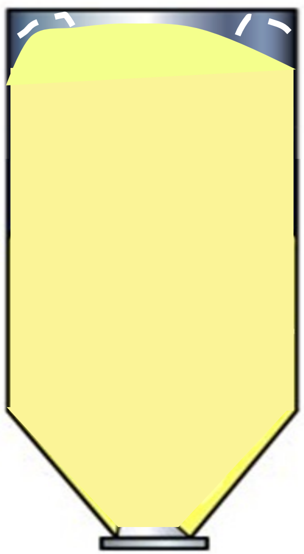
Funnel flow bins



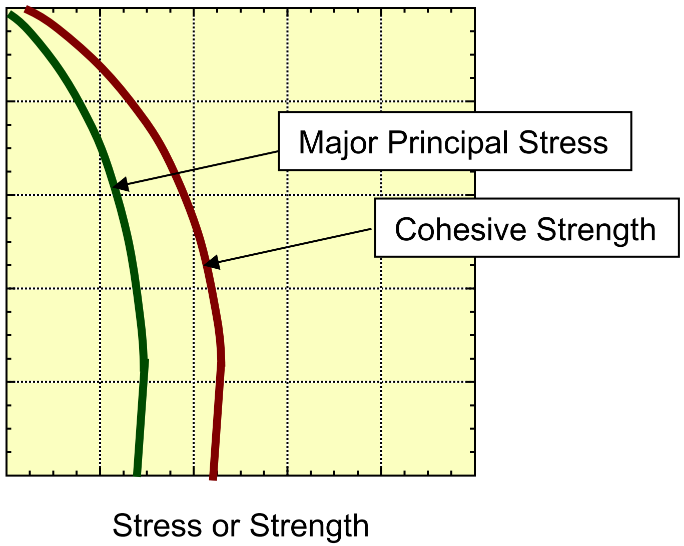
D = diameter of round outlet or *diagonal* of slotted outlet



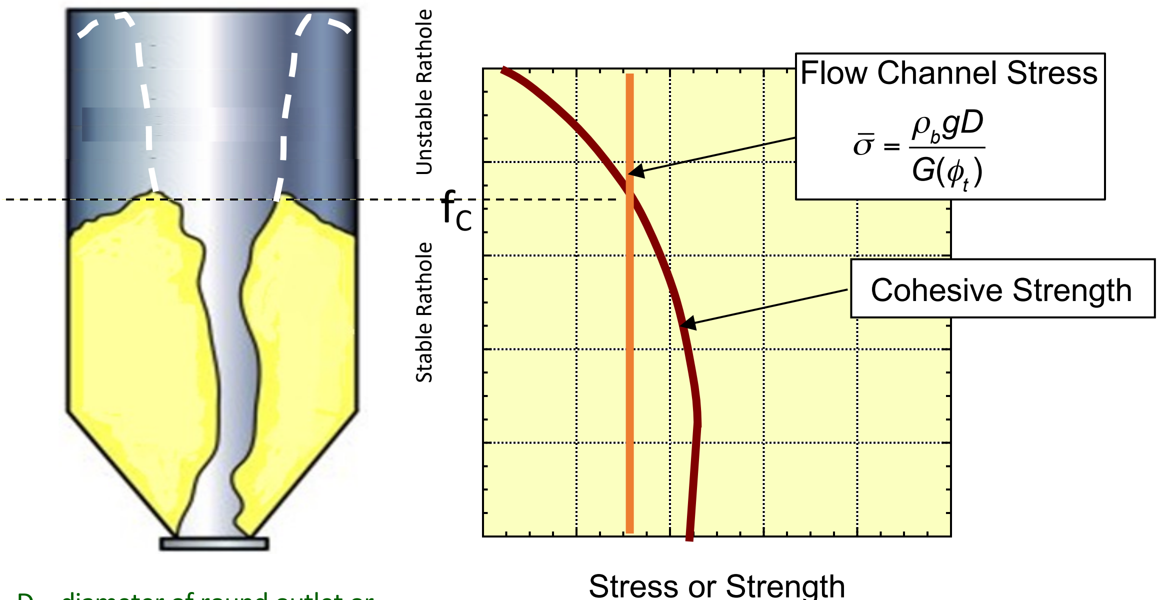
Funnel flow bins



D = diameter of round outlet or
diagonal of slotted outlet



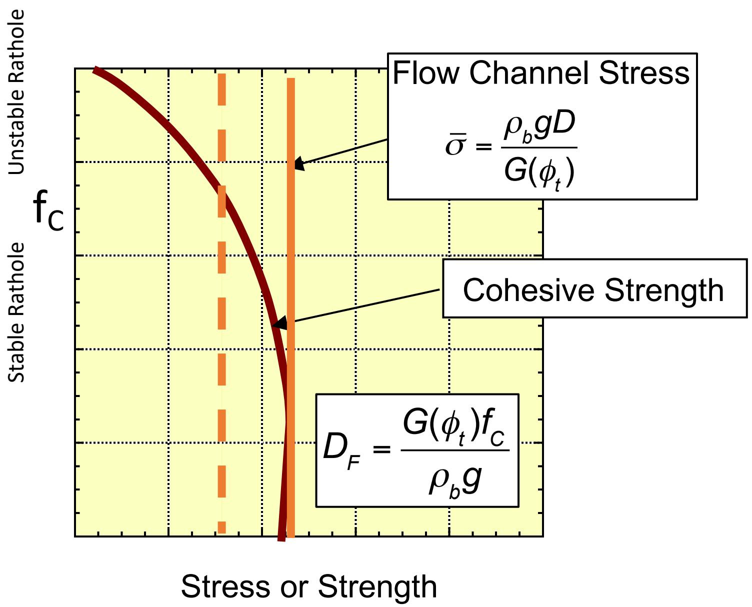
Funnel flow bins



D = diameter of round outlet or *diagonal* of slotted outlet

Funnel flow bins

D = diameter of round outlet or *diagonal* of slotted outlet

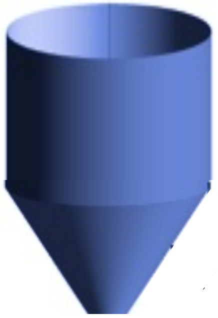


Funnel flow bin

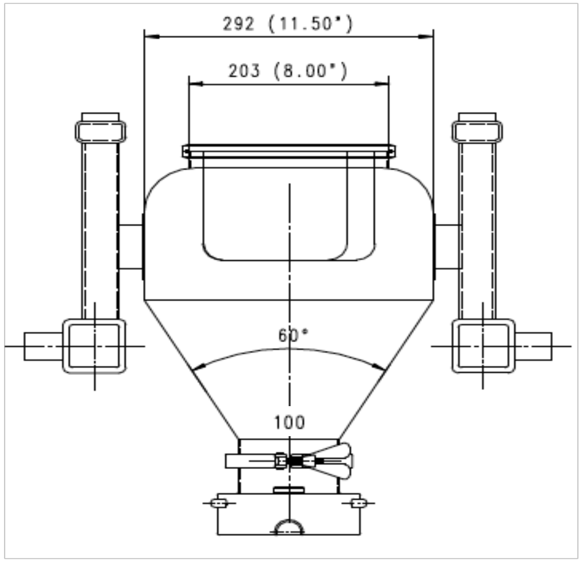
$$D_F = \frac{G(\phi_t)f_C}{\rho_b g}$$

= (2.8)(0.42)/[(780)(9.8)] = 0.15 m = 150 mm

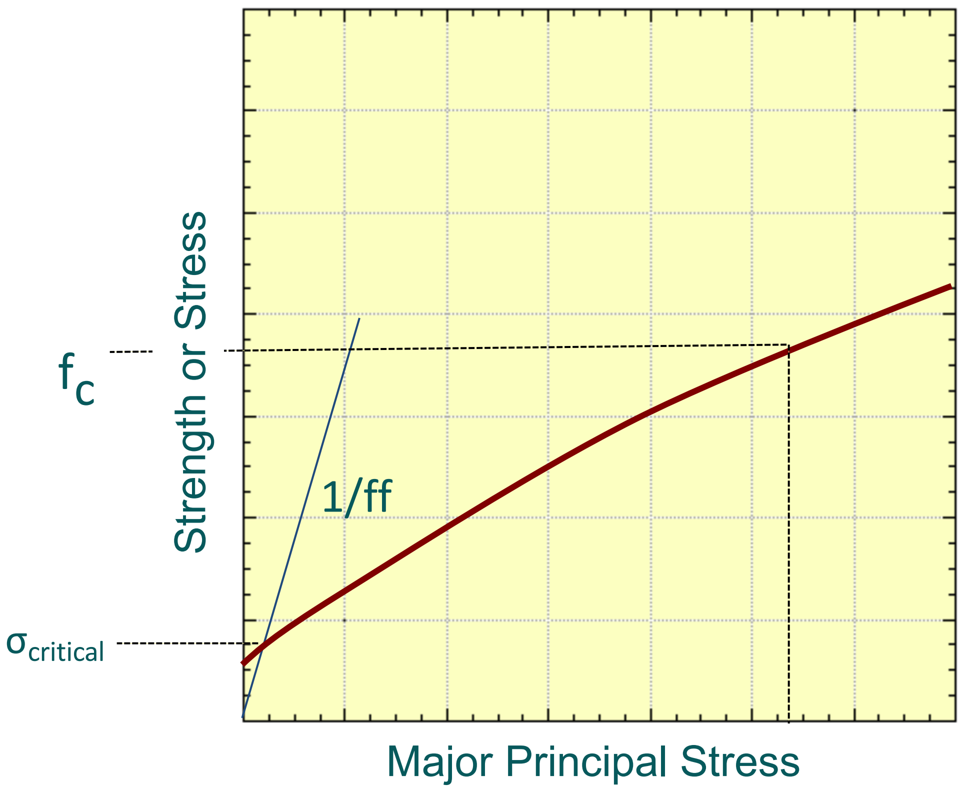
Critical rathole diameter: 150 mm



150 mm



Mass flow *vs.* funnel flow



To prevent arching in a mass flow bin:

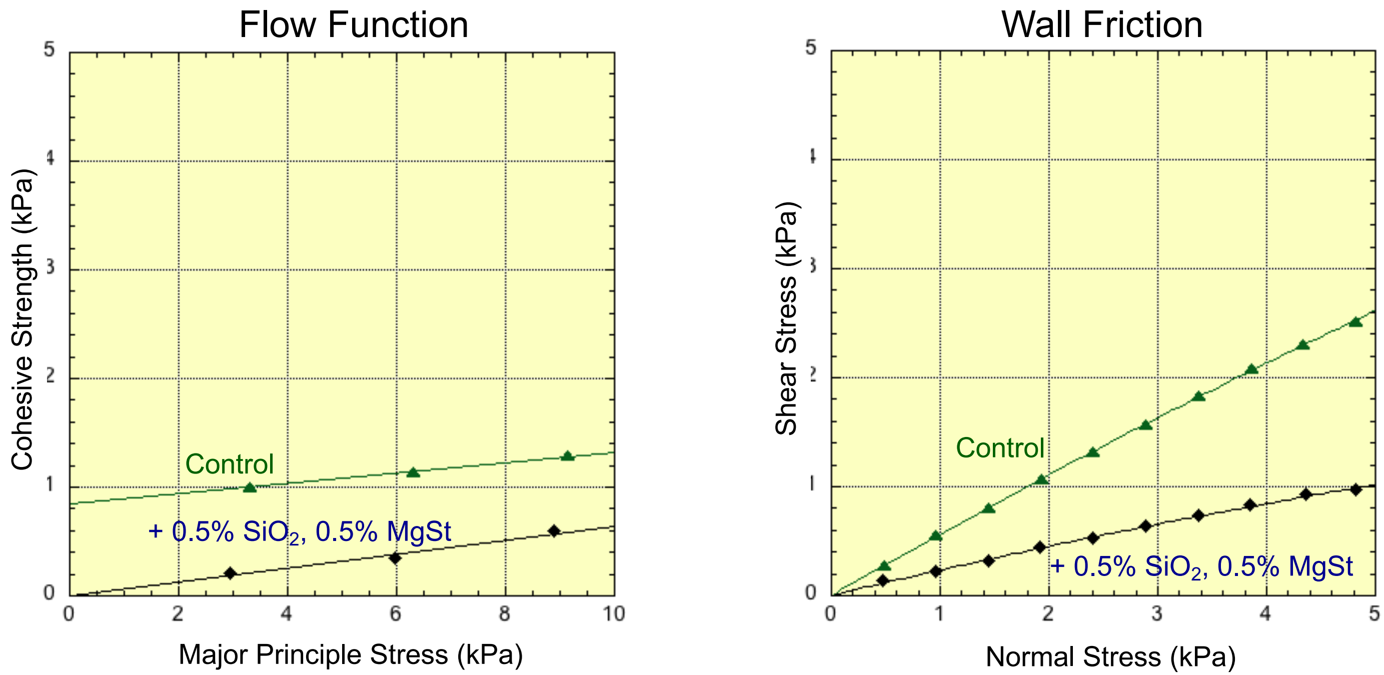
$$B_{min} = \frac{H(\theta')\sigma_{critical}}{\rho_b g}$$

To prevent a stable rathole in a funnel flow bin:

$$D_F = \frac{G(\phi_t)f_C}{\rho_b g}$$

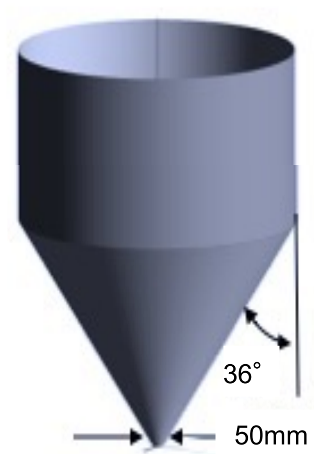
$$\sigma_v(z) = \frac{\rho_b g R_H}{k \tan\phi'}\left[1 - \exp\left(\frac{-k\tan\phi' z}{R_H}\right)\right]$$

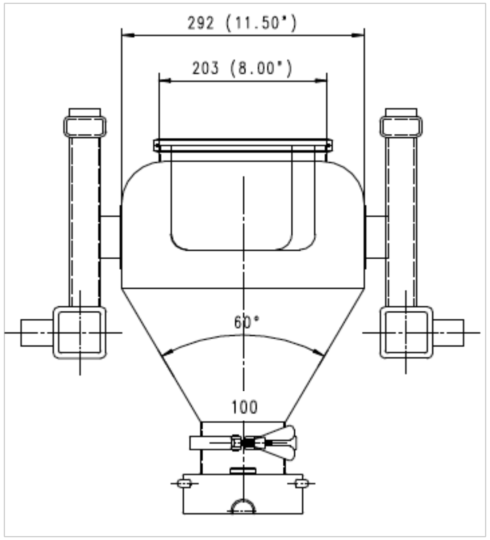
Add mag stearate



Add mag stearate

FFC = 15 – “Free-flowing”
Critical arching dimension: 50mm
Rec’d mass flow hopper angle: 36˚





Summary

- Flow properties
  - Cohesive strength
  - Internal friction
  - Compressibility
  - Wall friction
- Flowability metrics
  - Arching diameter
  - Ratholing diameter
  - Mass flow hopper angle

Summary

- Flow properties
  - Cohesive strength
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  - ~~*FFC*~~

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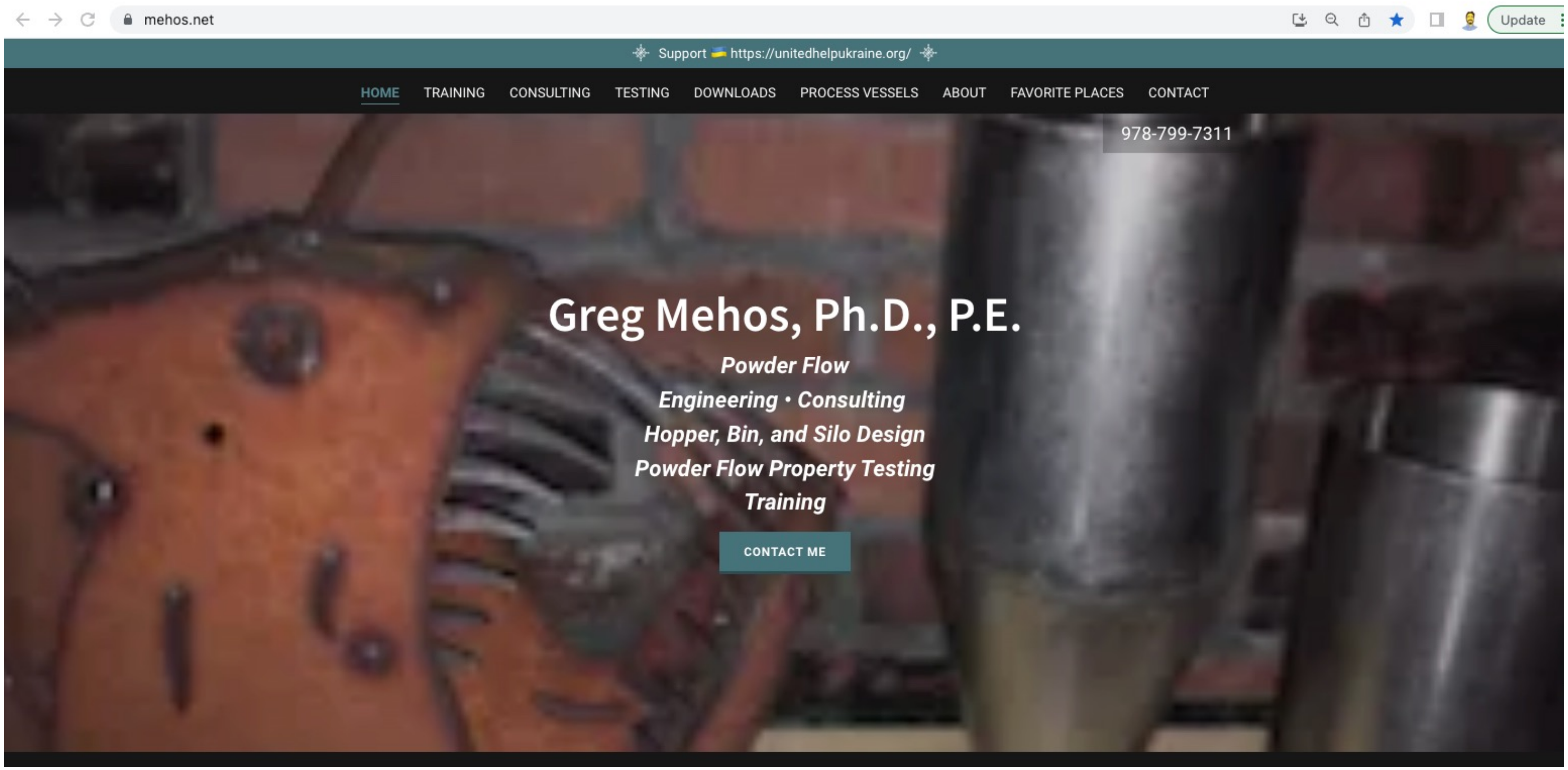

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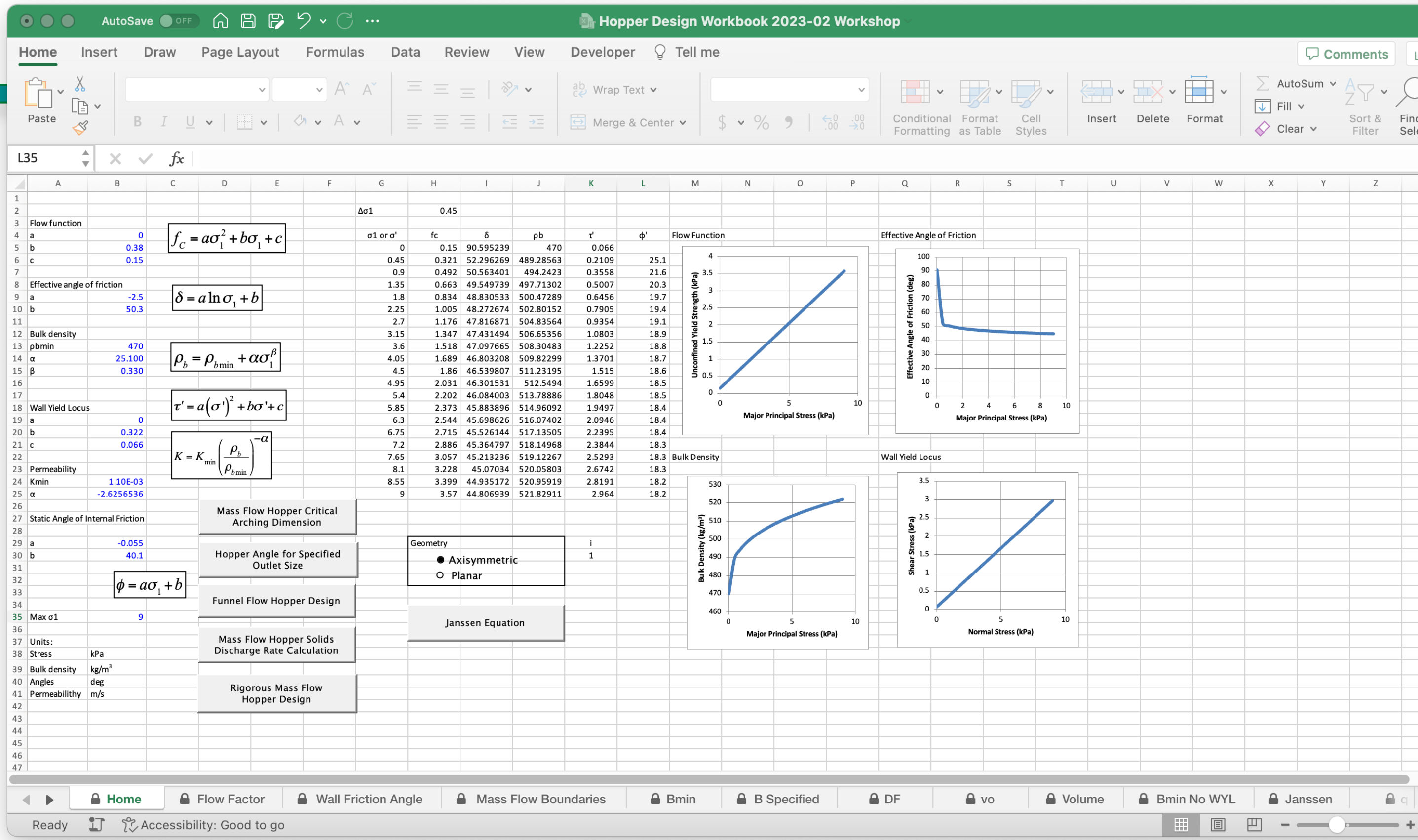

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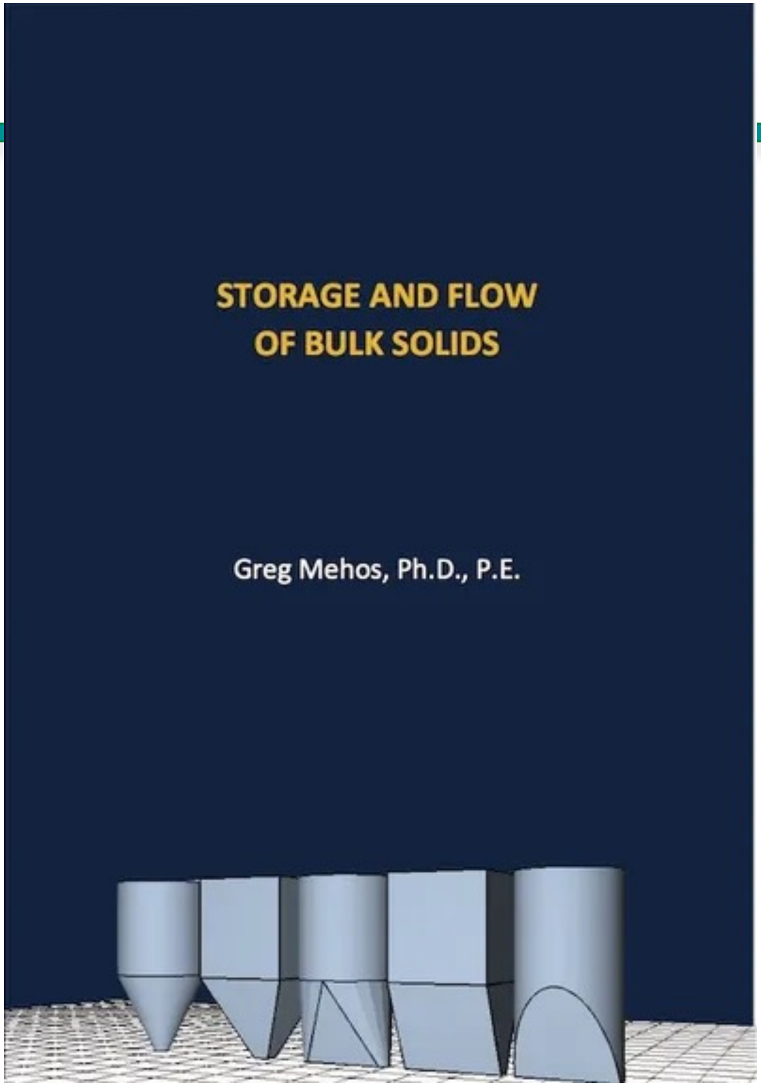
Hopper Design Files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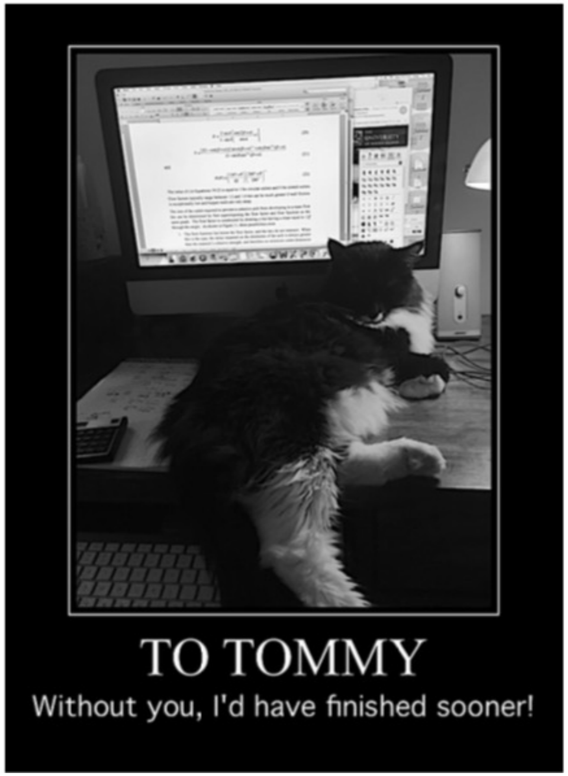
URI Notes

Password: Tommy



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