USING SOLIDS FLOW PROPERTY TESTING TO DESIGN MASS- AND FUNNEL-FLOW HOPPERS

This article describes how bulk solids testing can provide the information necessary to properly design a bulk solids storage vessel.

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Unless a hopper, bin, or silo is properly designed, flow problems such as arching, ratholing, and erratic discharge can occur. Fortunately, with the availability of automated test equipment, measuring a material's fundamental bulk solids flow properties necessary to design a storage vessel is straightforward. Using the test results to design a vessel can be tedious, however, as the associated equations that must be solved are complex, and finding a solution is an iterative process. Fortunately, spreadsheets and other computer tools allow the calculations to be performed somewhat effortlessly. This article summarizes the bulk solids testing that needs to be conducted and explains how the test results are used to design reliable hoppers, bins, and silos.

Fundamental bulk solids flow properties

The critical bulk solids flow properties to know are: 1) cohesive strength, 2) internal friction, 3) compressibility, 4) wall friction, and 5) permeability. Shear cell testers are used to measure cohesive strength, internal friction, compressibility, and wall friction. Permeability is determined by measuring the pressure drop that accompanies gas flow through a bed of the bulk solids in question.

To measure the *cohesive strength*, a powder sample is placed in the tester's cell and then presheared, during which the sample is consolidated by applying a normal stress and then shearing it until the measured shear stress is steady. Next, the shear step is conducted, in which the vertical compacting load is replaced with a smaller load. The sample is again sheared until it fails. The preshear and shear steps are repeated at the same consolidation level for a number of normal stresses. The yield locus is then determined by plotting the failure shear stress against normal stress, as shown in Figure 1. From the yield locus, the major principal stress σ_1, unconfined yield strength f_C, effective angle of friction δ, and kinematic angle of internal friction ϕ are determined. The bulk density ρ_b is also determined.

By conducting the test over a range of consolidation states, the relationship between the bulk material's major principal stress and cohesive strength (as defined

FIGURE 1

Yield locus from shear cell test

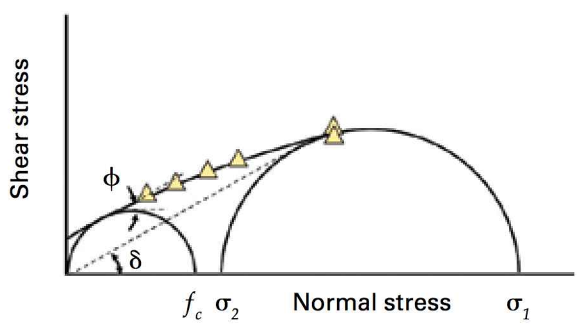
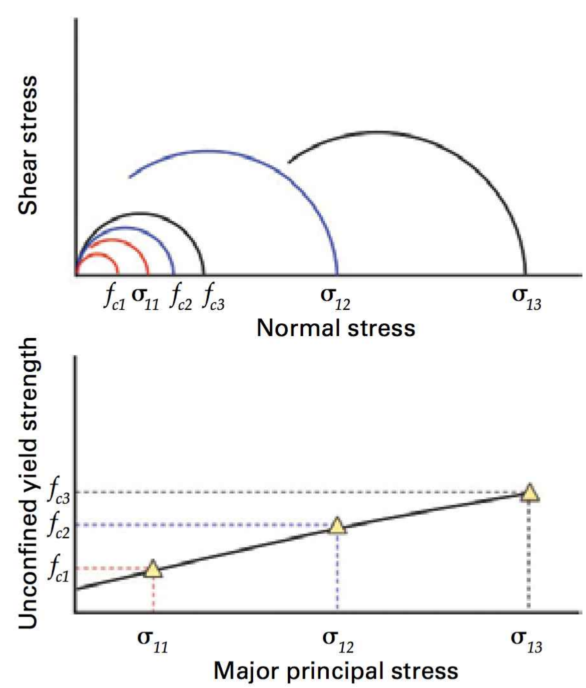


FIGURE 2

Construction of the flow function



by the unconfined yield strength) can be established. This relationship is called the material's *flow function*. Construction of the flow function from three yield loci is illustrated in Figure 2.

Wall friction is determined by measuring the stress required to slide a sample of powder along a coupon of wall material over a range of normal loads applied to the powder. The measured shear stress is plotted against normal load to give the wall yield locus, which is shown in Figure 3. The angle of wall friction ϕ' is the angle that's formed when a line is drawn from the yield locus to the origin.

Permeability is determined by passing a gas through a bed of powder contained in a cell as shown in Figure 4. The pressure drop between two locations of the bed and the gas flowrate are measured. From this, the permeability can be calculated using Darcy's Law

$$q_g = \frac{KA}{\rho_b g} \frac{\Delta P}{h} \quad (1)$$

where q_g is the volumetric gas flowrate, K is the permeability, A is the cross-sectional bed area, ΔP is the pressure drop, h is the distance between pressure measurements, and g is the gravity accelerator.

Bulk solids flow patterns

There are two general flow patterns that can occur when a bulk solid is discharged from a hopper: mass flow and funnel flow. In *mass flow*, the entire solids bed is in motion when the material is discharged from the outlet. In *funnel flow*, an active flow channel forms above the outlet with stagnant material remaining at the periphery. Funnel flow occurs when the walls of the hopper are not steep enough or have low enough friction to allow flow along them. Flow patterns are illustrated in Figure 5.

If a material is cohesive, has segregation tendencies, or readily cakes, a hopper should be designed for mass flow. Funnel flow is acceptable as long as segregation isn't a concern, the powder doesn't aerate, and the hopper outlet is large enough to prevent a stable rathole from forming.

Mass-flow hopper design

The hopper angle required to allow mass flow depends on the effective angle of friction, the wall friction angle, and the hopper's geometry. An analytical expression for the theoretical mass-flow boundary for conical hoppers is given by Equation 2[2]

$$\Theta' = 90° - ½ \cos^{-1}\left(\frac{1 - \sin\delta}{2 \sin\delta}\right) - \beta \quad (2)$$

FIGURE 3

Wall yield locus

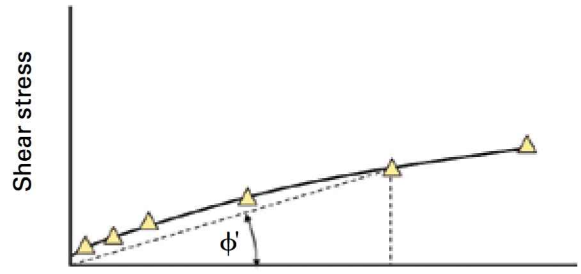


FIGURE 4

Permeability tester

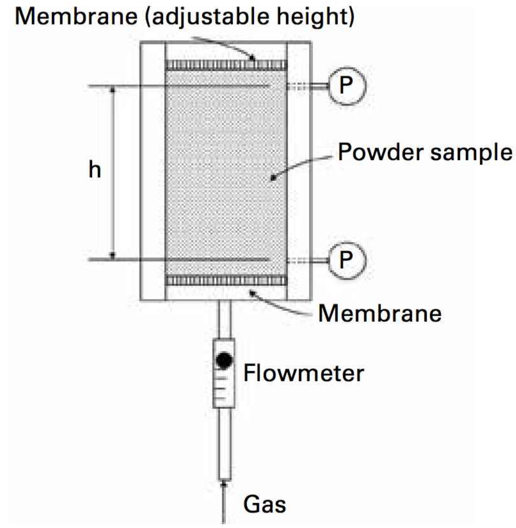
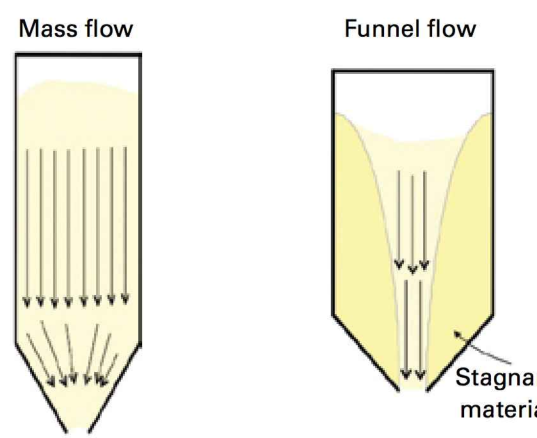


FIGURE 5

Bulk solids flow patterns



where Θ' is the hopper angle *referenced from vertical*, and

$$\beta = \tfrac{1}{2}\left(\phi' + \sin^{-1}\left(\frac{\sin\phi'}{\sin\delta}\right)\right) \qquad 3$$

When specifying a hopper angle for mass-flow conical hoppers, a safety factor of 2 to 3 degrees should be subtracted from the result of Equation 2. For hoppers with slotted outlets, the following equation can be used to calculate the recommended hopper angle[2]

$$\Theta' = \frac{\exp\left[\,3.75\,(1.01)^{(\delta-30°)/10}\,\right] - \phi'}{0.725\,(\tan\delta)^{1/5}} \qquad 4$$

for ϕ' less than δ -3 degrees. The outlet length must be at least three times its width (twice its width if the transition hopper's end walls are vertical). No safety factor applies; in fact, hoppers sloped 5 to 10 degrees greater than that suggested by Equation 4 will still provide mass flow.

The wall friction angle ϕ' depends on the effective angle of friction δ and the solids stress normal to the wall σ'. If the wall yield locus is linear, which is often true at low stresses found at a mass-flow hopper outlet, the wall yield locus can be described by

$$\tau' = a\sigma' + b \qquad 5$$

where τ' is the shear stress at the wall surface, and a and b are empirical constants determined from regression. The normal stress can then be calculated from

$$\sigma' = \frac{-\beta + \sqrt{\beta^2 - 4\alpha\gamma}}{2\alpha} \qquad 6$$

where

$$\alpha = a^2 + 1 \qquad 7$$

$$\beta = 2\,(ab - \sigma_{avg}) \qquad 8$$

and

$$\gamma = b_2 + \sigma_{avg}^2 - R^2 \qquad 9$$

where

$$R = \frac{\sigma_1 - \sigma_2}{2} \qquad 10$$

$$\sigma_{avg} = \frac{\sigma_1 + \sigma_2}{2} \qquad 11$$

and

$$\sigma_2 = \frac{1-\delta}{1+\delta}\,\sigma_1 \qquad 12$$

The wall friction angle is then calculated from

$$\phi' = \tan^{-1}\left(\frac{\tau'}{\sigma'}\right) \qquad 13$$

Jenike[1] postulated that there's a critical hopper outlet size B_{min} where the stress on the abutments of a cohesive arch that forms over the outlet is equal to the bulk solid's cohesive strength. This outlet dimension represents the minimum outlet size that prevents a stable cohesive arch from developing. Jenike defined the *flow factor*, or *ff*, as the ratio of the major consolidation stress σ_1 to the arch-supporting stress $\bar{\sigma}$:

$$ff = \frac{\sigma_1}{\bar{\sigma}} \qquad 14$$

The flow factor depends on the powder's effective angle of friction δ, the wall friction angle ϕ', and the hopper angle Θ'. Arnold and McLean[3] provide the flow factor's analytical expression

$$ff = \frac{Y\,(1 + \sin\delta)\,H\,(\Theta')}{2\,(X - 1)\,(\sin\Theta')} \qquad 15$$

where

$$X = \frac{2^i \sin\delta}{1 - \sin\delta}\left[\frac{\sin(2\beta + \Theta')}{\sin\Theta'} + 1\right] \qquad 16$$

$$Y = \frac{[\,2\,(1-\cos(\beta+\Theta'))]^i \sin\Theta'\,(\beta+\Theta')^{1-i} + \sin\beta\sin^{1+i}(\beta+\Theta')}{(1 - \sin\delta)\,\sin^{2+i}(\beta + \Theta')} \qquad 17$$

and

$$H\,(\Theta') = \left(\frac{130° + \Theta'}{65°}\right)^i \left(\frac{200° + \Theta'}{200°}\right)^{1-i} \qquad 18$$

The value of i in Equations 16-19 is equal to 1 for circular outlets and 0 for slotted outlets.

Alternatively, the following empirical equation can be used to calculate the flow factor

$$ff = \left[1.118 + \frac{0.285}{(\tan\delta)^{1.59}}\right]^{i} \left[1.125 + \frac{0.176}{(\tan\delta)^{2.90}}\right]^{1-i} \quad \mathbf{19}$$

Equation 19 is based on a relationship published by Johanson[4], who plotted flow factor values against effective angle of friction for wall friction angles greater than about 12 degrees and hopper angles near the mass-flow boundary. If a powder has exceptionally low wall friction, Equation 15 should be used to calculate the flow factor.

To determine B_{min}, the following procedure is used.

1. Estimate flow factor value. Typical values range from 1.2 to 1.7.
2. Plot flow factor and flow function values together. There are three possibilities, as shown in Figure 6.

a. The flow function lies below the flow factor and the two do not intersect. In this case, the arch-supporting stress is always greater than the material's cohesive strength, and specifying a minimum outlet dimension to prevent cohesive arching isn't necessary.

b. The flow factor and flow function intersect. The flow factor is first estimated. The solids stress at the outlet is then determined from the intersection of the flow factor and flow function. Using this value of σ_1, the effective angle of friction δ is determined from the cohesive strength test results. A new flow factor is then estimated using Equation 19. These steps are repeated until the solution converges. The minimum outlet dimension B_{min} is then calculated using Equation 20

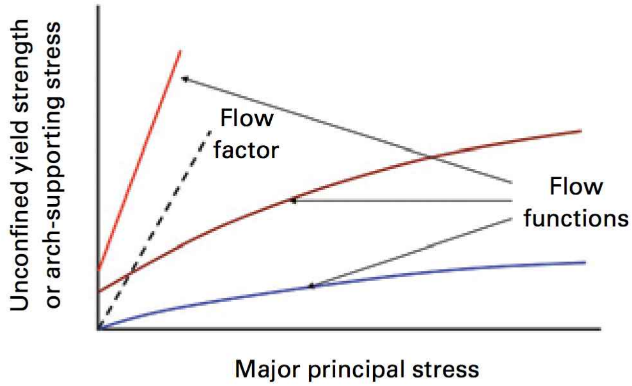
$$B_{min} = \frac{H(\Theta')\,\sigma_{crit}}{\rho_b g} \quad \mathbf{20}$$

where σ_{crit} is the unconfined yield strength or arch-supporting stress at the intersection of the flow function and flow factor. For design purposes, $H(\Theta')$ can be set equal to 2.3. Its value can be adjusted accordingly once the mass-flow hopper angle has been specified.

c. There's no intersection, and the flow function lies above the flow factor. Gravity flow will no longer be possible in a hopper with converging walls.

FIGURE 6

Unconfined yield strength or arch-supporting stress



To determine the recommended mass-flow hopper angle, the following procedure can be used:

1. The outlet dimension B, one greater than or equal to B_{min}, is specified.
2. The solids stress at the outlet is estimated from

$$\sigma_1 = \frac{ff\,\rho_b g B}{H(\Theta')} \quad \mathbf{21}$$

with $H(\Theta')$ estimated to equal 2.3 and setting ρ_b equal to a value near the material's minimum bulk density.

3. The geometry is selected, and Equation 2 or 4 is used to estimate the recommended mass-flow hopper angle using values of δ and ϕ' determined from the cohesive strength and wall friction test results respectively. (A safety factor of 2 to 3 degrees is used if the proposed hopper is conical.)
4. Equation 18 is used to update $H(\Theta')$ using the estimated value of Θ'.
5. An updated estimate of σ_1 is found from Equation 21. Note that Equation 21 is implicit in σ_1 because the bulk density ρ_b is a function of the major principal stress.
6. Using Equations 5-13, the estimate of ϕ' is updated, and an updated estimate of δ is determined from the cohesive strength test results.
7. Steps 3-6 are repeated until the solution converges.

In addition to being large enough to prevent arching, the hopper's outlet size must allow the desired discharge rate to occur. Equation 22 gives the solids velocity v at the outlet of a mass-flow hopper:[5]

$$v_o = \sqrt{\frac{Bg}{2(i+1)\tan\Theta'}\left(1 + \frac{1}{\rho_{bo} g}\left.\frac{dP}{dz}\right|_o\right)} \quad \mathbf{22}$$

where dP/dz is the gas pressure gradient and the subscript o denotes the hopper outlet. Gu *et al.*[6] derived a relationship between the air and solids flowrates that when combined with Darcy's Law gives

$$\left.\frac{dP}{dz}\right|_o = \frac{v_o g \rho^2_{bo}}{K_o}\left(\frac{1}{\rho_{bmp}} - \frac{1}{\rho_{bo}}\right) \quad \mathbf{23}$$

The subscript o denotes the hopper outlet and ρ_{bmp} is the material's bulk density at a location inside the hopper where gas pressure is at a minimum. Combining Equations 22 and 23 yields a quadratic

$$\left[\frac{2\,(i+1)\,tan\Theta'}{Bg}\right]v^2_o + \left[\frac{1}{K_o}\left(1 - \frac{\rho_{bo}}{\rho_{bmp}}\right)\right]v_o - 1 = 0 \quad \mathbf{24}$$

Funnel-flow hopper design

For funnel-flow hoppers, the outlet must be large enough to prevent a stable rathole from forming. The critical rathole diameter is calculated by first determining the maximum major consolidating stress σ_1 on the bulk solid. The maximum stress can be estimated by the Janssen equation:

$$\sigma_1 = \frac{\rho_b g R_H}{k\,tan\phi'}\left[1 - exp\left(\frac{-k\,tan\phi' h}{R_H}\right)\right] \quad \mathbf{25}$$

where R_H is the hydraulic radius of the cylinder, h is the solids depth in the cylinder section, and k is the Janssen coefficient, which is approximately 0.4 for most powders.

The critical rathole diameter D_F is the diameter of a conical funnel-flow hopper's outlet or the diagonal of a planar funnel-flow hopper's outlet that must be exceeded to ensure that a rathole collapses. D_F can be calculated from:[1]

$$D_F = \frac{G(\phi)\,f_c}{\rho_b g} \quad \mathbf{26}$$

where $G(\phi)$ is a function provided by Jenike[1]. McGlinchey[7] provides the following analytical expression for $G(\phi)$:

$$G(\phi) = 4.3\,tan\phi \quad \mathbf{27}$$

Example calculation

Consider a powder having the flow properties shown in Figures 7a. through 7d. Regression of the data gives the relationships shown in Table I.

The flow function empirical relationship was found by fitting the data to a quadratic, fixing the intercept equal to a value obtained by linear extrapolation of the two lowest data points.

First, design a mass-flow hopper using the following steps. (Figure 8 is a flow chart of the design procedure.)

1. Estimate *ff*. Choose $ff = 1.3$.

2. Determine σ_1 from the intersection of the flow function and a line through the origin with a slope equal to 1/1.3. By setting $f_C = \overline{\sigma}$, the solution to the simultaneous equations Equation E.1 and $\overline{\sigma} = \sigma_1/1.3$ gives $\sigma_1 = 0.26$ kPa.

3. Calculate δ. Substitution of $\sigma_1 = 0.26$ kPa into Equation E.2 gives δ = 41.7 degrees.

4. Update flow factor. Solving Equation E.2 with δ = 41.7 degrees gives $ff = 1.46$.

5. Update σ_1 from intersection of the flow function and the updated flow factor gives 0.30 kPa.

6. Update δ. Solving Equation E.3 with updated value of σ_1 gives δ = 41.7 degrees.

7. Solving Equation E.2 gives $ff = 1.46$. The solution has converged.

TABLE I

Regression results

Flow function: $f_C = 0.177 + 0.0939\,\sigma_1 - 0.00177\,\sigma_1^2$	**E.1**
Effective angle of friction: $\delta = 41.7 - 0.157\,\sigma_1$	**E.2**
Kinematic angle of internal friction: $\phi = 35.3 - 0.0312\,\sigma_1$	**E.3**
Bulk density: $\rho_b = 303.6 + 39.77\,\sigma_1^{0.517}$	**E.4**
Wall yield locus: $\tau' = 0.0395 + 0.269\,\sigma'$	**E.5**
Permeability: $K = 0.022$ at material's minimum bulk density	

Units: stress, kPa, bulk density, kg/m³, internal friction, degrees, permeability, and meters/second.

8. Calculate σ_{crit}. $\sigma_{crit} = \sigma_1/ff = 0.30/1.46 = 0.20$ kPa.
9. Find B_{min}. From Equation E.4, $\rho_b = 325$ kg/m³. Setting $H(\Theta') = 2.3$ and solving Equation 20 gives $B_{min} = 0.12$ meters (4.9 in.).
10. Specify the outlet diameter. For this example, choose $B = 0.15$ meters (5.8 in.).
11. Estimate σ_1. From Equation 21 after setting $H(\Theta') = 2.3$, $ff = 1.3$, and $\rho_b = 304$ kg/m³, $\sigma_1 = 0.34$ kPa.
12. Update δ and ϕ'. From Equation E.2, $\delta = 41.6$ degrees. Solving Equations 6-13 gives $\phi' = 21.6$ degrees.
13. Estimate recommended mass-flow hopper angle. From Equation 2 after subtracting a 3-degree safety factor, $\Theta' = 20.3$ degrees.
14. Update ff. Solution of Equation 19 gives $ff = 1.39$.
15. Update σ_1. Solution of Equation 21 using $H(20.3$ degrees$) = 2.31$ gives $\sigma_1 = 0.39$ kPa.
16. Update δ and ϕ'. From Equation E.2, $\delta = 41.6$ degrees. Solving Equations 6-13 gives $\phi' = 21.6$ degrees.
17. Update recommended mass-flow hopper angle. From Equation 4 after subtracting a 3-degree safety factor, $\Theta' = 21.6$ degrees.
18. Calculate ff. Updated $ff = 1.40$.
19. Update σ_1. Solution of Equation 21 with $H(21.6$ degrees$) = 2.33$ gives $\sigma_1 = 0.39$ kPa. Solution has converged.

Our recommended mass-flow conical hopper has an 8-inch-diameter outlet (minimum arching diameter is 5.8 inches) and walls sloped 22 degrees from vertical if fabricated using the same material used in the wall friction test.

To confirm that the recommended mass-flow hopper will allow the desired steady discharge rate, solve Equation 22. The discharge rate depends on the dimensions of the storage vessel's cylinder section. For this example, specify a 4-foot-diameter, 15-foot-tall cylinder. For design purposes, estimate ρ_{bmp} to equal the bulk density of the material at the bottom of the cylinder.[8] Assuming a wall friction angle of 17 degrees and an average bulk density of 380 kg/m³, the Janssen equation (Equation 25) gives a solids stress at the bottom of the cylinder equal to 8.4 kPa. At this stress, $\rho_b = 423$ kg/m³. Solving Equation 24 gives $v_o = 0.081$ m/s. The solids discharge rate is then equal

FIGURE 7

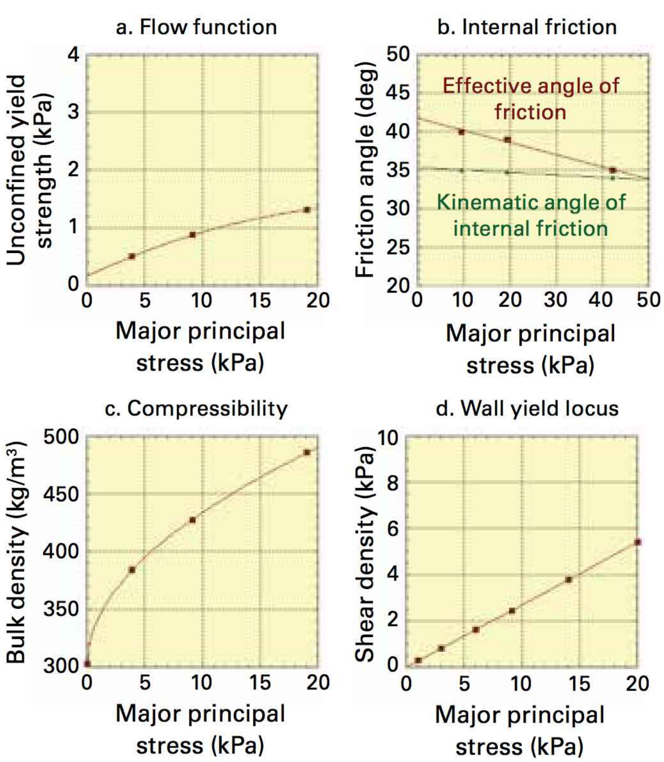
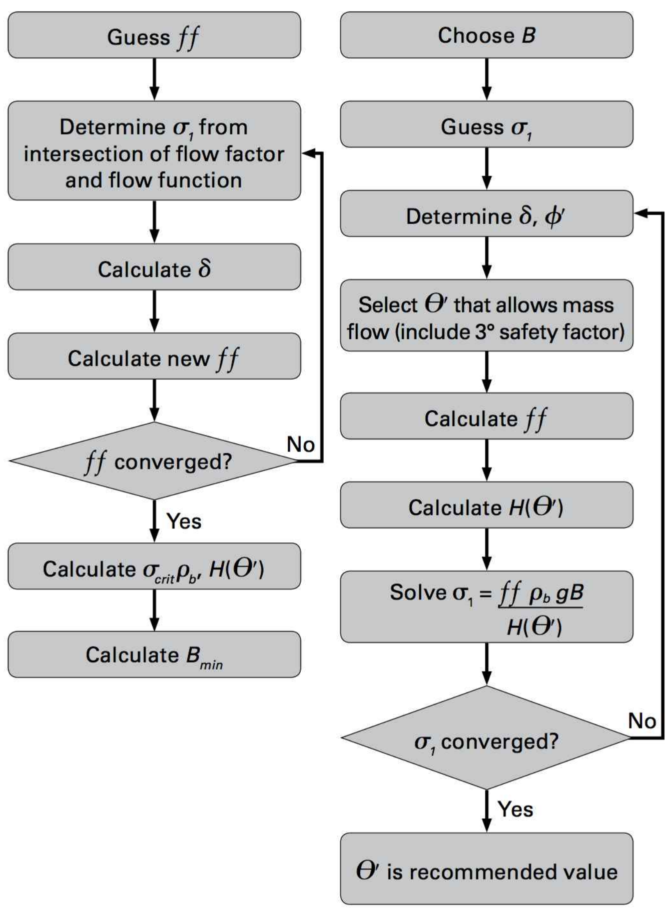


FIGURE 8

Flow chart for calculating critical arching dimension (left) and recommended mass-flow hopper angle (right).



to (0.081 m/s) $(0.023^2\pi/4\ m^2)(304\ kg/m^3)(3600\ s/hr)$ = 2,900 kg/hr (3.2 ton/hr).

Now consider a funnel-flow hopper. The critical rathole diameter depends on the cylinder dimensions. From Equations E.1 and E.3, at σ_1 = 8.4 kPa, f_C = 0.84 kPa and ϕ = 35 degrees, respectively. From Equation 27, $G(\phi)$ = 3.0. Solving Equation 26 for D_F gives a critical rathole diameter of 0.61 m (24 in.).

A funnel-flow conical hopper may not be practical due to the large outlet diameter required to prevent stable ratholes from developing. However, a hopper section that transitions from the cylinder to a slotted outlet may be an option provided that the diagonal of the outlet is greater than 0.61 m. The width of the outlet must be large enough to prevent bridging. Jenike[1] recommends using a flow factor of 1.7 to determine the critical arching dimension. For a transition hopper beneath a 1.2-meter-diameter, 4.5-meter-tall cylinder, and filled with bulk material, the outlet width should be greater than or equal to 0.18 m (6.9 in.).

Editor's note: If you're interested in software that solves hopper design equations, please contact the author.

References

1. Jenike, A., Storage and Flow of Solids – Bulletin 123, University of Utah, Salt Lake City, Utah, 1964.
2. Arnold, P.C., A.G. McLean, and A.W. Roberts, Bulk Solids: Storage, Flow, and Handling, TUNRA Publications, 1989.
3. Arnold, P.C. and A.G. McLean, "Improved Analytical Flow Factors for Mass-Flow Hoppers", Powder Techn., 15, 2, 279 (1976).
4. Johanson, J., "Design of Bins and Hoppers", Chapter 21 of Kulwiec, R. (ed.), Materials Handling Handbook, John Wiley and Sons, Hoboken, NJ, 1985.
5. Schulze, D., "Powders and Bulk Solids: Behavior, Characterization, Storage and Flow," Springer, Berlin, 2008.
6. Gu, Z.H., P.C. Arnold, and A.G. McLean, "Prediction of Flow Rate from Mass Flow Bins with Conical Hoppers", Powder Techn., 72, 39 (1992).
7. McGlinchey, Bulk Solids Handling – Equipment Selection and Operation, Blackwell Publishing Company, Ames, IA, 2008.
8. Johanson, K., "Successfully Dealing with Erratic Flow Rates", Powder Pointers, 3, A (2009).

For further reading

Find more information on this topic in articles listed under "Solids flow" and "Storage" in *Powder and Bulk Engineering*'s article index in the December 2019 issue or the article archive on *PBE*'s website, www.powderbulk.com.

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