

TECHNICAL DIFFICULTIES

**PLEASE
STAND
BY**





Flow and Storage of Powders

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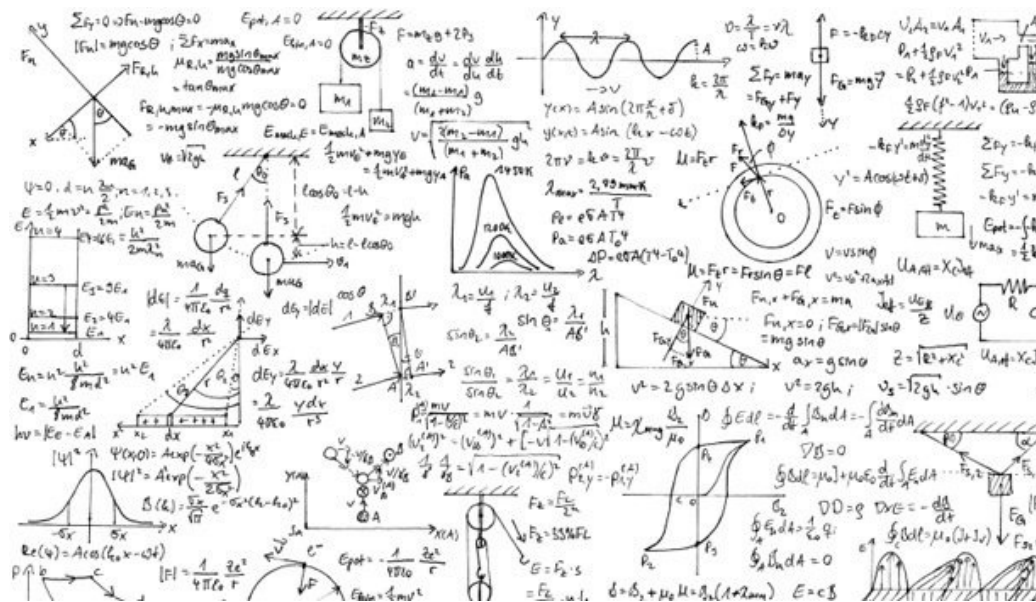
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Flow and Storage of Powders



Flow and Storage of Powders



Flow and Storage of Powders



Flow and Storage of Powders



Goals

- Understand *fundamental* powder flow properties
- Predict flow behavior of powders
- Know how to design/modify equipment for reliable powder handling



Goals



Goals



Definitions

Bulk solid



“Powder”

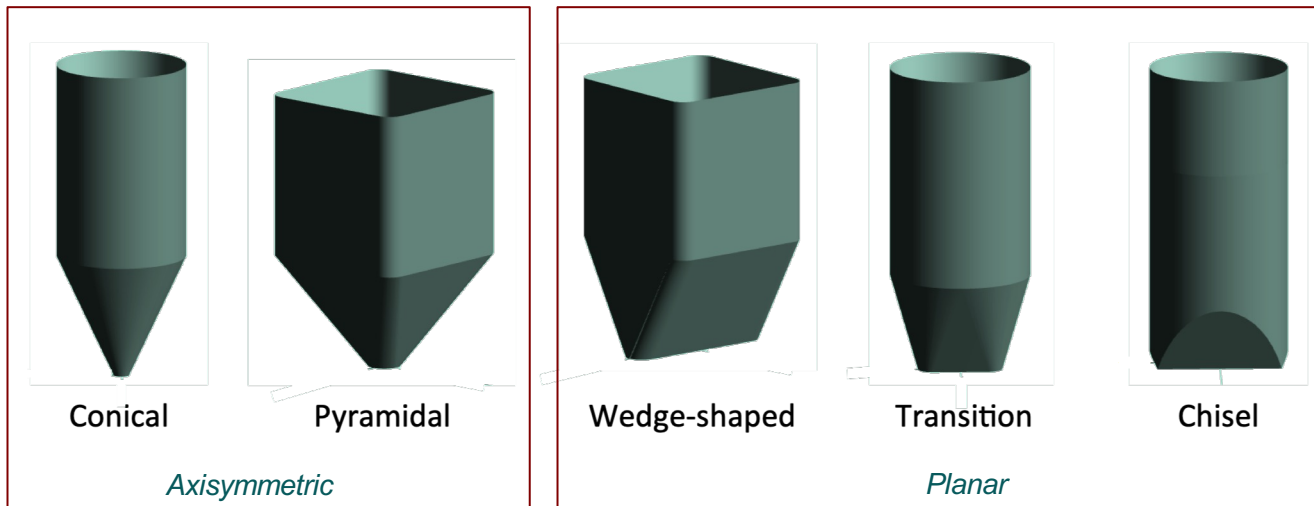
Definitions

Biomass



Definitions

Hopper, bin, or silo



Liquids vs. solids

	Liquids	Solids
Friction	Frictionless	Frictional
Compressibility	Nearly incompressible	Generally compressible
Isotropy	Isotropic	Anisotropic



Classification of liquids

Newtonian	Stress is proportional to strain rate
Viscoelastic	Combination of viscous and elastic behaviors
Rheoplectic	Viscosity increases over time
Thixotropic	Viscosity decreases over time
Pseudoplastic	Viscosity increases with increased stress
Dilatant	Viscosity decreases with increased stress



Classification of bulk solids

“Normal” Stable and consistent behavior

- Very predictable, even, and uniform nature
- Insensitive to handling and environment
- Invariable with time and place
- Easy to accommodate and control

“Neurotic” Awkward to handle

- Behavior varies widely with circumstances
- Unstable in state and condition
- Sensitive to surroundings and handling
- Prone to run amok and lose mobility

“Schizophrenic” Behavior changes completely

- Absorbs liquor, hardens, or dries out
- Sensitive to heat, aggregates easily
- Forms strong, unwelcome bonds

“Masochistic” Suffers readily

- Fragile, delicate, easily breaks down
- Degrades, sensitive to contamination
- Deteriorates rapidly, has to be handled with care

“Paranoid” Hypersensitive to own condition

- Sensitive to size, shape, appearance, purity
- Alarmed by unquantifiable concerns
- Plagued with constraints or quality considerations
- Obsessed by regulation, bounds of acceptance

“Sadistic” Aggressive to surroundings

- Abrasive, toxic, hot, or inflames readily
- Unpleasant to touch or be near, irritates
- Fouls or contaminates the locality, corrosive

“Plane nasty” Obstinate, dangerous, hazardous

Credit: Lyn Bates!



Flow problems

- Ratholing



Flow problems

- Ratholing
- Arching/bridging



Bridging



Flow problems

- Ratholing
- Arching
- Caking



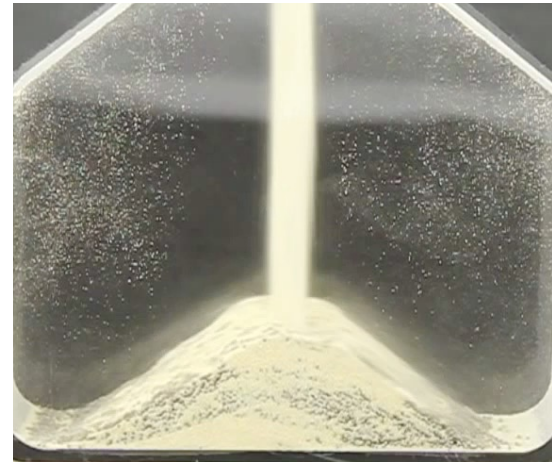
Flow problems

- Ratholing
- Arching
- Caking
- **Flooding**



Flow problems

- Ratholing
- Arching
- Caking
- Flooding
- Segregation



Flow problems

- Ratholing
- Arching
- Caking
- Flooding
- Segregation
- Limiting discharge rate



Solids flow patterns

- Funnel flow
- Mass flow



Funnel flow



Mass flow



Mass flow



Funnel flow



No flow



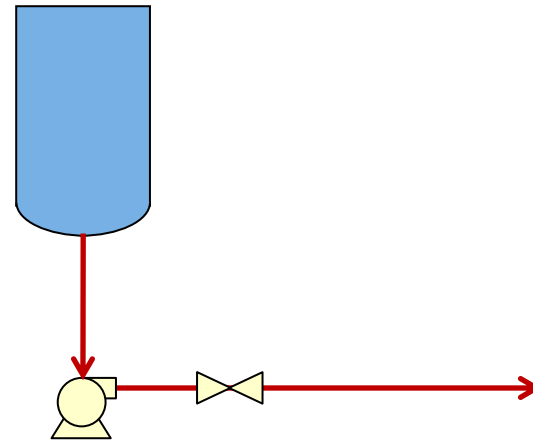
Preventing flow problems

- Ensure properly sized outlet
- Design/modify to allow mass flow
- Need powder flow properties



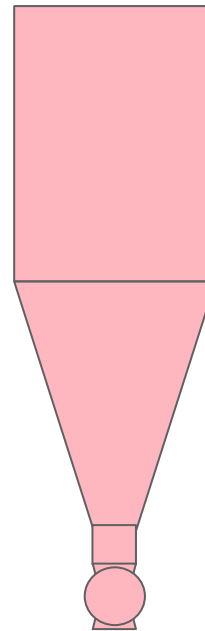
Fundamental properties

- Viscosity
- Density
- Vapor pressure



Fundamental properties

- Cohesive strength
- Internal friction
- Compressibility
- Wall friction
- Permeability



Andrew Jenike



Fundamental properties

- Cohesive strength
 - Internal friction
 - Compressibility
 - Wall friction
 - Permeability
- Allows calculation of hopper outlet dimensions required to prevent flow obstructions or predict flow stoppages



Fundamental properties

- Cohesive strength
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- Allows calculation of hopper outlet dimensions required to prevent flow obstructions or predict flow stoppages
- Used to calculate recommended mass flow hopper angles or predict flow pattern



Fundamental properties

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- Allows calculation of hopper outlet dimensions required to prevent flow obstructions or predict flow stoppages
- Used to calculate recommended mass flow hopper angles or predict flow pattern
- Used to calculate solids discharge rates





Test equipment

- Shear cell testers

- Cohesive strength
- Internal friction
- Compressibility
- Wall friction



Jenike &
Johanson



Dietmar
Schulze



Brookfield
Engineering



Anton
Paar



Freeman
Technology



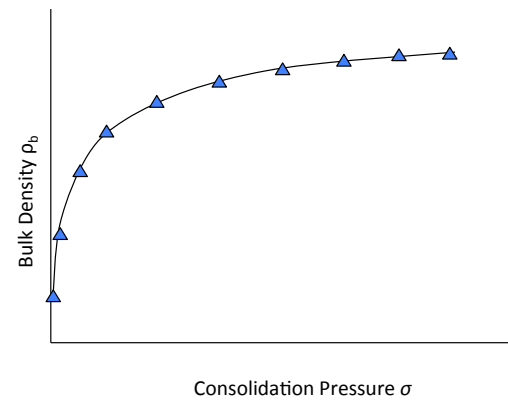
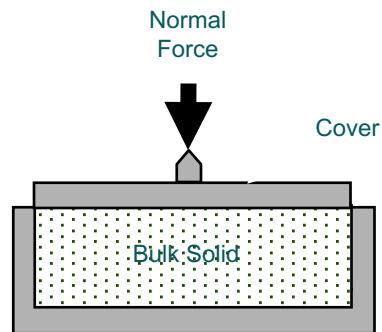
E&G
Associates

- Permeability testers

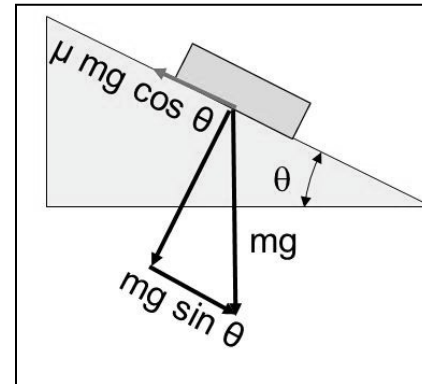
Compressibility



Compressibility



Wall friction

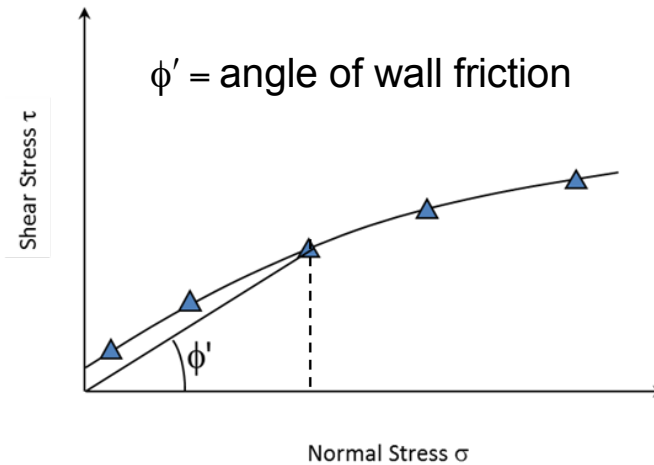
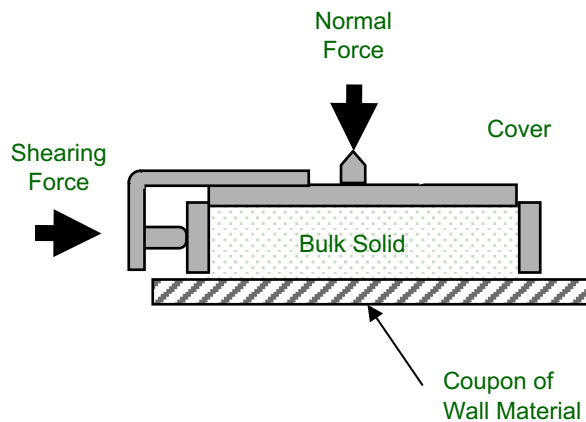


$$mg \sin \theta = \mu mg \cos \theta$$

$$\mu = \tan \theta$$



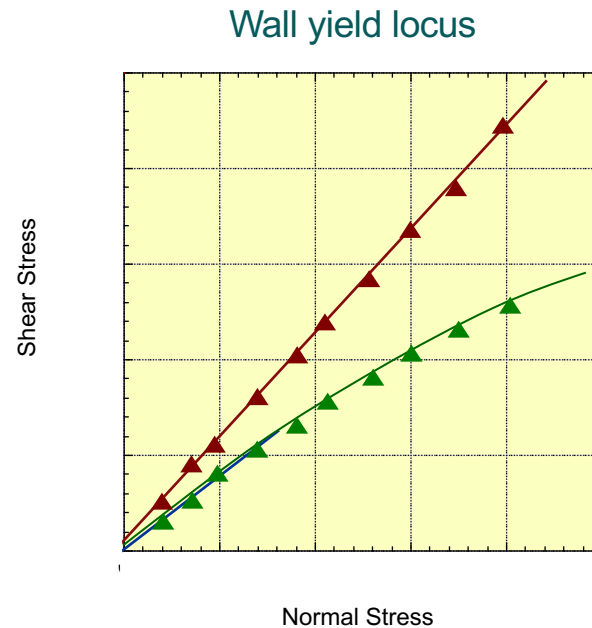
Wall friction



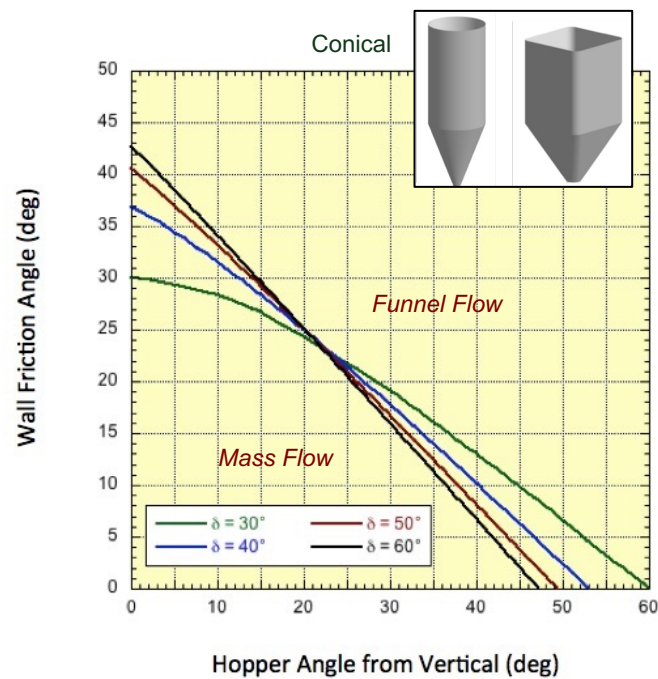
$$\tan \phi' = \frac{\text{Shear Stress}}{\text{Normal Stress}} = \mu$$



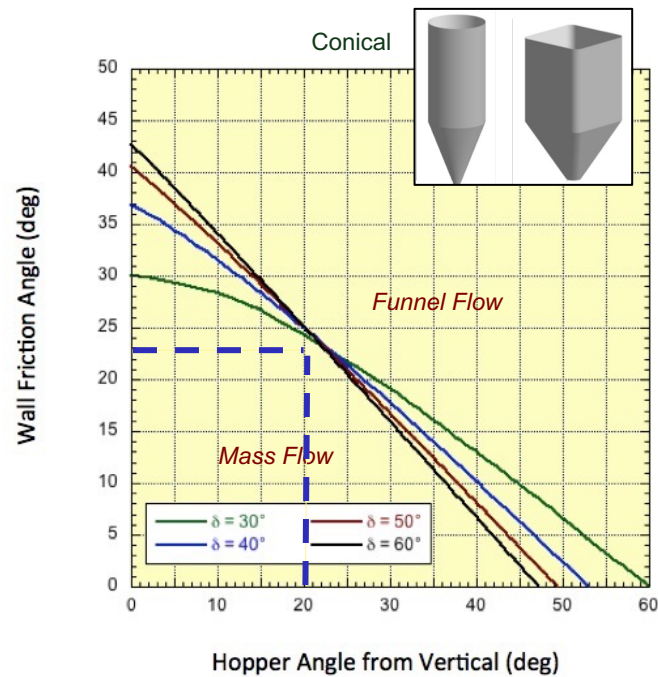
Wall friction



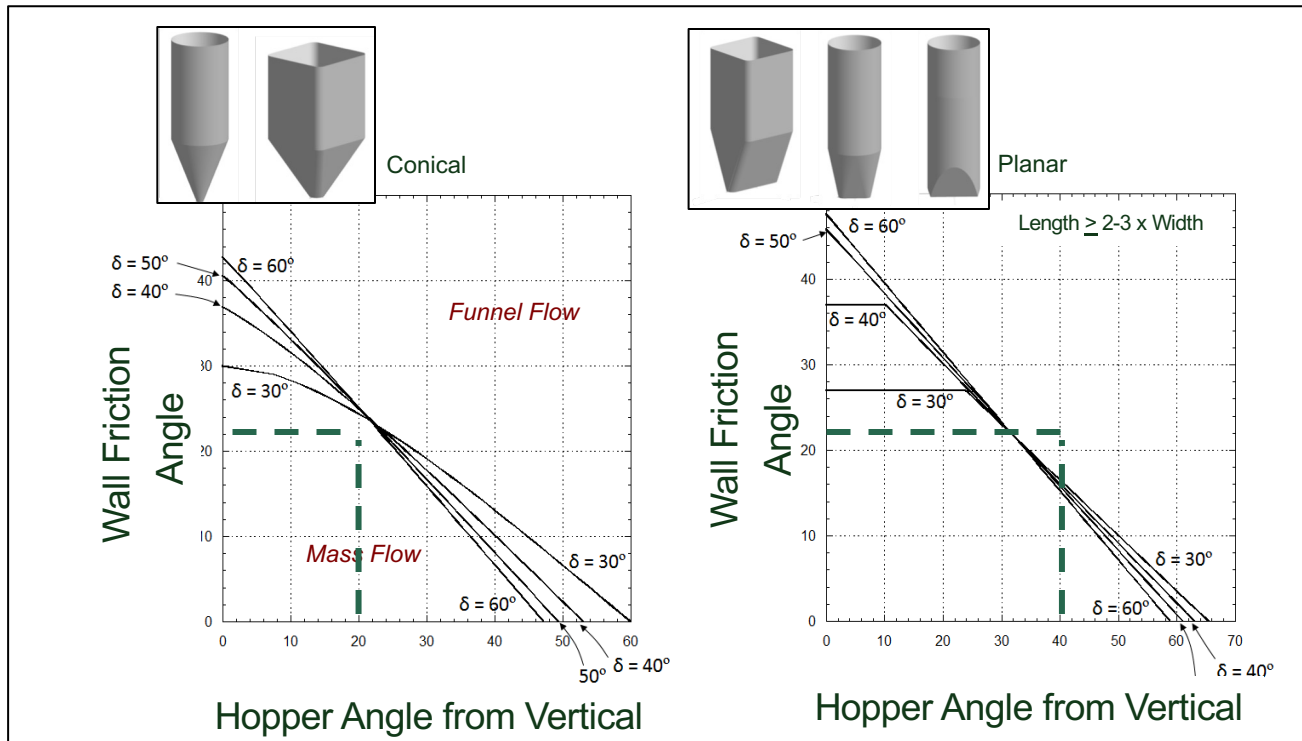
Mass flow hopper angle



Mass flow hopper angle



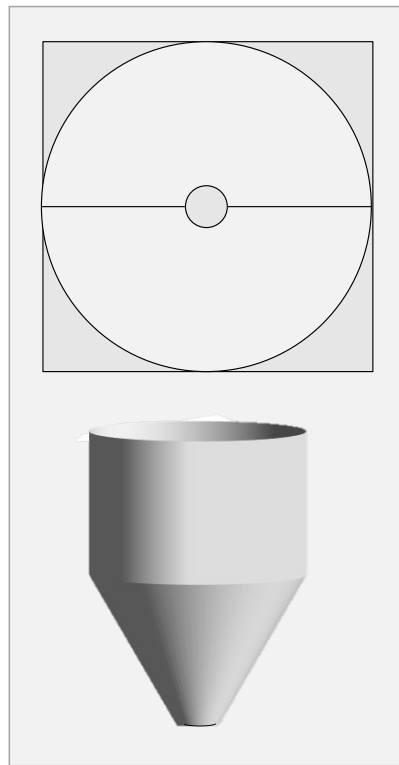
Mass flow hopper angles



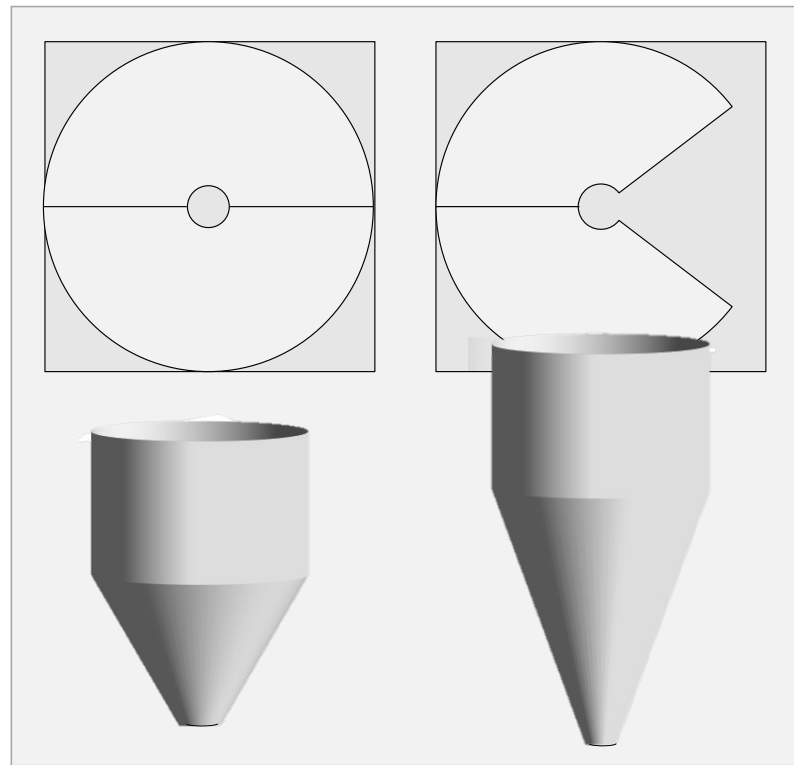
From Jenike Bulletin 123



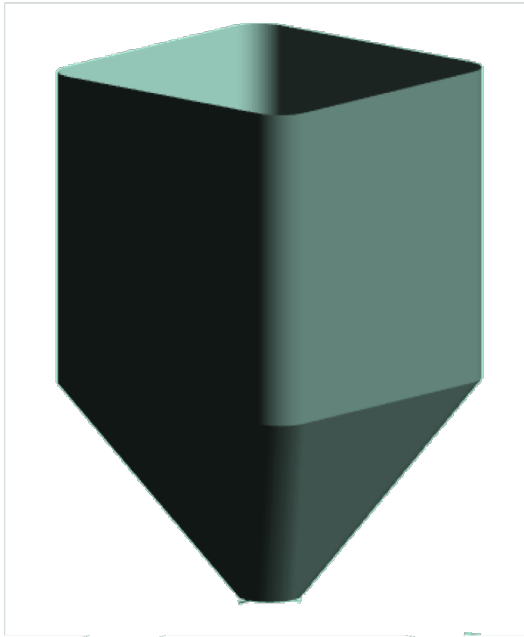
Mass flow hopper angle



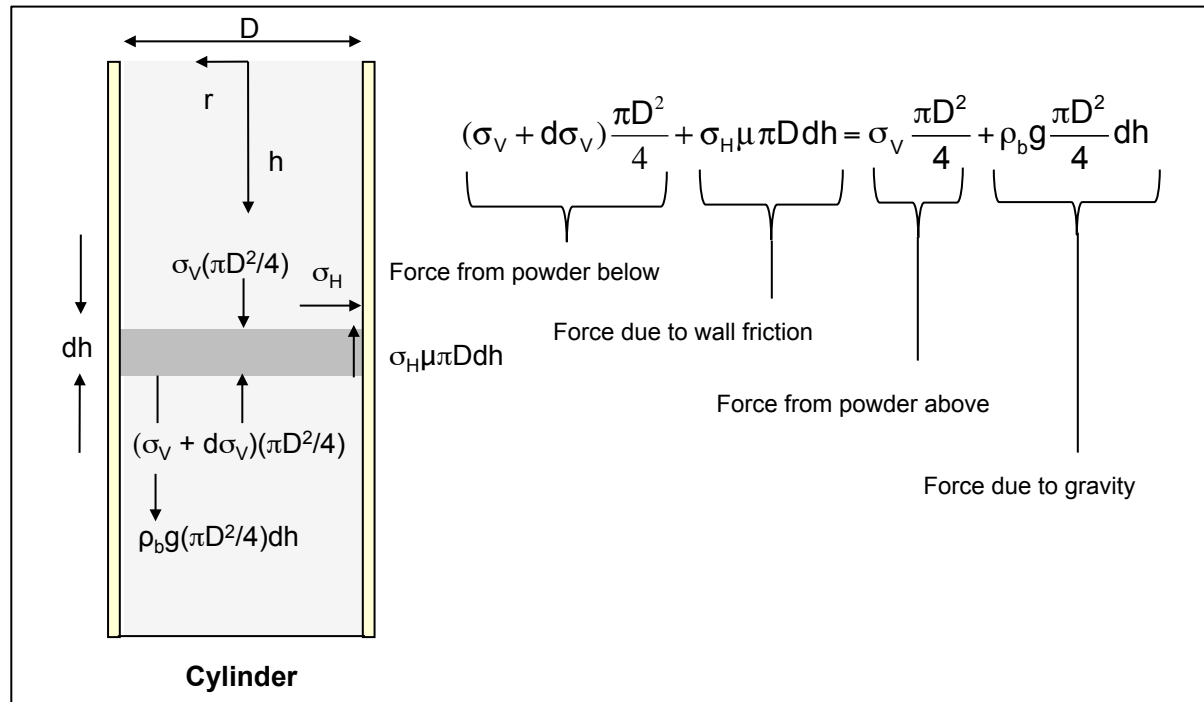
Mass flow hopper angle



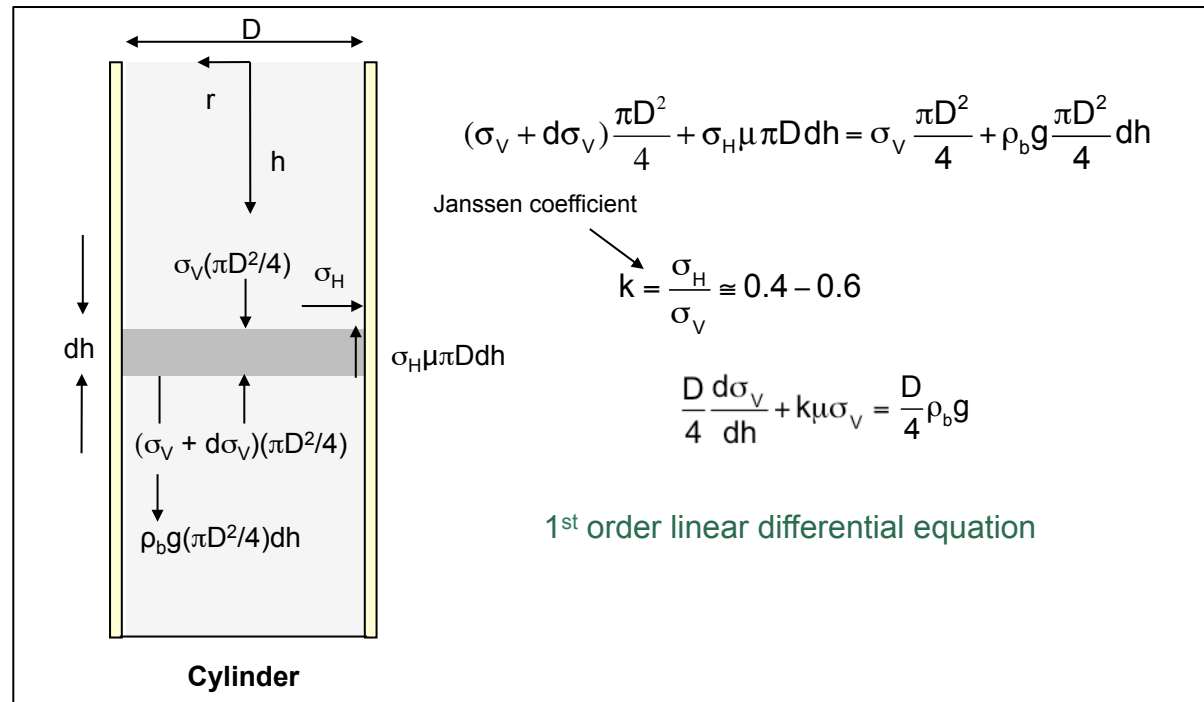
Valley angles



Pressure distribution



Pressure distribution



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Integrating factor

From Wikipedia, the free encyclopedia
(Redirected from [Integration factor](#))

In mathematics, an **integrating factor** is a function that is chosen to facilitate the solving of a given equation involving [differentials](#). It is commonly used to solve [ordinary differential equations](#), but is also used within [multivariable calculus](#) when multiplying through by an integrating factor allows an [inexact differential](#) to be made into an [exact differential](#) (which can then be integrated to give a [scalar field](#)). This is especially useful in thermodynamics where [temperature](#) becomes the integrating factor that makes [entropy](#) an exact differential.

Contents [\[hide\]](#)

- Use in solving first order linear ordinary differential equations
 - Example
- General use
- References
- See also

Use in solving first order linear ordinary differential equations [\[edit \]](#)

Integrating factors are useful for solving [ordinary differential equations](#) that can be expressed in the form

$$y' + P(x)y = Q(x)$$

The basic idea is to find some function $M(x)$, called the "integrating factor," which we can multiply through our [DE](#) in order to bring the left-hand side under a common derivative. For the canonical first-order, [linear differential equation](#) shown above, our integrating factor is chosen to be

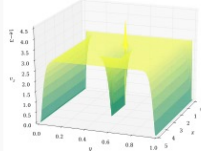
$$M(x) = e^{\int_0^x P(s)ds}$$

In order to derive this, let $M(x)$ be the integrating factor of a first order, linear differential equation such that multiplication by $M(x)$ transforms a partial derivative into a total derivative, then:

- $$M(x)(y' + P(x)y)$$

partial derivative

Differential equations



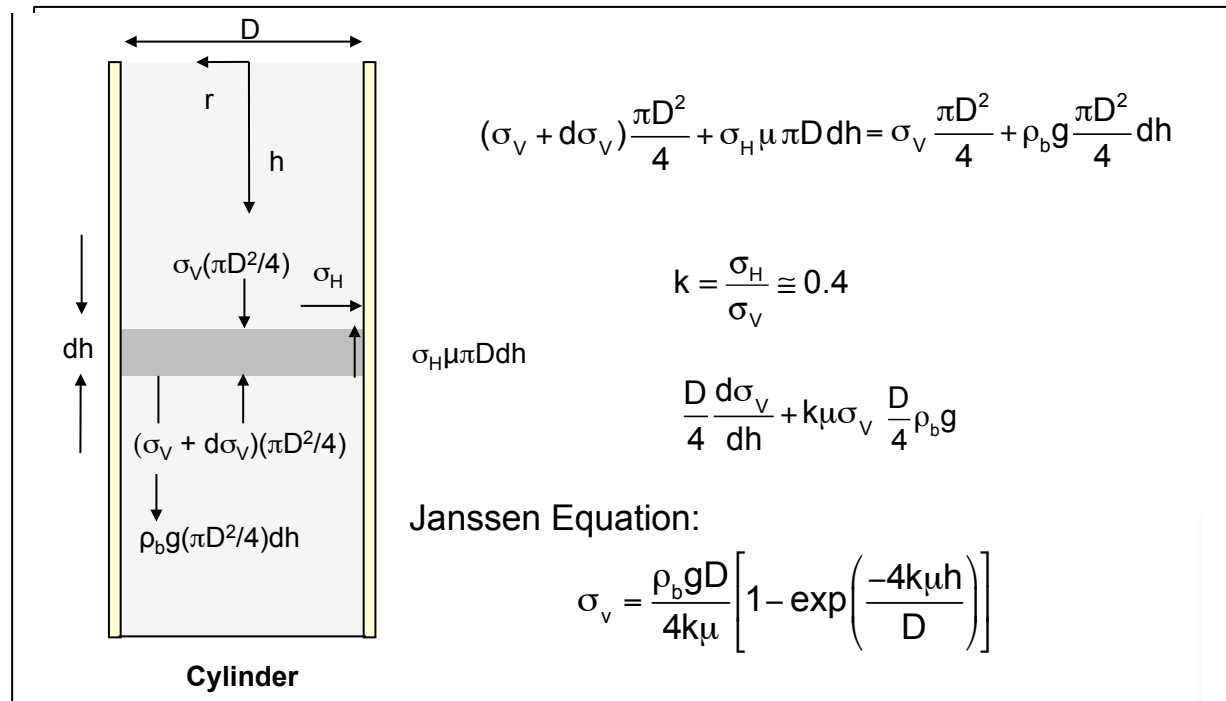
Navier-Stokes differential equations used to simulate airflow around an obstruction.

Scope	[show]
Classification	
Types	[show]
Relation to processes	[show]
Solution	
General topics	[show]
Solution methods	[show]
People	[show]

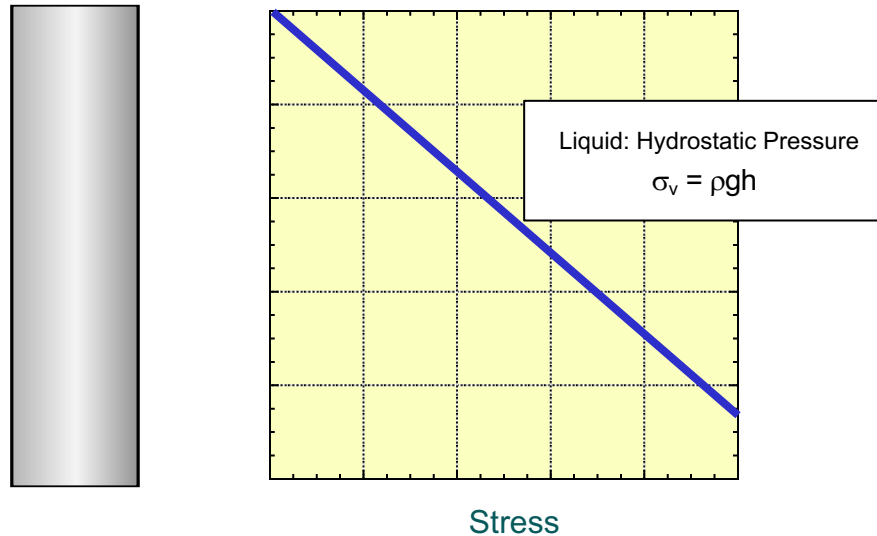
V · T · E



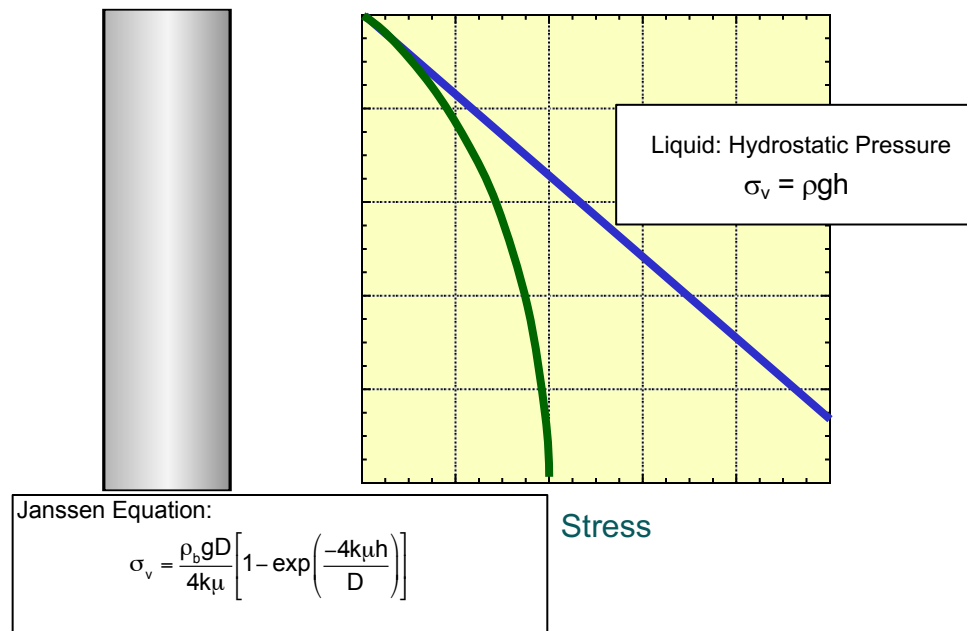
Pressure distribution



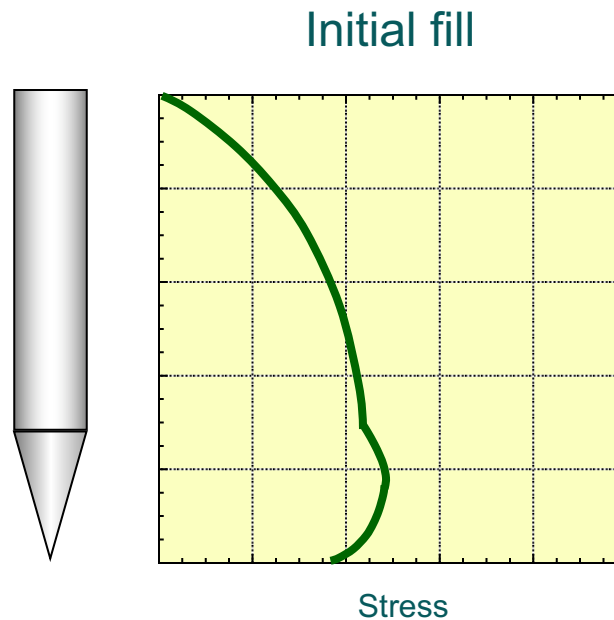
Pressure distribution



Pressure distribution

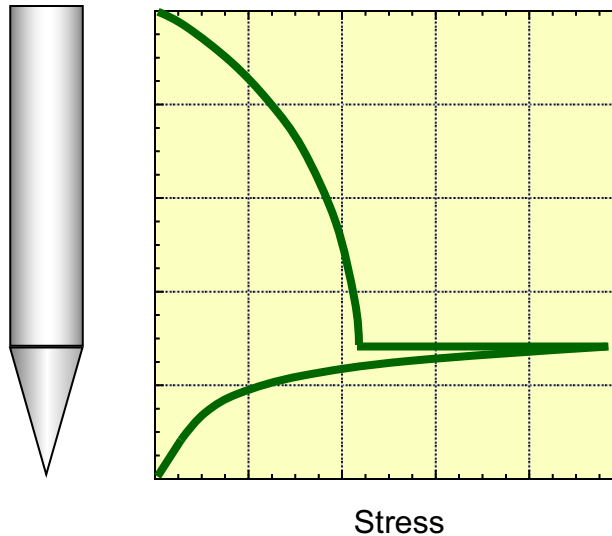


Pressure distribution

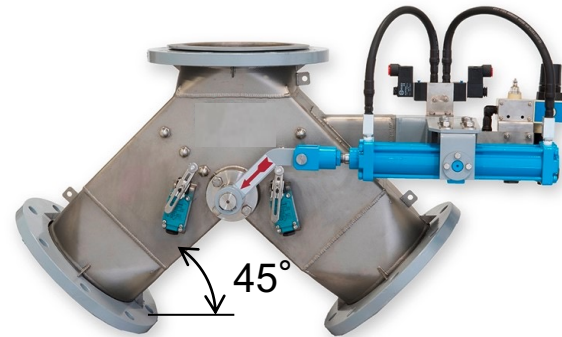


Pressure distribution

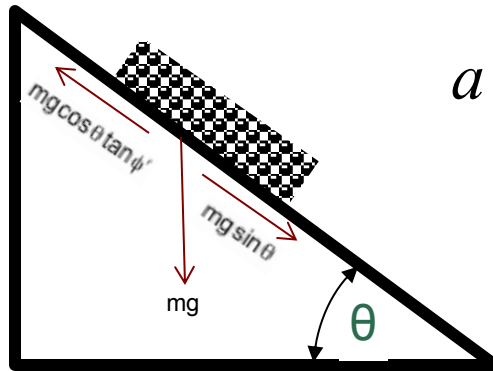
After discharge (mass flow)



Transfer chutes and diverter valves



Transfer chutes

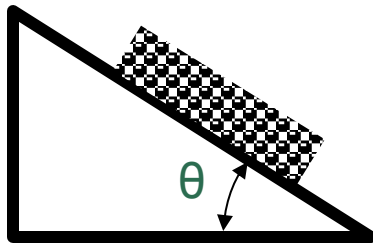


$$\begin{aligned} a &= g(\sin \theta - \cos \theta \tan \phi') \\ &= g \cos \theta (\tan \theta - \tan \phi') \end{aligned}$$

For dependable flow, $\theta' = \phi' + 5^\circ$

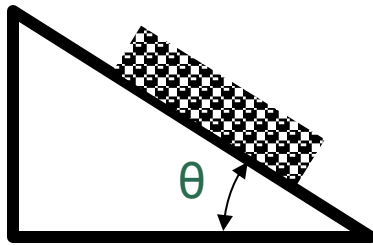


Transfer chutes



$$a = g \cos \theta (\tan \theta - \tan \phi')$$

Transfer chutes

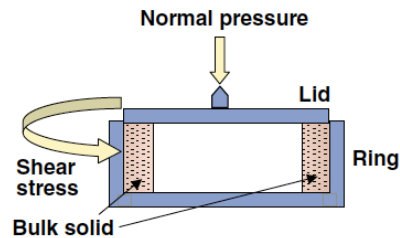


$$a = g \cos \theta (\tan \theta - \tan \phi')$$

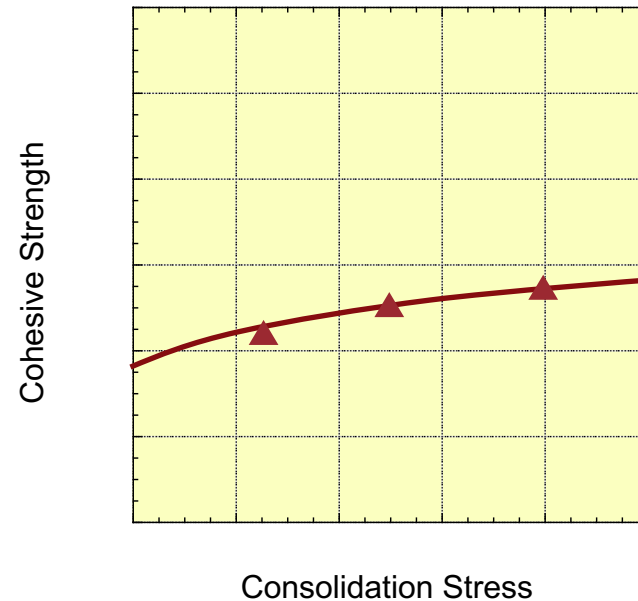
Cohesive strength



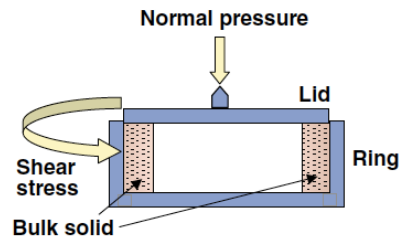
Cohesive strength



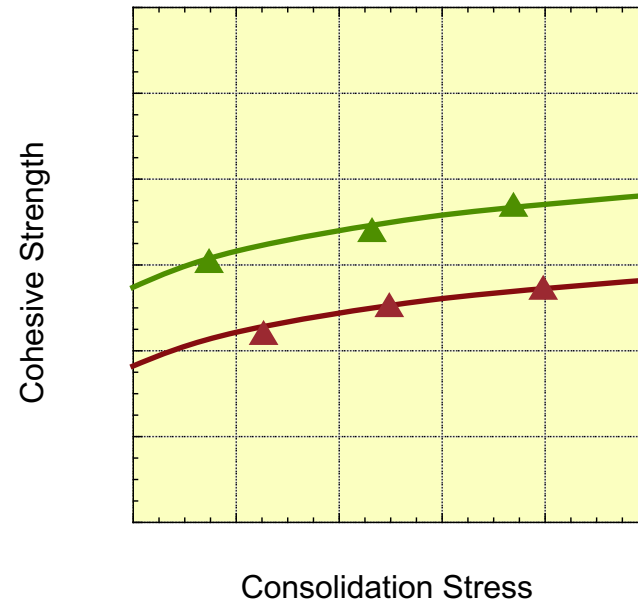
Flow Function



Cohesive strength



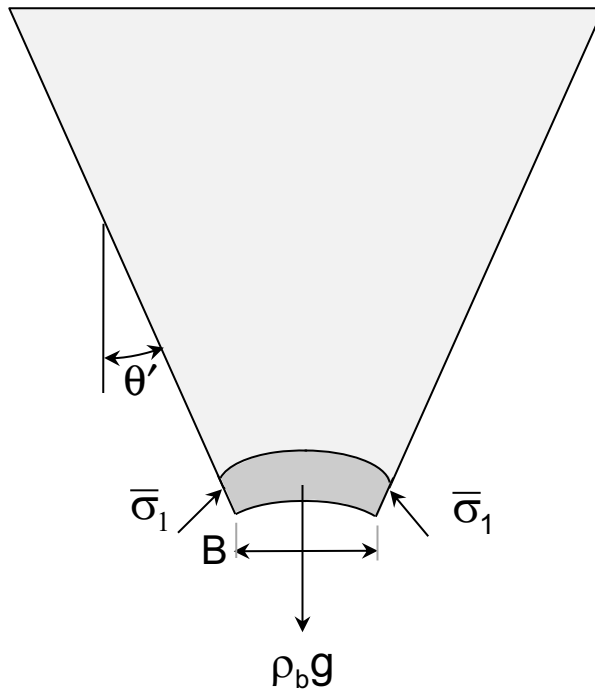
Flow Function



Arch stress



Arch stress

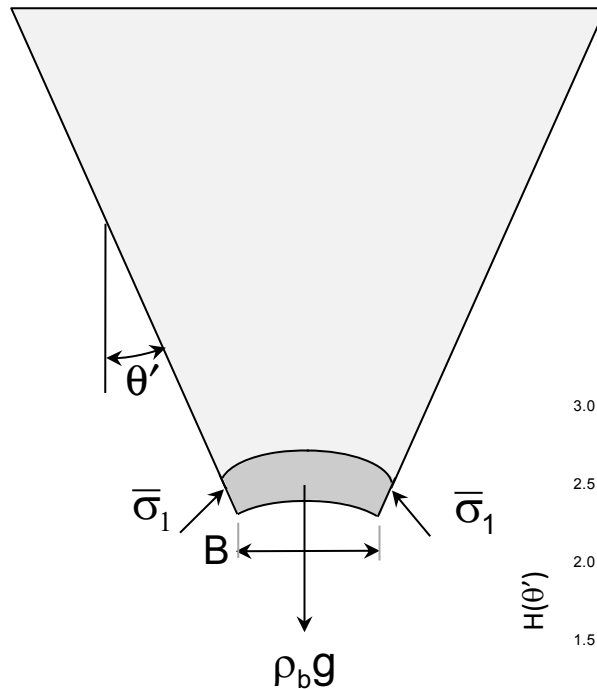


Stress on abutments of arch:

$$\bar{\sigma}_1 \approx \frac{\rho_b g B}{2}$$

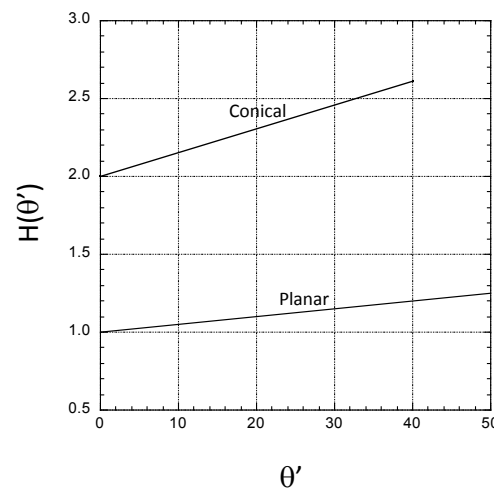


Arch stress



Stress on abutments of arch:

$$\bar{\sigma}_1 = \frac{\rho_b g B}{H(\theta')}$$



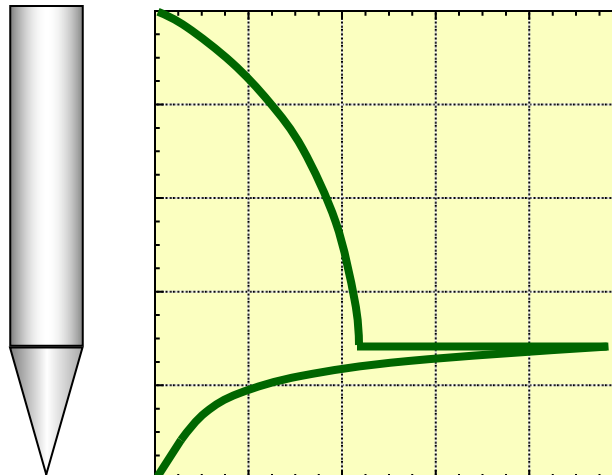
Conical: $H(\theta') \approx 2$

Slot: $H(\theta') \approx 1$



Pressure distribution

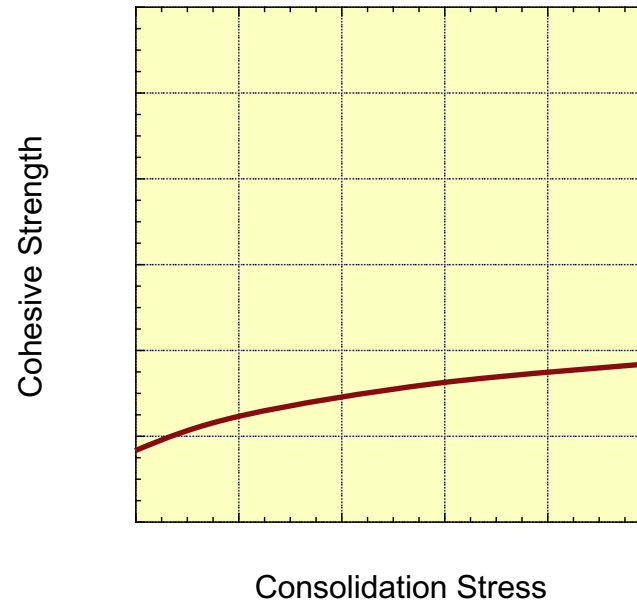
After discharge (mass flow)



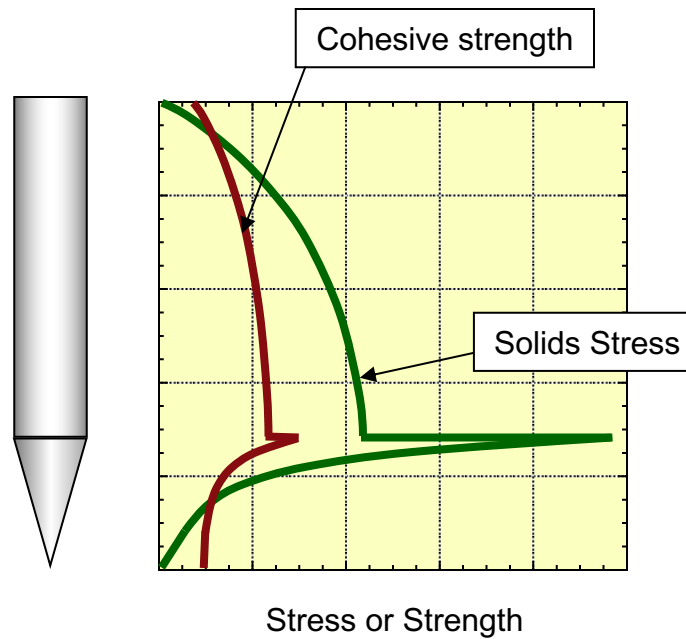
Consolidation Stress



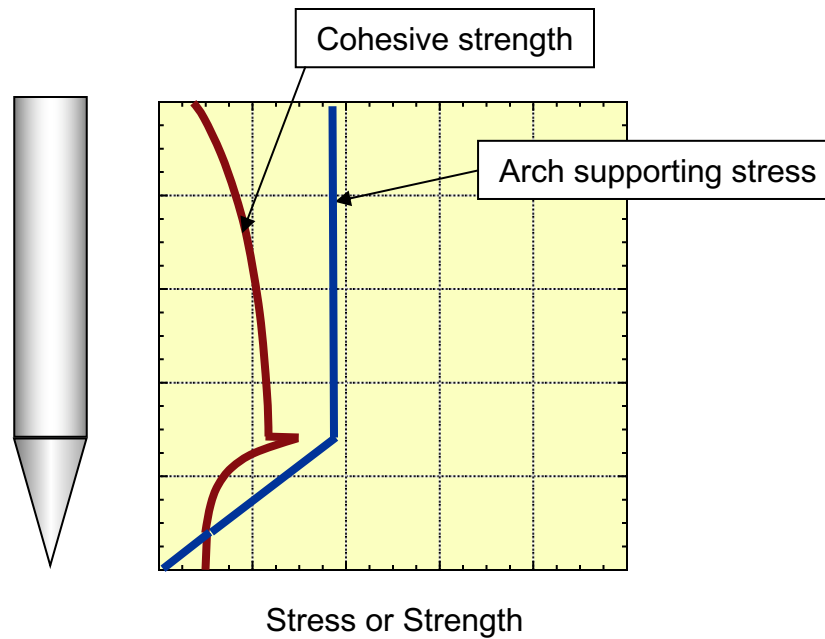
Cohesive strength



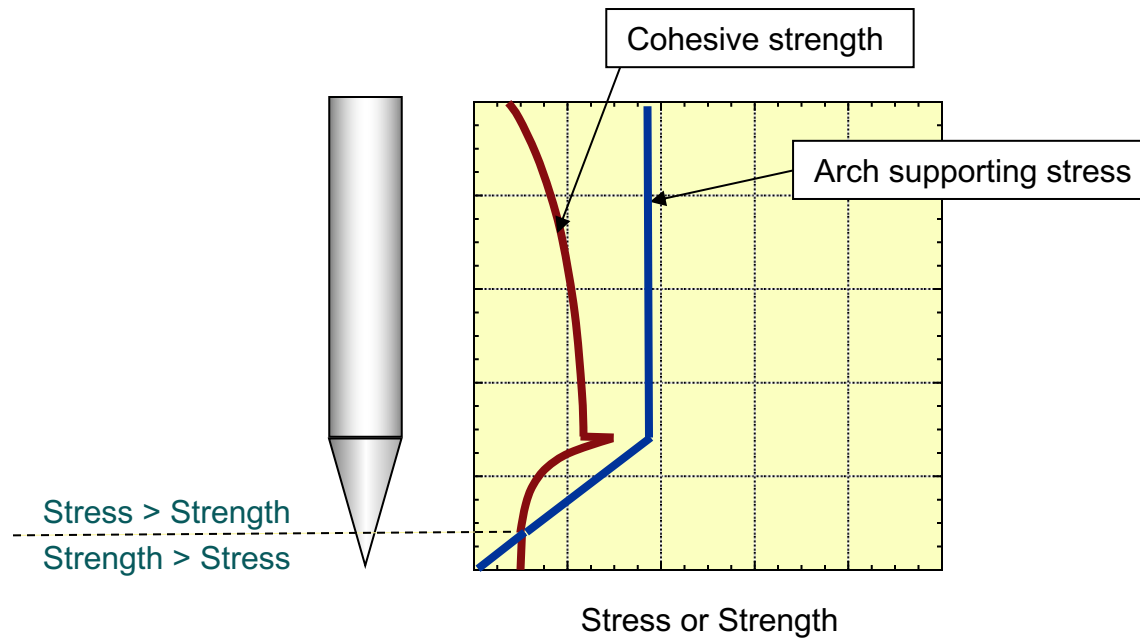
Cohesive strength



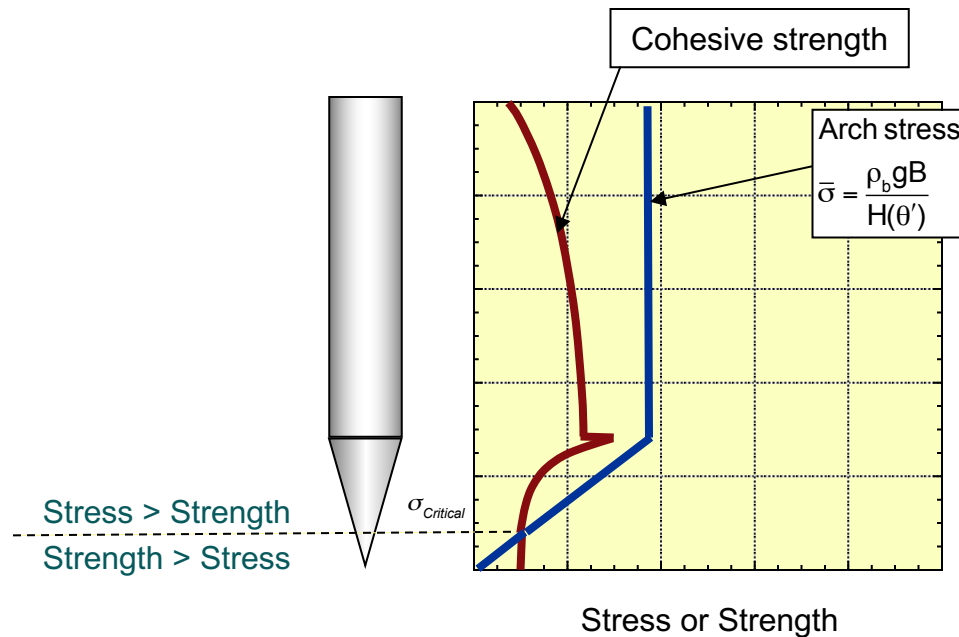
Arch stress



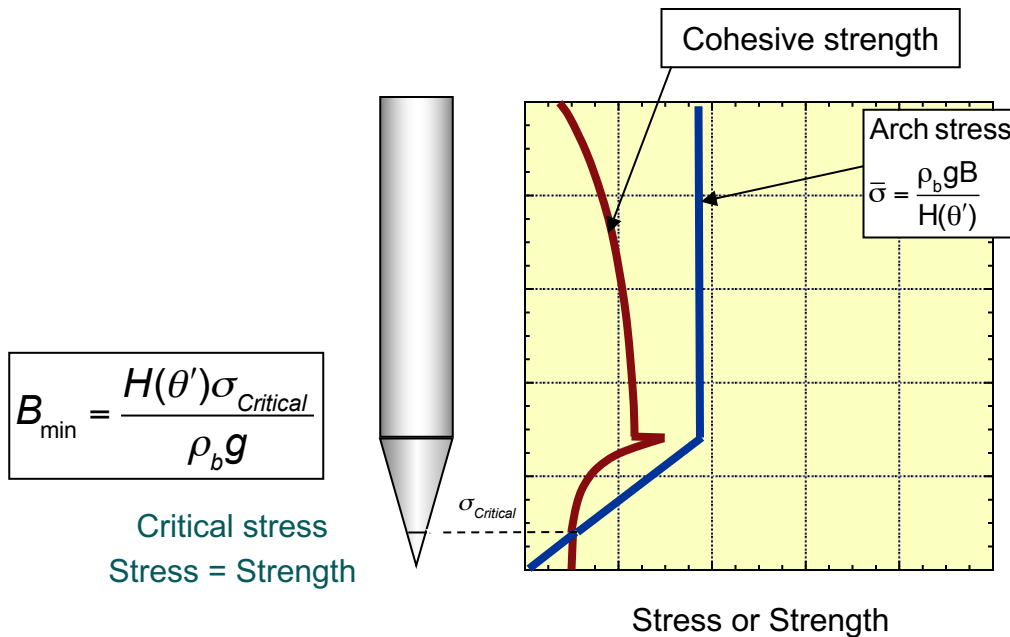
Arch stress



Critical arching diameter



Critical arching diameter



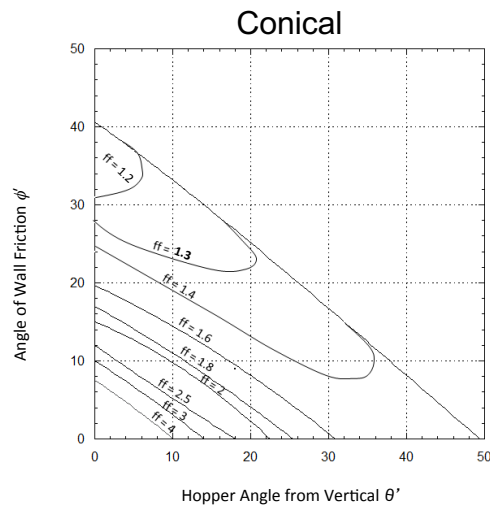
Flow factor

- Flow function (FF)
 - Relationship between consolidation stress and cohesive strength
- Flow factor (ff)
 - Ratio of consolidation stress and arch supporting stress

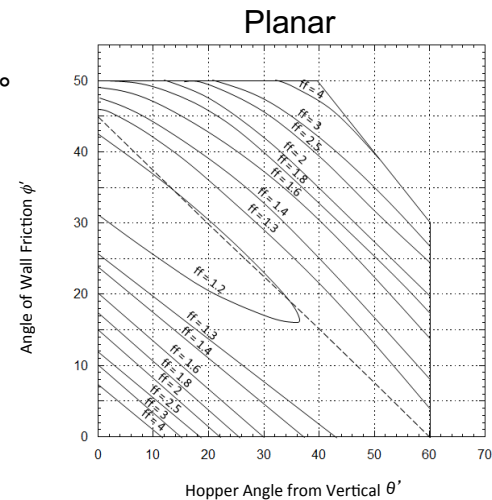
$$ff = \frac{\text{consolidation stress}}{\text{arch stress}}$$



Flow factor



$\delta = 50^\circ$

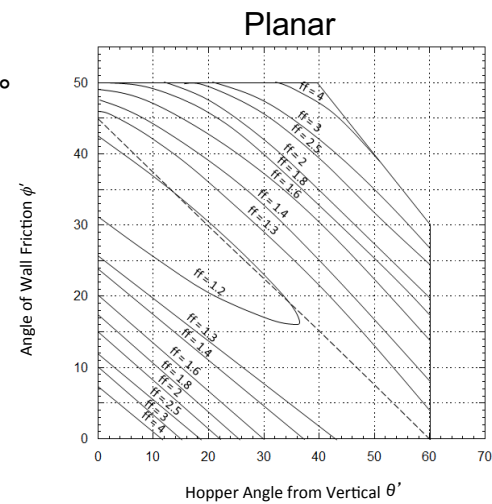
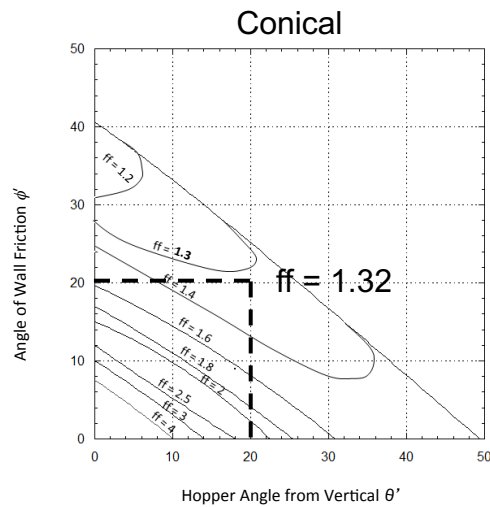


From Jenike Bulletin 123

$$ff = \frac{\text{consolidation stress}}{\text{arch stress}}$$



Flow factor

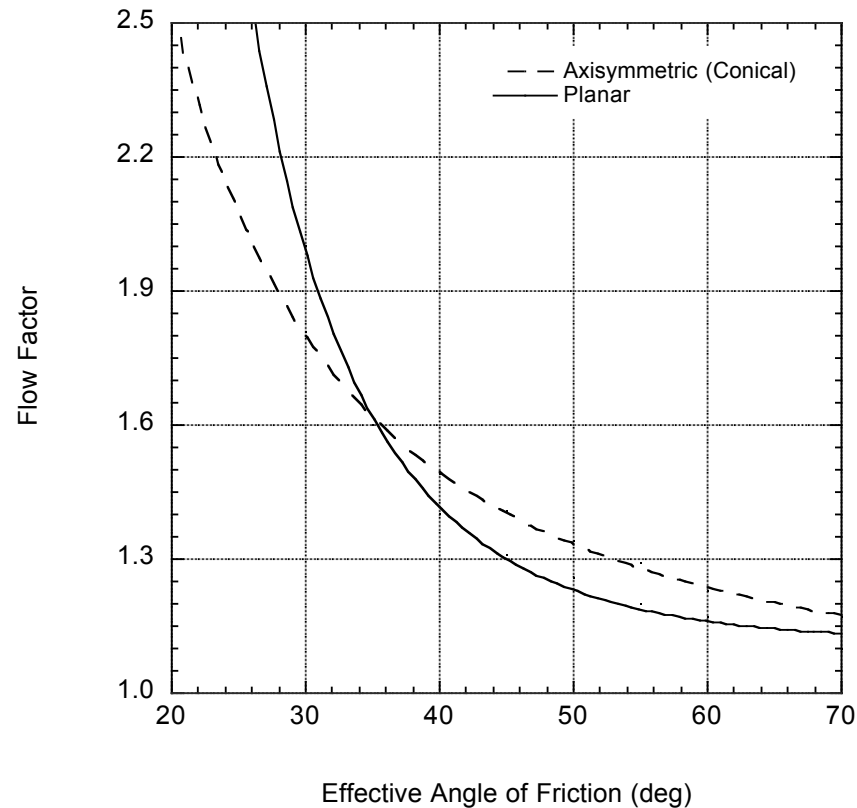


From Jenike Bulletin 123

$$ff = \frac{\text{consolidation stress}}{\text{arch stress}}$$



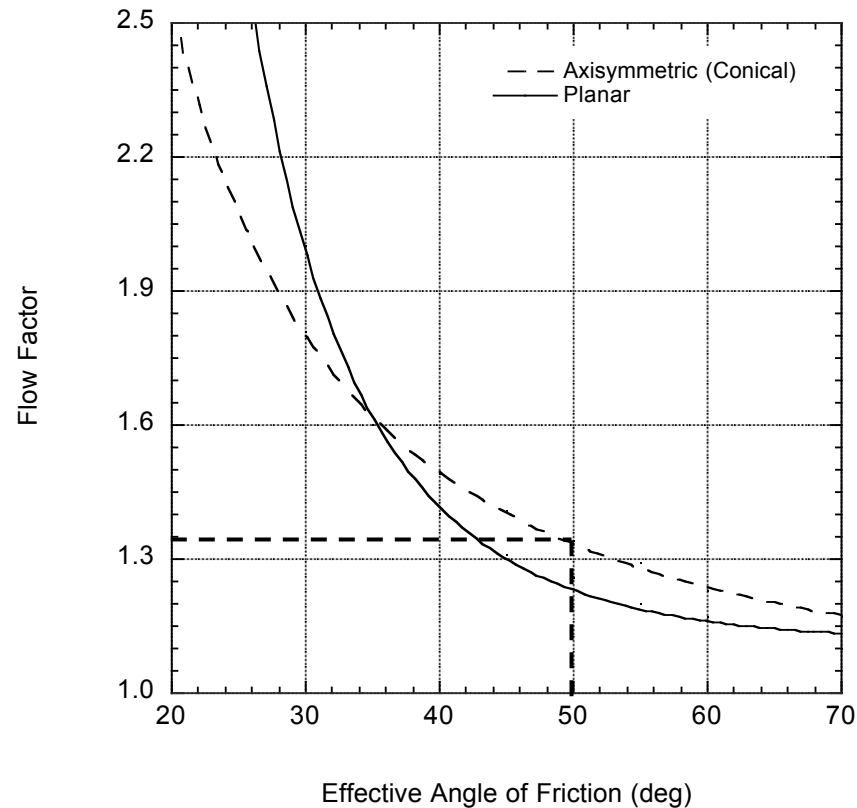
Critical flow factor



from Jerry Johanson's chapter in McGlinchey, Bulk Solids Characterization, 2005



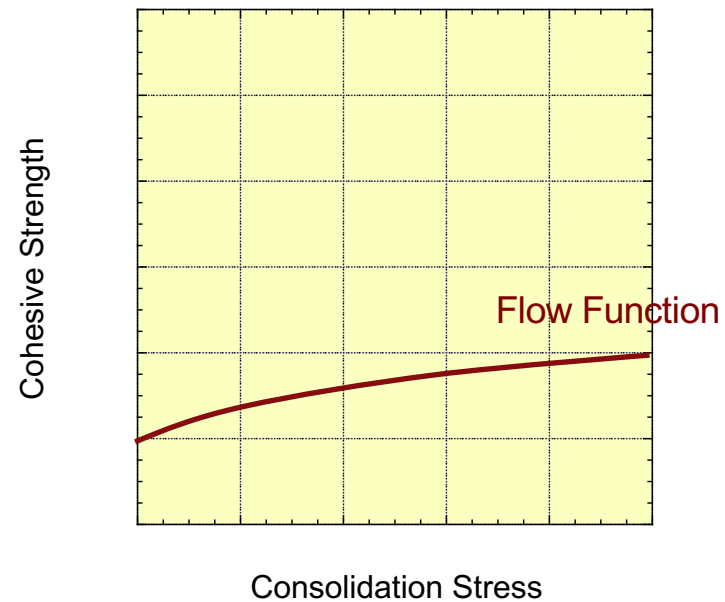
Critical flow factor



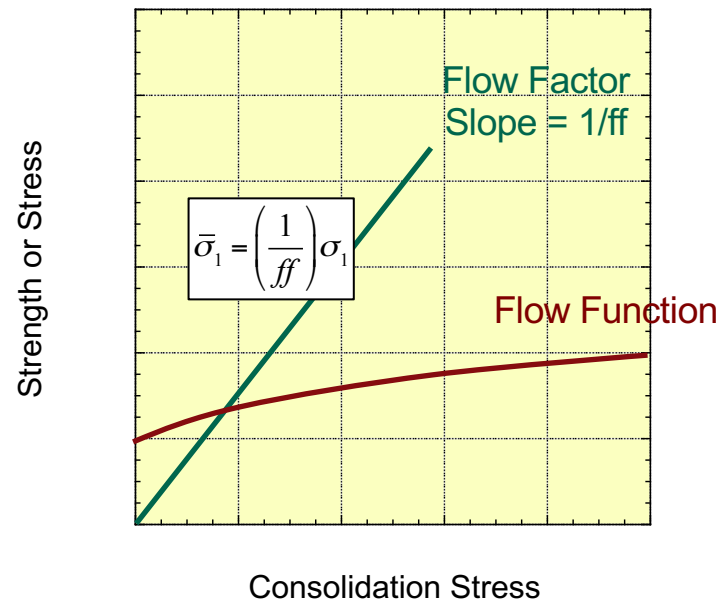
from Jerry Johanson's chapter in McGlinchey, Bulk Solids Characterization, 2005



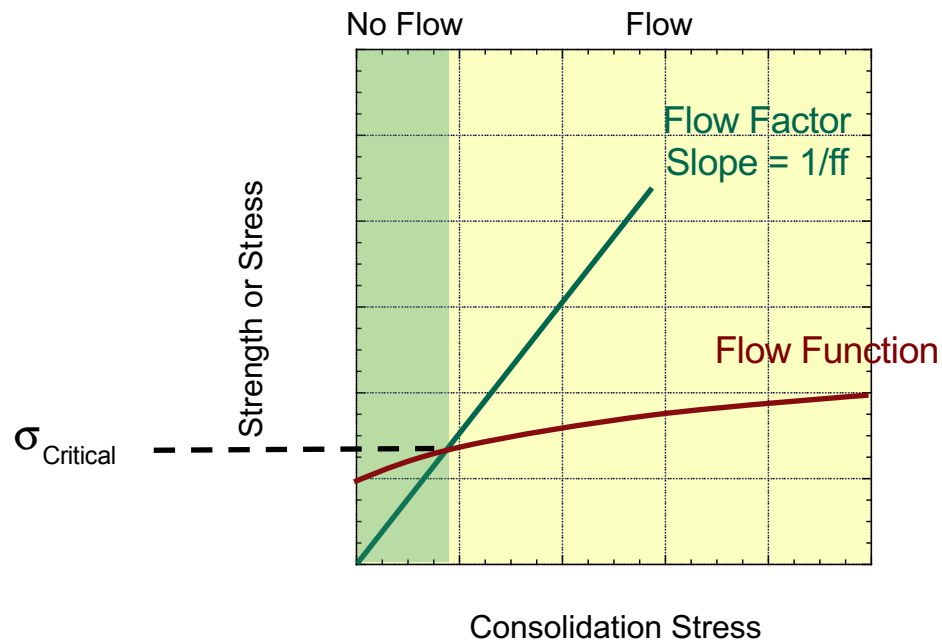
Flow function



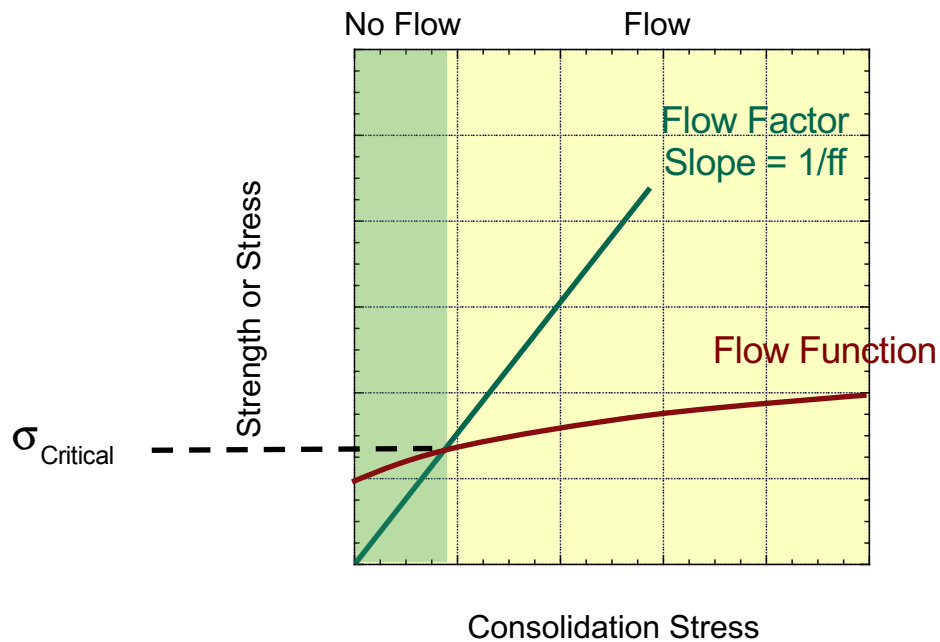
Flow factor



Critical stress



Critical stress

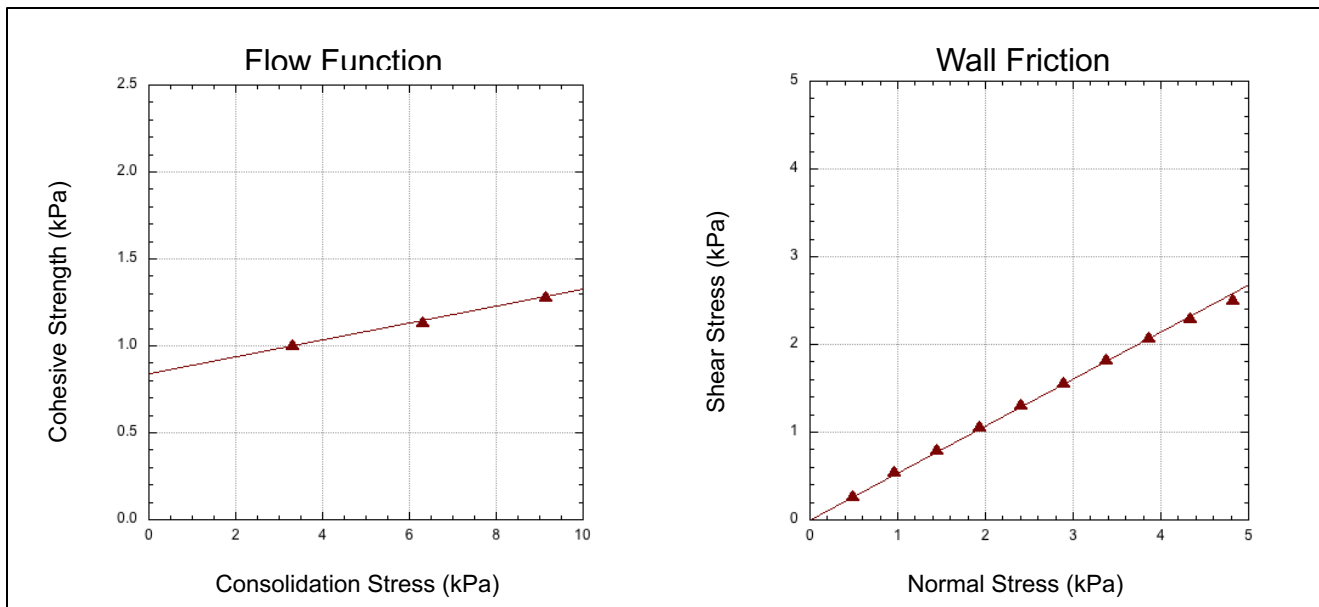


$$B_{\min} = \frac{H(\theta')\sigma_{\text{Critical}}}{\rho_b g}$$

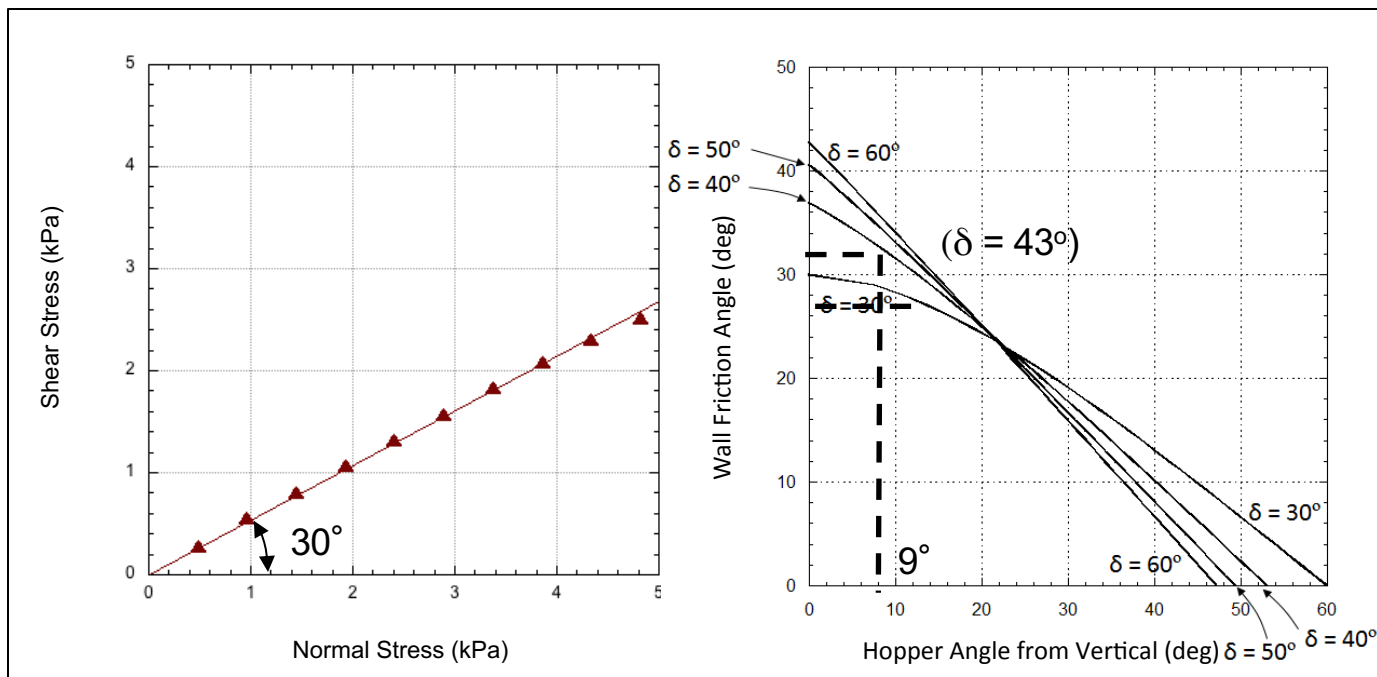


Example

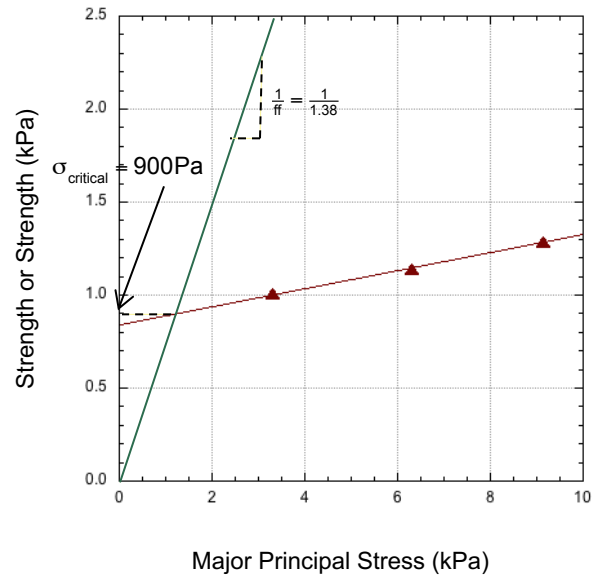
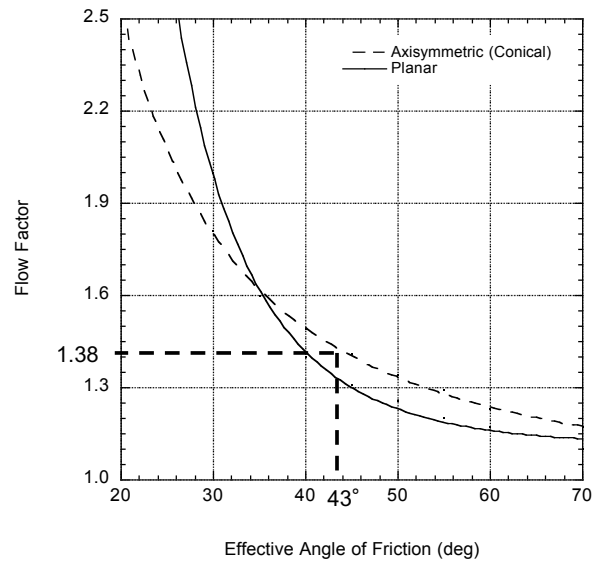
Design a hopper that reliably handles 67/29/4 blend of acetaminophen/Avicel/hydroxypropyl cellulose



Example

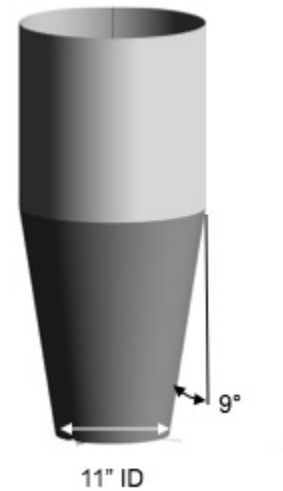


Example

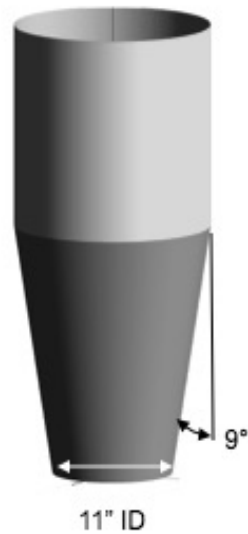


Example

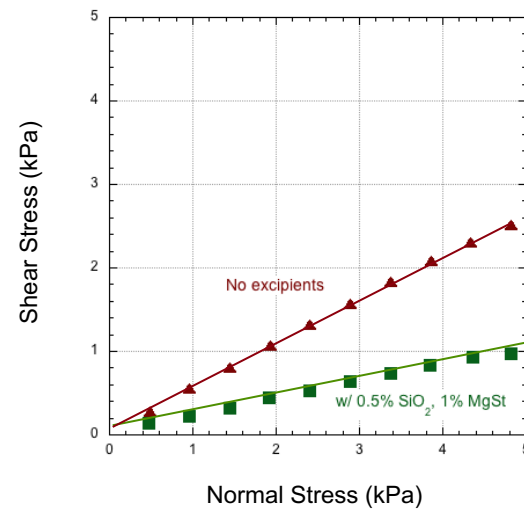
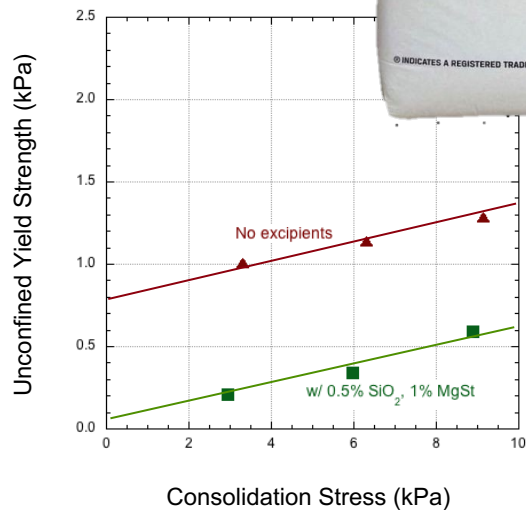
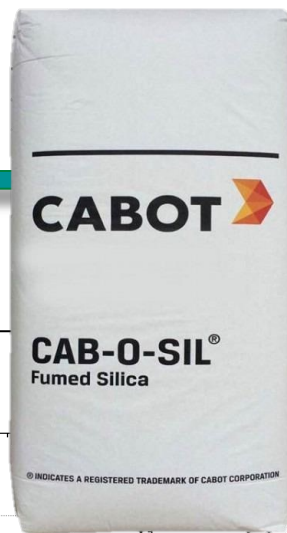
$$B_{\min} = \frac{H(\theta')\sigma_{\text{Critical}}}{\rho_b g} = \frac{(2.14)(900)}{(690)(9.8)} = 0.28 \text{ m} = 11 \text{ in.}$$



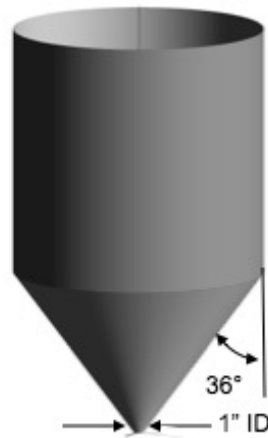
Example



Example



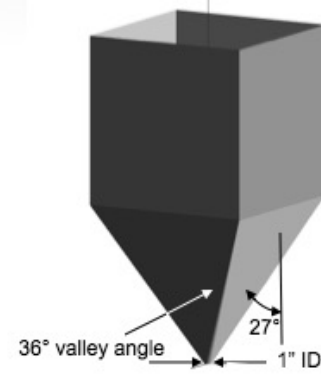
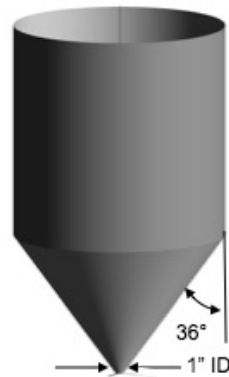
Example



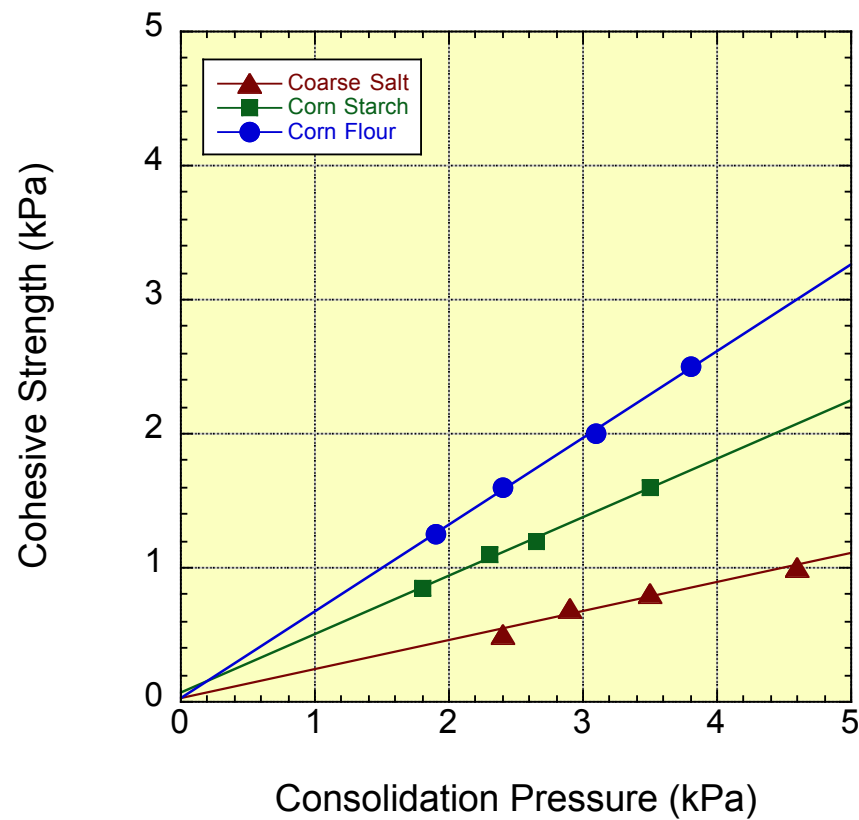
Note 🎵: need to confirm outlet size will allow required discharge rate



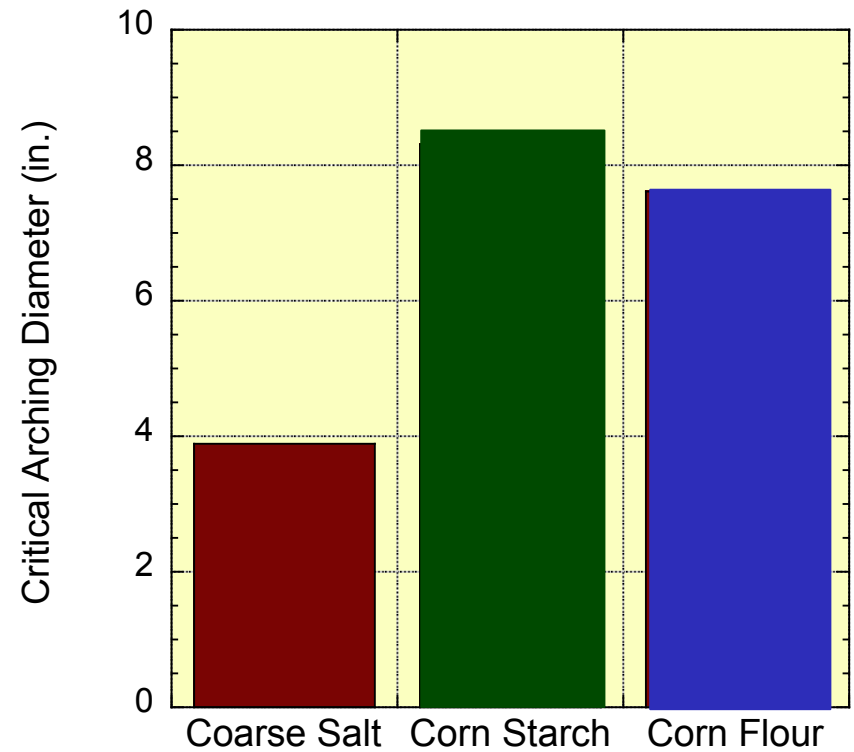
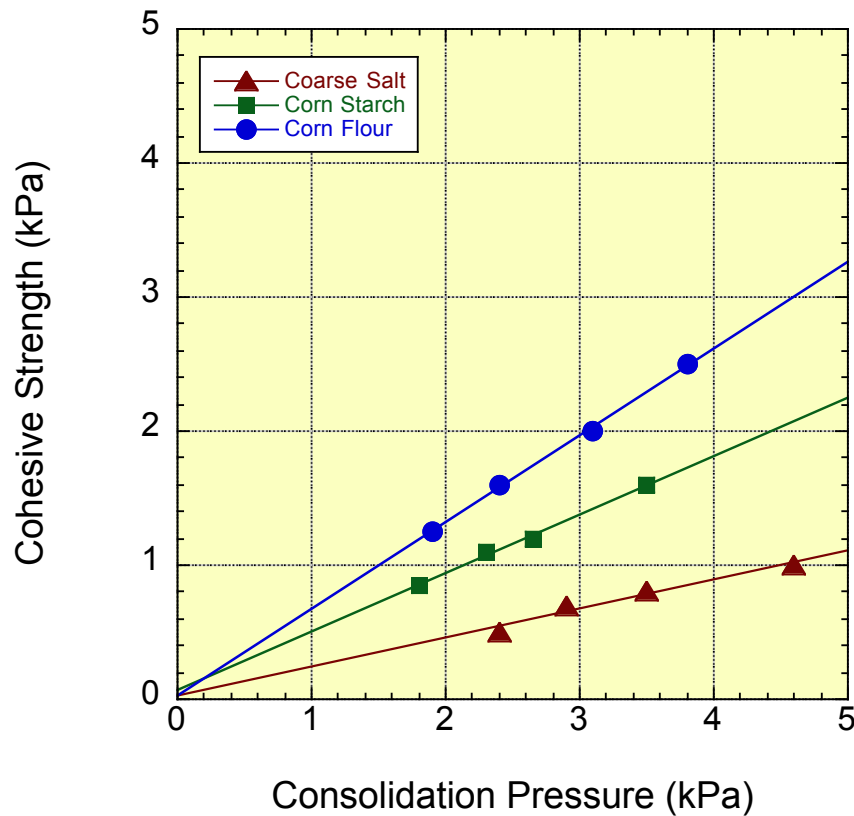
Example



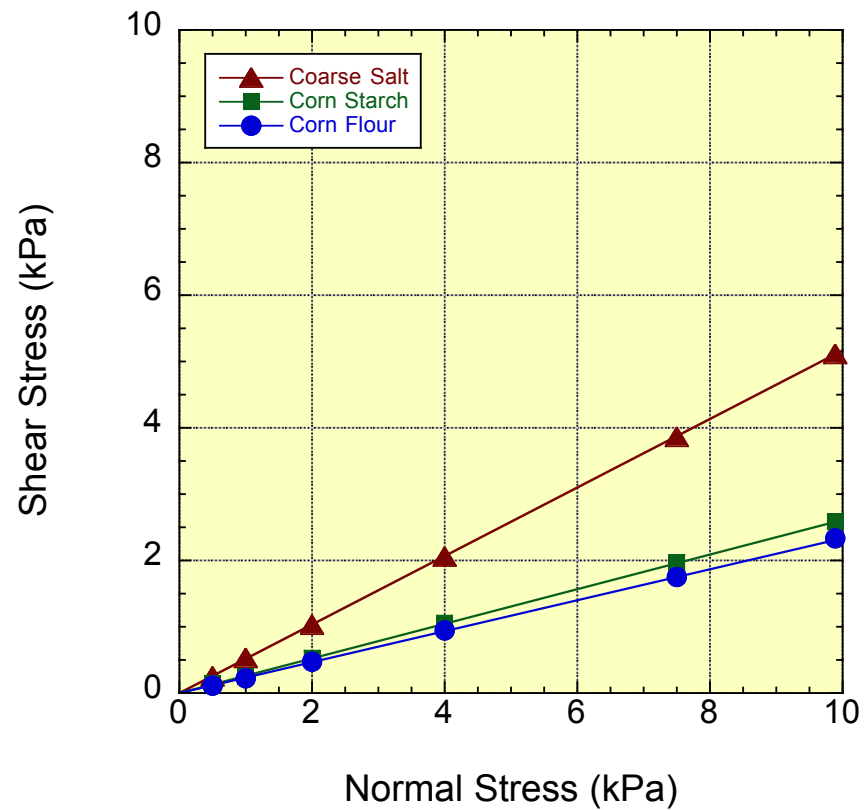
Example



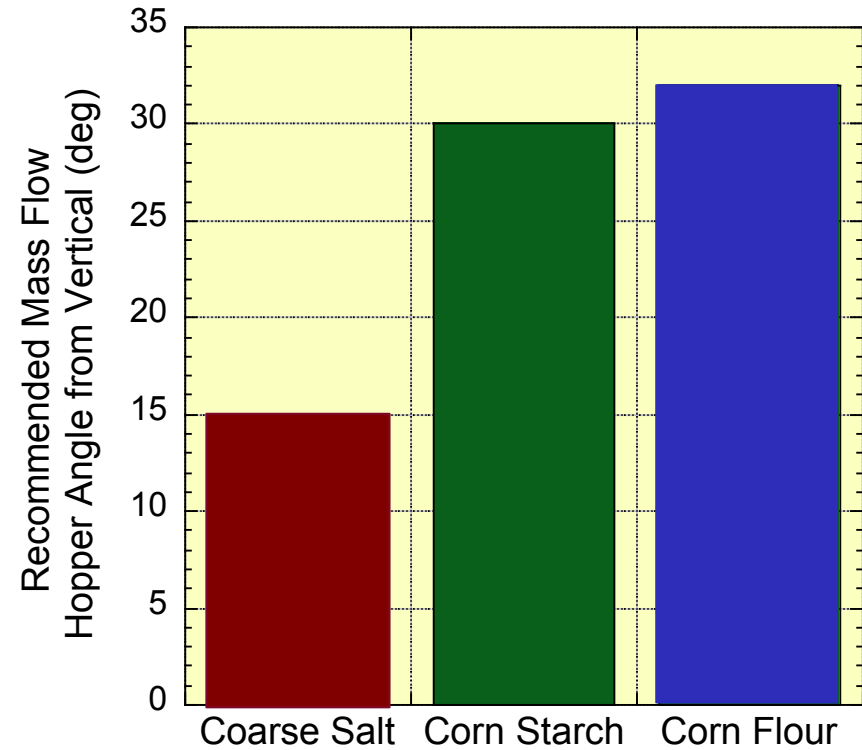
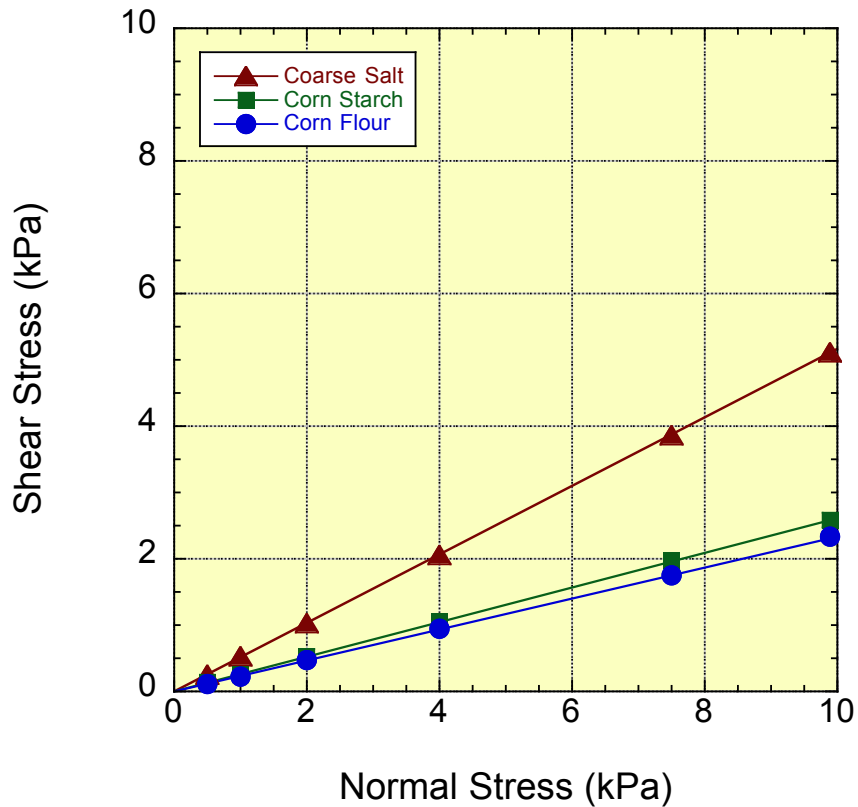
Example



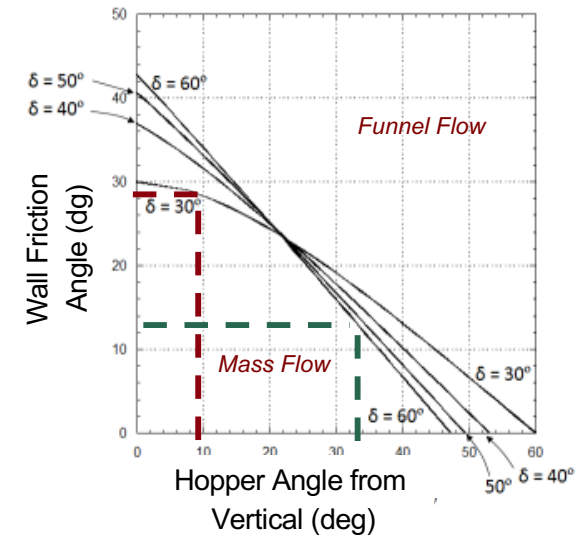
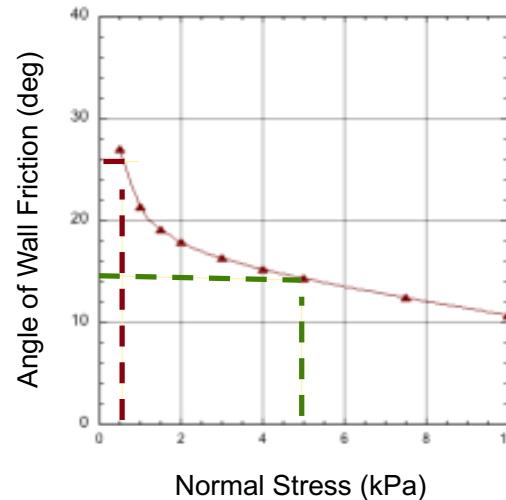
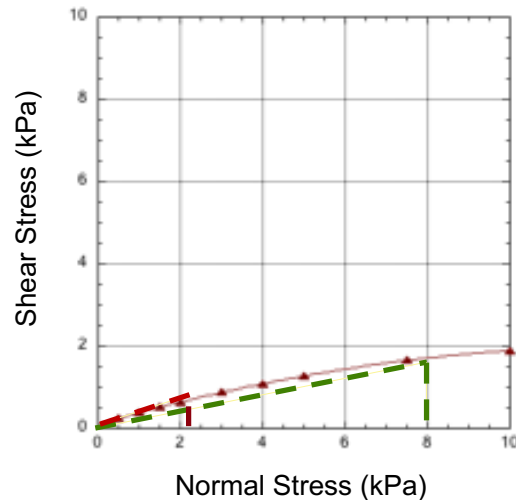
Example



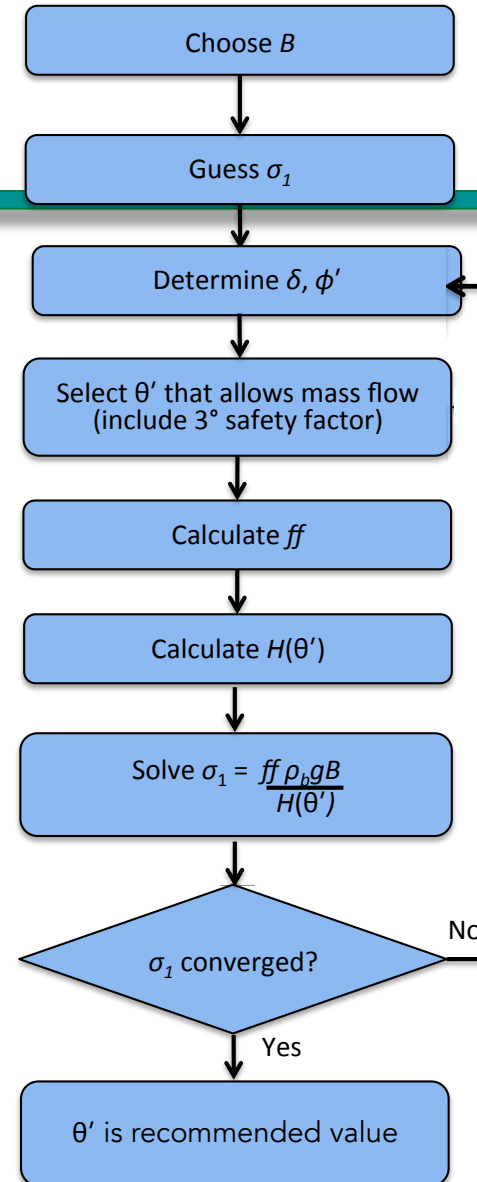
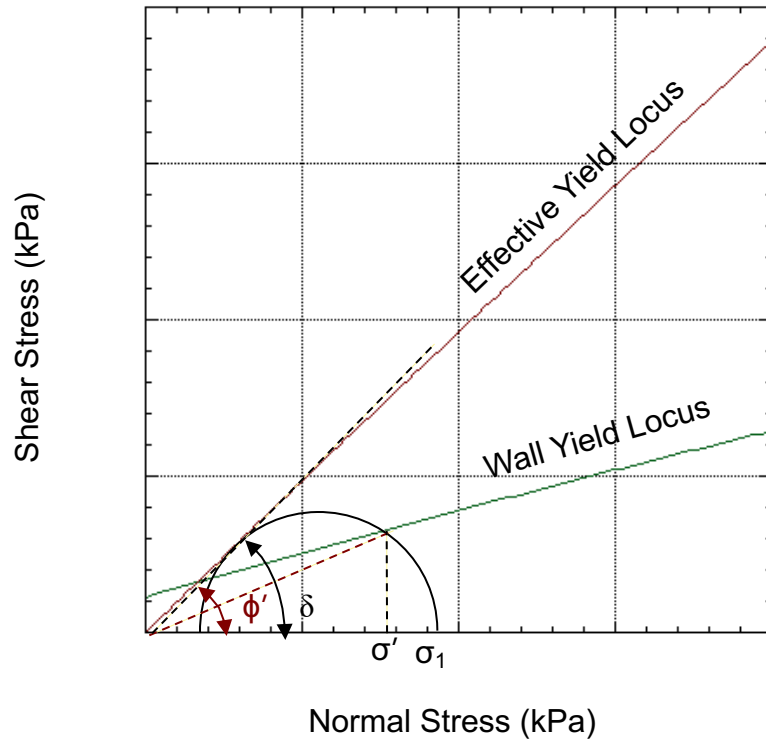
Example



Mass flow hopper angle



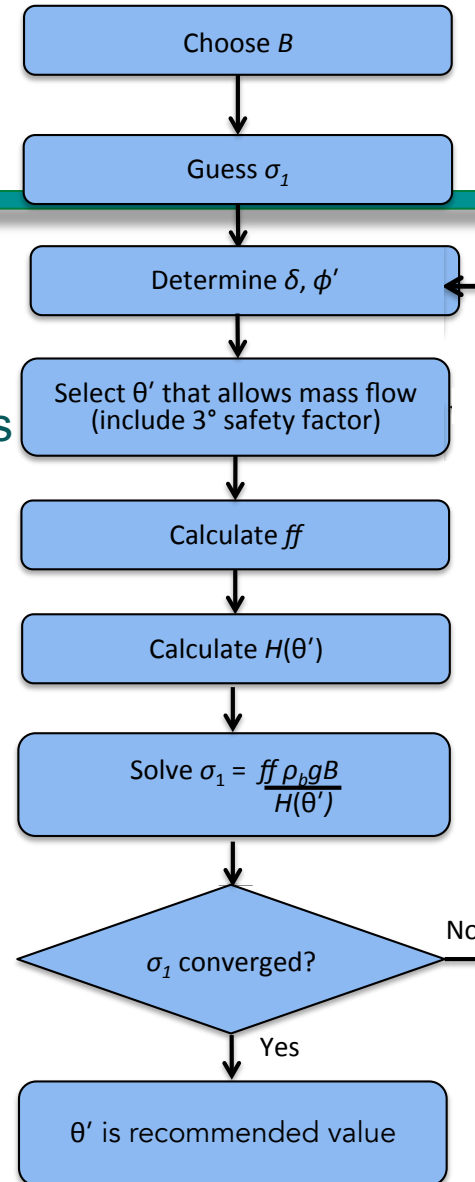
Mass flow hopper angle



Mass flow hopper angle

TUNRA

The University of Newcastle Research Associates
Shortland, NSW 2307, Australia



Flow factors

$! = 1$: conical, $! = 0$: planar, slotted outlet

Mass flow hopper angles

Conical

$$\theta' = 90^\circ - \frac{1}{2} \cos^{-1} \left(\frac{1 - \sin \delta}{1 + \sin \delta} \right) - \beta$$

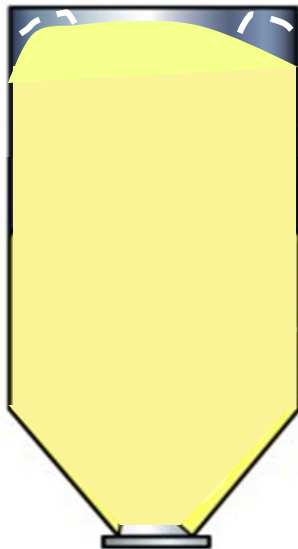
Planar, slotted outlet

$$2\beta = \phi' + \sin^{-1} \left(\frac{\sin \phi'}{\sin \delta} \right)$$

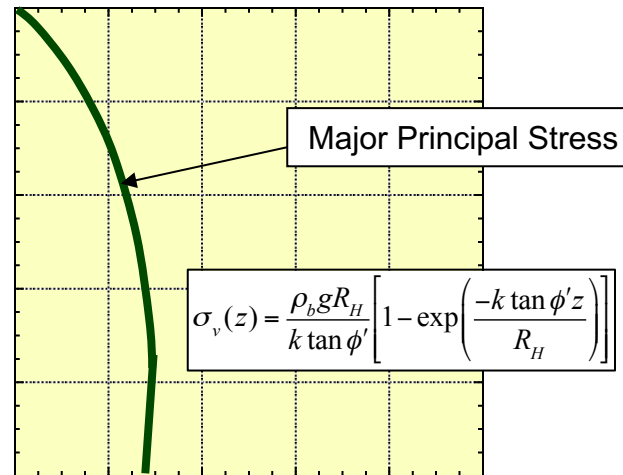
$$\theta' = \exp[3.75(1.01)^{-(\phi' - 30)^\circ/10}] - \phi'_{1/5}^{0.725(\tan \delta)}$$



Funnel flow bins

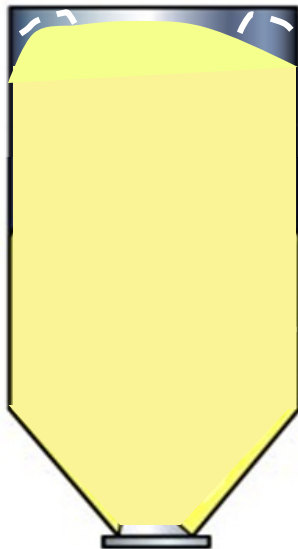


D = diameter of round outlet or
diagonal of slotted outlet

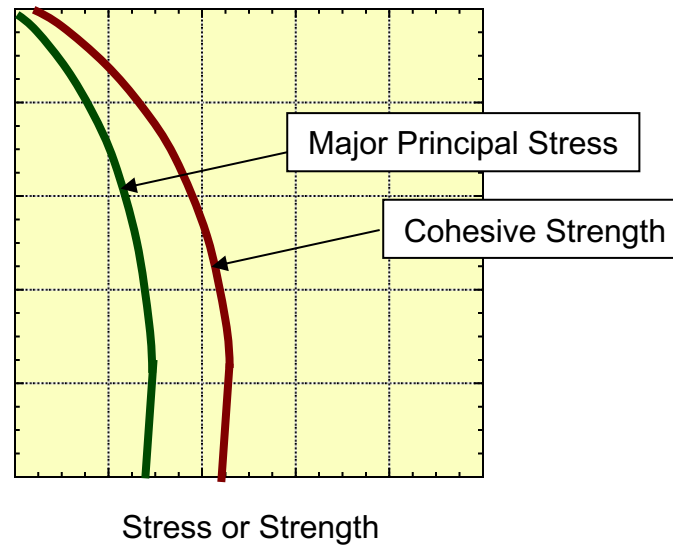


Stress

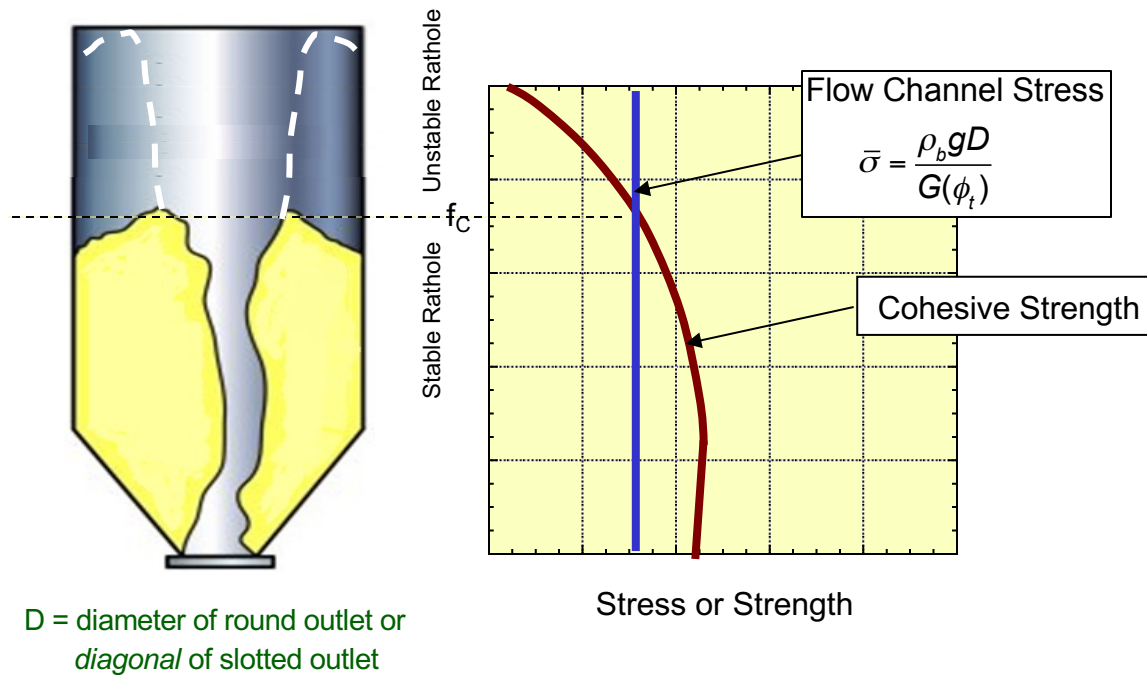
Funnel flow bins



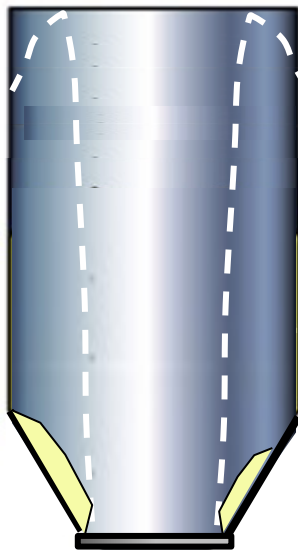
D = diameter of round outlet or
diagonal of slotted outlet



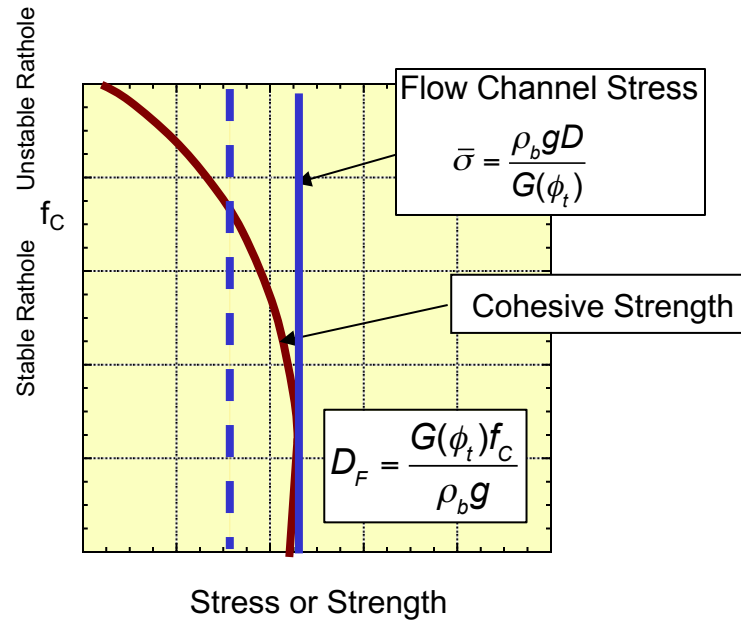
Funnel flow bins



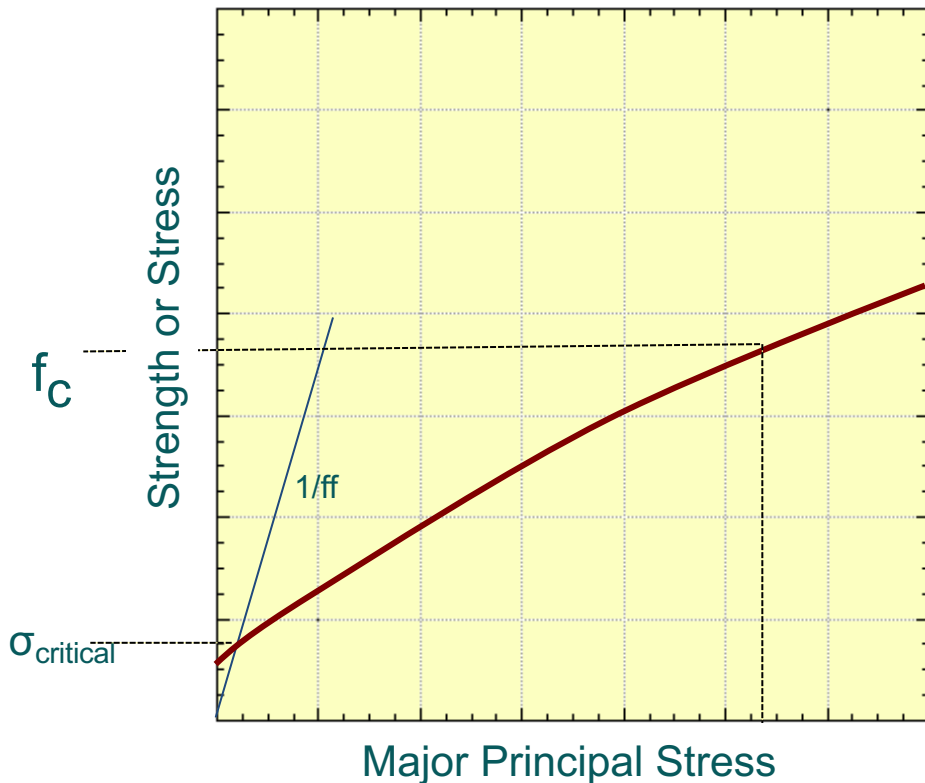
Funnel flow bins



D = diameter of round outlet or
diagonal of slotted outlet



Mass flow vs. funnel flow



To prevent arching in a mass flow bin:

$$B_{min} = \frac{H(\theta')\sigma_{critical}}{\rho_b g}$$

To prevent a stable rathole in a funnel flow bin:

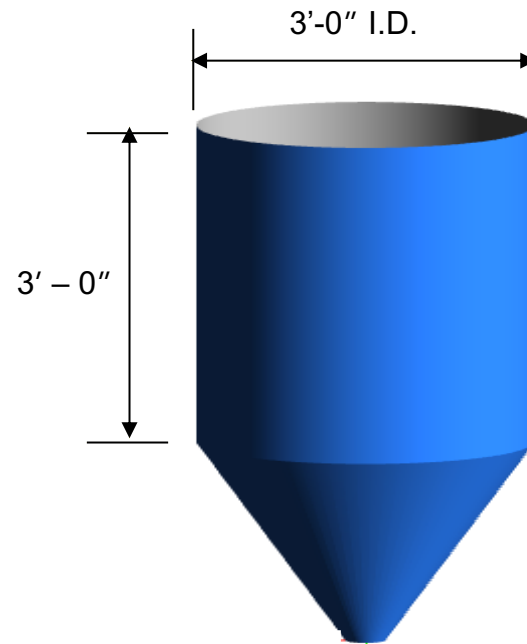
$$D_F = \frac{G(\phi_t)f_c}{\rho_b g}$$

$$\sigma_v(z) = \frac{\rho_b g R_H}{k \tan \phi'} \left[1 - \exp\left(\frac{-k \tan \phi' z}{R_H}\right) \right]$$

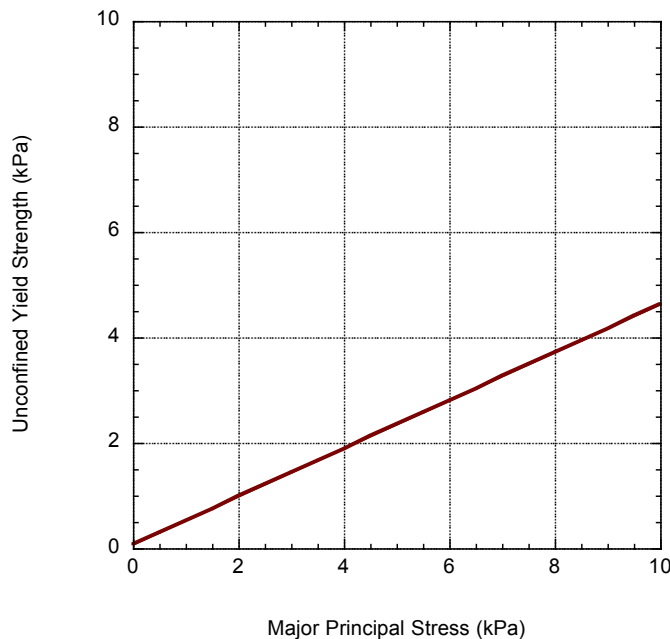


Funnel flow hopper design

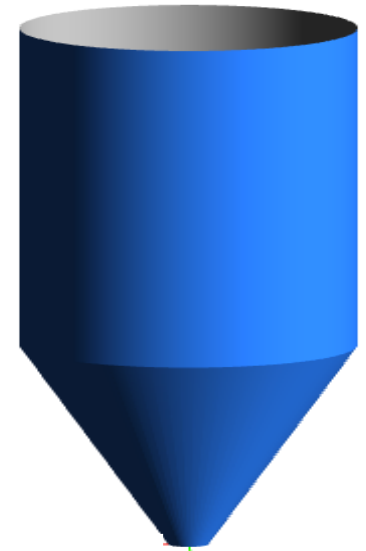
Critical rathole diameter depends on dimensions of cylinder section



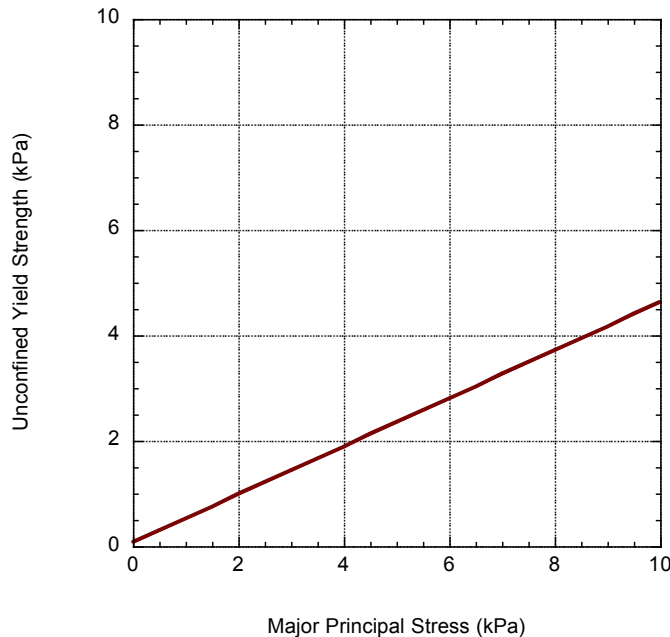
Funnel flow hopper design



$$D_F = \frac{G(\phi_t)f_c}{\rho_b g}$$
$$\sigma_1 = \frac{\bar{\rho}_b g R_H}{k \tan \phi'} \left[1 - \exp\left(\frac{-k(\tan \phi')h}{R_H} \right) \right]$$

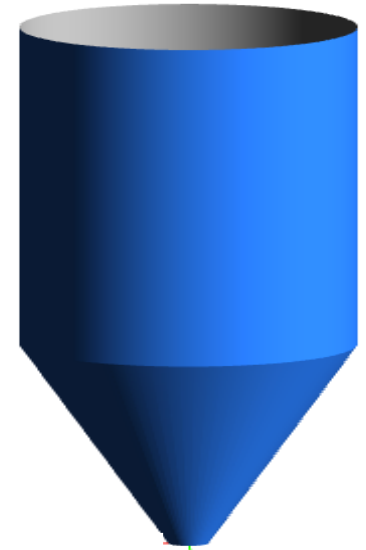


Funnel flow hopper design



$$D_F = \frac{G(\phi_t) f_C}{\rho_b g}$$

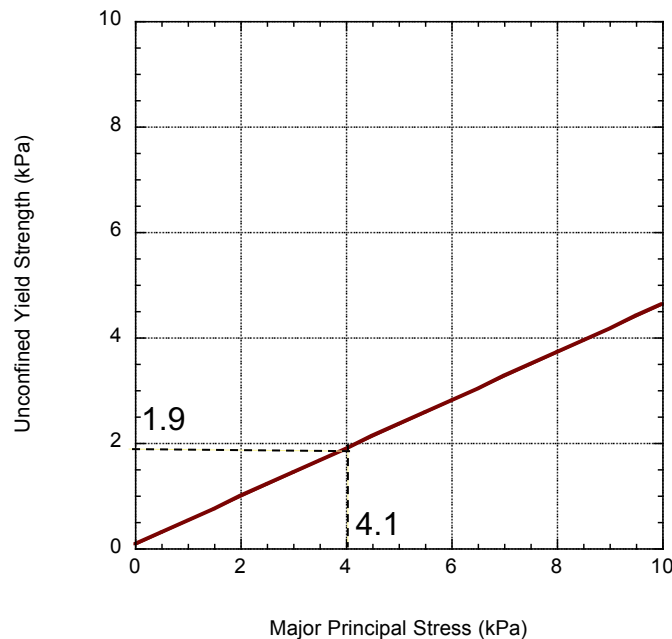
$$\sigma_1 = \frac{\bar{\rho}_b g R_H}{k \tan \phi'} \left[1 - \exp \left(\frac{-k(\tan \phi') h}{R_H} \right) \right]$$



$$\sigma_1 = \frac{(615)(9.8)(0.914 / 4)}{(0.4) \tan(21^\circ)} \left[1 - \exp \left(\frac{-(0.4) \tan(21^\circ)(0.914)}{0.914 / 4} \right) \right] = 4100 \text{ Pa}$$

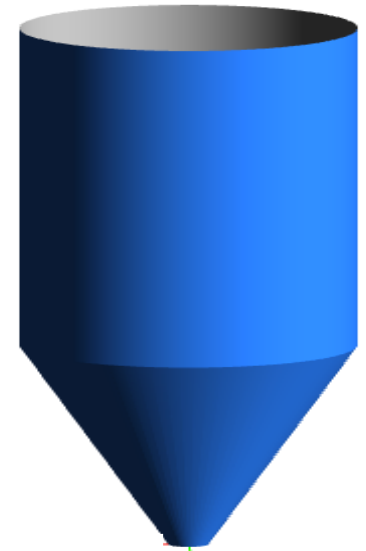


Funnel flow hopper design

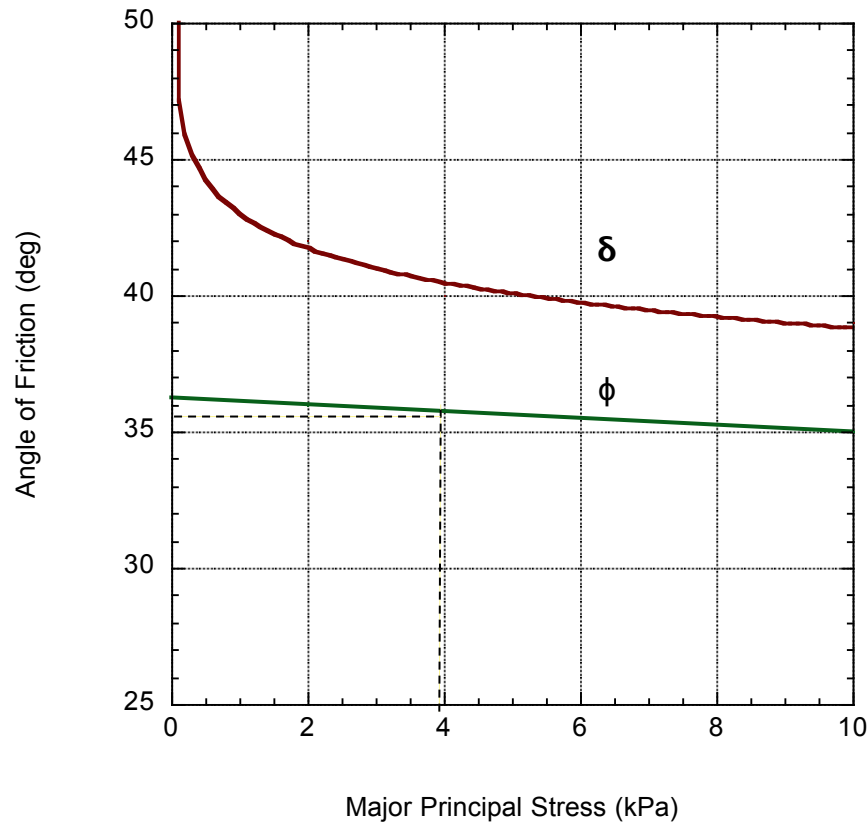


$$D_F = \frac{G(\phi_t)f_C}{\rho_b g}$$

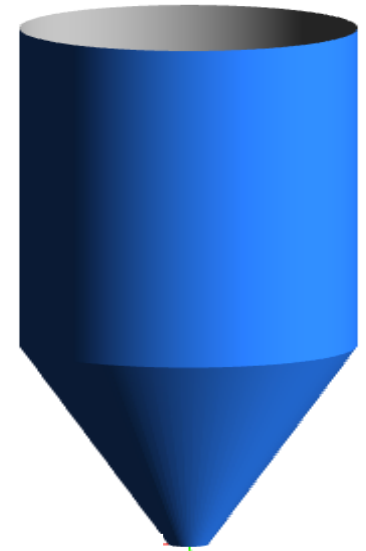
$$f_C = 1.9 \text{ kPa}$$



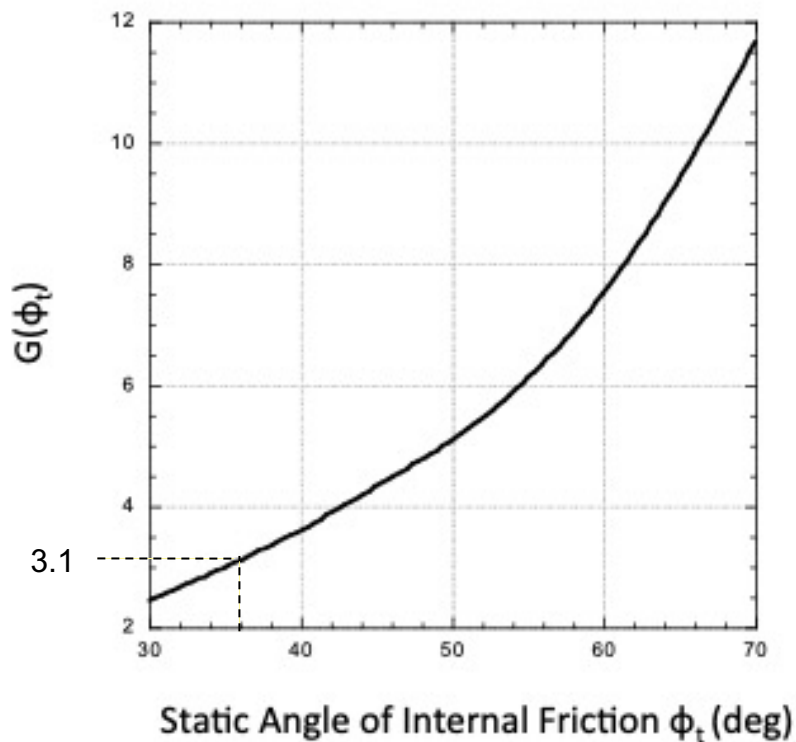
Funnel flow hopper design



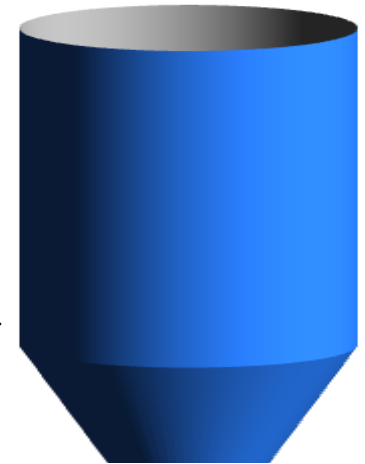
$$D_F = \frac{G(\phi_t)f_C}{\rho_b g}$$



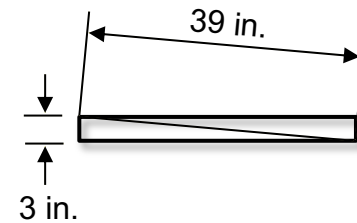
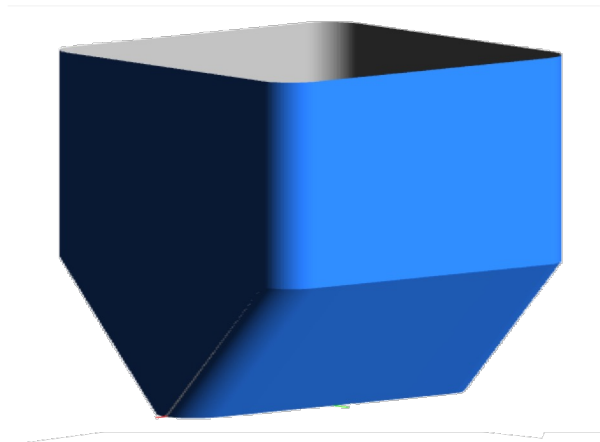
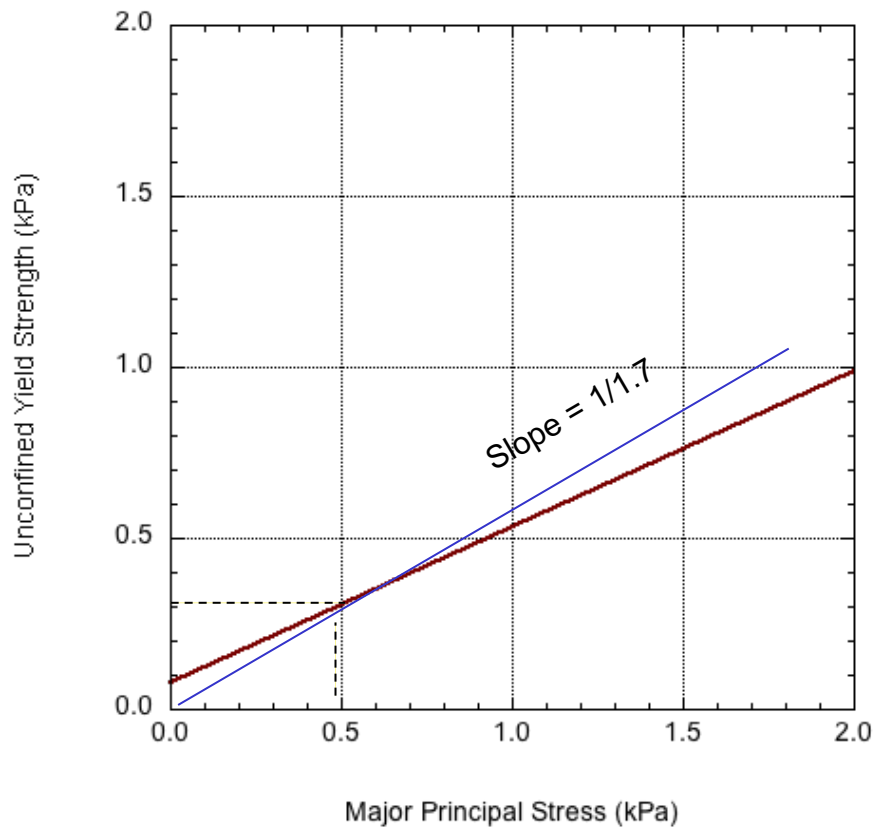
Funnel flow hopper design



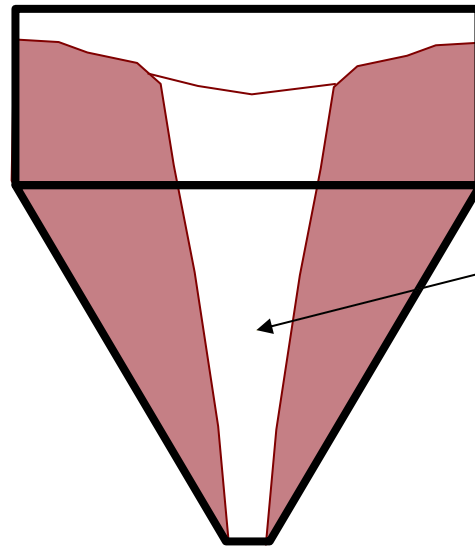
$$D_F = \frac{G(\phi_t) f_c}{\rho_b g}$$
$$= \frac{(3.1)(1.9)}{(618)(9.8)} = 1.0 \text{ m} = 39 \text{ in.}$$



Funnel flow hopper design



Flow channel



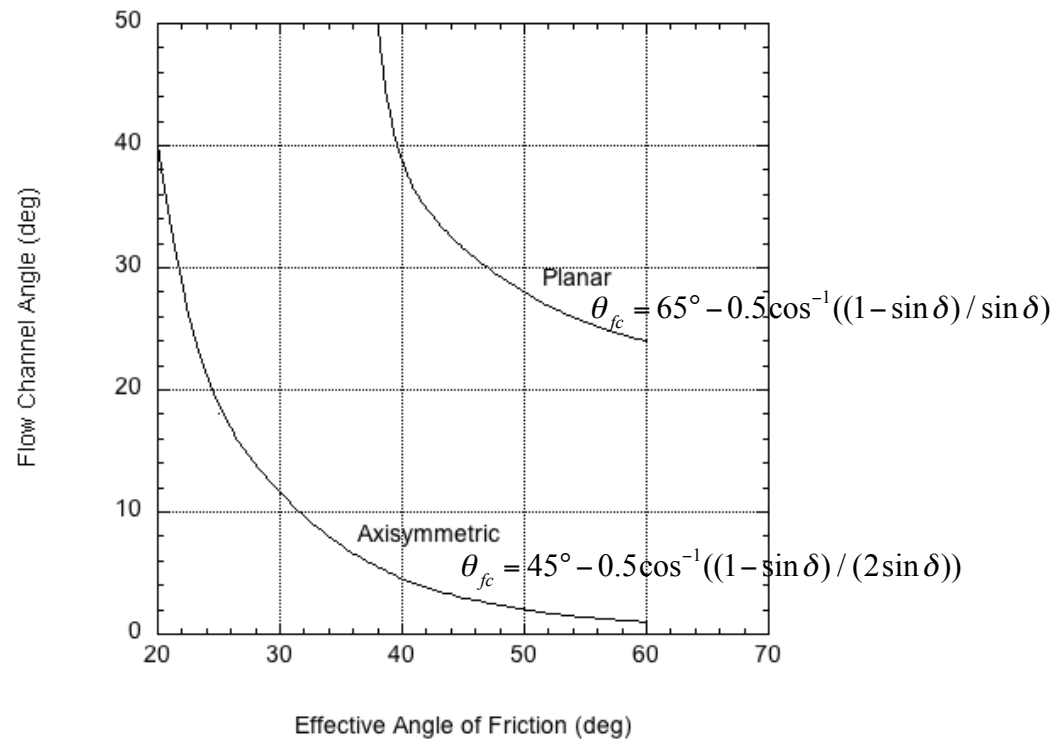
Flow Channel



Flow channel



Flow channel

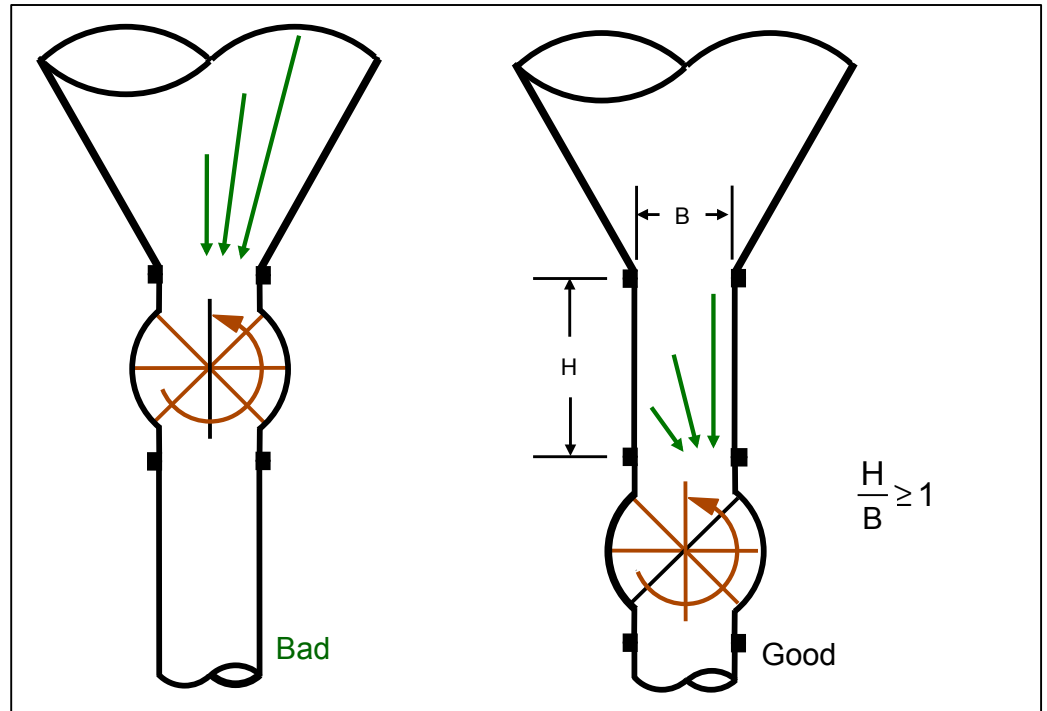
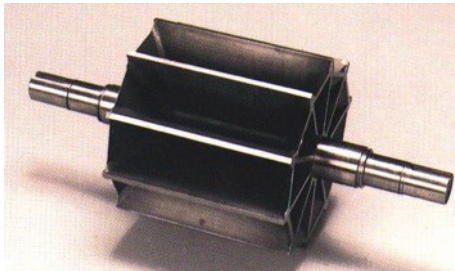


Feeders

- Used to modulate the discharge rate of a powder from a hopper, bin, or silo
- Outlet must be large enough to prevent flow obstruction and allow desired discharge rate
- Slide gates should never be used to control discharge rates



Rotary valves

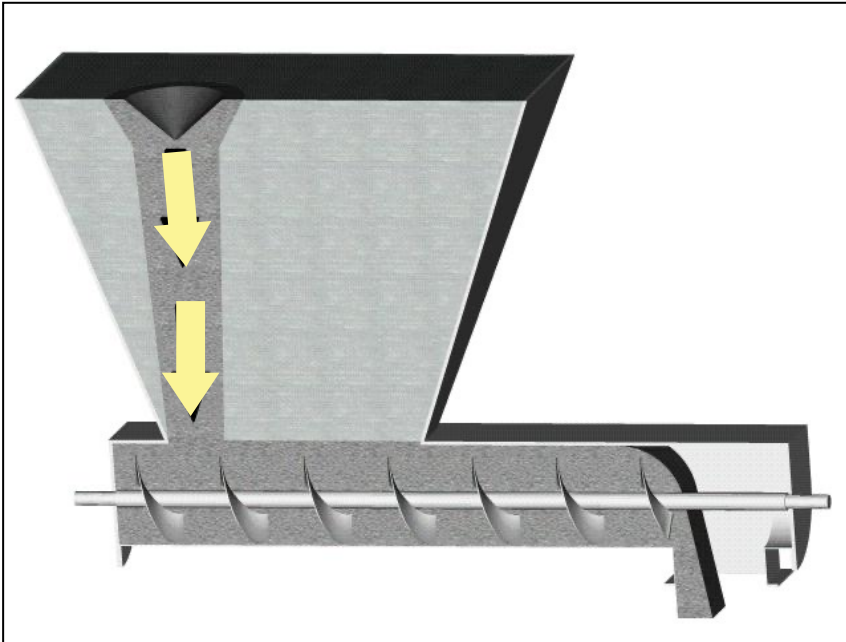


Rotary valves

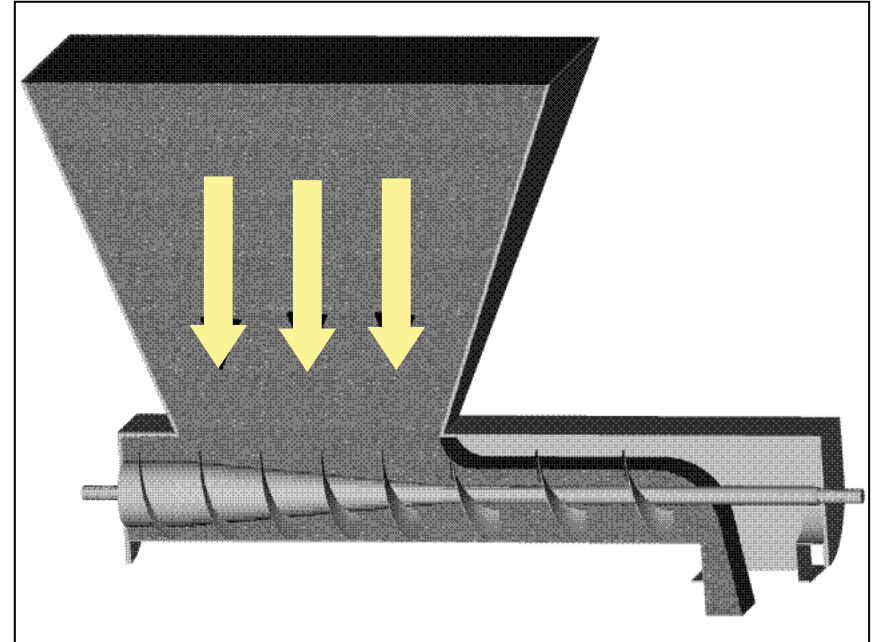


- Typical speed: 15-25 rpm
- Maximum speed: 35-40 rpm

Screw feeders



Constant diameter, constant pitch



Tapered shaft diameter, increasing pitch

Constant pitch and shaft diameter



Jenike & Johanson



Tapered shaft and increasing pitch



Jenike & Johanson

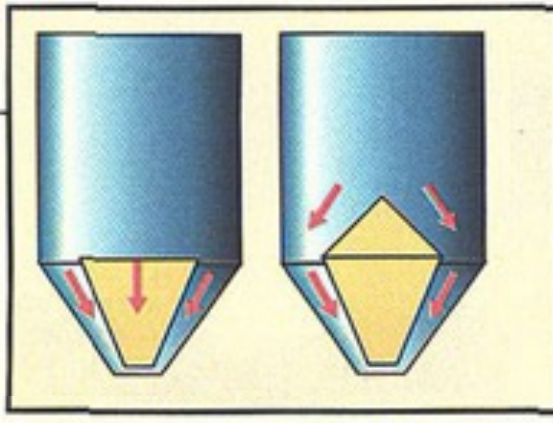


Tricks for handling neurotic powders

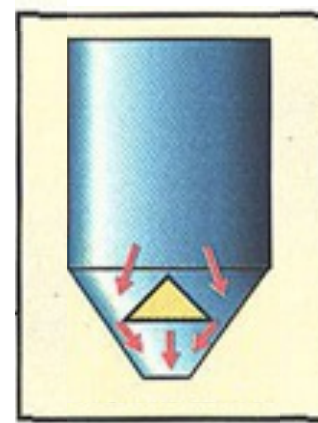
- Inserts
- Converging/Diverging walls
- Agitation
- Air assistance



Inserts

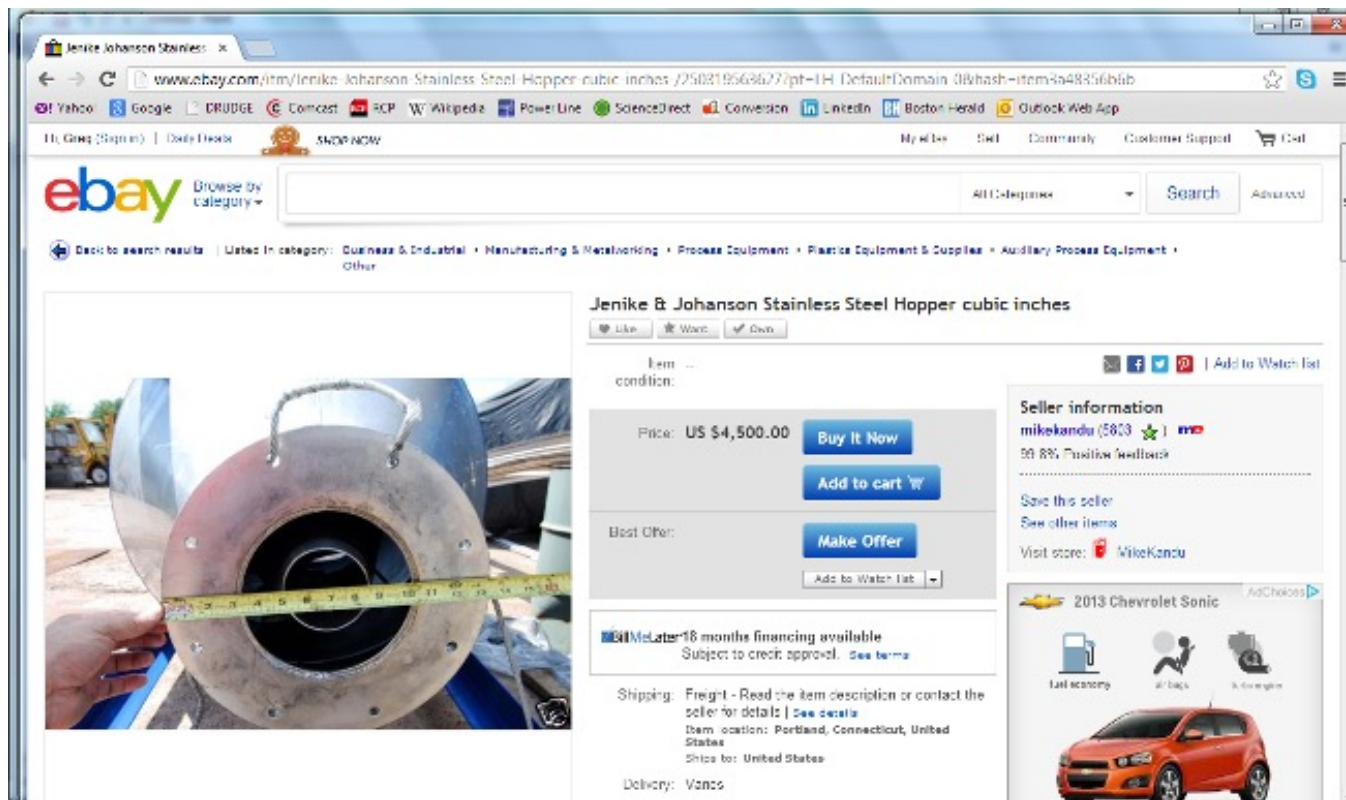


Cone-in-cone

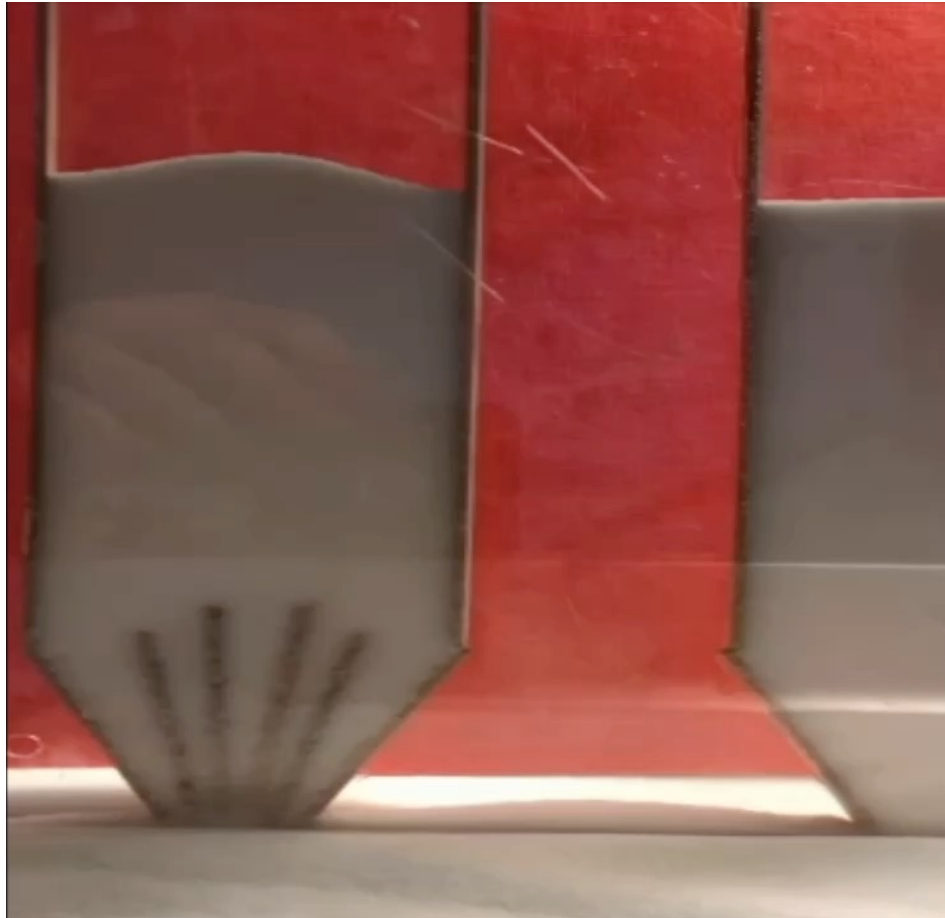


Inverted cone

Inserts



Inserts^2



Converging/diverging



Diamondback



Stacked wedges
a.k.a. LynFlow



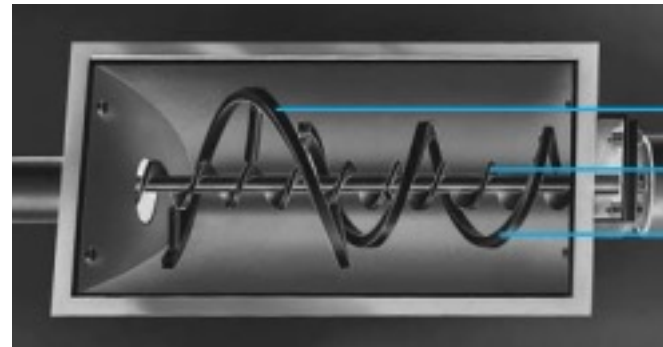
Converging/diverging



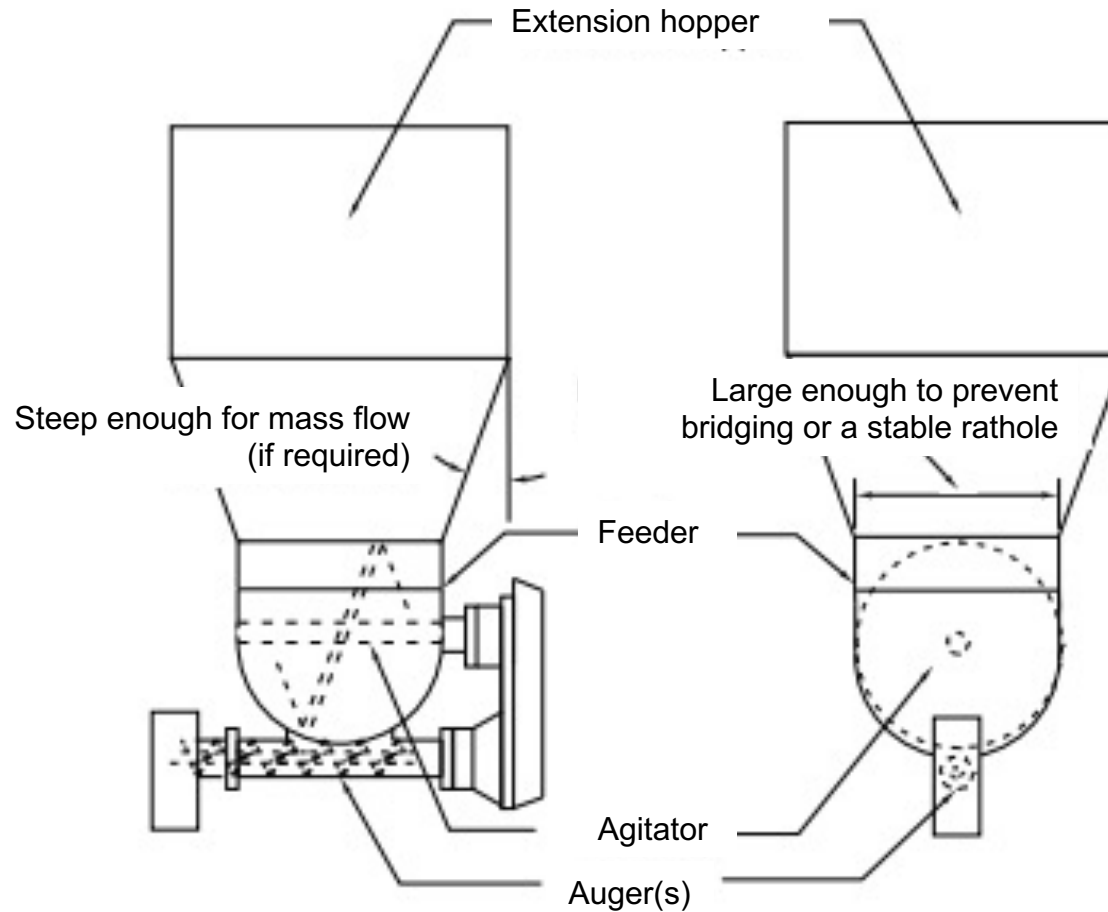
“D-tron”



Agitated feeders



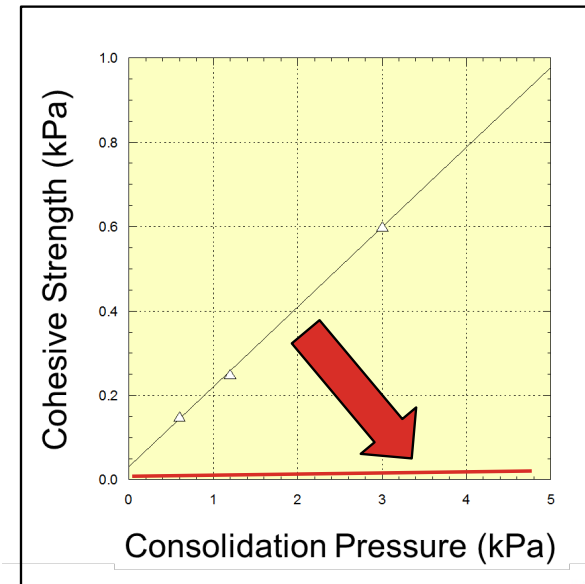
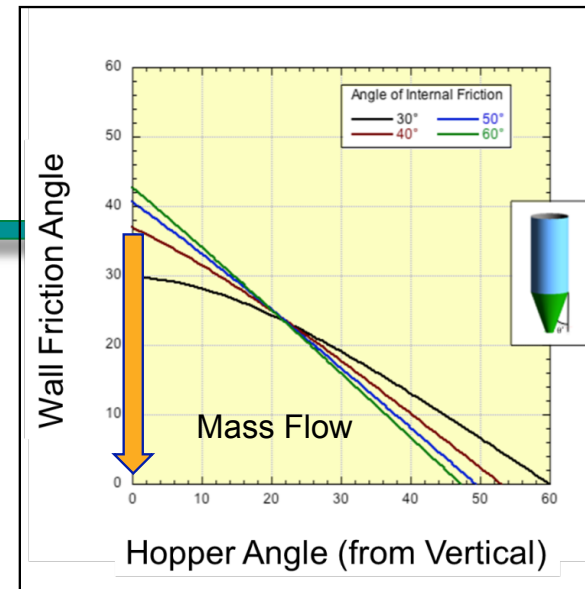
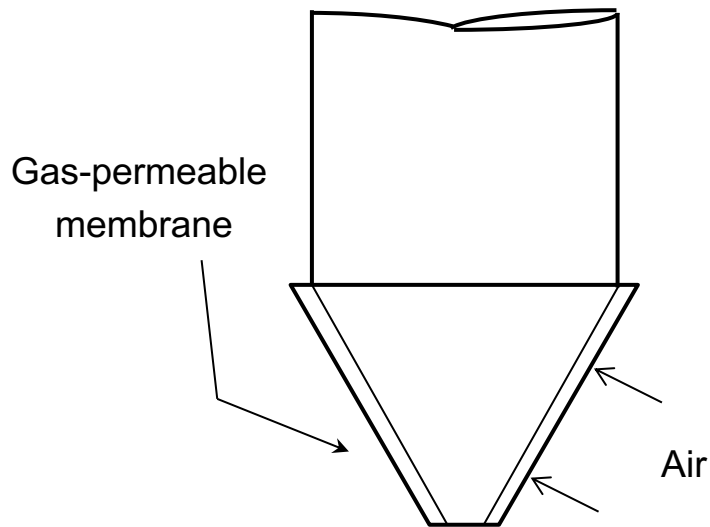
Agitated feeders



Air assistance



Air assistance



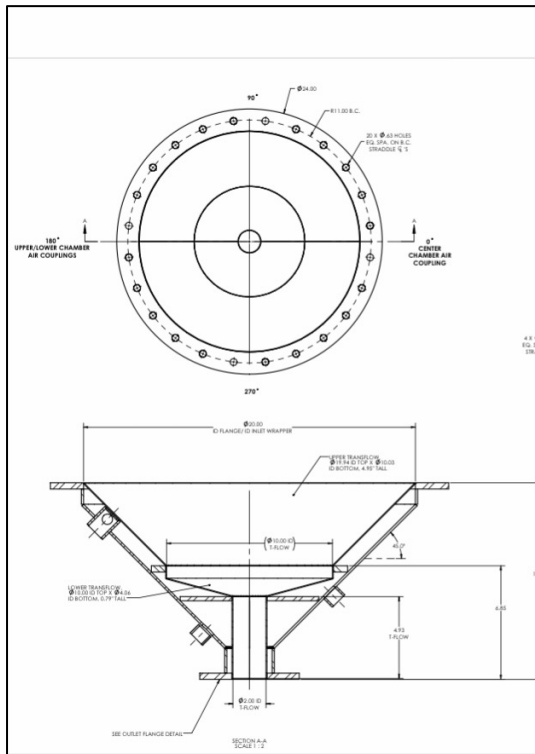
Air assistance



Air assistance



Air assistance



Vibrators

a) no vibration

Material	Fill Level	A	B	C	D	E	F	G	Cylinder
Lactose FastFlo (1.11)	1/2 Height	●	●	●	●	●	●	●	
	Full Height	●	●	●	●	●	●	●	●
MCC Avicel PH200 (1.19)	1/2 Height	●	●	●	●	●	●	●	
	Full Height	●	●	●	●	●	●	●	●
MCC Avicel PH102 (1.26)	1/2 Height	●	●	●	●	●	●	●	
	Full Height	●	●	●	●	●	●	●	●
MCC Avicel PH101 (1.42)	1/2 Height	●	●	●	●	●	●	●	
	Full Height	●	●	●	●	●	●	●	●
Starch + Silicone Oil (1.73)	1/2 Height	●	●	●	●	●	●	●	
	Full Height	●	●	●	●	●	●	●	●
Acetaminophen -USP APAP (2.03)	1/2 Height	●	●	●	●	●	●	●	
	Full Height	●	●	●	●	●	●	●	●



Vibrators

a) no vibration

Material	Fill Level	A	B	C	D	E	F	G	Cylinder
Lactose FastFlo (1.11)	1/2 Height	●	●	●	●	●	●	●	
	Full Height	●	●	●	●	●	●	●	●
MCC Avicel PH200 (1.19)	1/2 Height	●	●	●	●	●	●	●	
	Full Height	●	●	●	●	●	●	●	●
MCC Avicel PH102 (1.26)	1/2 Height	●	●	●	●	●	●	●	
	Full Height	●	●	●	●	●	●	●	●
MCC Avicel PH101 (1.42)	1/2 Height	●	●	●	●	●	●	●	
	Full Height	●	●	●	●	●	●	●	●
Starch + Silicone Oil (1.73)	1/2 Height	●	●	●	●	●	●	●	
	Full Height	●	●	●	●	●	●	●	●
Acetaminophen -USP APAP (2.03)	1/2 Height	●	●	●	●	●	●	●	
	Full Height	●	●	●	●	●	●	●	●

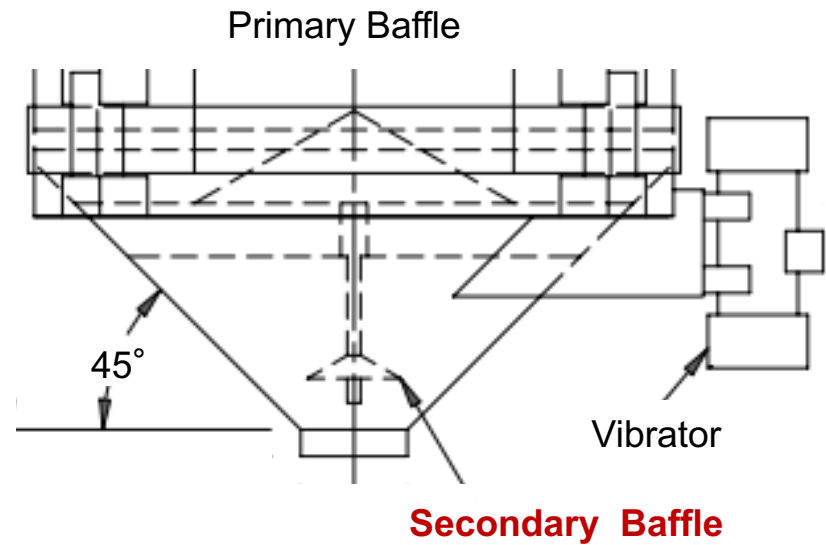
b) with vibration

Material	Fill Level	A	B	C	D	E	F	G	Cylinder
Lactose FastFlo (1.11)	1/2 Height	●	●	●	●	●	●	●	
	Full Height	●	●	●	●	●	●	●	●
MCC Avicel PH200 (1.19)	1/2 Height	●	●	●	●	●	●	●	
	Full Height	●	●	●	●	●	●	●	●
MCC Avicel PH102 (1.26)	1/2 Height	●	●	●	●	●	●	●	
	Full Height	●	●	●	●	●	●	●	●
MCC Avicel PH101 (1.42)	1/2 Height	●	●	●	●	●	●	●	
	Full Height	●	●	●	●	●	●	●	●
Starch + Silicone Oil (1.73)	1/2 Height	●	●	●	●	●	●	●	
	Full Height	●	●	●	●	●	●	●	●
Acetaminophen -USP APAP (2.03)	1/2 Height	●	●	●	●	●	●	●	
	Full Height	●	●	●	●	●	●	●	●

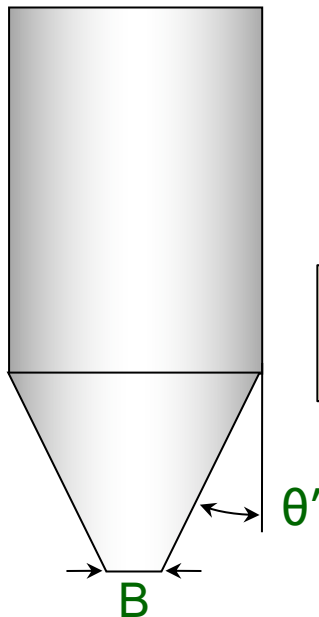
● Mass Flow ● Funnel Flow ● Rathole ● Bridging



Bin activators



Solids discharge rates



$$\frac{2(m+1)\tan\theta'}{Bg}v_o^2 = 1 - \frac{2(m+1)\cos\theta'\sin(\beta+\theta')f_c}{\rho_b g B} + \frac{1}{\rho_{bo}g} \frac{dP}{dz} \Big|_o$$

$$\left[\frac{2(m+1)\tan\theta'}{Bg} \right] v_o^2 + \left[\frac{1}{K_o} \left(1 - \frac{\rho_{bo}}{\rho_{bmp}} \right) \right] v_o + \frac{2(m+1)\cos\theta'\sin(\beta+\theta')f_c}{\rho_b g B} - 1 = 0$$

Permeability

Mehos, G., “Maximum Solids Discharge Rates from Hoppers”, *Chem. Engr. Res. Des.*, 191, 564 (2023).



Summary

- Flow obstructions
 - Arching and ratholing
 - Erratic discharge
- Flow properties
 - Cohesive strength
 - Compressibility
 - Wall friction
- Design
 - Outlet
 - Hopper angle

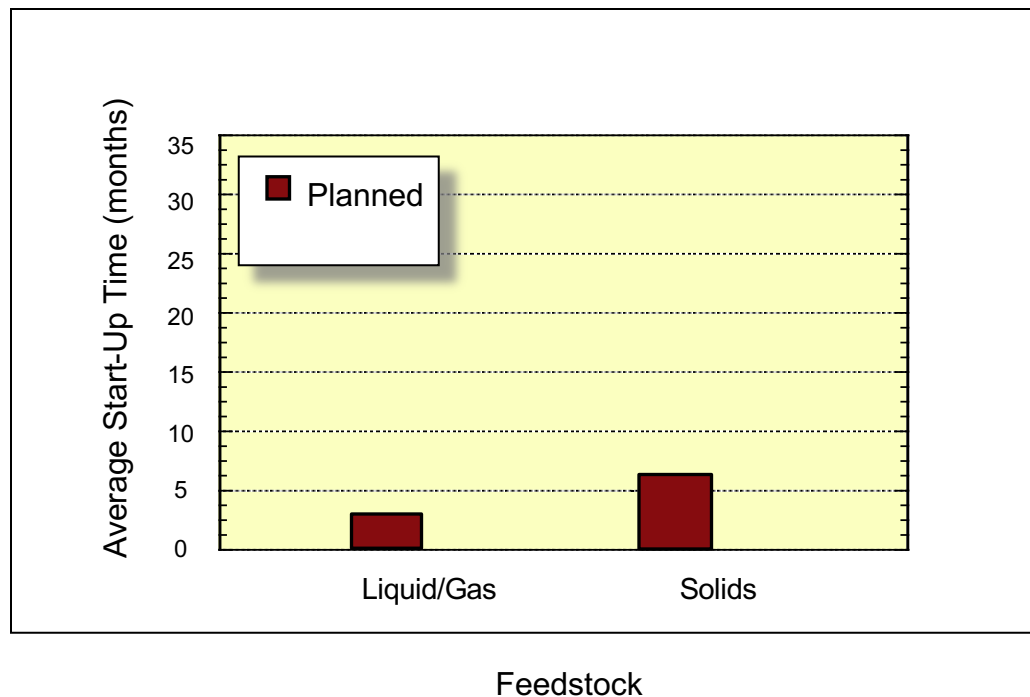


Rand Corporation Study

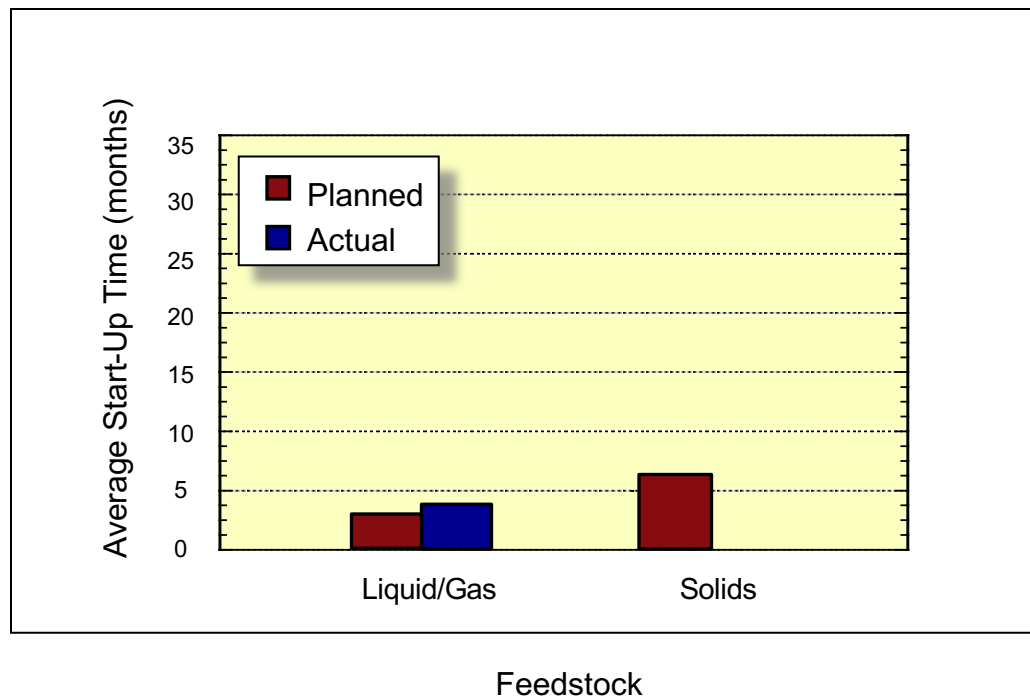
- Study by independent organization
- 40 solids processing plants in U.S. and Canada
- Six-year duration



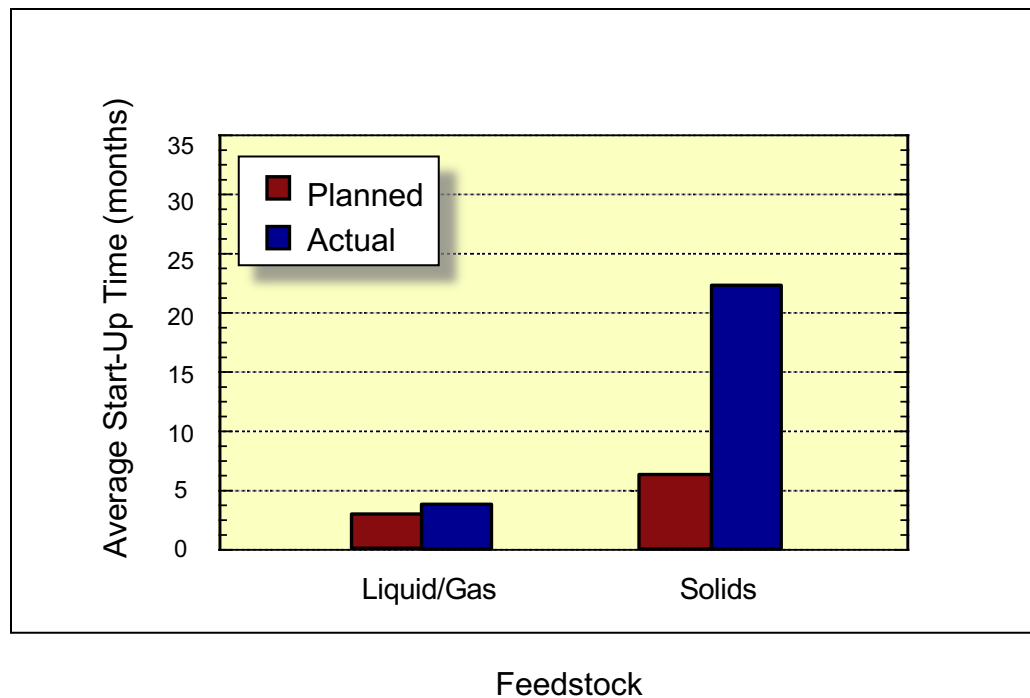
Planned vs. actual start-up



Planned vs. actual start-up



Planned vs. actual start-up



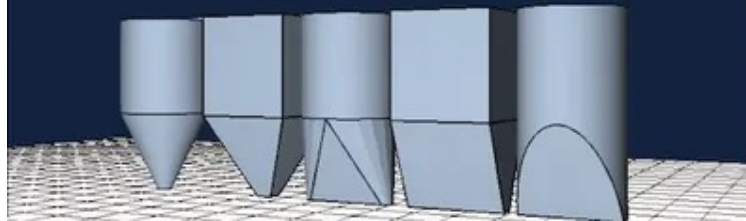
Rand conclusions

- 80% experienced solids handling problems
- Average startup time *ca.* 20 months (vs. 4 mo. for fluids)
- Typical performance 40 to 50% of design



STORAGE AND FLOW OF BULK SOLIDS

Greg Mehos, Ph.D., P.E.



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Flow and Storage of Powders

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