Solids Velocity Profiles in Hoppers

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Segregation can be induced when a free-flowing powder having a distribution of particle sizes is transferred into a hopper. As a pile is formed, larger particles may slide toward the hopper walls while fine particles percolate through the bed and concentrate beneath the fill point. The result is side-to-side or sifting segregation.

Sifting segregation can be mitigated if the hopper is designed for mass flow. For mass flow, the hopper walls must be steep enough and low enough in friction to allow flow along the walls when the powder is discharged. Because all the powder is in motion, it tends to re-mix as it flows toward the outlet. Andrew Jenike determined recommended mass flow hopper angles based on the powder's effective angle of friction δ and the angle of wall friction ϕ', properties that can be determined by shear cell testing. Recommended hopper angles for mass flow are given in Figures 1 and 2 for axisymmetric and planar hoppers, respectively.

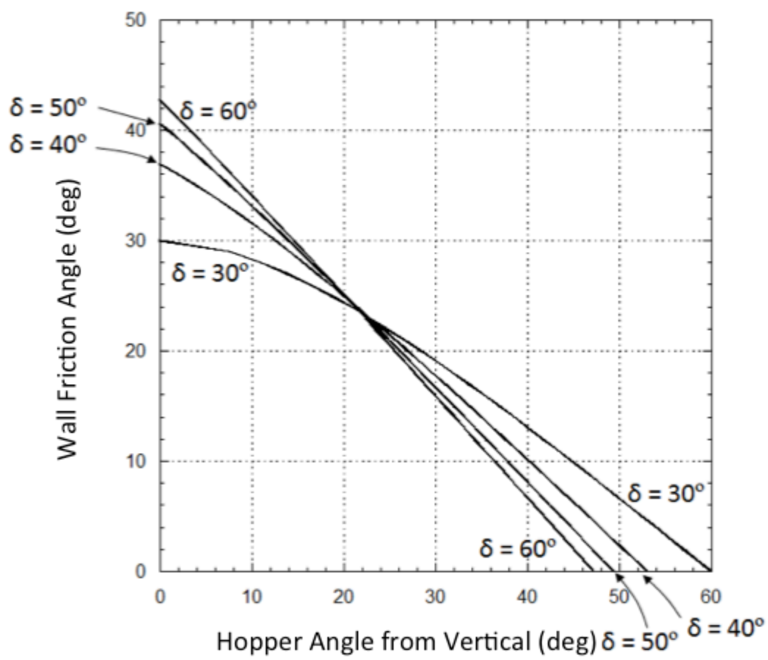

Figure 1. ***Theoretical*** **mass flow hopper angles for hoppers with round or square outlets.** ***Note: a minimum safety factor of 2 to 3° should be used.***

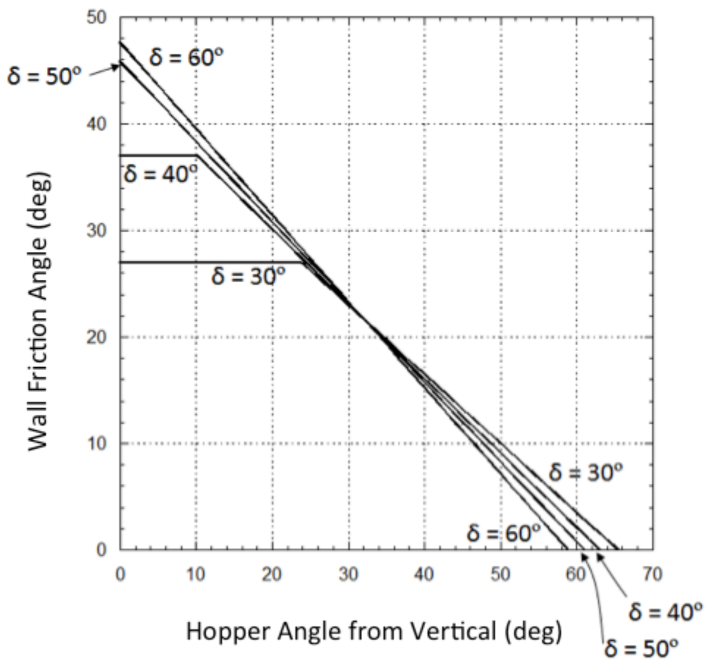

Figure 2. Recommended mass flow hopper angles for wedge-shaped hoppers.

Segregation may still occur in mass flow, as the degree of segregation depends on the solids velocity profile that develops inside the hopper section. If the velocity profile is severe, segregation can be significant.

Andrew Jenike realized that a radial velocity field was compatible with the radial stress field function that he solved when calculating flow factors. In his Bulletin 108, he solved the following equations to determine the velocity profile:

$$\frac{d\psi}{d\theta} = -1 - \frac{ms\sin\delta(1+\sin\delta)(\cot\theta\sin 2\psi + \cos 2\psi - 1) + \cos\theta - \sin\delta\cos(\theta + 2\psi) + s\cos^2\delta}{2s\sin\delta(\cos 2\psi - \sin\delta)} \quad (1)$$

$$\frac{ds}{d\theta} = \frac{s\sin 2\psi + \sin(\theta + 2\psi) + ms\sin\delta[\cot\theta(1+\cos 2\psi) - \sin 2\psi]}{\cos 2\psi - \sin\delta} \quad (2)$$

$$\psi(\theta') = \frac{1}{2}\left[\phi' + \sin^{-1}\left(\frac{\sin\phi'}{\sin\delta}\right)\right] + 90° \quad (3)$$

$$\psi(90°) = 90° \quad (4)$$

$$\frac{V}{V_0} = \exp\left[-(2+m)\int_0^{\theta'} \tan(2\psi)\, d\theta\right] \quad (5)$$

where δ is the effective angle of friction, θ is the radial coordinate, ϕ' is the wall friction angle, ψ is the angle between the direction of the major principle stress and the radial coordinate ray, m is equal to 0 or 1 for planar flow and axisymmetric flow, respectively, s is the radial stress function, V is the radial velocity, and V_0 is the centerline velocity. Equations 3 and 4 describe the boundary conditions for Equation 1, which means that the set of equations present a split boundary problem. Because computers were not readily available at the time, Jenike had to solve the equations by hand. In his Bulletin 108, he provided the solutions in graph form only for δ equal to 50°, which are shown in Figures 3 and 4 for uniaxial and planar geometries, respectively.

Fortunately, computers allow Jenike's equations to be readily solved for combinations of effective angle of friction, wall friction angle, and hopper angle that allow mass flow. Velocity profiles are plotted against dimensionless distance r/R, *i.e*, the normalized distance from the hopper wall, in Figures 5 and 6 for conical hoppers with angles of 20° and 30° from vertical, respectively.

The ratio of the radial velocity at the wall V' to the centerline velocity V_0 is a good metric for the uniformity of the solids velocity. It can be considered the span of the solid velocity. Figures 7 through 14 are plots of V'/V_0 *vs.* hopper angle θ' for different values of the wall friction angle ϕ'.

Figures 14 – 22 plot the ratio of the average solids velocity to the centerline velocity. To minimize sifting segregation that occurs during filling of a hopper, it is best to design a mass flow hopper that allows a high average solids velocity to centerline velocity ratio.

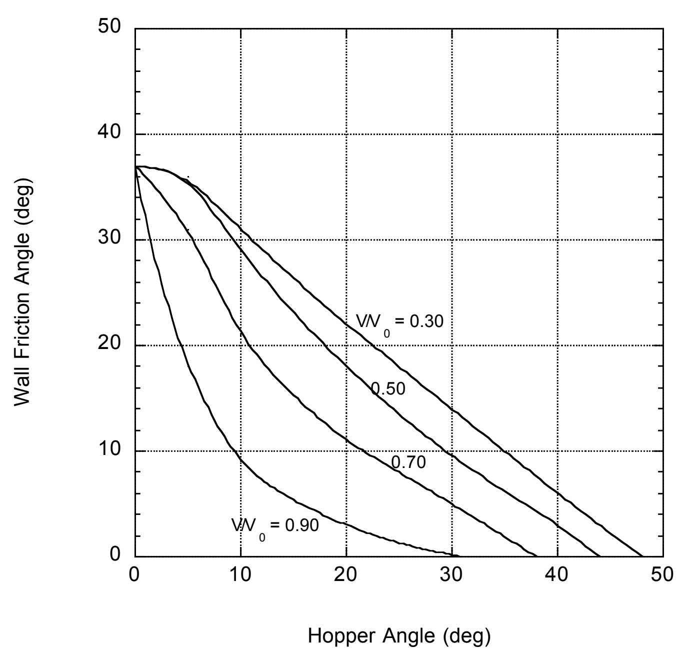

Figure 3. Solids velocity profiles for δ = 50°, uniaxial flow.

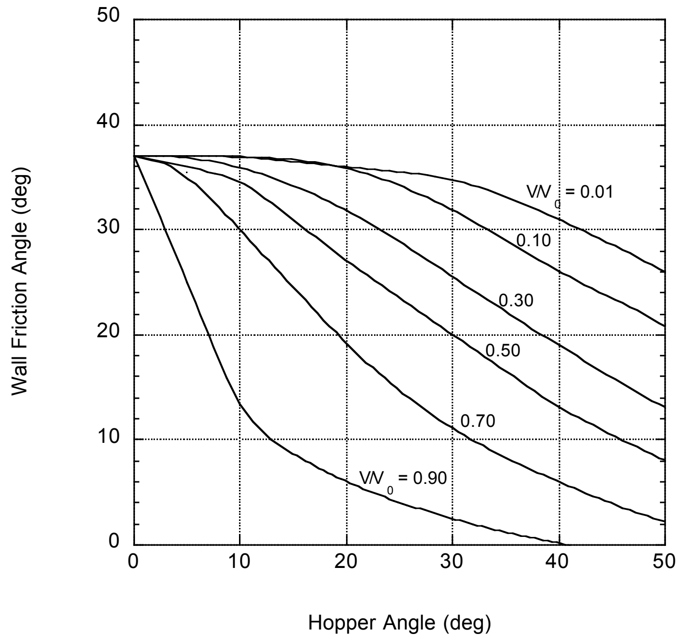

Figure 4. Solids velocity profiles for δ = 50°, planar flow.

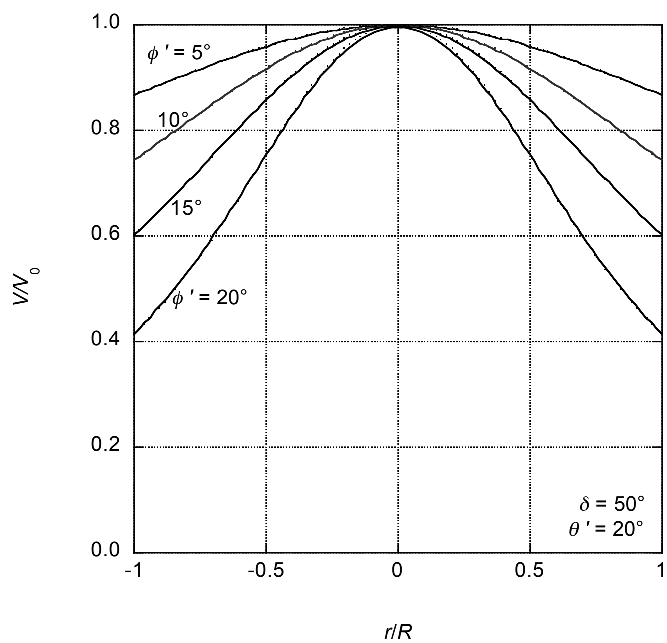

Figure 5. Velocity profiles for δ = 50°, θ = 20°, axisymmetric flow.

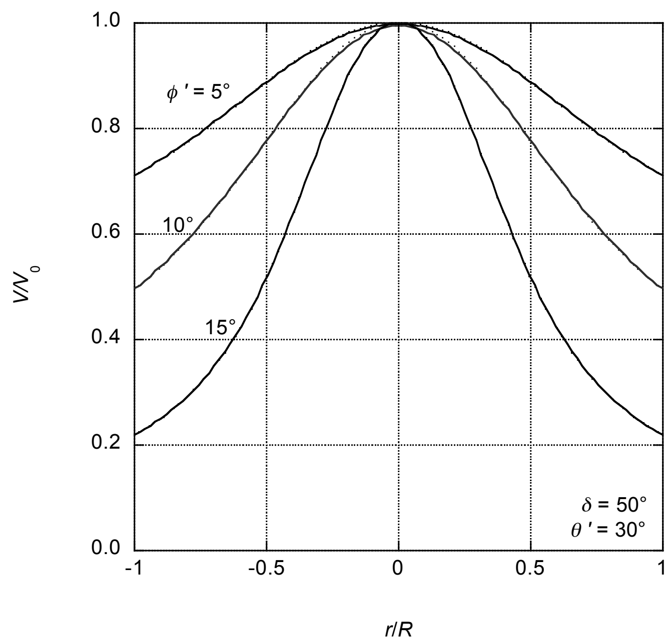

Figure 6. Velocity profiles for $\delta = 50°$, $\theta = 30°$, axisymmetric flow.

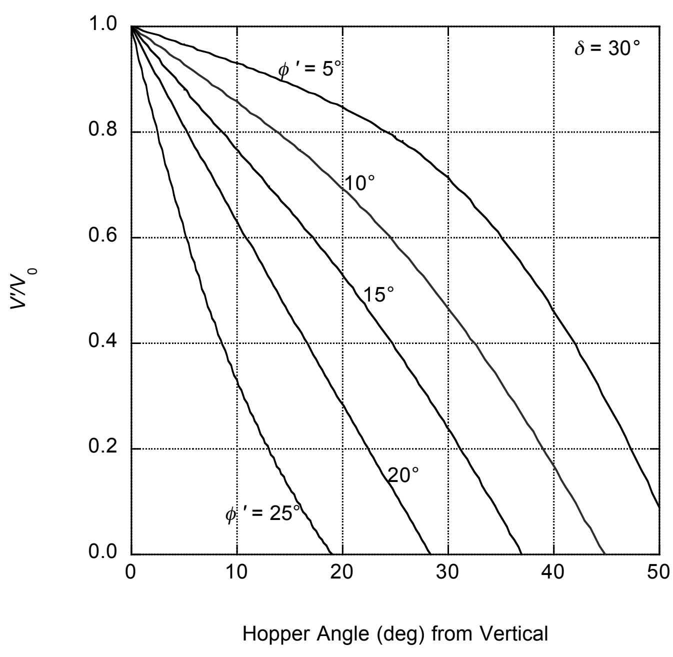

Figure 7. Wall-centerline velocity ratio, $\delta = 30°$, axisymmetric flow.

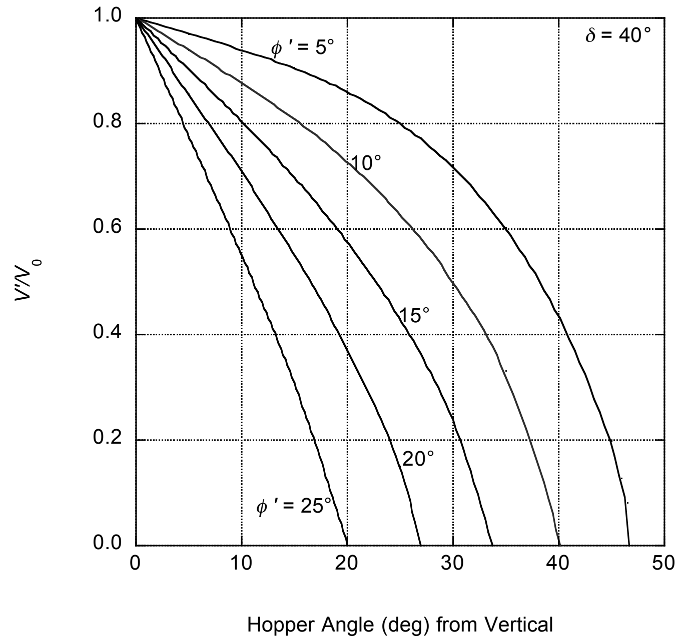

Figure 8. Wall-centerline velocity ratio, δ = 40°, axisymmetric flow.

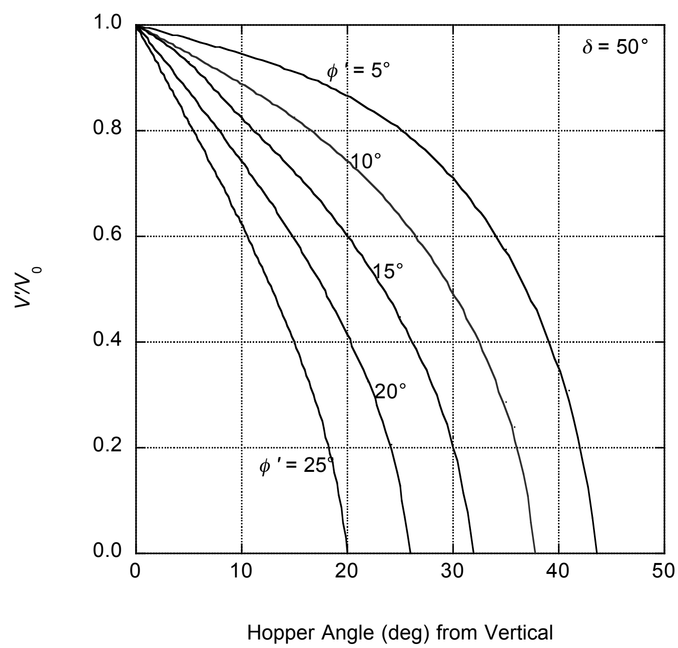

Figure 9. Wall-centerline velocity ratio, δ= 50°, axisymmetric flow.

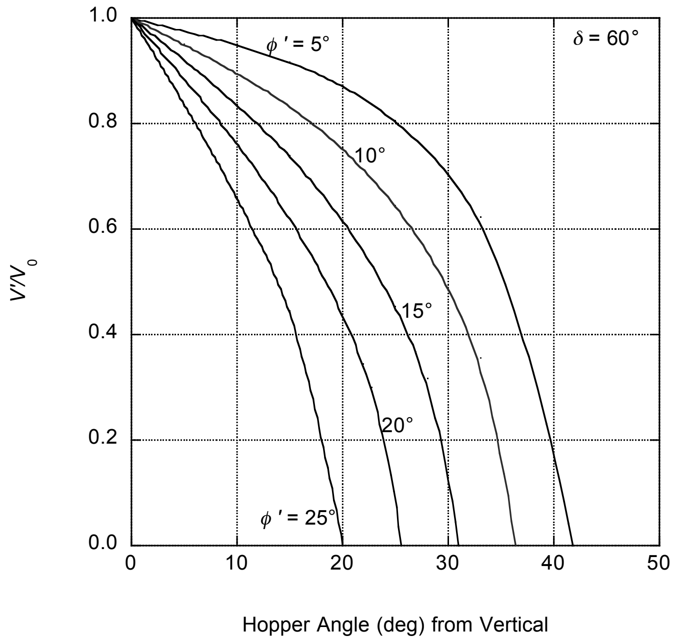

Figure 10. Wall-centerline velocity ratio, δ = 60°, axisymmetric flow.

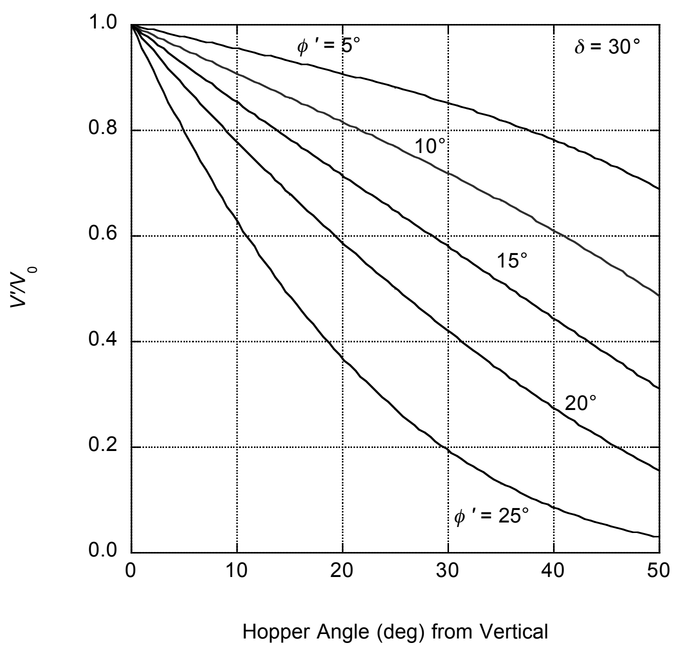

Figure 11. Wall-centerline velocity ratio, δ = 30°, planar flow.

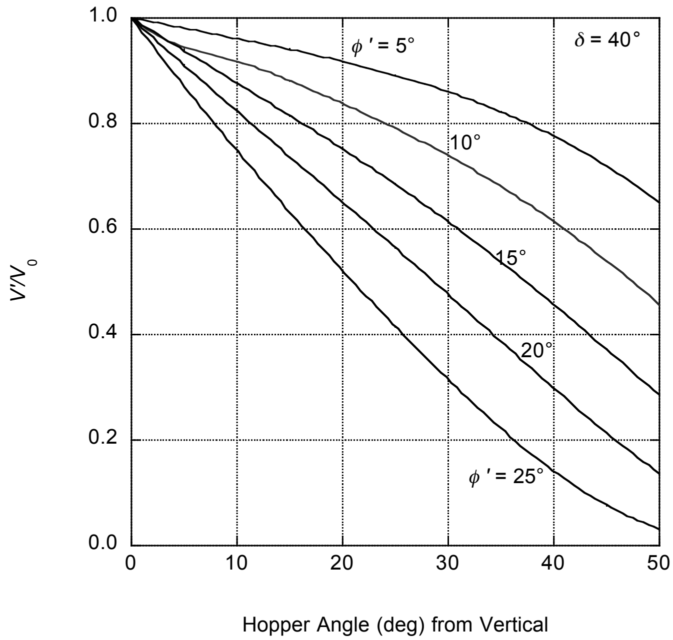

Figure 12. Wall-centerline velocity ratio, δ = 40°, planar flow.

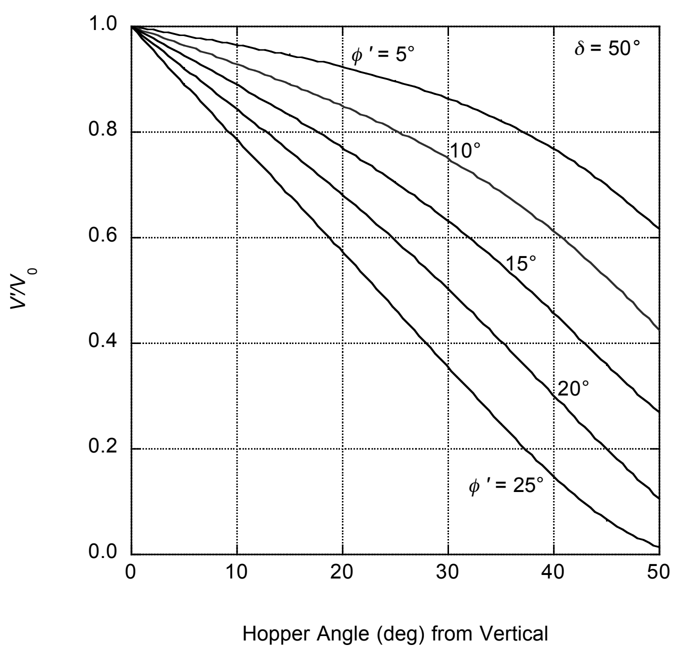

Figure 13. Wall-centerline velocity ratio, δ= 50°, planar flow.

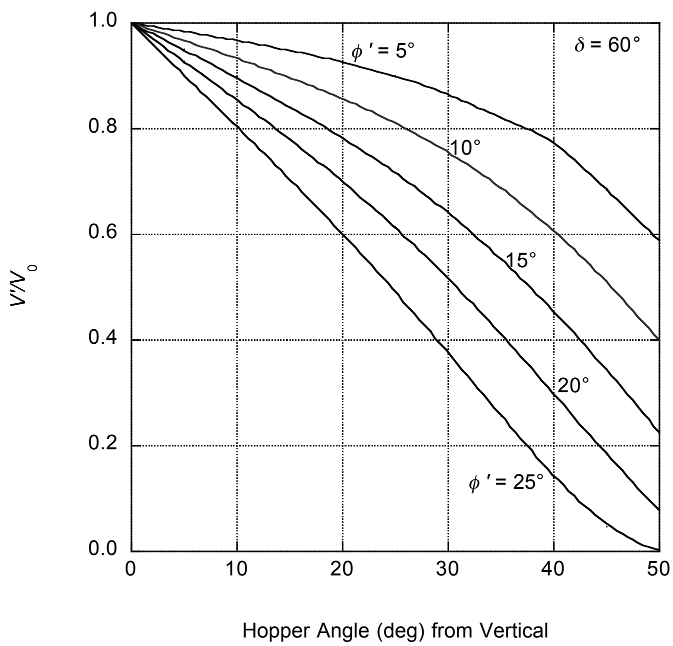

Figure 14. Wall-centerline velocity ratio, δ = 60°, planar flow.

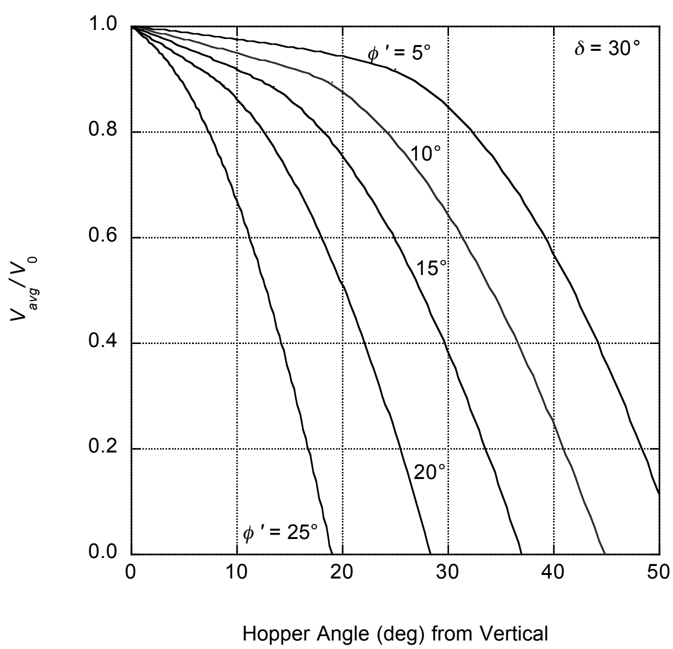

Figure 15. Average-centerline velocity ratio, δ = 30°, axisymmetric flow.

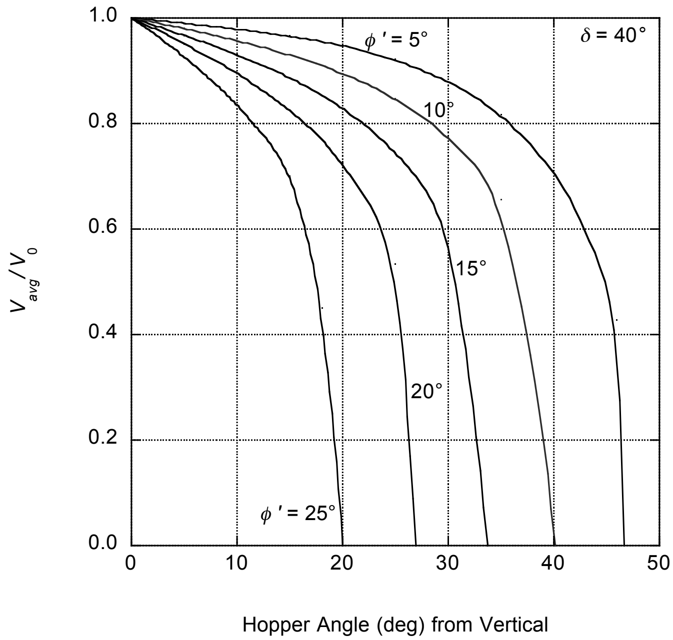

Figure 16. Average-centerline velocity ratio, δ = 40°, axisymmetric flow.

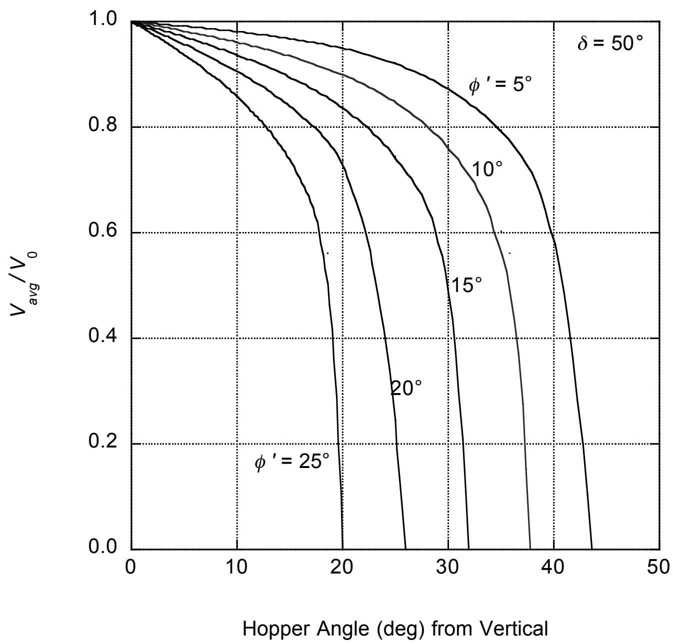

Figure 17. Average-centerline velocity ratio, δ = 50°, axisymmetric flow.

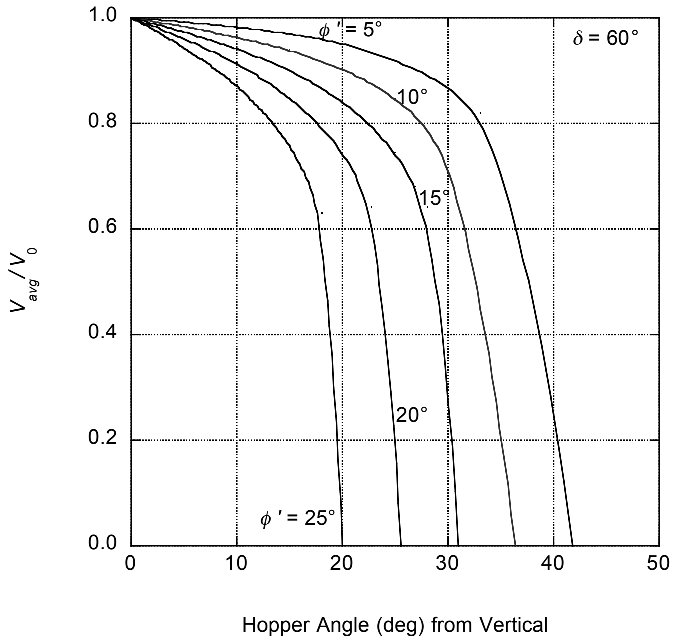

Figure 18, Average-centerline velocity ratio, δ = 60°, axisymmetric flow.

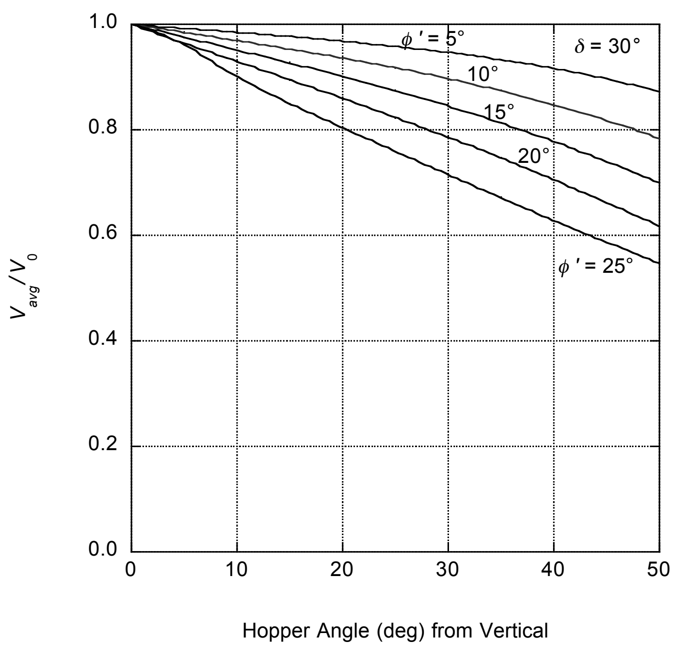

Figure 19. Average-centerline velocity ratio, δ = 30°, planar flow.

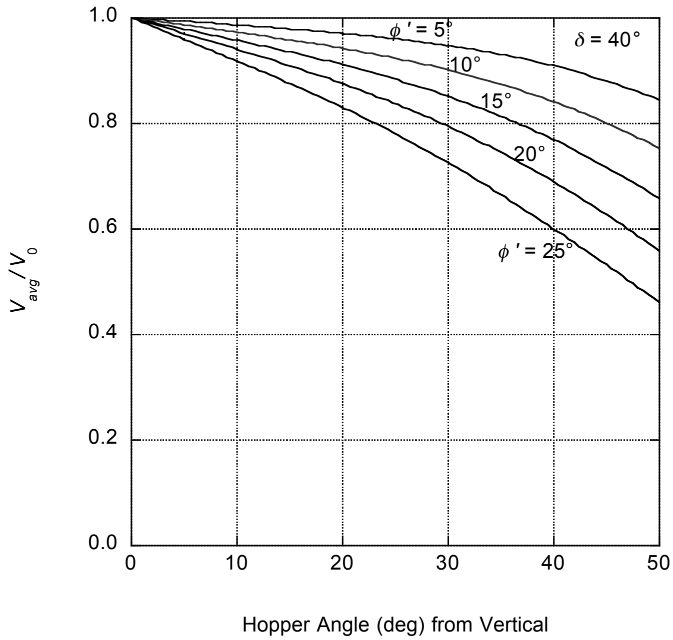

Figure 20. Average-centerline velocity ratio, δ = 40°, planar flow.

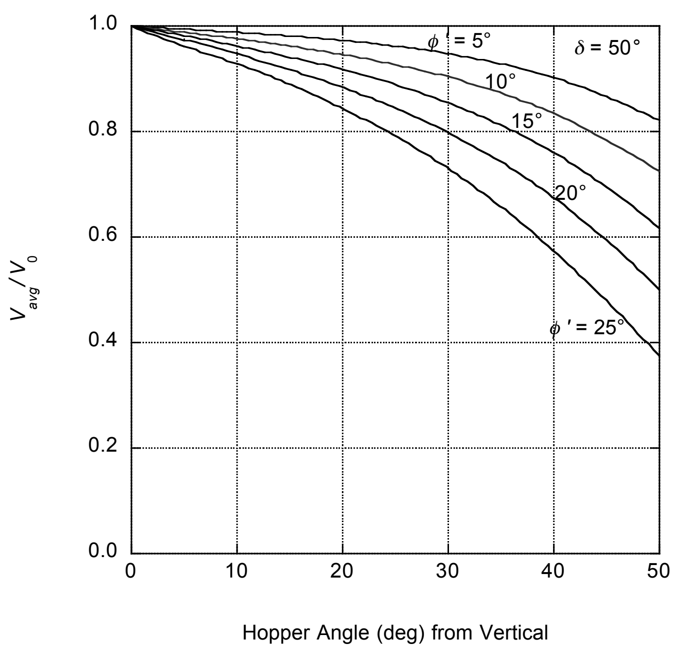

Figure 21. Average-centerline velocity ratio, δ = 50°, planar flow.

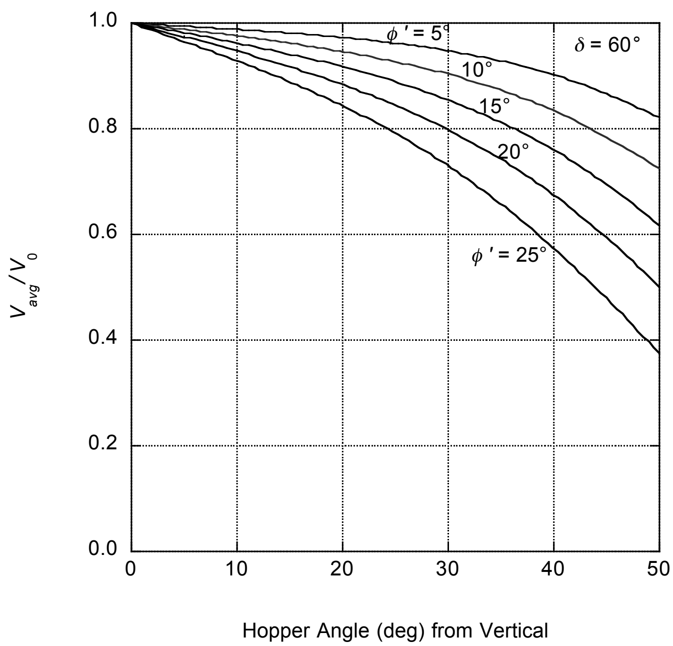

Figure 22. Average-centerline velocity ratio, δ = 60°, planar flow.

In mass flow, the material in the center of a converging hopper than the material near the walls. Because a surcharge repose angle is formed when a hopper is filled, the material in the center of the hopper has to travel further than material along the periphery. These two factors tend to compensate each other if the hopper is filled from the center. The angle of repose of a powder can therefore be used as a gauge of how uniform the velocity profile must be to get an acceptable reduction in segregation.