Interpreting the Power Spectrum and Integrated Spectrum

Power Spectrum

The Fourier analysis 'fits' the trend data to a series of sine wave cycles of different periods and amplitudes. The cycle can be defined by its period (T) and amplitude (a).

Its' important to recognize that the power spectrum only fits the trend data to cycles at harmonics of the fundamental period. Let's say that you collect data for 1024 seconds (say sampling at 1 second intervals for 1024 samples). The Fourier analysis will try to fit the trend data to a 1024 second cycle (fundamental period), followed by a 512 second cycle (2^{nd} harmonic), followed by a 341 second cycle (3rd harmonic) – all the way to 2 seconds (the 512^{th} harmonic).

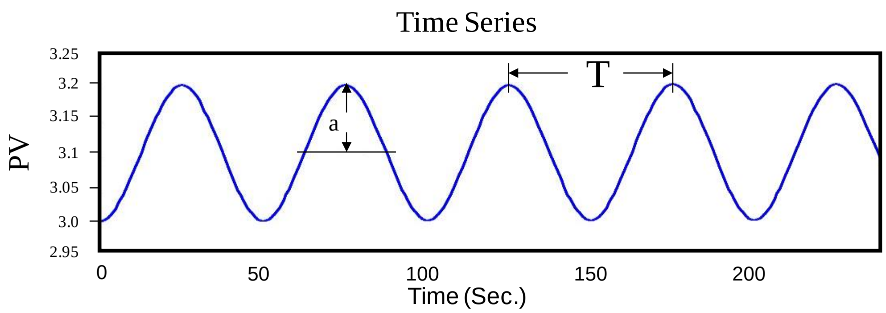
The power spectrum is a powerful tool to assist in identifying the source of variability. If two variables have strong cycles at the same period, it is *possible* that one variable is causing the cycle in the other variable. It is also *possible* that a third variable is affecting both. It is also possible that there is no inter-dependency.

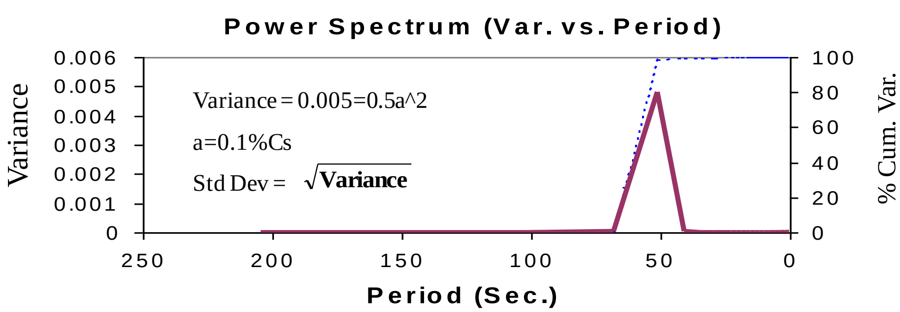
It's important to keep in mind that additional testing is required to confirm a cause and effect relationship between variables that have common cycles. These tests include:

- Stop the cycle in one process and see if it disappears from the second process. This test is most applicable if the cycle is controller induced – and you can simply put the loop into manual mode
- Conduct a coupling test to determine the dynamic relationship between the 2 variables. This will make clear the impact of a cycle in one variable on another and you can accurately estimate the impact a cycle will have on the related variable.

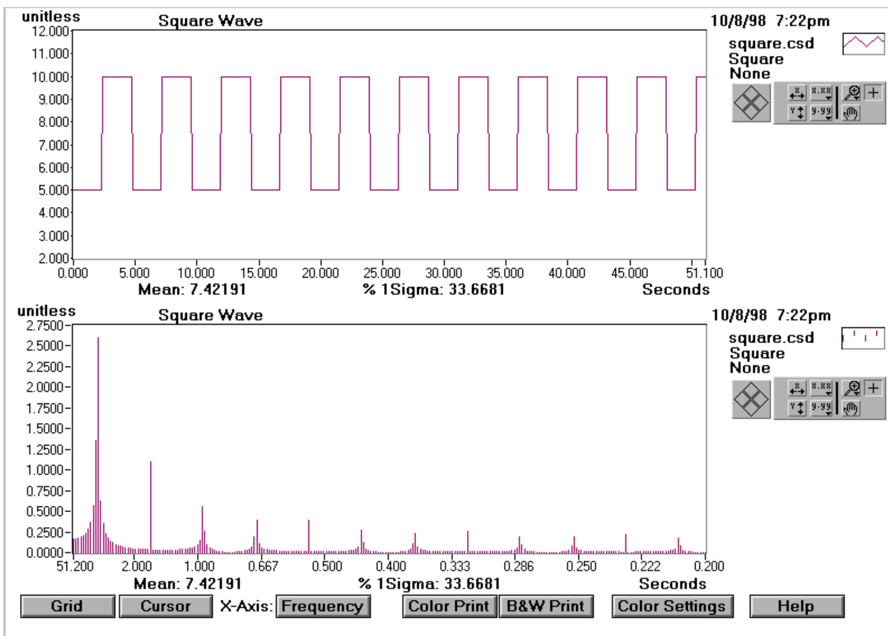
Limitations of the Power spectrum

There are limitations in the power spectrum and the results need to be interpreted with care. The limitations of the power spectrum include the following.



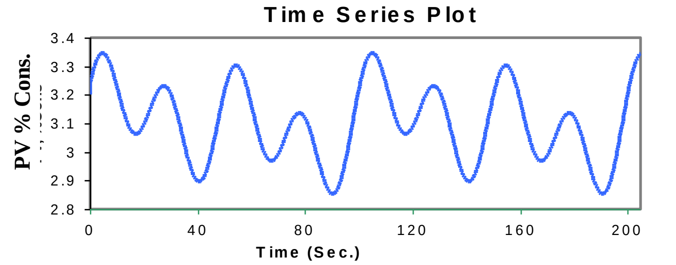


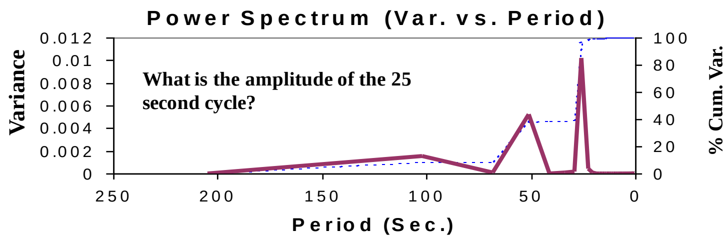
- ***Limitation 1*** – only the harmonics of the fundamental period are evaluated. Obviously the range of cycles that be examined for the case study above is between 1024 seconds and 2 seconds. But what happens if there is an actual cycle between the harmonics – say a 750 second cycle. The variance contributed by this cycle will be divided between the 1024 second harmonic and the 512 second harmonic. Slow drifting in the data will show up as variance at the fundamental period.   So – it's really important to take power spectrum 'peaks' slower than the 10^{th} harmonic with a large grain of salt.
- ***Limitation 2*** – not all process cycles are sine wave cycles. The power spectrum will add sine wave cycles at the harmonics of the dominant cycle to better fit the series of sine waves to a square wave. In the example below the 50 second square wave cycle produces 'false' cycles at harmonics of the actual cycle period. It's sometime neccesary to take the faster cycles in the power spectrum with a grain of salt.



Integrated (Cumulative) Spectrum

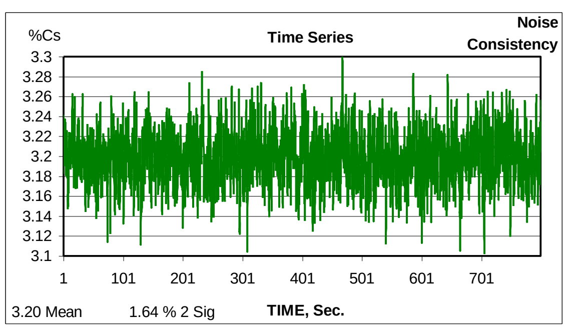
The cumulative spectrum plots the percent of total variance versus cycle period. This is a good way to judge the percent contribution of the individual cycles. The 25 second cycle contributes approximately 60% of the variance in the example below.

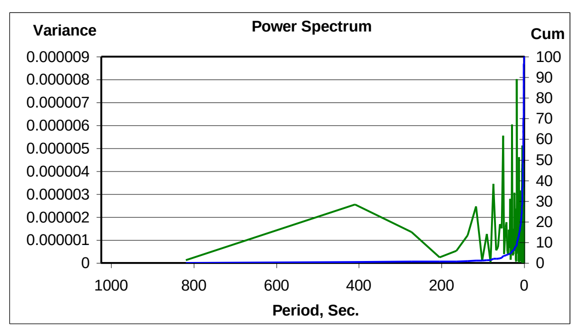


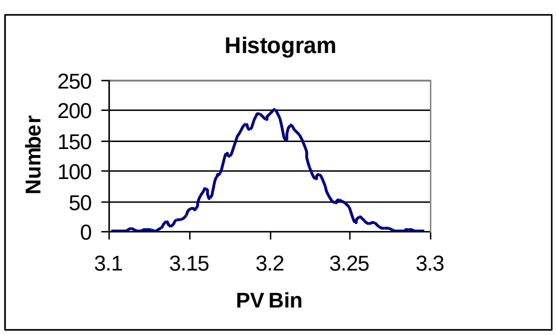


The integrated spectrum is useful because it indicates the relative contribution of each cycle to the total variance. As such it is useful in establishing priorities for variability reduction. An individual cycle that contributes 5% of the total variance is usually worth pursuing.

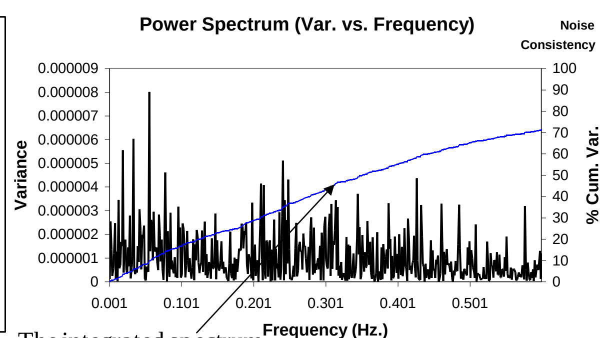
White Noise (usually associated with sensor noise) produces an integrated spectrum with a relatively straight line. That is, relatively equally variance contribution at all frequencies. Higher noise levels increases the difficulty in identifying ‘real’ process cycles.







The white noise signal is normally distributed



The integrated spectrum shows approximately equal variance contributed at all frequencies

Tips on using the power spectrum / integrated spectrum tools

- Plan your data collection run carefully. The sample period and duration of the data collection will define the cycles that can be identified accurately.
  - If you are looking for a 100 second cycle the duration of the data collection run should be at least 1024 seconds and ideally 2048 seconds. Remember that the cycle period can only be identified accurately after the 10^{th} (or even 20^{th}) harmonic. It is usually a good idea to ignore power spectrum cycles that are slower than the 10^{th} harmonic.
  - The fastest cycle that can be identified is 2 times the sample period. Ideally the sample period should be 10 times faster than the cycle period of interest. For example, if you are trying to identify a 2 second cycle the sample period should be 0.2 seconds or faster.
- Look for power spectrum peaks that are faster than the 10^{th} harmonic and that contribute a significant amount of the total variance (integrated spectrum). An individual cycle that contributes 5% or more of the total variance is usually considered significant. Often, cycles of this magnitude can be seen in the trend plot.
- You need to be aware that the power spectrum will report false cycles at harmonics of the dominant period for square wave, saw-tooth, and pulse type cycles. Square wave cycles are generated by control valve stiction or poor sensor resolution.
- The power spectrum is a good tool to assist indentifying sources of variability. Proving a cause and effect relationship between 2 variables with the same cycle period(s) requires additional testing. The additional tests might involve stopping the cycle in the variable of interest or conducting a coupling test (step process A and observe the response of Process B).
- One good test of control loop performance is minimal process variability slower than 10 times the controller cutoff period (2*π*(ƛ+ϴD)). For example the cutoff period for a well tuned consistency loop is 90 seconds. If a significant amount of the consistency variability is slower than 900 seconds, the control loop performance should be evaluated.
- The measured variability consists of 'real' process variation and sensor noise. Sensor noise typically produces a white noise power spectrum (equal variance at all frequencies). Sensor noise increases the difficulty in identifying cycle periods.
- Filtering will mask high frequency cycles in the process. Remove filtering at the sensor or controller prior to data collection if you are attempting to identify high frequency cycles.