On the Asymmetry of Parts and Wholes

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Arithmetic is the branch of mathematics dealing with the properties and fundamental operations of nonnegative real numbers, including addition, subtraction, multiplication and division. The mastery of Arithmetic is critical for problem-solving and foundational for understanding higher-level math. As such, any mistake made in our understanding of arithmetic could, conceivably, propagate through all other branches of mathematics and science. Said another way:

"It ain't what you don't know that gets you into trouble. It's what you know for sure that just ain't so."

-Mark Twain

This insight, though colloquial in expression, captures a perennial philosophical hazard: the uncritical acceptance of self-evident truths. As such, I will examine one of the most universally accepted axioms in arithmetic:

1⁄2 + 1⁄2 = **1**

The focus on this paper is not to dwell on the numerical value of $\frac{1}{2}$ (0.5), as what is asserted about $\frac{1}{2}$ can just as easily be applied to any other fraction or act of division that produces a remainder. While mathematics is under no obligation to reproduce what we see in the physical world, divergences should be understood and explainable by virtue of a difference in the underlying assumptions. Mathematics serves as a language of abstract relationships, providing idealized structures that allow us to model and predict patterns. The equation $\frac{1}{2} + \frac{1}{2} = 1$ represents perfect abstract equality in this formal system. However, when we apply this idealized relationship to physical transformations—cutting a ball, fusing atoms, or dividing an apple—we encounter a fundamental mismatch. The physical world introduces costs, entropy, and identity transformations that pure mathematics abstracts away. This paper does not challenge the internal consistency of arithmetic itself, but rather examines the consequences of uncritically applying these idealizations to material reality. When we assert $\frac{1}{2} + \frac{1}{2} = 1$ in the physical realm, we silently erase the energetic and entropic costs of division and unification; costs that may hold profound implications for our understanding of fundamental physics.

Example 1: A Solid Ball

Imagine a solid ball. Consider how it bounces when dropped. Consider its surface area. Consider the way it dissipates heat. Now, imagine you cut that ball in half.



How do the halves bounce when dropped? Consider the surface area of the two halves of the ball. Has it increased, decreased, or stayed the same? Consider the way the halves would dissipate heat. Does dissipation occur faster, slower or at the same rate as before the cut was made?

Now consider the fact that energy was added to the ball to cut it in half. Given these differences in the ball's properties after the cut was made, was the mathematical identity of the ball preserved? That is to say, if the surface area of two halves of a ball are greater than the surface area of 1 ball, can it truly be said that there is an equality relating one mathematical expression (2/2) to another mathematical expression (1), such that 2/2 and 1 produce the same value for all values of the variables within this domain of discourse?

Finally, consider the fact that energy could be added to the halves of the ball to bind them back together, restoring the surface area to the halves.

As such, I assert that:

½ + ½ ≠ 1

But rather:

 $\frac{1}{2} + \frac{1}{2} + \varepsilon = 1$

 $\frac{1}{2} = 1 - \frac{1}{2} - \varepsilon$

Where ε = the energy required to divide or restore the halves of the ball back to 1 solid ball. Unity is not free.

Example 2: Helium Fusion

Hydrogen is comprised of 1 proton, 1 neutron and 1 electron. Helium is comprised of 2 protons, 2 neutrons and 2 electrons. Hydrogen contains exactly half the number of protons, neutrons and electrons that Helium does, yet the fusion of Helium isn't as simple as H + H = He.



Additionally, the mass of Hydrogen is 1.00784 u. Being comprised of exactly twice the protons, neutrons, and electrons of Hydrogen, it would be reasonable to expect that the mass of would be twice that of Hydrogen (2.01568 u). But since we know that it requires 4 Hydrogen atoms to fuse 1 Helium, we might be tempted to expect a mass 4 times greater than Hydrogen (4.03136 u). Instead, we find that Helium has a mass of 4.002602 u. This phenomenon is known as "mass defect", and arises from the fact that some of the mass of the Helium atom is converted into energy, as described by Einstein's equation E=mc². But there is more that must be considered than a simple accounting of lost mass.

	Hydrogen	Helium
Mass	1.00784 u	4.002602
Reactive	Yes	No
Structure	Diatomic	Monatomic
Boiling Point	-252.8 °C	-268.9 °C
Melting Point	-259.14 °C	-272.2 °C

Not only is the fusion process from Hydrogen to Helium nonlinear as $\frac{1}{2} + \frac{1}{2} = 1$ would have us believe, but a new identity also emerges from fusion. The whole is not just the sum of its parts. Addition is only applicable to values ≥ 1 , used in the summation of separate identities without structural change. Fusion is required for values <1, where incomplete identities merge, incurring structural cost. It would be as if Tangerine + Tangerine = Apple.

Example 3: The Apple

Consider an apple. It is estimated that a 100-gram apple contains 10²⁵ atoms (based on its composition and average molar mass). If we cut an apple in half, we would recognize the halves as halves of an apple based on common characteristics. However, imagine we cut the apple into 10²⁵ pieces. Each "piece" would be an atom, no longer identifiable as "apple".

Theorem: Fusion Precedes Addition

Let {ui} be a finite set of entities, where each ui possesses a structural identity coefficient relative to full system coherence, 0<ui≤1.

Then:

• If ∃ ui<1 for any i, the combination of the set results in a fusion operation, not a pure additive operation.

True addition, where: F(u1,u2,...,un)=Σ(u1,u2,...,un) only holds when: ∀i, ui=1 That is, addition is only valid for fully unified entities; otherwise, a structural reconciliation process — fusion — must occur, incurring entropy increase and/or structural adjustment.

Proof:

- 1. Definitions:
- Define the identity coefficient ui as a normalized measure of the completeness of each entity with respect to system unity, where:
 - $ui=1 \Rightarrow$ entity is fully formed, structurally independent.
 - \circ 0<ui<1 ⇒ entity is partial, incomplete, requiring fusion to achieve full unity.
- Define two operations:
 - \circ Σ(u1,u2,...,un) = naive addition (sum without structural change).
 - F(u1,u2,...,un) = fusion operation (sum with structural unification process, incurring entropy shift or identity loss).
- 2. Case 1: Existence of Incompleteness Suppose there exists at least one uj such that uj<1. Since uj is structurally incomplete:
- It cannot exist independently without external support or synthesis.
- Combining uj with any other entity requires structural integration.
- Structural integration necessarily introduces energy transformation (binding energy, mechanical stress) and/or entropy redistribution (localized order increase with global entropy rise).

Thus:

 $F(u1,u2,...,un) \neq \Sigma(u1,u2,...,un)$ because the resulting structure is not the simple arithmetic sum of its constituents — losses, transformations, and hidden costs must be accounted for.

- 3. Case 2: Completeness of All Entities Suppose ∀i, ui=1. Each entity is:
- Structurally complete.
- Capable of maintaining independent identity without external binding.
- Capable of combining without internal restructuring costs.

Thus, naive addition applies: $F(u1,u2,...,un)=\Sigma(u1,u2,...,un)$ and no fusion-induced entropy or structural transformation is required.

4. Conclusion: Addition without fusion is only possible if all participating entities possess full structural unity.

Therefore:

Fusion is the primitive operation. Addition is the emergent operation of unified structures.

Conclusion: The Cost of Unity

The assumption that $\frac{1}{2} + \frac{1}{2} = 1$, though arithmetically convenient, embeds a profound structural error when applied beyond idealized numbers into physical systems. This misunderstanding reverberates through some of the deepest unresolved tensions in physics.

The Lorentz transformation, essential for reconciling the constancy of the speed of light across reference frames, introduces a denominator:

 $(1-v^2/c^2)$

that approaches infinity as velocity approaches c. In doing so, it reveals that time, mass, and energy behave nonlinearly at high velocities, the system resists simple addition of velocities or masses. Just as partial identities below unity cannot naively add, so too do reference frames *fuse* rather than simply *sum* near relativistic limits.

Similarly, the universe presents multiple points of effective infinity, black hole singularities, cosmological horizons, and the asymptotic behavior of time near the speed of light. Yet these infinities are embedded in an otherwise finite structure. This paradox mirrors the incomplete fusion of systems assumed to have additive behavior, when in fact they fracture under entropic tension.

The forward arrow of time, too, emerges not from an arbitrary law but from entropy-driven fusion: every act of unification, from particle collisions to galaxy formation, creates irreversible structure and releases entropy.

Time flows forward because unity has a cost, and once paid, it cannot be unspent.

Dark matter and dark energy may likewise reflect systemic misinterpretations born from additive assumptions.

• Dark matter could be the gravitational memory of incomplete unifications, structural tension embedded in the fabric of space itself, unseen but exerting force.

• Dark energy could arise from the hidden cost of maintaining large-scale coherence across an expanding universe, not from a mystical external force but from the internal necessity of paying for unity at cosmic scales.

In each case, the universe behaves not as a perfect sum, but as a living record of fusion, loss, and transformation.

Thus, many so-called "problems" in modern physics, from the infinity paradoxes to the missing mass problem, may not be problems at all. They are the natural consequence of mistaking incomplete systems for complete ones, and of overlooking the energy and entropy required to maintain unity.

Fusion is the primitive operation. Addition is the emergent illusion.

Correcting this fundamental misunderstanding may not only clarify the mathematics of parts and wholes but illuminate the structure, evolution, and destiny of the universe itself.

The cost of unity is not a flaw, it is the fabric itself.

Time marches forward because entropy demands it.

Mass defects are not mere anomalies; they are ledgers of what was paid to become whole. Dark matter may be nothing more than gravity remembering what identity forgot. And dark energy, the breath between unifications, may simply be the debt of assuming addition where fusion was required.

We have not misunderstood the universe because it is complex. We have misunderstood it because we trusted that $\frac{1}{2} + \frac{1}{2} = 1$ was always true.

I assert that it never was.