**Applications for Mathematics in Robotics**

<https://www.roboticsbook.org/S13_math.html>

<https://library.fiveable.me/introduction-autonomous-robots/unit-1/coordinate-systems-transformations/study-guide/8NJfsX8MqKiLY8bE>

<https://www.roboticscareer.org/news-and-events/news/62741>

This challenging applied math master’s thesis was written with the aid of performance enhancing supplements, including artificial intelligence. Math and physics theses are tough, because you either have to create a new theorem, or write an expository thesis. It is difficult to create a new theorem in math or physics, although it is quite easy to come up with a novel hypothesis in the social sciences. Even in engineering, you can focus on modern applications, such as sodium batteries or perovskite solar panels in electrical engineering. We are just beginning research on sodium batteries and perovskite solar panels, so there is a lot of material to work with in that area. A class on the History of Mathematics teaches us, however, that math is an ancient discipline, and the most innovative research has already been performed. Even string theory in physics, a novel concept in the 1970s, has not been proven, and as Leonard said on the Big Bang Theory, physics research is a dead end. These are just views on math and physics from an engineer. I am sure that anyone smart enough to complete a PhD thesis in math or physics has the brainpower to create new theorems, but I am sticking to engineering and PV Magazine.

The field of robotics utilizes mathematics from diverse disciplines such as: linear algebra, calculus, probability and statistics, and optimization techniques. Robotic systems are modeled, sensor data is analyzed, movement is planned, and decisions made by and for robots all with the direct aid of mathematics. Linear algebra is used to create models of robot systems, and to represent robot positions, orientations, and transformations using vectors and matrices, all within a specific coordinate system, such as cartesian, cylindrical, polar, or spherical. Calculus is used to optimize the robotic system's performance by minimizing errors and reducing cycle time by calculating derivatives for velocity and acceleration, and integrals for motion planning. Probability and statistics are used to analyze sensor data, optimize robotic processes, deal with uncertainty, and make informed decisions.

How these math concepts are applied:

* **Kinematics:**

Using geometric relationships to determine the position and orientation of a robot's end effector based on joint angles.

* **Dynamics:**

Analyzing forces and torques acting on a robot to plan precise movements.

* **Path Planning:**

Calculating optimal trajectories for a robot to navigate through a given environment.

* **Control Theory:**

Designing algorithms to regulate a robot's behavior and maintain stability.

**Linear Algebra in Robotics**

Vectors and matrices to represent robot positions, orientations, and transformations.

In robotics, linear algebra is heavily used in areas like coordinate transformations, kinematics analysis, control systems design, motion planning, data analysis for perception, and optimization algorithms, where concepts like matrices, vectors, eigenvalues, and eigenvectors are crucial for representing and manipulating robot movements and sensor data within different reference frames.

In robotics, robots use coordinate systems and transformations to understand their environment and move around. Coordinate systems provide a framework for representing positions, orientations, and movements in space. Different systems, like Cartesian and polar coordinates, suit various situations and geometries.

Transformations allow robots to convert between coordinate systems and perform actions like rotation and translation. Homogeneous coordinates and transformation matrices simplify these operations, enabling robots to plan and execute complex movements accurately. Understanding these concepts is essential for autonomous robot navigation and manipulation.

 $\left[\begin{matrix}1&0&0&0\\0&cosθ&-sinθ&0\\0&sinθ&cosθ&0\\0&0&0&1\end{matrix}\right]$

Key linear algebra applications in robotics:

* **Coordinate Transformations:**Using transformation matrices to convert between different coordinate systems (robot base, joint space, world coordinates) for accurate motion planning and control.
* **Kinematics Analysis:**Solving forward and inverse kinematics problems to determine robot joint angles based on desired end-effector positions and vice versa, often involving matrix operations to calculate joint angles.
* **Jacobian Matrices:**Calculating the Jacobian matrix to analyze the relationship between joint velocities and end-effector velocities, important for motion control and trajectory planning.
* **Least Squares Optimization:**Employing linear least squares techniques to solve problems like camera calibration, sensor fusion, and state estimation where minimizing errors in measurements is crucial.
* **Eigenvalue Analysis:**Analyzing the eigenvalues and eigenvectors of a system to understand its stability and principal components, which is relevant in robot control and motion planning.
* **Data Projection:**Utilizing linear projections to reduce dimensionality of sensor data (like point clouds) for efficient processing and analysis.
* **Control System Design:**Designing feedback control systems for robots using linear algebra to model system dynamics and calculate control gains.

Specific research areas within robotics where linear algebra plays a significant role:

* **Robot manipulation:**Precise manipulation of objects with multiple degrees of freedom, requiring accurate kinematic analysis and control using linear algebra.
* **Autonomous navigation:**Using linear algebra for obstacle avoidance, path planning, and localization in complex environments.
* **Visual serving:**Utilizing linear algebra for camera calibration and visual feedback control to guide robot movements based on visual information.
* **Machine learning for robotics:**Applying linear regression, principal component analysis (PCA), and other linear algebra techniques for data analysis in robotic perception and decision making.

In robotics, coordinate transformations, achieved through the use of transformation matrices, are crucial for accurately converting points between different coordinate systems like the robot base, joint space, and world coordinates, enabling precise motion planning and control by allowing the system to understand and manipulate a robot's position and orientation across different reference frames.

Key Concepts:

* **Coordinate Systems:**
	+ **Robot Base:** Represents the origin at the robot's base, with axes aligned to the robot's initial orientation.
	+ **Joint Space:** Describes the position of each joint in the robot, usually expressed as angles.
	+ **World Coordinates:** A fixed, external reference frame used to represent positions and orientations in the surrounding environment.
* **Transformation Matrix:**A mathematical matrix that represents a combination of rotations and translations, allowing the conversion of coordinates from one frame to another.

How it works:

* 1. **Defining Transformation Matrices:**
	+ **Individual Joint Transformations:** For each robot joint, a transformation matrix is calculated based on its current joint angle, link length, and joint offset, describing the change in coordinates from the previous link to the current one.
	+ **Homogeneous Coordinates:** To represent both translation and rotation in a single matrix, a fourth dimension is added to the coordinate vector, enabling efficient calculations.
* 2. **Forward Kinematics:**
	+ **Multiplying Matrices:** To find the position and orientation of the robot's end-effector in world coordinates, the transformation matrices of each joint are multiplied sequentially (from base to tip) to obtain the final transformation matrix.
	+ **Calculating End-Effector Position:** By multiplying the end-effector's coordinates in joint space with the forward kinematics transformation matrix, you get its coordinates in world coordinates.
* 3. **Inverse Kinematics:**
	+ **Solving for Joint Angles:** To determine the required joint angles to reach a desired end-effector position in world coordinates, the inverse kinematics process is used, which involves inverting the forward kinematics transformation matrix to solve for joint angles.

Applications:

* **Motion Planning:**By transforming desired trajectories from world coordinates to joint space, the robot can plan a sequence of joint movements to reach a target position.
* **Collision Detection:**By transforming the robot's geometry into world coordinates, potential collisions with obstacles can be detected.
* **Sensor Data Processing:**Sensor readings from the robot's environment can be transformed into the robot's reference frame for further analysis.

Key Points:

* Transformation matrices are a powerful tool for representing complex robot motions and manipulating coordinates across different reference frames.
* Understanding the relationship between joint angles and the resulting end-effector position is critical for accurate motion planning and control.
* Accuracy of the robot's motion depends on the correct calculation and application of transformation matrices.

**Calculus**

Derivatives for calculating velocities and accelerations, integrals for motion planning.

If an object's position is given by the function x(t) = t^2, then its velocity v(t) is the derivative of x(t) which is 2t, and its acceleration a(t) is the derivative of v(t) which is 2; meaning that the acceleration is a constant value of 2 regardless of time, while the velocity changes depending on the time value "t".

Explanation:

* **Position function:** x(t) = t^2
* **Velocity function (derivative of position):** v(t) = d/dt [x(t)] = d/dt [t^2] = 2t
* **Acceleration function (derivative of velocity):** a(t) = d/dt [v(t)] = d/dt [2t] = 2

Key points:

* Velocity is the rate of change of position, which is calculated by taking the derivative of the position function with respect to time.
* Acceleration is the rate of change of velocity, which means it is the derivative of the velocity function with respect to time.

**Trigonometry**

For calculating angles and distances within a robot's workspace.

**Probability and Statistics**

To analyze sensor data, deal with uncertainty, and make informed decisions.

**Robotic Optimization**

To find the best possible solutions for robot movements and control strategies.

Optimization is considered a branch of applied mathematics, specifically focused on finding the best possible solution from a set of available alternatives, often by maximizing or minimizing a function under certain constraints.

Key points about optimization:

* **Focus on maxima and minima:**

The core concept is to find the highest or lowest value of a function, which can involve techniques from calculus.

* **Constraint handling:**

Optimization problems often involve constraints, meaning limitations on the possible solutions.

* **Applications:**

Optimization is widely used in various fields like economics, engineering, computer science, machine learning, and operations research.

Examples of optimization problems in robotics include: trajectory planning to minimize travel time while avoiding obstacles, optimal grasp selection for manipulation tasks, energy-efficient robot locomotion, multi-robot task allocation, robot joint angle optimization for precise movements, and scheduling of robot operations in a manufacturing environment; all aiming to find the best possible solution within constraints like joint limits, workspace boundaries, and power consumption.

Specific scenarios:

* **Motion planning:**
	+ Finding the shortest path for a robot to navigate a complex environment while avoiding collisions.
	+ Optimizing the trajectory of a robot arm to reach a target point with maximum speed and smoothness.
	+ Planning a collision-free path for multiple robots operating in the same space.
* **Grasping and manipulation:**
	+ Selecting the optimal hand configuration to securely grasp an object with varying shapes and textures.
	+ Optimizing the force applied during manipulation to avoid damaging delicate objects.
* **Robot design:**
	+ Designing a robot with optimal joint angles and link lengths for a specific task.
	+ Minimizing the weight of a robot while maintaining necessary strength and dexterity.
* **Energy efficiency:**
	+ Finding the most energy-efficient gait for a walking robot.
	+ Optimizing robot motion to minimize power consumption during a task.
* **Cooperative robotics:**
	+ Assigning tasks to multiple robots in a team to maximize overall efficiency.
	+ Coordinating the movements of multiple robots to achieve a common goal.

Key points about optimization in robotics:

* **Constraints:**

Most robotics optimization problems involve constraints like joint limits, workspace boundaries, and safety considerations.

* **Objective function:**

The goal is to minimize or maximize a specific function depending on the desired outcome (e.g., minimizing travel time, maximizing grasping force).

* **Optimization algorithms:**

Various algorithms are used depending on the complexity of the problem, including gradient descent, dynamic programming, evolutionary algorithms, and mixed integer programming.