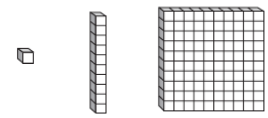
Grade 4 Mathematics
Key Concepts and Skills
A Parents' Guide

Below are the key mathematics concepts that we hope your child will master by the end of Grade 4.

Grade 4 is the year that we solidify your child's number sense in relation to working with larger numbers and with fractions and decimals. We hope they:

- have strong visual images for quantities,
- understand place value and the relationships between the places,
- can easily compare quantities,
- can represent quantities with objects and drawings (e.g., Base 10 materials),
- understand and use open number lines to represent numbers,
- notice and describe numeric and computational patterns,
- can create fractional units,
- can represent fractions on open number lines, and
- can compare fractions with like numerators or like denominators.



Grade 4 is the year that your child extends his or her use of a variety of addition and subtraction strategies and representations to add and subtract fractions. Multiplication strategies and representations are extended to 2-digit by 2-digit multiplication and simple fraction multiplication. Your child is also introduced to partial quotient division. Your child should be very flexible in the strategies used to solve whole number and fraction problems. She or he should be able to solve a problem in more than one way.

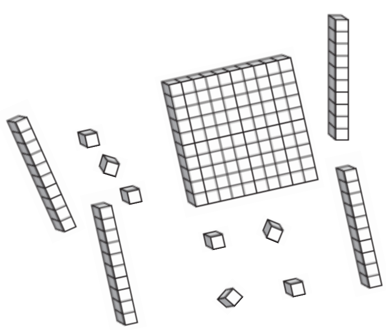
Grade 4 is also the year that your child is introduced to angle measure, types of lines (parallel, perpendicular), types of angles (obtuse, acute, right) and use these attributes to classify shapes.

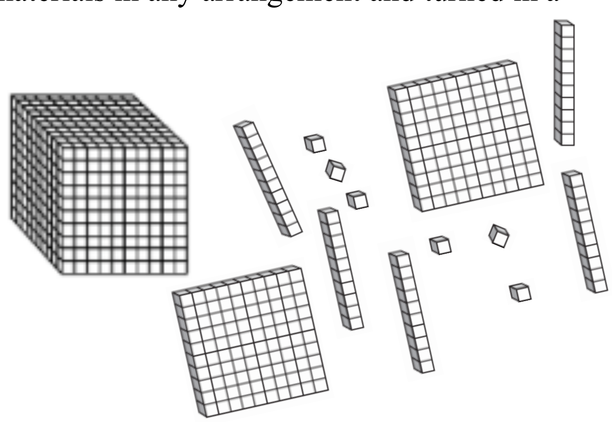
Number Concepts

Subitize Numbers

Subitize may be an unfamiliar term but many of us do this without knowing its name. For example, when you play a game involving dice, do you recognize a 6 without counting the pips? That is subitizing, recognizing and naming a quantity without counting the individual objects.

1. Your child should be able to subitize Base 10 materials in any arrangement and turned in a variety of ways. For example,



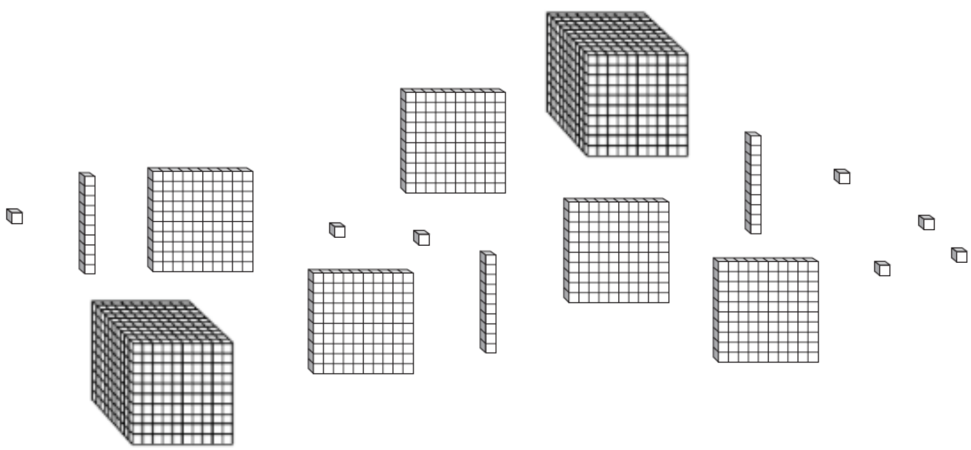


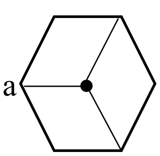
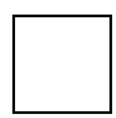
2. Your child should be able to generate a pattern when given a rule (e.g., double) and describe characteristics of the pattern. For example, every number in a doubling pattern will be even.

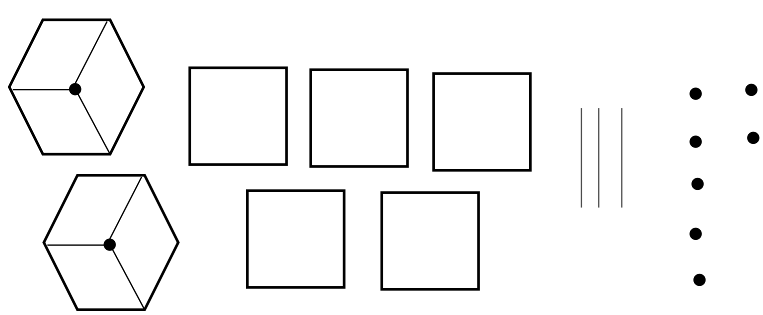
Place Value

3. Your child should be able to correctly read and write numbers between 1 and 100,000 using numerals (e.g., 1235, 3056, 999, 19023), number words (e.g., one thousand, two hundred thirty-five; three thousand, fifty-six; nine hundred ninety-nine; nineteen thousand, twenty-three), and expanded form (e.g., 1235 = 1000 + 200 + 30 + 5 OR 1 x 1000 + 2 x 100 + 3 x 10 + 5 x 1; 3056 = 3000 + 50 + 6 OR 3 x 1000 + 0 x 100 + 5 x 10 + 6 x 1; 999 = 900 + 90 + 9 OR 9 x 100 + 9 x 10 + 9 x 1; 19023 = 10,000 + 9,000 + 20 + 3 OR 1 x 10000 + 9 x 1000 + 0 x 100 + 2 x 10 + 3 x 1). He or she should understand that commas are helpful when included in numbers beyond 1000 but are not required. Both 19023 and 19,023 are correct.

4. Your child should be able to represent numbers between 1 and 10,000 using base 10 materials and the related drawings. Suppose the number is 2537, your child could represent using base 10 materials as shown below (**Note:** Your child should understand that the position of the base 10 materials is not important. The materials themselves have the value of 1000s, 100s, 10s, and 1s).



Your child can then draw a  for 1000, a  for 100, a | for 10, and a • for 1.



5. Your child should be able to describe the result on the different places when adding or subtracting by 1, 10, 100, 1000, 10000. For example, adding 1000 affects the thousands place but the hundreds, tens, and ones places do not change.

6. Your child should be able to describe the difference between putting a number such as 5 in the ones place versus the tens place. If placed in the ones place it is worth 5. If placed in the 10s place it is worth 50, 10 times as much as 5. If placed in the hundreds place it is 100 times 5 and 10 times 50.

7. Your child should be able to represent quantities in a variety of ways using 1000s, 100s, 10s and 1s. For example, 2164 can be represented with 2 thousands, 1 hundred, 6 tens and 4 ones, OR 21 hundreds, 4 tens and 24 ones, OR 1 thousand, 10 hundreds, 16 tens and 4 ones, etc.

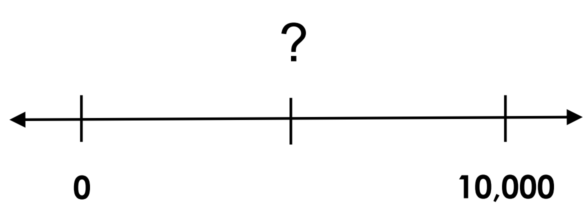
Thousands	**Hundreds**	**Tens**	**Ones**
2	1	6	4
	21	4	24
1	10	16	4
		216	4
			2164

This flexibility in representing quantities with thousands, hundreds, tens, and ones is important for understanding the steps in computation. For example, if we subtract 982 from 2164 we use the 3rd row of the above chart when we "regroup". We think of an equivalent way to make 2164 using thousands, hundreds, tens, and ones (1 thousand, 10 hundreds, 16 tens, and 4 ones). This helps your child understand the traditional procedures.

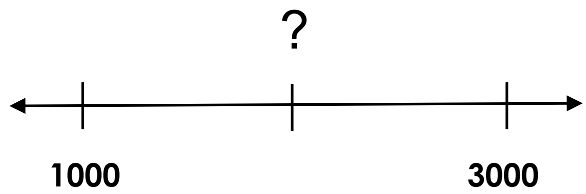
$$\begin{array}{r} {\scriptstyle 1\ \ 10\ \ 1} \\ \not{2}\ \not{1}\ 6\ 4 \\ -\ \ 9\ 8\ 2 \\ \hline \end{array}$$

Open Number Lines

8. Your child is expected to be able to identify points on an "open" number line. An open number line is a number line on which only some of the hash marks are given. We use benchmarks such as halfway points to find the values on the number line. For example, in the number line below, the endpoints 0 and 10000 are given. Your child is asked to find the halfway point, 5000.



Your child may be asked to solve problems with "0" as one of the endpoints. He or she may also be asked to find the missing values when the starting number is a number other than 0. For example, in the problem below, your child is asked to find the value that is halfway between 1000 and 3000. (2000)



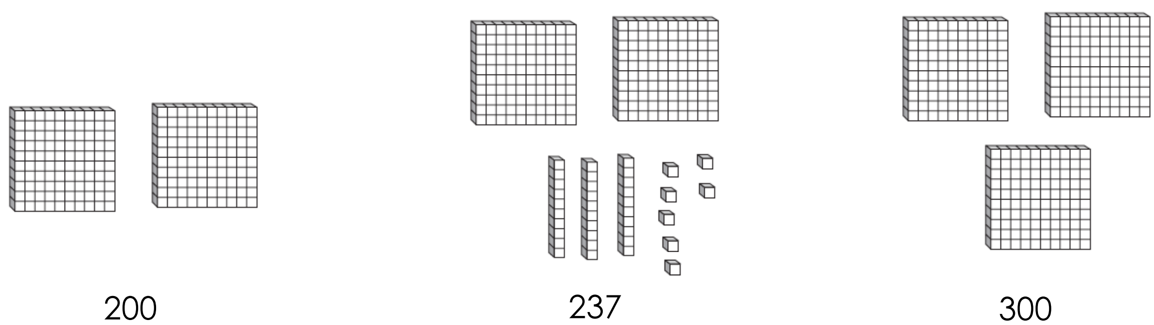
Comparing Quantities

9. Given 2 numbers, your child should be able to tell which is larger, smaller, or the same. He or she should be able to prove using base 10 materials, drawings, and showing on an open number line.

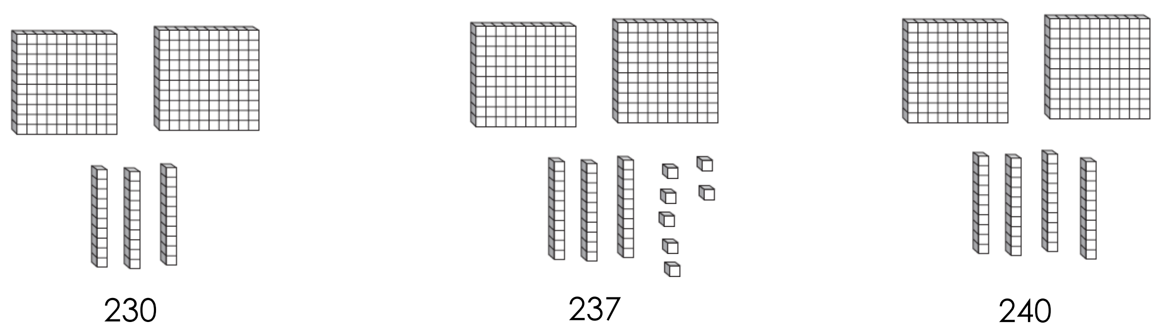
10. Your child should be able to correctly use the symbols >, <, =.

11. Your child should be able to represent the 100s and the 10s that a 3-digit number is between with base 10 materials, drawings, number lines, and numerals. He or she should then be able to identify the nearest hundred (round to the nearest 100) and the nearest ten (round to the nearest 10). For example, 237 is between 200 and 300 (hundreds it is between) and 230 and 240 (tens it is between).

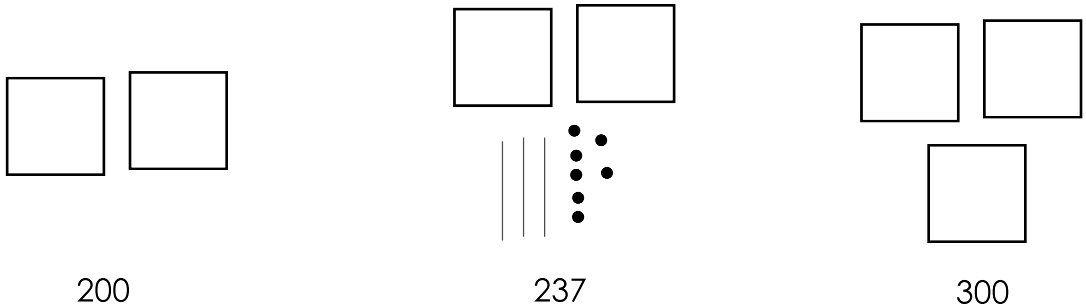
With base 10 materials—the hundreds that 237 is between:



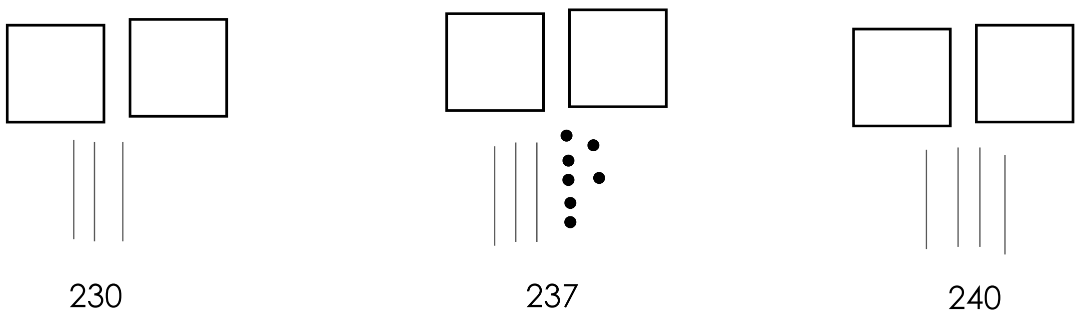
With base 10 materials—the tens that 237 is between:



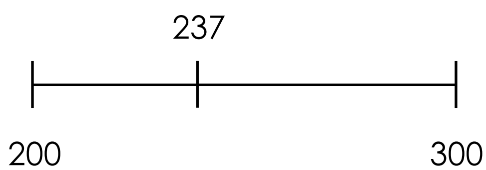
With drawings—the hundreds that 237 is between:



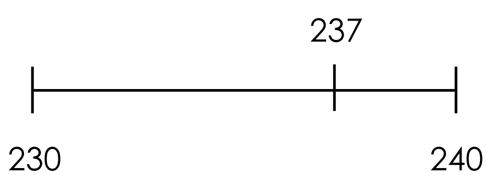
With drawings—the tens that 237 is between:



On an open number line—the hundreds that 237 is between:



On an open number line—the tens that 237 is between:

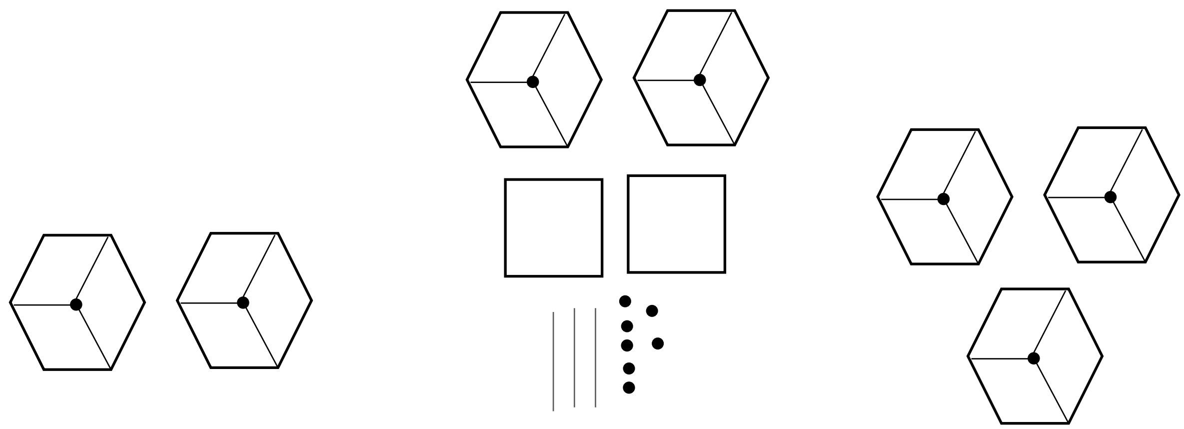


Rounding to the nearest hundred, 237 rounds to 200.

Rounding to the nearest ten, 237 rounds to 240.

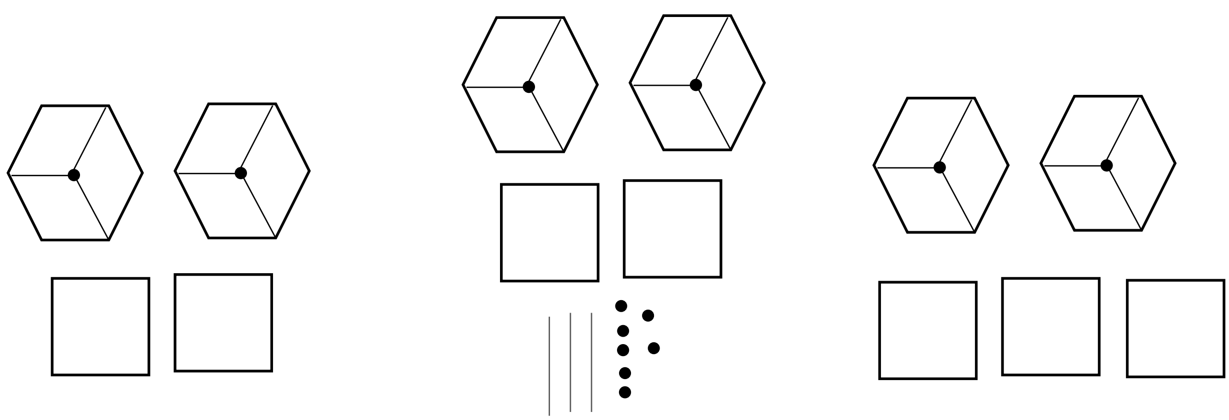
12. Your child should be able to represent the 1000s, 100s, and the 10s that a 4-digit number is between with drawings, number lines, and numerals. He or she should then be able to identify the nearest hundred (round to the nearest 100) and the nearest ten (round to the nearest 10). For example, 2237 is between 2000 and 3000 (thousands it is between), it is between 2200 and 2300 (hundreds it is between), and 2230 and 2240 (tens it is between).

With drawings—the thousands that 2237 is between:



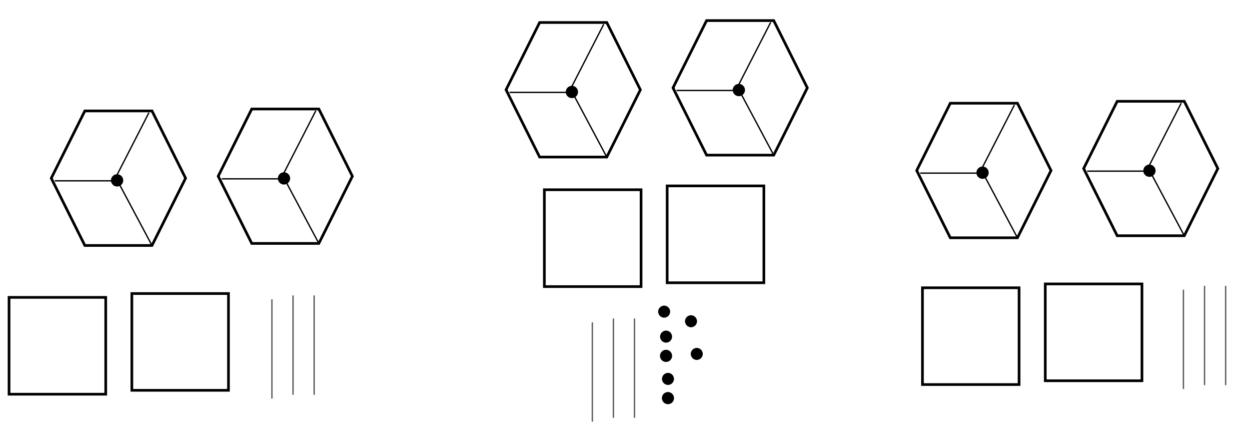
2000 2237 3000

With drawings—the hundreds that 2237 is between:



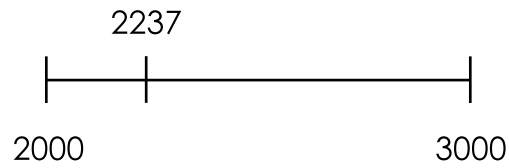
2200 2237 2300

With drawings—the tens that 2237 is between:

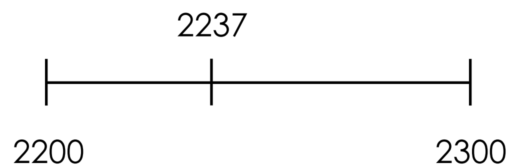


2230 2237 2240

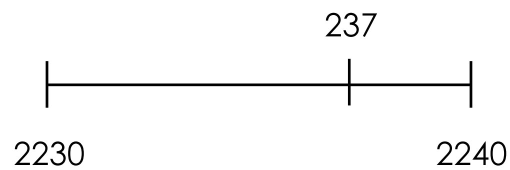
On an open number line—the thousands that 2237 is between:



On an open number line—the hundreds that 2237 is between:



On an open number line—the tens that 2237 is between:



Rounding to the nearest thousand, 2237 rounds to 2000.

Rounding to the nearest hundred, 2237 rounds to 2200.

Rounding to the nearest ten, 2237 rounds to 2240.

13. Your child understands that when rounding a number at the midpoint, we round up. For example, if we are rounding 65 to the nearest ten we know that 65 is the midpoint between 60 and 70. Therefore we round up to 70. If we are rounding 250 to the nearest hundred we know that 250 is the midpoint between 200 and 300. Therefore, we round up to 300. If we are rounding 3500 to the nearest thousand we know that 3500 is the midpoint between 3000 and 4000. Therefore, we round up to 4000.

Fractions

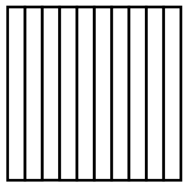
Creating Fractional Units

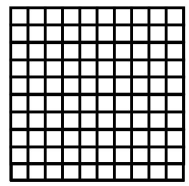
In grade 3 your child was asked to cut a whole into 2, 4, 8, 3, or 6 equal pieces and name the pieces. In grade 4, your child is asked to cut wholes into 2, 4, 8, 3, 6 12, 5, 10 and 100 equal pieces. Cutting a whole into 10 and 100 equal pieces provides a link to decimals.

1. Your child should be able to cut a rectangular region (shown below) into 2, 4, 8, 3, 6, or 12 equal pieces and draw a picture to match.

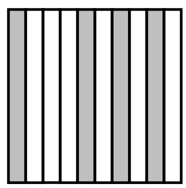
   This drawing will be very useful as the children learn to add, subtract, multiply, and divide fractions.

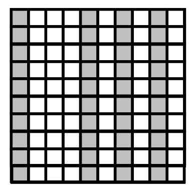
2. Your child should be able to represent tenths and hundredths using the rectangular regions shown below.





3. Your child should be able to link tenths and hundredths. For example,



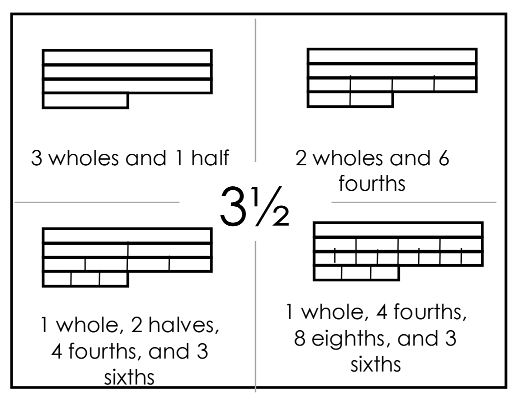


$\frac{4}{10}$ is equivalent to $\frac{40}{100}$

4. Your child can cut an interval from 0 to 1 (1 to 2, 2 to 3, and so on) on a number line into 2, 4, 8, 3, 6, 10, or 12 equal sections.

5. Your child should understand that a fraction can be separated into parts (with the same denominator) in a variety of ways. For example,
   $\frac{3}{4} = \frac{1}{4} + \frac{2}{4}$ OR
   $\frac{3}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$

6. Your child should be able to represent a fraction in a variety of ways using wholes, halves, thirds, fourths, sixths, eighths, twelfths, etc. For example, 3½ can be represented with 3 wholes and 1 half, OR 2 wholes and 6 fourths, OR 1 whole, 2 halves, 4 fourths, and 3 sixths, etc.



3½

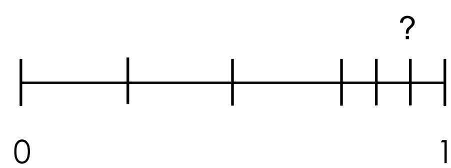
wholes (1s)	halves ($^1/_2$)	thirds ($^1/_3$)	fourths ($^1/_4$)	sixths ($^1/_6$)	eighths ($^1/_8$)	twelfths ($^1/_{12}$)
3	1					
2			6			
1	2		4	3		
1			4	3	8	
		3			4	24

To solve $6\,{}^1/_6 - 2\,{}^5/_6$ we need to think of an equivalent way to make $6\,{}^1/_6$ that would give us more sixths. We could make it with 5 wholes and 7, one-sixth pieces, $5\,{}^7/_6$, $5\,{}^7/_6$ is equivalent to $6\,{}^1/_6$. We are building the foundation for this understanding through the use of equivalency charts.

Naming Fractional Units

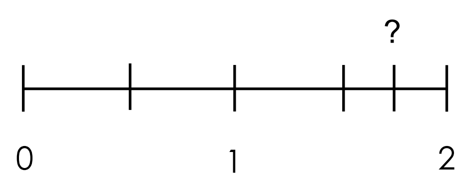
7. Many of us learned to name a fraction such as ¾ as three-fourths or 3 out of 4. It is important for your child to learn to name fractions in 3 ways: three-fourths, 3 out of 4, and 3 one-fourth pieces. In this last way of naming the fraction, your child learns that the numerator, 3, gives the number of unit fraction pieces (in this case one-fourth). A unit fraction is a fraction with the numerator of 1. This way of reading the fraction will help your child compare fractions and compute with fractions.

8. Your child can label fractions represented on a number line between 0 and 1. For example,



Your child may first notice that the distance from 0 to 1 was divided into 4 equal sections. The last section was divided into 3 equal sections or twelfths. The missing value would be $\frac{11}{12}$.

9. Your child can label fractions represented on a number line beyond 1. For example,



Your child may first notice that the distance from 1 to 2 was divided into 2 equal sections. The last section was divided into 2 equal sections or fourths. The missing value would be $\frac{3}{4}$.

Comparing Fractional Units

10. Your child should be able to compare fractions with the **same denominator**. For example,

$$\frac{3}{8} \qquad \frac{5}{8}$$

We'll ask your child, "Which is bigger 3, one-eighth pieces or 5, one-eighth pieces?" Understanding that you have more one-eighth pieces if you have 5 one-eighth pieces then five-eighths is greater than three-eighth. Your child can defend that five-eighths is greater than three-eighths using fraction pieces or drawings.

11. Your child should be able to compare fractions with the **same numerator**. For example,

$$\frac{3}{8} \qquad \frac{3}{6}$$

We'll ask your child, "Which is bigger: 3, one-eighth pieces or 3, one-sixth pieces?" Understanding that since a one-sixth piece is bigger than a one-eighth piece, your child can defend that 3, one-sixth pieces would be greater than 3, one-eighth pieces using fraction pieces or drawings.

12. In grade 4, your child is expected to compare fractions with different numerators and denominators. (Remember we are working with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.) This can be done by finding common numerators, common denominators, or comparing to 0, ½, or 1.

<u>Example 1:</u> To compare

$$\frac{3}{5} \qquad \frac{6}{8}$$

we can find a common numerator. In this case we can think of $^6/_8$ as ¾ OR we can think of 3/5 as 6/10. In both cases we have a common numerator. If we use ¾ for $^6/_8$,

$$\frac{3}{5} \qquad \frac{3}{4}$$

we can think, "Which is bigger… 3, one-fifth pieces or 3, one-fourth pieces?" Since $^1/_5$ is smaller than ¼ we know that 3, one-fifth pieces would be smaller than 3, one-fourth pieces.

If we change $^3/_5$ to its equivalent $^6/_{10}$

$$\frac{6}{10} \qquad \frac{6}{8}$$

we can think, "Which is bigger…6, one-tenth pieces or 6, one-eighth pieces?" Since $^1/_{10}$ is smaller than $^1/_8$ we know that 6, one-tenth pieces would be smaller than 6, one-eighth pieces.

Example 2: To compare ¾ and $^5/_8$ we can find a common denominator.

$$\frac{3}{4} \qquad \frac{5}{8}$$

In this case we can think of ¾ as $^6/_8$.

$$\frac{6}{8} \qquad \frac{5}{8}$$

We can think, "Which is bigger…6, one-eighth pieces or 5, one-eighth pieces. Since you have more one-eighth pieces with 6, one-eighth pieces, $^6/_8$ must be greater.

Example 3: To compare $^2/_5$ and $^5/_8$ we can compare each fraction to ½.

$$\frac{2}{5} \qquad \frac{5}{8}$$

Two-fifths ($^2/_5$) is less than one-half. We know this because 2 is less than half of 5. Another way to think about this is since double 2 is 4 we know that we need to double more than 2 to get to 5.

Five-eighths is more than one-half. We know this because half of 8 is 4. So 5 is more than half of 8. This means $^5/_8$ is more than ½. Therefore, $^2/_5$ is less than $^5/_8$.

13. Your child understands that on a number line, the number to the right is larger. He or she can use this understanding to compare fractions on a number line.

14. Your child is also expected to use the symbols >, <, and = correctly. To help with this, we have your child put 2 dots next to the larger value and 1 dot next to the smaller number.

$$\frac{3}{8} \;\cdot\; : \;\frac{3}{6}$$

Your child then connects the dots.

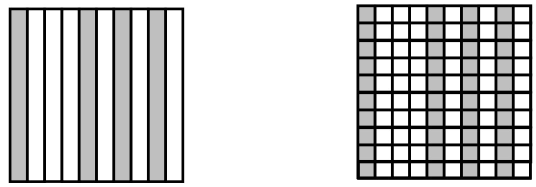
$$\frac{3}{8} < \frac{3}{6}$$

Comparing Decimal Fractions

15. Your child should understand that comparing .3 and .5 is the same as comparing $\frac{3}{10}$ and $\frac{5}{10}$ (like denominators). 3, one-tenth pieces is fewer than 5, one-tenth pieces. Likewise, comparing .03 and .05 is the same as comparing $\frac{3}{100}$ and $\frac{5}{100}$ (like denominators). 3, one-hundredth pieces is less than 5, one-hundredth pieces.

16. Your child should understand that comparing .3 and .03 is the same as comparing $\frac{3}{10}$ and $\frac{3}{100}$ (like numerators). Since one-tenth is larger than one-hundredth, 3, one-tenth pieces would be greater than 3, one-hundredth pieces.

17. Your child should understand that comparing 0.4 and 0.37 is the same as comparing $\frac{4}{10}$ and $\frac{37}{100}$. To compare your child uses his or her understanding that $\frac{4}{10}$ is equivalent to $\frac{40}{100}$. To compare 0.4 and 0.37 he or she can compare 0.40 and 0.37 (like denominators). For example,



$\frac{4}{10}$ is equivalent to $\frac{40}{100}$

Since $\frac{40}{100}$ is greater than $\frac{37}{100}$, $\frac{4}{10}$ is greater than $\frac{37}{100}$. Therefore, 0.4 > 0.37 or .4 > .37 .

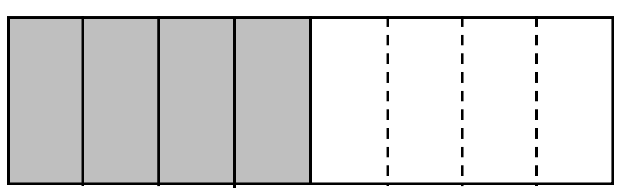
Equivalent Fractions

18. In Grade 4 your child should understand that we can divide the same region into smaller equal-sized pieces. For example, we can begin with ½.

We can cut the halves into 2 equal pieces. We see that 2, one-fourth pieces is equivalent to one-half.

$\frac{1}{2} = \frac{2}{4}$

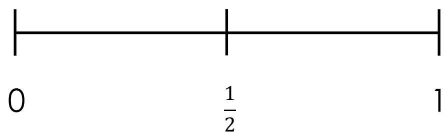
We can cut the fourths into 2 equal pieces. We see that 4, one-eighth pieces is equivalent to one-half.



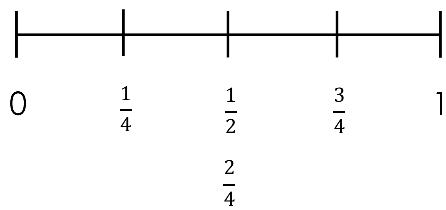
$\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$

Your child should understand how to create equivalent fractions using fraction pieces or drawings. He or she should notice and explain the relationship between $\frac{1}{2}$ and $\frac{4}{8}$ ($\frac{4}{8}$ can be obtained by multiplying the numerator and denominator of $\frac{1}{2}$ by the common factor 4).

19. Your child understands that we can name the same location on a number line in more than one way. For example, suppose we cut the interval from 0 to 1 into 2 equal sections.

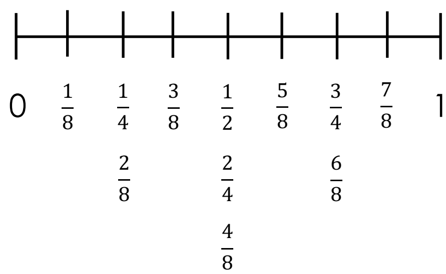


We then cut the interval from 0 to 1 into 4 equal sections.



$\frac{1}{2}$ and $\frac{2}{4}$ name the same location on the number line so $\frac{1}{2}$ is equivalent to $\frac{2}{4}$.

Suppose we repeat and cut the interval from 0 to 1 into 8 equal sections.



$\frac{1}{4}$ and $\frac{2}{8}$ name the same location on the number line so $\frac{1}{4}$ is equivalent to $\frac{2}{8}$.

$\frac{1}{2}$, $\frac{2}{4}$, and $\frac{4}{8}$ name the same location on the number line so they are equivalent.

$\frac{3}{4}$ and $\frac{6}{8}$ name the same location on the number line so $\frac{3}{4}$ is equivalent to $\frac{6}{8}$.

Your child should understand that equivalent fractions on a number line name the same location. She or he should be able to represent equivalent fractions on a number line and defend that the fractions are equivalent.

Addition & Subtraction

Word Problems

1. Your child should be able to use a variety of addition and subtraction strategies to solve multi-step word problems.

Addition Strategies

2. Your child is expected to be able to add fractions with like denominators using a variety of strategies and representations, including fraction strips, drawings, properties, and equations.

Bridge to a Whole (Associative Property)

The bridge to 10 strategy is a powerful addition strategy for your child to know. It was introduced to your child in grade 1. He or she learned that another way to think of 9 + 4 is as 10 + 3 or 8 + 6 as 10 + 4. In grade 2 we used this strategy to solve problems such as 38 + 16 knowing that it is the same as 40 + 14. In grade 3, your child learned that this strategy is really the Associative Property as he or she used it to solve problems such as 538 + 197, linking it to 535 + 200. For example, to solve 538 + 197 we thought of 538 as 535 + 3. Instead of associating the 3 with the 535 we associate it with 197,

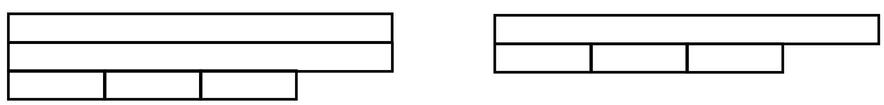
$$538 + 197 = (535 + 3) + 197 = 535 + (3 + 197). \text{ So,}$$

$$538197 = 535 + 200 = 735$$

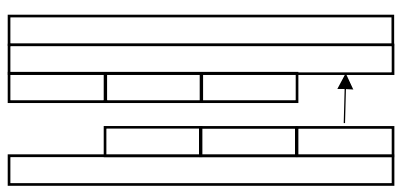
In grade 4 your child will extend this idea to adding fractions.

3. In grade 4 your child should be able to use the Bridge to a Whole (Associative Property) to add fractions with like denominators. An important component of using this strategy with fractions is the drawing of pictures.

<u>Example 1:</u> $2\frac{3}{4} + 1\frac{3}{4}$. I could draw a picture like the one below.



But if we rotate the image for $1\frac{3}{4}$ and place it under $2\frac{3}{4}$ it is much easier to see how one of the fourths can slide into the empty space of $2\frac{3}{4}$ to complete the whole (bridge to a whole).



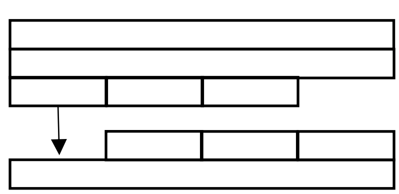
Note: It doesn't matter which $\frac{1}{4}$ piece we slide to complete the whole.

We are calling it Bridge to a Whole but it is the Associative Property. To find $2\frac{3}{4} + 1\frac{3}{4}$ we think of $1\frac{3}{4}$ as $\frac{1}{4} + 1\frac{2}{4}$. So $2\frac{3}{4} + 1\frac{3}{4} = 2\frac{3}{4} + (\frac{1}{4} + 1\frac{2}{4})$. Instead of associating the $\frac{1}{4}$ with $1\frac{2}{4}$, we associate it with $2\frac{3}{4}$.

$$2\tfrac{3}{4} + (\tfrac{1}{4} + 1\tfrac{2}{4}) = (2\tfrac{3}{4} + \tfrac{1}{4}) + 1\tfrac{2}{4} = 3 + 1\tfrac{2}{4}$$

$$2\tfrac{3}{4} + 1\tfrac{3}{4} = 4\tfrac{2}{4} \text{ or } 4\tfrac{1}{2}$$

<u>Example 2:</u> $2\frac{3}{4} + 1\frac{3}{4}$. We also could have chosen to "complete the whole" for the $\frac{3}{4}$ in $1\frac{3}{4}$.



In this example, we would think of 2 ¾ as $2\frac{2}{4} + \frac{1}{4}$. So,

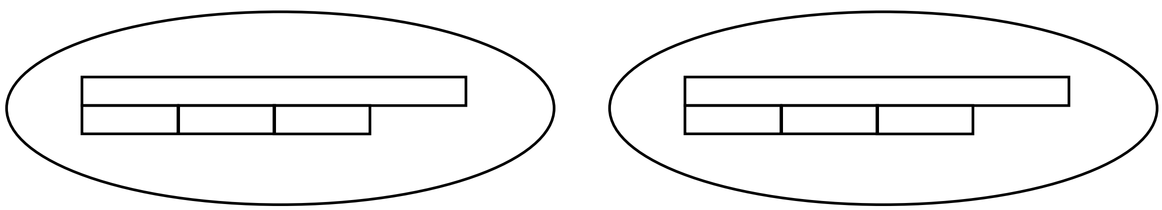
$$
\begin{aligned}
2\tfrac{3}{4} + 1\tfrac{3}{4} &= (2\tfrac{2}{4} + \tfrac{1}{4}) + 1\tfrac{3}{4} \\
&= 2\tfrac{2}{4} + (\tfrac{1}{4} + 1\tfrac{3}{4}) \\
&= 2\tfrac{2}{4} + 2 \\
&= 4\tfrac{2}{4}
\end{aligned}
$$

Doubling Strategies

Doubles and near doubles (doubles plus 1, doubles minus 1, doubles plus 2, doubles minus 2) are important addition strategies for your child to know. In grade 1, your child learned to double the numbers 1 to 9 (e.g., double 8 is 16, double 6 is 12). In grade 2 we used these strategies for doubling 2-digit numbers. In grade 3 we used these strategies to quickly solve problems such as 199 + 199. Your child can think of this problem as double 200 minus 2, 56 + 56 as double 50 plus double 6 or 112. In grade 4 we use this same strategy to solve 2 x 199 or 2 x 56. We can also use the doubling strategy to double fractions less than one and mixed numbers.

4. Your child is expected to be able to double mixed numbers and record using pictures, an addition equation, and a multiplication equation.

   Example 1: Double 1¾.



Double 1¾ can be written as, 1¾ + 1¾ or 2 x 1¾ . We can describe what we have as 2 groups of 1, or double 1, put together with 2 groups of ¾, or double ¾.

So $2 \times 1\frac{3}{4} = 2 \times 1 + 2 \times \frac{3}{4}$.

This is the Distributive Property and is linked to a whole number times a mixed number when doing fraction multiplication.

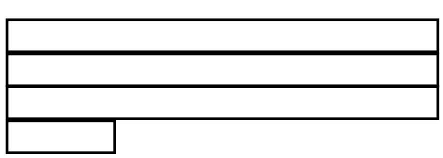
Subtraction Strategies

5. Your child should be able to use the round and adjust strategy to subtract mixed numbers with common denominators. He or she should be able to represent with fraction strips, drawings, and equations.

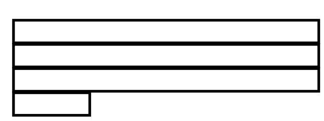
   Example 1: 3¼ – 1¾

   Begin with showing 3¼ using fraction kit materials (with drawings).

Materials

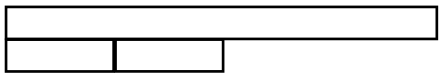


Drawings

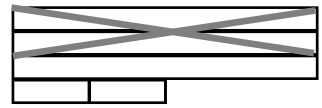


To remove 1¾, we remove 2 and then give back ¼. We rounded 1¾ to 2 and then needed to adjust (give ¼ back) because we took away one-fourth too many. The illustrations below show the changes in our materials and drawing after using this strategy.

Materials



Drawings



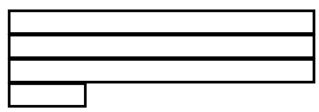
I removed 2 wholes and gave back ¼. (Note: It doesn't matter which of the wholes are removed or crossed out.)

3¼ – 1¾ = 3¼ – 2 + ¼

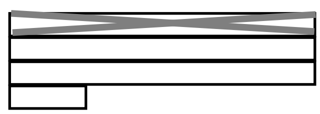
6. Your child should be able to decompose a whole to subtract mixed numbers with common denominators. He or she should be able to represent with drawings and equations.

Example 1: 3¼ - 1¾

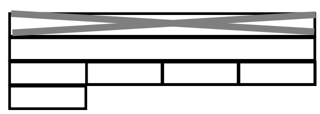
Begin by drawing 3¼.



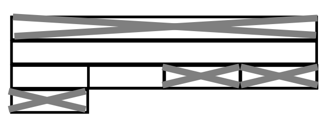
We think of 1¾ as 1 and ¾ (1 + ¾). First remove 1.



We don't have ¾ to remove so we "cut" a whole into fourths (Note: It doesn't matter which whole we cut into fourths).



We "remove" 3, one-fourth pieces and we are left with the answer (Note: It doesn't matter which of the one-fourth pieces we remove).



3¼ – 1¾ = 3¼ – 1 – ¾ = $1\,{}^{2}/_{4}$ or 1½

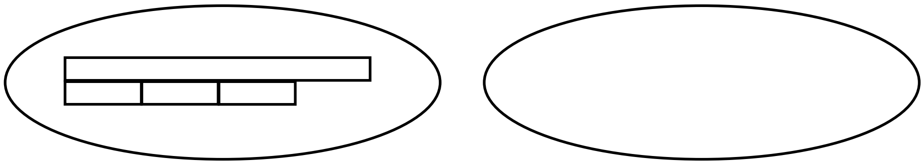
7. Your child should be able to solve subtraction problems by connecting to addition (missing addend). Using this strategy we think of problems such as 3¼ – 1¾ as how many more than 1¾ is 3¼ or how many do we need to add to 1¾ to get to 3¼.

1¾ + ___ = 3¼

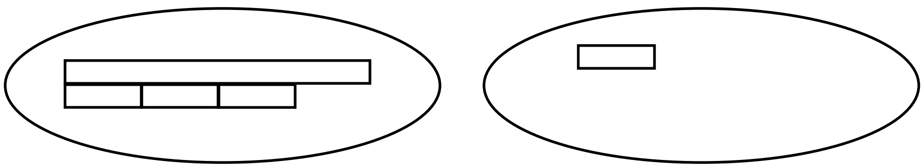
8. Your child should be able to solve missing addend problems by finding the missing part. He or she should be able to represent with fraction pieces, drawings, and equations.

<u>Example 1:</u> $1\frac{3}{4}$ + ____ = $3\frac{1}{4}$

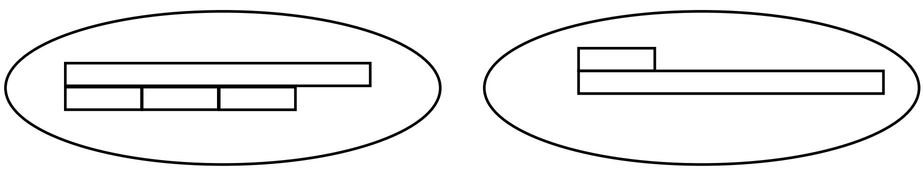
Begin with representing the part you know ($1\frac{3}{4}$) on one section (this can be a plate or sheet of paper). We will put the pieces for the missing part on the other section.



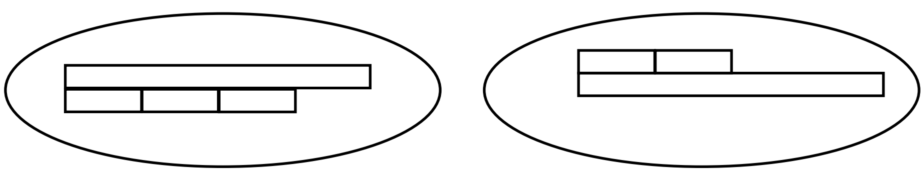
On the other section we add fourths and 1s until we reach the total ($3\frac{1}{4}$). For example, if we add $\frac{1}{4}$, we have a total of 2. (**Note:** I don't have to begin by adding fourths. I could add a 1 first until I get close to the goal number.)



We add a whole and we have a total of 3.



We then add a $\frac{1}{4}$ and we have a total of $3\frac{1}{4}$.



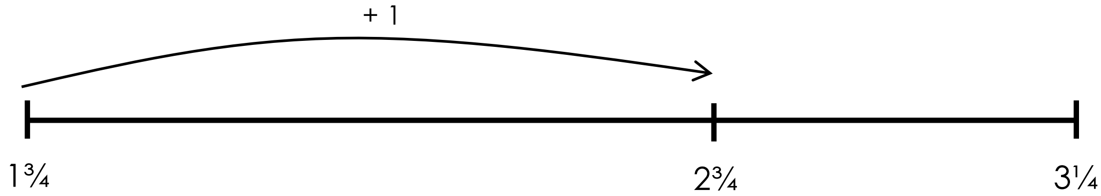
The missing part is $1^{2}/_{4}$ or $1\frac{1}{2}$.

$$1\tfrac{3}{4} + \boxed{1^{2}/_{4}} = 3\tfrac{1}{4} \quad \text{so} \quad 3\tfrac{1}{4} - 1\tfrac{3}{4} = \boxed{1^{2}/_{4}} \quad \text{or } 1\tfrac{1}{2}$$

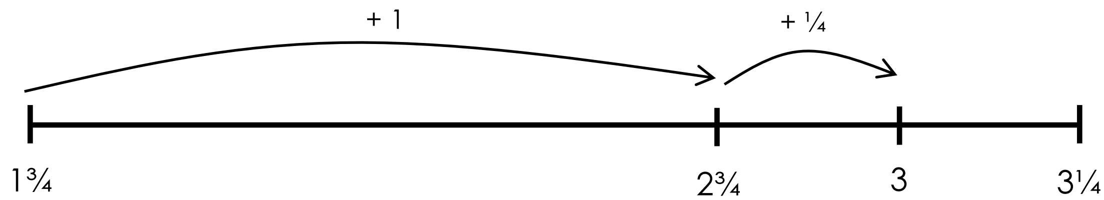
9. Your child should be able to solve missing addend problems by counting up on an open number line. He or she should be able to represent with drawings and equations.

<u>Example 1:</u> $1\frac{3}{4}$ + ____ = $3\frac{1}{4}$

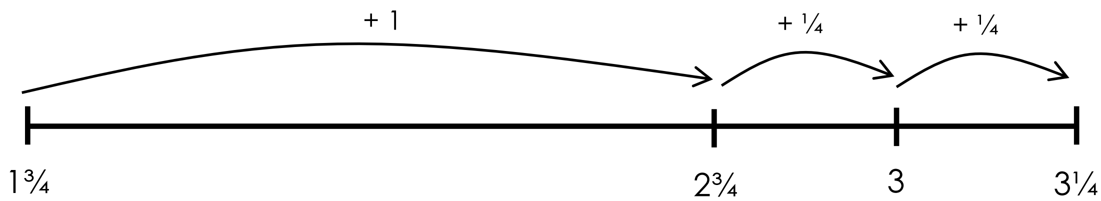
We can make a jump of 1 to get to $2\frac{3}{4}$ (Count up 1).



We can make a jump of ¼ to get to 3 (Count up ¼).



We can make a jump of ¼ to get to 3¼ (Count up ¼).



We combine the jumps to get the missing addend.

$$1\tfrac{3}{4} + \underline{\mathbf{1 + \tfrac{1}{4} + \tfrac{1}{4}}} = 3\tfrac{1}{4}$$

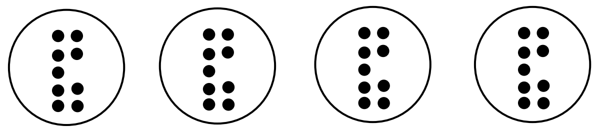
$$1\tfrac{3}{4} + \boxed{1\tfrac{2}{4}} = 3\tfrac{1}{4} \quad \text{so} \quad 3\tfrac{1}{4} - 1\tfrac{3}{4} = \boxed{1\tfrac{2}{4}} \text{ or } 1\tfrac{1}{2}$$

Multiplication and Division

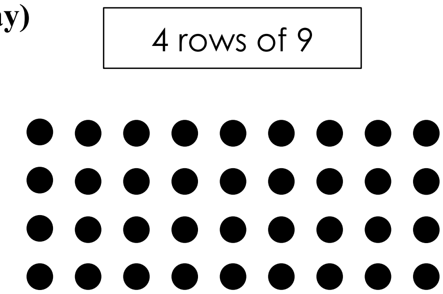
Multiplication Concepts

1. For many of us, learning multiplication involved memorizing the multiplication facts. Although fact fluency is important, your child is expected to understand all of the ways that we represent multiplication: equal groups, equal rows (array), equal jumps, area, and as multiplicative comparisons. For example,

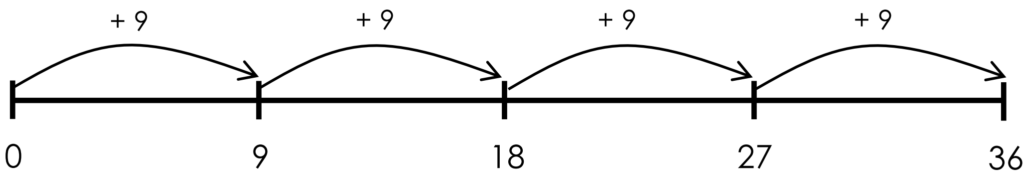
Equal Groups 4 groups of 9



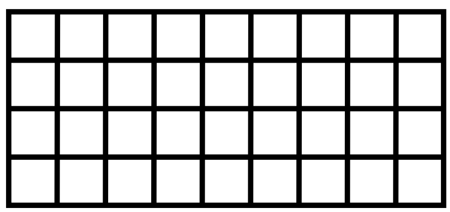
Equal Rows (Array)



Equal Jumps

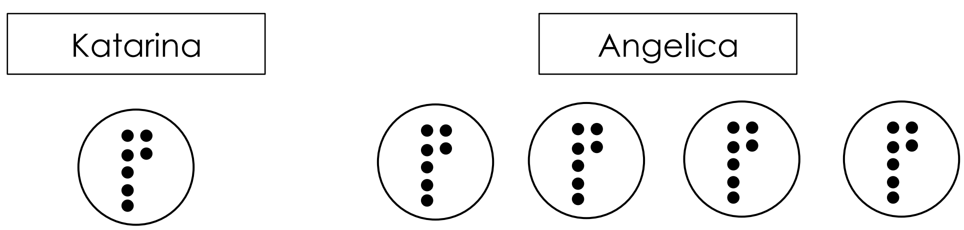


Area



Multiplicative Comparison

Katarina has started to collect shells. She has 7 shells. Her older sister Angelica has 4 times as many shells.



2. Your child should know the language associated with each multiplication model. For example,
    - equal groups of,
    - equal rows of,
    - equal jumps of,
    - “by” (when working with area), or
    - times as many.

   For a problem such as 4 x 9, we could read as:
    - 4 groups of 9,
    - 4 rows of 9,
    - 4 jumps of 9,
    - 4 by 9, or
    - 4 times as many as 9.

3. Your child should understand that multiplication is Commutative. That is the order in which we multiply numbers will not change the product. For example, 9 x 5 is the same as 5 x 9. 2 x 9 x 5 is the same as 2 x 5 x 9.

4. Your child should understand and explain the patterns that occur when multiplying a one-digit number by multiples of 10 or 100. For example, 9 x 20 is the same as 9 x (2 x 10) or (9 x 2) x 10. We can first find the fact we know, 9 x 2, and then multiply by 10. For 9 x 50 we could first find 9 x 5 and then multiply by 10. For 9 x 400,

$$\begin{aligned} 9 \times 400 &= 9 \times (4 \times 100) \\ &= (9 \times 4) \times 100 \\ &= 36 \times 100 \\ &= 3600 \end{aligned}$$

This is the Associative Property.

5. Your child should understand and name the numbers (factors) that are multiplied together to make a value between 1 and 100.

6. Your child should understand that a number between 1 and 100 is a multiple of its factors. For example, 2 factors for 35 are 5 and 7 because 5 x 7 = 35. Thirty-five is a multiple of 5 (one of its factors) and it is a multiple of 7.

7. Your child should understand that a number is prime if it has exactly 2 factors. For example, 7 is prime because the only factors are 1 and 7. Your child should understand that a number is composite if it has more than 2 factors. For example, 4 is a composite number because its factors are 1, 2, and 4. Your child should understand that 1 is neither prime nor composite because it has only 1 factor, 1.

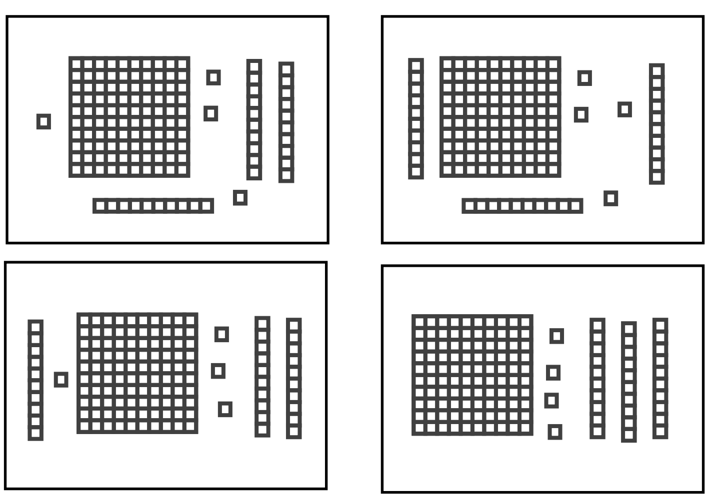
Equal Grouping Model

8. Your child should understand and use the Distributive Property to find the product of 2 numbers using an equal grouping model. He or she should be able to represent with base 10 materials, drawings, and equations.

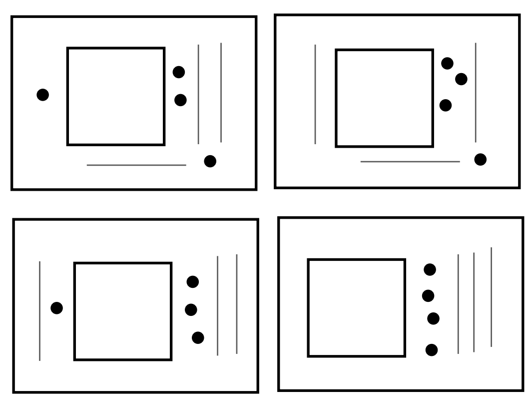
<u>Example 1:</u> 4 x 134

4 x 134
4 groups of 134

<u>Materials</u>



<u>Drawing</u>



To represent 4 groups of 134 we used 4 groups of 100 (4 x 100), 4 groups of 30 (4 x 30), and 4 groups of 4 (4 x 4). This is the Distributive Property.
To make 134 with the Base 10 materials we thought of it as 100 + 30 + 4 (expanded form).

$$4 \times 134 = 4 \times (100 + 30 + 4) = (4 \times 100) + (4 \times 30) + (4 \times 4)$$

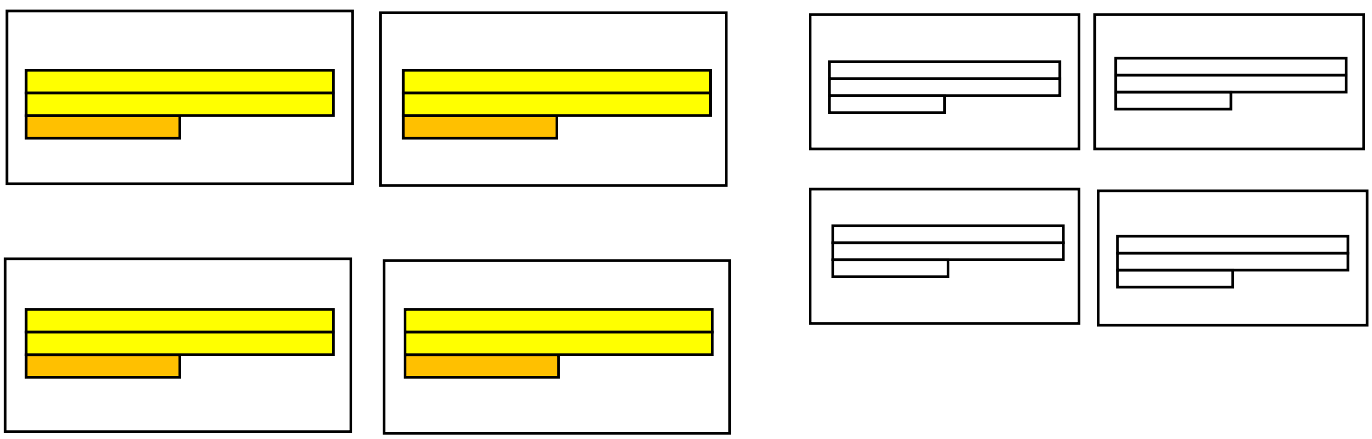
4 groups of 134 is the same as 4 groups of 100 put together with 4 groups of 30 put together with 4 groups of 4.

Note: In (4 x 100) + (4 x 30) + (4 x 4) parentheses are not required but are helpful. For this problem your child would be correct if he or she wrote, 4 x 100 + 4 x 30 + 4 x 4.

<u>Example 2</u> 4 x 2½

4 x 2½
4 groups of 2½

Materials Drawing



To represent 4 groups of 2½ we used 4 groups of 2 (4 x 2) and 4 groups of ½ (4 x ½). This is the Distributive Property.
To make 2½ with the fraction kit materials we thought of it as 2 + ½ (expanded form).

$$4 \times 2\tfrac{1}{2} = 4 \times (2 + \tfrac{1}{2}) = (4 \times 2) + (4 \times \tfrac{1}{2})$$

4 groups of 2½ is the same as 4 groups of 2 put together with 4 groups of ½.
Note: In (4 x 2) + (4 x ½) parentheses are not required but are helpful. For this problem your child would be correct if he or she wrote, 4 x 2 + 4 x ½.

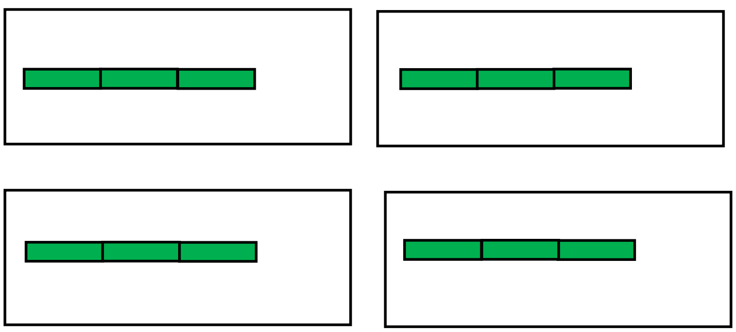
$$\begin{aligned} 4 \times 2\tfrac{1}{2} = 4 \times (2 + \tfrac{1}{2}) &= (4 \times 2) + (4 \times \tfrac{1}{2}) \\ &= 8 + 2 \\ &= 10 \end{aligned}$$

9. Your child should understand and use an equal grouping model representation of the Associative Property to find the product of a whole number times a fraction less than one.

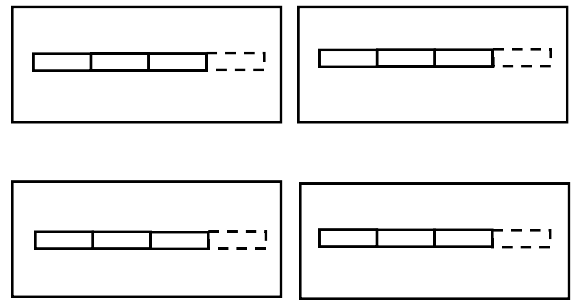
Example 1: 4 x ¾.

4 x ¾
4 groups of ¾

Materials



Drawing



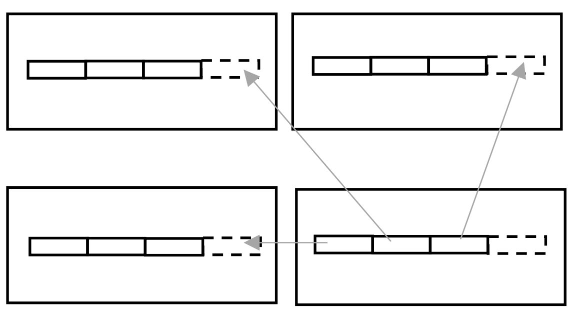
3, one-fourth pieces is needed for each group (3 x ¼).
4 groups of 3, one-fourth pieces is the same as 4 x (3 x ¼).
Instead of associating the 3 with the ¼ we can associate it with the 4, (4 x 3) x ¼.

4 x ¾ = (4 x 3) x ¼ = 12 x ¼.

12, one-fourth pieces can be used to make 3 wholes.

Students can also find this answer by noticing that if they take the 4^{th} group of ¾, they can use each of the ¼ pieces to complete the whole for the other 3 groups.

4 x ¾ = 3



10. Your child should understand and connect partial products and the Distributive Property.

Example 1: If we think of 4 x 134 as 4 x 100 + 4 x 30 + 4 x 4, then 4 x 134 = 400 + 120 + 16. 400, 120, and 16 would be the partial products. You may see this written vertically as,

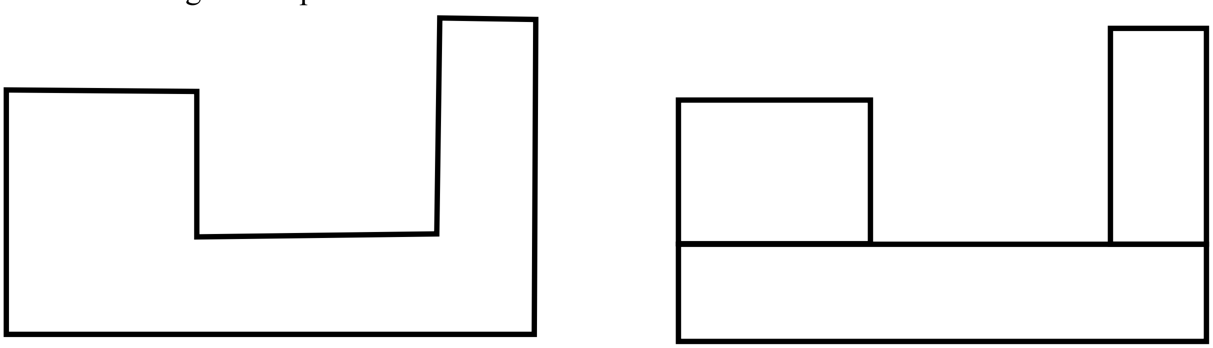
134		134
x 4		x 4
400	OR	16
120		120
+16		+400
536		536

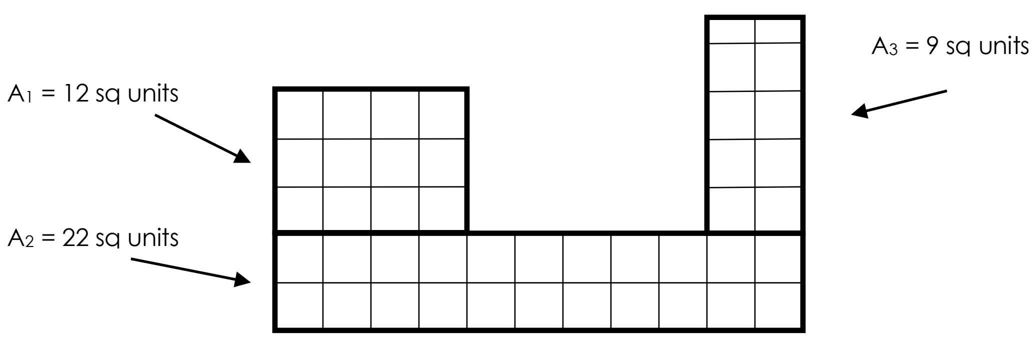
When we use place value or the Distributive Property the order of each partial product will not affect the answer (Commutative Property). When your child is taught the arithmetic procedure in 5th grade that we learned as children, we have them examine a way to collapse the 3 steps (3 partial products).

11. Your child should understand that although the totals are the same for problems such as 1 x 7 and 7 x 1, the pictures will be different.

Area Model

12. Your child should understand that shapes can be divided into smaller pieces. He or she can find the area of each of those pieces and then add them together to get the total area of a shape.
    Example: To find the area of the shape below, it can be cut into rectangles as shown on the right. We can then find the area of each rectangle, combine those areas and obtain the area of the original shape.





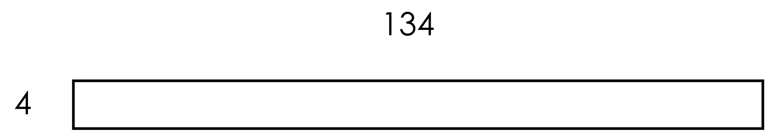
Combining the areas, A_1, A_2, and A_3

$$A_1 + A_2 + A_3 = 12 + 22 + 9 \text{ sq units} = 43 \text{ sq units}$$

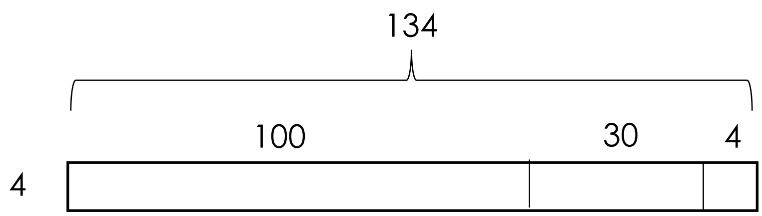
13. Your child should use their understanding of adding areas to find the total area of a shape for 1-digit by up to 4-digit and 2-digit by 2-digit multiplication.

<u>Example 1</u>   4 x 134

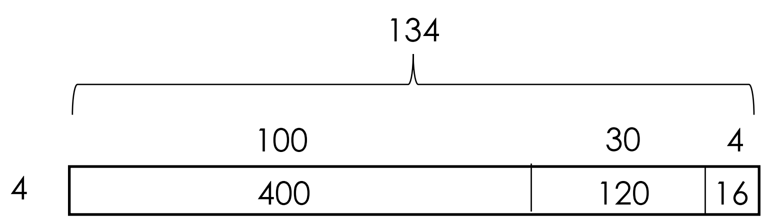
4 x 134
4 by 134



Cut the area into smaller sections using expanded form (use place value). Thinking of 134 as 100 + 30 + 4 we'll have 3 rectangles: one with the dimensions 4 by 100, one with the dimensions 4 by 30, one with the dimensions 4 by 4.



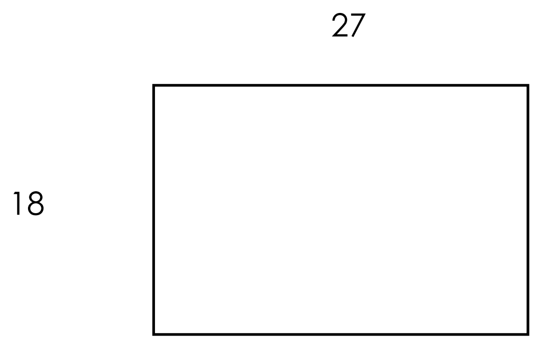
4 x 134 = 4 x (100 + 30 + 4)
OR (4 x 100) + (4 x 30) + (4 x 4).
This is the Distributive Property.



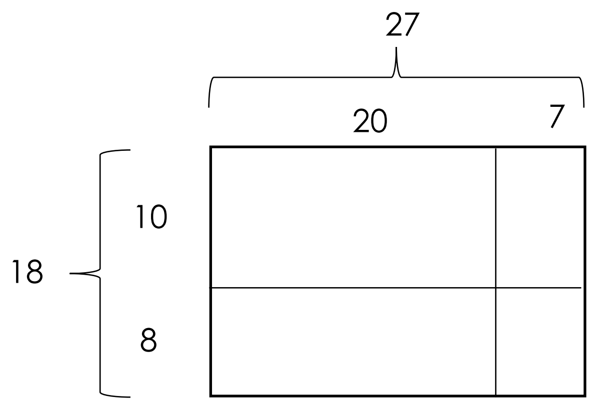
4 x 134 = 4 x (100 + 30 + 4)
= (4 x 100) + (4 x 30) + (4 x 4)
= 400 + 120 + 16
= 536

Example 2: 18 x 27

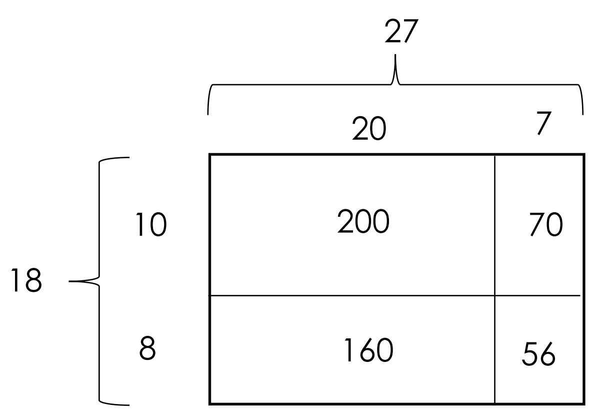
18 x 27
18 by 27



Cut the area into smaller sections using expanded form (use place value). Thinking of 18 as 10 + 8 and 27 as 20 + 7 we'll have 4 rectangles: one with the dimensions 10 by 20, one with the dimensions 10 by 7, one with the dimensions 8 by 20, and one with the dimensions 8 by 7.



18x 27 = (10 + 8) x (20 + 7)
OR (10 x 20) + (10 x 7) + (8 x 20) + (8 x 7)
This is the Distributive Property.



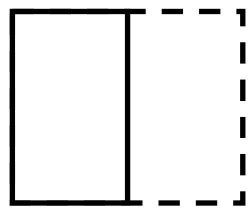
18x 27 = (10 + 8) x (20 + 7)
= (10 x 20) + (10 x 7) + (8 x 20) + (8 x 7)
= 200 + 70 + 160 + 56
= 270 + 216
= 486

Example 3: $^1/_3$ x $^1/_2$

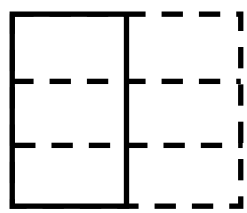
We can use the area model to represent problems such as $^1/_3$ x $^1/_2$. We can read this as one-third by one-half or one-third of one-half.

- We begin with a rectangular whole represented with a dashed line.

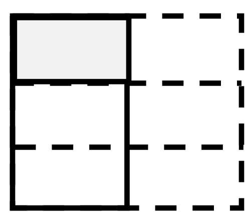
- We cut the whole into 2 equal parts and put a solid line around one-half.



- We now want to cut the halves into 3 equal pieces. Show with a dashed line extended across the whole.



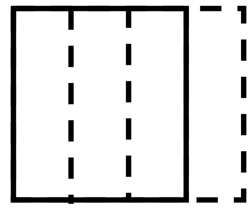
- Shade in a third of the half. We see that $^1/_3$ x $^1/_2$ is $^1/_6$.



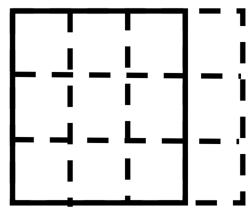
Example 4: $^2/_3$ x $^3/_4$.

- We begin with a rectangular whole represented with a dashed line.

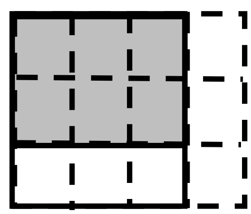
- We cut the whole into 4 equal parts and put a solid line around three-fourths.



- We now want to cut the fourths into 3 equal pieces so that we can find two-thirds of three-fourths. Show with a dashed line extended across the whole.

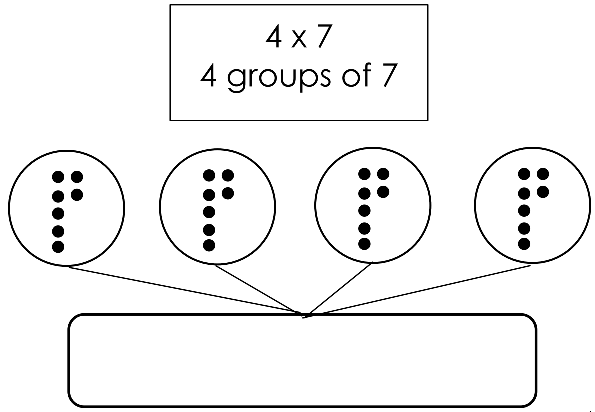


- Shade in a two-thirds of the three-fourths. We see that $^2/_3$ x $^3/_4$ is $^6/_{12}$.

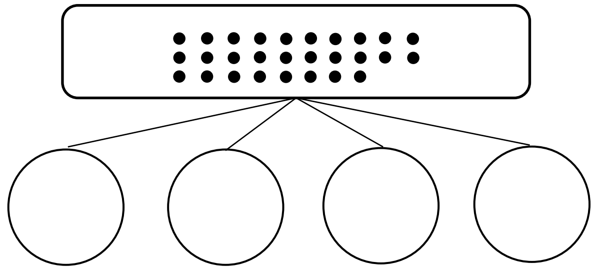


Division

14. Your child should understand that multiplication and division are inverse operations. That is, your child should understand that:
    - if multiplication is combining equal groups, division is separating into equal groups or pulling off equal groups of a given number.
    - if multiplication is combining equal rows, then division is separating into equal rows or pulling off equal rows of a given number.
    - if multiplication is combining equal jumps, then division is separating into equal jumps.
    - if multiplication is finding the area when given the dimensions of a rectangle, then division is finding an unknown side length when the area and one side length is known.



28 ÷ 4
28 separated into 4 equal groups



15. In grade 4, your child is expected to understand and use the different representations and language for division. For example,
    - separating into equal groups,
    - how many groups of ___ are in ___,
    - separating into equal rows,
    - how many rows of ___ are in ___,
    - separating into equal jumps,
    - how many jumps of ___ are in ___.

    For a problem such as 24 ÷ 6, we could read as:
    - 24 separated into 6 equal groups,
    - how many groups of 6 are in 24,
    - 24 separated into 6 equal rows,
    - how many rows of 6 are in 24,
    - 24 separated into 6 equal jumps,
    - how many jumps of 6 are in 24.

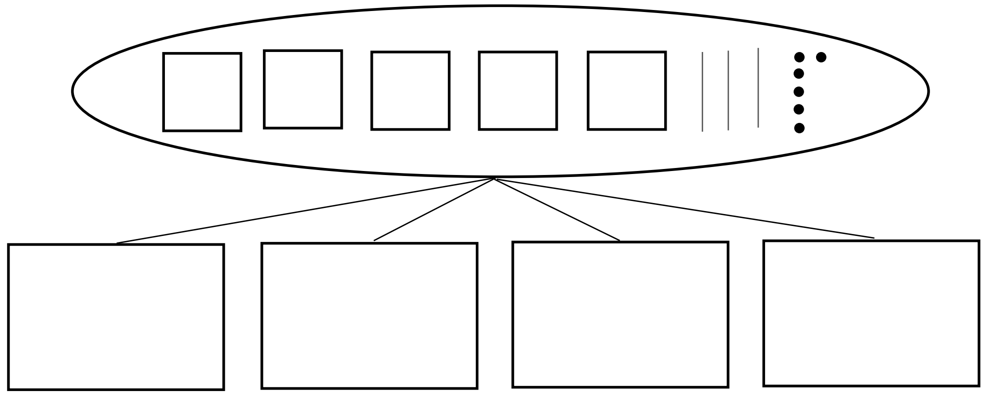
16. He or she is also expected to use his or her knowledge of multiplication to divide. That is, to find 24 ÷ 6 your child can think, "What times 6 is 24?" (missing factor). This is helpful when using the area model to divide. For example, 6 by ___ is 24.

Division using the equal grouping model and partial quotients

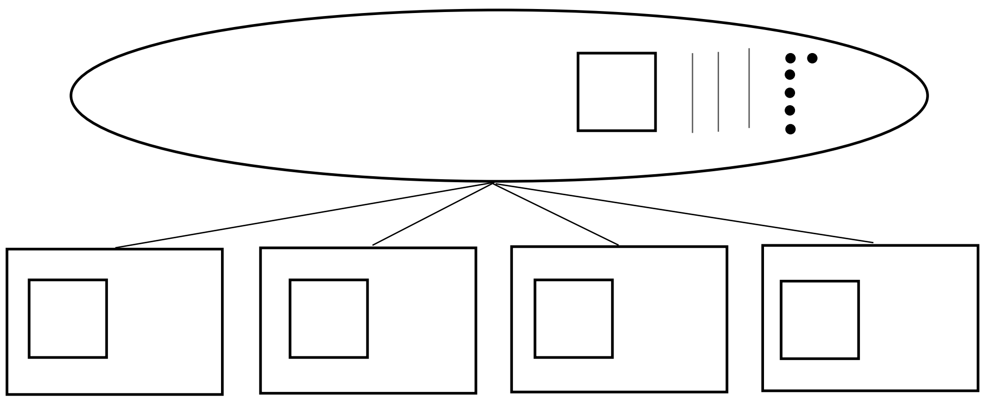
17. Your child should be able to represent division using the equal grouping model, dividing up to 4-digit numbers by a 1-digit number.

    <u>Example 1</u>: 536 ÷ 4 using an equal grouping model

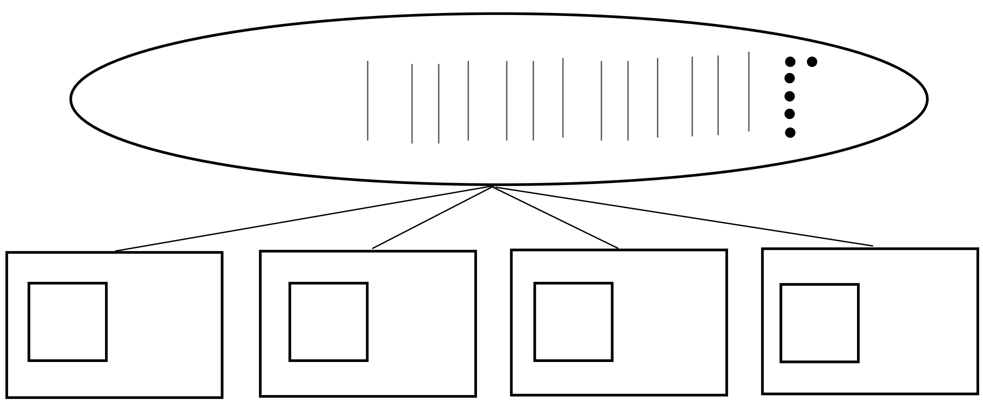
    - Begin by translating the division problem into language that helps bring imagery. When dividing by a single digit number it is still efficient to think of division as separating into equal groups. We can translate 536 ÷ 4 as 536 separated into 4 equal groups.



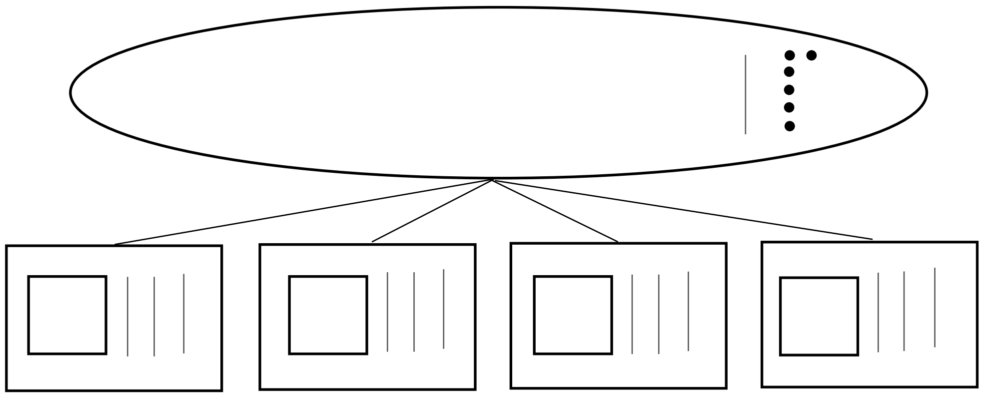
- We can first put 100 in each group.



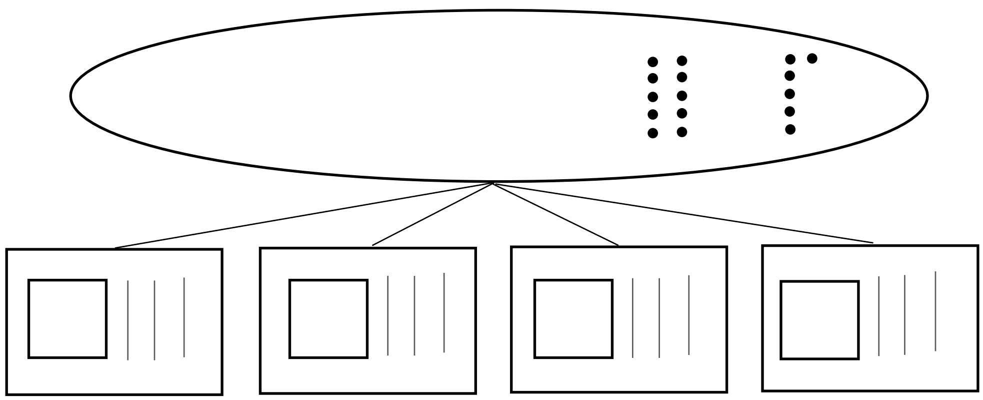
- We know that an equivalent way to think of 1 hundred and 3 tens is 13 tens.



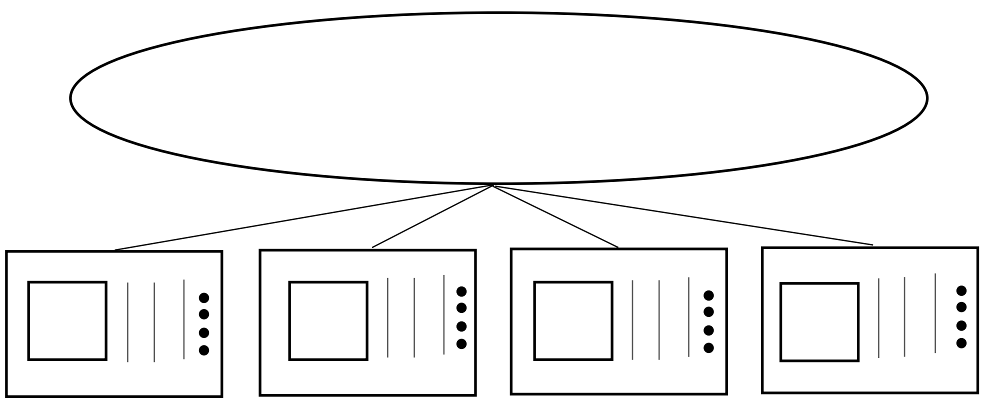
- We can separate the 13 tens into the 4 groups by placing 3 tens in each group.



- We know that an equivalent way of thinking of 1 ten and 6 ones is as 16 ones.



- We can separate the 16 ones into the 4 groups by placing 4 ones in each group.



These are the actual steps that are taken when doing the long division procedure, but we begin with understanding how the quantities are being distributed.

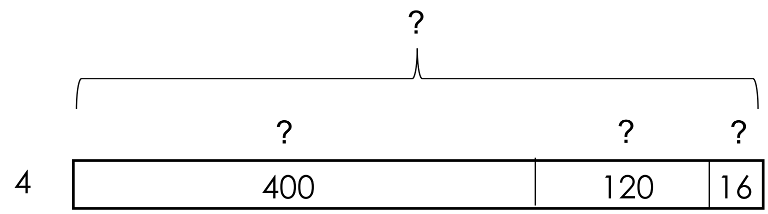
Division Using the Area Model & Partial Quotients

18. Your child should be able to represent division using the area model, dividing up to a 4-digit number by a 1-digit number or 2-digit number.

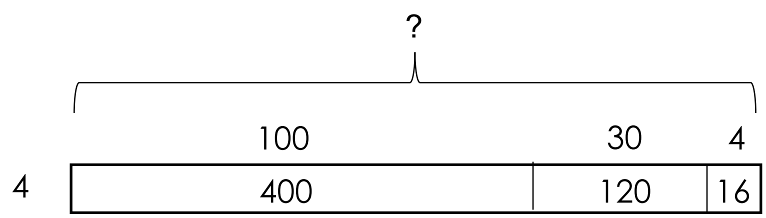
Example 1: 536 ÷ 4 using the area model.

We can also use an area model to find the partial quotients. If we use the area model to find 536 ÷ 4, we think of 536 as the total area and 4 as one of the side lengths. Our goal is to find the other side length.

- We cut the area into multiples of the side length (in this case 4). One possibility is shown below.

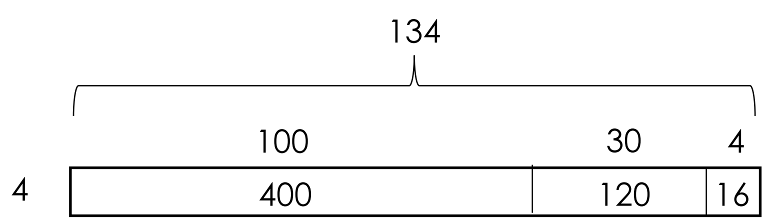


- To find the missing values we think, 4 times what will give 400, 4 times what will give 120, and 4 times what will give 16.
- We know that 4 times 100 is 400, 4 times 30 is 120, and 4 times 4 is 16.

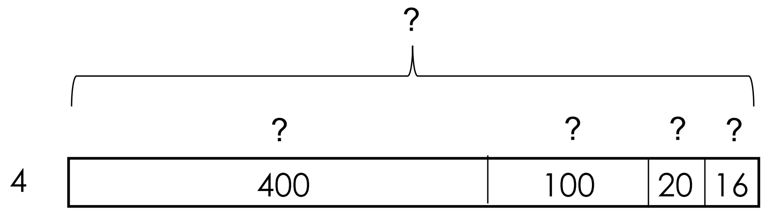


- We add the partial quotients to get the total. So,

536 ÷ 4 is the same as 4 x ___ = 536
4 x ____ = 400
4 x ____ = 120
4 x ____ = 16
So 536 ÷ 4 = 100 + 30 + 4 = 134



Note: We can separate the area into any multiple of 4. See below for another example of a way we could separate the area into multiples of 4. Remember, we need the total area to equal 536 and for each partial area to be multiples of the dividend (4).



In this case we would find,

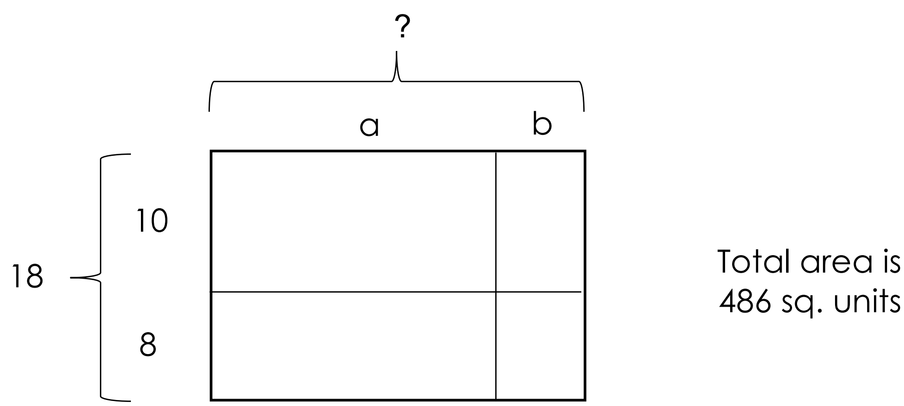
4 x ____ = 400
4 x ____ = 100
4 x ____ = 20
4 x ____ = 16

So 536 ÷ 4 = 100 + 25 + 5 + 4 = 134

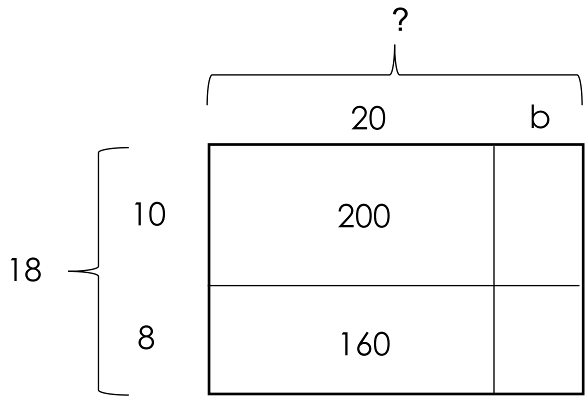
<u>Example 2</u>: 486 ÷ 18 using the area model.

To use the area model to solve 486 ÷ 18, we think of 486 as the total area and 18 as one of the side lengths. We know that the other side length must be a double-digit number. Since 10 times 18 (the first 2-digit number) is 180, we know it cannot be a number less than 10. A single-digit number would be too small. The goal is to determine the areas (partial quotients) that we need to separate 486 into to find the missing 2-digit side length.

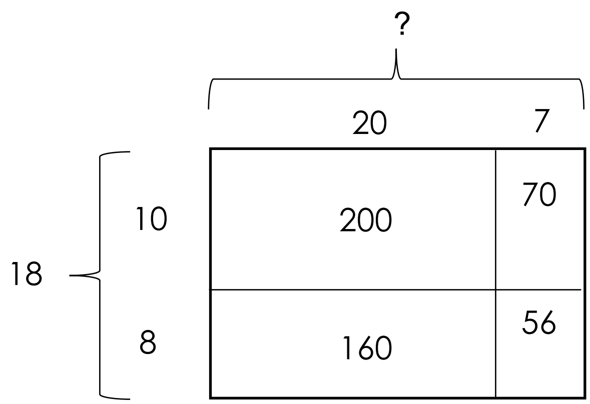
- We begin with what we know.



- Because we have been using expanded form of a number for our dimensions we know that "a" is a multiple of 10. We can try 10, 20, or 30. If we try 30, we will have a 10 by 30 rectangle and an 8 by 30 rectangle. Since 300 + 240 is greater than our total area, 30 will not work. Ten is too low because 10 by 10 combined with 10 by 8 is 180. We are 306 short of our total area. Let's try substituting 20 for "a".



- To determine the value for "b", your child needs to understand that the ending digit of a multi-digit product is the ending digit obtained when multiplying the numbers in the ones place. In this problem, 486 ends in 6. We know that the number in the bottom right corner must end in 6. That means that "b" must either be 2 or 7 since 2 x 8 = 16 and 7 x 8 = 56. Since we are 126 away from 486, 2 will be too low. Try 7.



$$200 + 70 + 160 + 56 = 486$$

so $486 \div 18 = 27$

Word Problems

19. Your child should be able to represent and solve multiplication and division multi-step word problems.

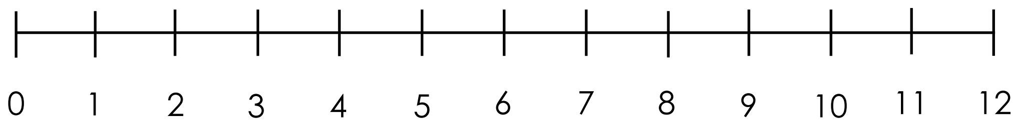
Measurement Concepts

Time

1. Your child should be able to accurately tell time to the nearest minute on an analog and a digital clock. An analog clock is the clock that is divided into 12 equal sections. Sometimes the sections are numbered 1 to 12, with 12 at the top of the clock. Sometimes only some of the sections are labeled (e.g., 3, 6, 9, and 12). Your child should understand that the numbers on the clock represent 2 different time measures. First, the numbers represent the number of hours that have passed since midnight or since noon. It takes 1 hour for the hour hand to move from one number to the next. The numbers also represent multiples of 5 minutes.

2. As your child works with angle measure in grade 4, he or she should be able to determine that the angle formed between 2 consecutive numbers on a clock is 30 degrees. Your child should understand that the angle formed, from the center, between 12 and 3 is 90 degrees. (Note: This is true for any 15-minute intervals. For example, 1 to 4, 2 to 5, 6 to 9 all form right angles.) Since there are 3 angles formed by the numbers between 12 and 3 (12 and 1, 1 and 2, 2 and 3), 90 divided by 3 is 30. Therefore, each angle is 30 degrees.

3. Your child should understand that an analog clock is a number line that wraps on itself. If we "unwrap" the clock face we would have something similar to the number line show below.



   At 12 we are back at 0. He or she should be able to place times on a number line to solve word problems.

4. Your child should be able to solve addition, subtraction, multiplication, and division multi-step word problems involving time.

Money

5. In 4th grade your child is expected to be able to:
   - name each of the coins regardless of the image on the head or tail,
   - name the value of each coin,
   - find and name other information available on the coins (date, United States of America, In God We Trust, value, etc.),
   - name the total when given a collection of mixed coins and bills,
   - show multiple ways to make the same value,
   - correctly write money values using $ and ¢ symbols, and
   - solve money word problems.

   Note: Remember that when using the ¢ symbol we do not use a decimal point. Think of ¢ as representing the value of a penny. We have 50¢ but not **.**50¢. The latter is the equivalent of half a penny. That is not part of US currency. We can also show 50 cents as a part of a dollar. A dollar is the equivalent of 100 cents. 50 cents is the equivalent of fifty hundredths of a dollar, or $**.**50. So to record 50 cents we can write it as 50¢ or $**.**50. We would write 25 cents as either 25¢ or $**.**25, 10 cents as 10¢ or $**.**10, 5 cents as 5¢ or $**.**05, and 1 cent as 1¢ or $**.**01 (one-hundredth of a dollar).

Measurement Conversion

6. Your child should understand the relationship between units of measure. For example, 1 foot is equivalent to 12 inches; 1 yd is equivalent to 3 feet or 36 inches; 1 gallon is equivalent to 4 quarts; 1 quart is equivalent to 2 pints; 1 pint is equivalent to 2 cups; 1 quart is equivalent to 4 cups; etc.

7. Your child should be able to use the relationship between units of measure to convert measures. For example, when given a measure in gallons, your child can also write it in pints, quarts, cups, etc.

8. Your child should be able to use addition, subtraction, multiplication, and division to solve word problems involving distances, liquid volumes, and masses of objects. This includes problems involving simple fractions and decimals and giving answers in a smaller unit of measure when given a larger unit (e.g., giving the answer in inches when the problem is given in feet).

Area & Perimeter

9. Your child should understand the difference between area (number of square units in a shape) and perimeter (number of linear units to “outline” the shape).

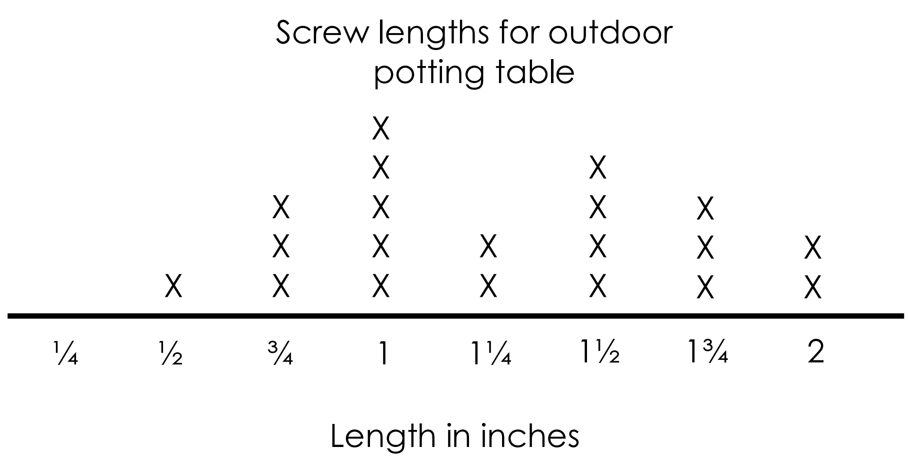
10. Your child should solve word problems involving area and perimeter.

Represent and Interpret Data

11. Your child should be able to draw a bar graph with a scale of more than 1. For example, your child should be able to draw a bar graph in which each square represents 5 people.

12. Your child should be able to solve one- and two-step word problems about a scaled bar graph or line plot.

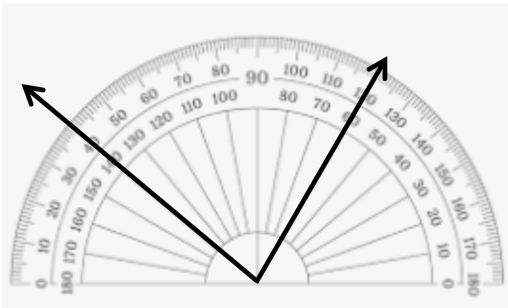
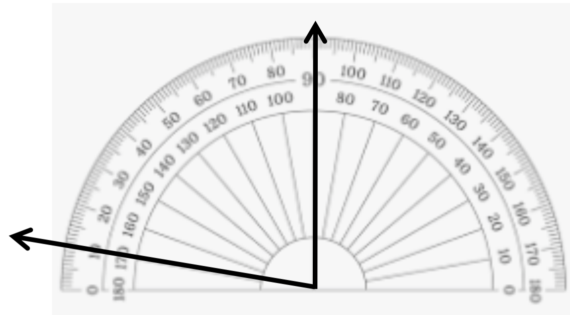
13. Your child should be able to measure a collection of items to the nearest quarter of an inch and create a line plot to show the data. For example,



Angle Measure

14. Your child should understand and describe the difference between a right angle, an acute angle, and an obtuse angle. Your child should be able to use simple “tools” such as the corner of a sheet of paper to identify right angles.

15. Your child should understand and use a protractor to measure angles. He or she should understand that the angle does not need to be aligned with 0 to measure but does have a specific location for the vertex of the angle.

16. Your child should understand that a circle is 360 degrees. He or she should connect this concept to the protractor and angle measure.

17. Your child should be able to solve simple problems involving adding angle measures or finding the missing angle.

Geometry

1. Your child should understand that shapes can be sorted in many different ways and be able to sort shapes into a group with a given attribute. For example, we can sort shapes by the number of sides, types of lines (parallel or perpendicular), types of angles (acute, right, obtuse).

2. Your child should be able to draw points, lines, line segments, rays, and angles.

3. In grade 4 your child is expected to identify symmetrical shapes and name lines of symmetry. A line of symmetry is the line on which we can "fold" the shape and each half of the shape will overlap exactly. This type of symmetry is called bilateral symmetry.