# Parent Information
# Grade 5

# Subtraction Strategies and Drawings

Our goals for your child are for her or him to be:

- playful with numbers,
- flexible in the way she or he adds or subtracts,
- efficient and accurate when she or he adds or subtracts.

Many of us learned a single strategy for adding numbers and a related strategy for subtracting numbers. We often heard the words "carry" and "borrow". These strategies always work but they aren't always the most efficient. Your child learns the regrouping procedure as another way to add and subtract in grade 4. However, he or she is also expected to be able to use the variety of strategies that were taught in Grades 2 and 3 and apply them to work with fractions and mixed numbers.

Please see the Grade 2 or Grade 3 Home-based Math Activities document for descriptions of whole number subtraction strategies. Those strategies can extend to multi-digit subtraction problems.

In grade 4, we examined mixed number subtraction for problems with like denominators. We began with strategies using the fraction kit manipulatives. If your child struggles with fractions, it is important for him or her to first represent the problem with the actual materials, draw a picture to match, then write the related equations. In grade 5 we extend these strategies to solve mixed number subtraction problems with unlike denominators.

## Subtraction as Removal

### Round and Adjust

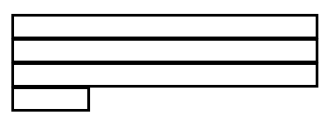
This strategy is one of the most efficient strategies for solving problems such as $3\frac{1}{4} - 1\frac{5}{6}$.

Example 1: $3\frac{1}{4} - 1\frac{5}{6}$

Begin with showing $3\frac{1}{4}$ using fraction kit materials (with materials and as a drawing).

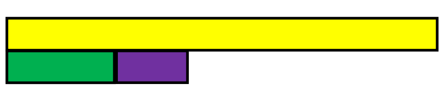
Materials

Drawings

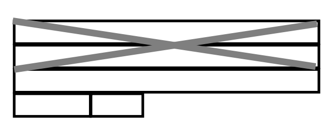


To remove $1\frac{5}{6}$, we remove 2 and then give back $\frac{1}{6}$. We rounded $1\frac{5}{6}$ to 2 and then needed to adjust (give $\frac{1}{6}$ back) because we took away one-sixth too many. The illustrations below show the changes in our materials and drawing after using this strategy.

Materials



Drawings



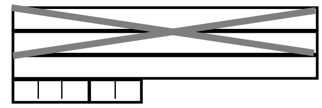
I removed 2 wholes and gave back $\frac{1}{6}$. (Note: It doesn't matter which of the wholes are removed or crossed out.)

$$3\frac{1}{4} - 1\frac{5}{6} = 3\frac{1}{4} - 2 + \frac{1}{6}$$

Your child then needs to determine the size of the unit fraction piece that fits evenly into both fourths and sixths. In this case it is twelfths.

Materials

Drawings



$$3\frac{1}{4} - 1\frac{5}{6} = 3\frac{1}{4} - 2 + \frac{1}{6} = 1\frac{5}{12}$$

**Decompose a Whole**

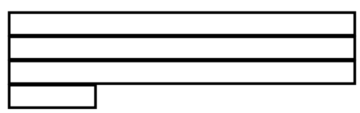
This strategy uses 2 different understandings.

- The first is that we can represent mixed numbers as parts and wholes. For example, 3¼ is 3 and ¼. When written as 3¼ = 3 + ¼, it is called expanded form of a number. Grade 4 students are expected to be able to write numbers in expanded form.
- The second understanding is that we can split a whole into fractional parts in many different ways.

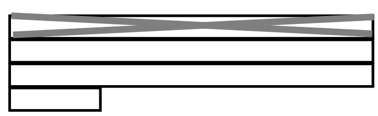
Let's use this strategy for $3\frac{1}{4} - 1\frac{5}{6}$ .

Example 1: $3\frac{1}{4} - 1\frac{5}{6}$

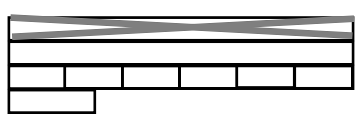
Begin with drawing 3¼.



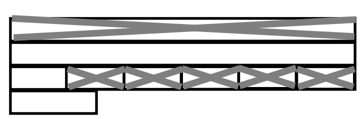
We think of $1\frac{5}{6}$ as 1 and $\frac{5}{6}$. First remove 1.



We don't have $\frac{5}{6}$ to remove so we "cut" a whole into sixths (Note: It doesn't matter which whole we cut into sixths).



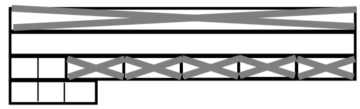
We "remove" 5, one-sixth pieces and we are left with the answer (Note: It doesn't matter which of the one-sixth pieces we remove).



$$3\frac{1}{4} - 1\frac{5}{6} = 3\frac{1}{4} - 1 - \frac{5}{6}$$

When we removed the whole and $\frac{5}{6}$ from a second whole we are left with $1\frac{1}{4} + \frac{1}{6}$ .

Your child then needs to determine the size of the unit fraction piece that fits evenly into both fourths and sixths. In this case it is twelfths.



$$3\tfrac{1}{4} - 1\tfrac{5}{6} = 3\tfrac{1}{4} - 1 - \tfrac{5}{6} = 1\tfrac{1}{4} + \tfrac{1}{6}$$
$$= 1\tfrac{3}{12} + \tfrac{2}{12}$$
$$= 1\tfrac{5}{12}$$

## Subtraction as Comparison (How Many More/Fewer)—Missing Addend

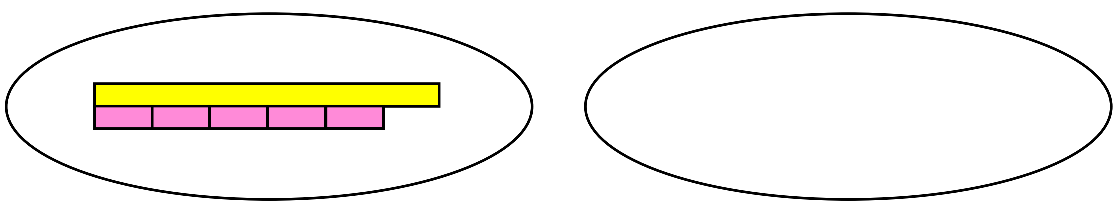
This strategy is very important for solving fraction subtraction problems. Using this strategy we think of problems such as $3\frac{1}{4} - 1\frac{5}{6}$ as how many more than $1\frac{5}{6}$ is $3\frac{1}{4}$ or how many do we need to add to $1\frac{5}{6}$ to get to $3\frac{1}{4}$.

$$1\tfrac{5}{6} + \_\_\_ = 3\tfrac{1}{4}$$

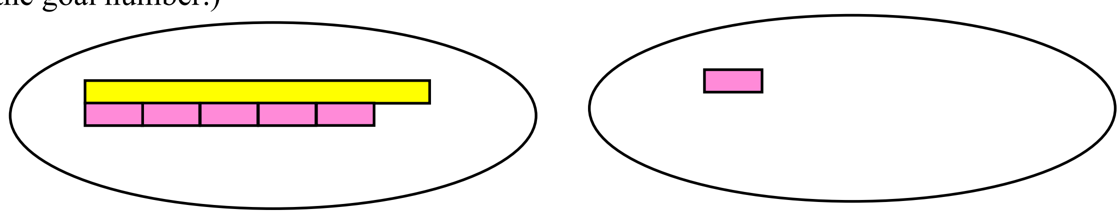
Part 1 shows a missing part strategy for finding the answer using our fraction strip model. Part 2 shows a counting on strategy using a number line.

### Think of Subtraction as How Many More—Missing Addend—Part 1-Finding the Missing Part

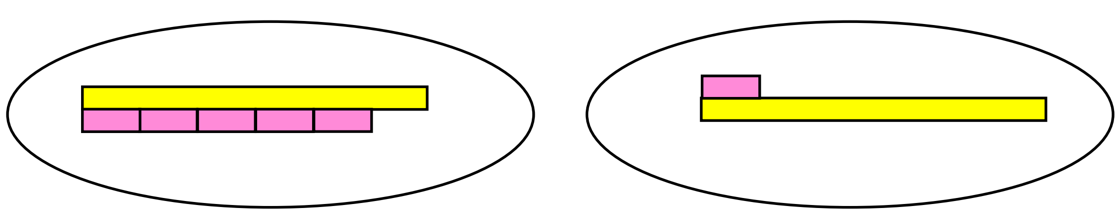
To solve $1\frac{5}{6} + \_\_\_ = 3\frac{1}{4}$ (or $3\frac{1}{4} - 1\frac{5}{6}$) using a missing part strategy, use 2 plates or sheets of paper and the fraction kit pieces. Begin with representing the part you know ($1\frac{5}{6}$) on one section (this can be a plate or sheet of paper). We will put the pieces for the missing part on the other section.



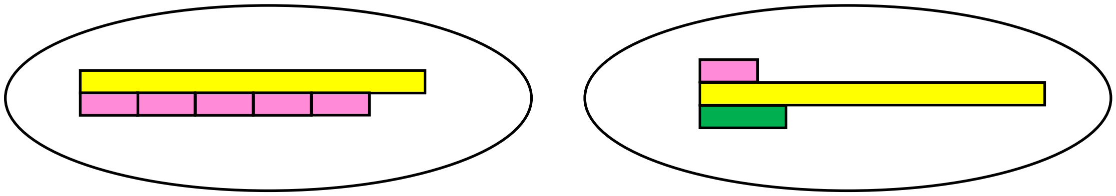
For our problem we add sixths, fourths and 1s until we reach the total ($3\frac{1}{4}$). For example, if we add $\frac{1}{6}$, we have a total of 2. (**Note:** We don't have to begin by adding sixths. We could add wholes first until we get close to the goal number.)



We add a whole and we have a total of 3.



We then add a ¼ and we have a total of 3¼.



The missing part is $1\frac{5}{12}$.

$$1\tfrac{5}{6} + \boxed{\tfrac{1}{6} + 1 + \tfrac{1}{4}} = 3\tfrac{1}{4}$$

$$\tfrac{1}{6} + 1 + \tfrac{1}{4} = \tfrac{2}{12} + 1 + \tfrac{3}{12}$$

$$1\tfrac{5}{6} + \boxed{1\tfrac{5}{12}} = 3\tfrac{1}{4} \quad \text{so} \quad 3\tfrac{1}{4} - 1\tfrac{5}{6} = \boxed{1\tfrac{5}{12}}$$

## <u>**Think of Subtraction as Missing Addend—Part 2—Counting Up on a Number Line**</u>

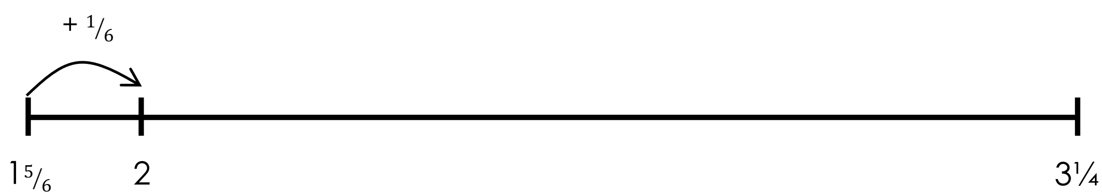
To solve $1\frac{5}{6}$ + ___ = 3¼ (or 3¼ – $1\frac{5}{6}$) using a counting on strategy on a number line, we begin with an open number line with the start number on the left and the goal number on the right.


The goal is to find the distance between $1\frac{5}{6}$ and 3¼ by making jumps.

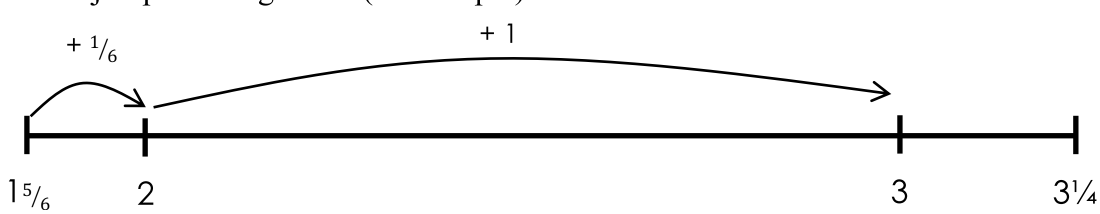
**Note:** There are many different ways to "jump" from $1\frac{5}{6}$ to 3¼.

<u>Example 1:</u> $1\frac{5}{6}$ + _____ = 3¼

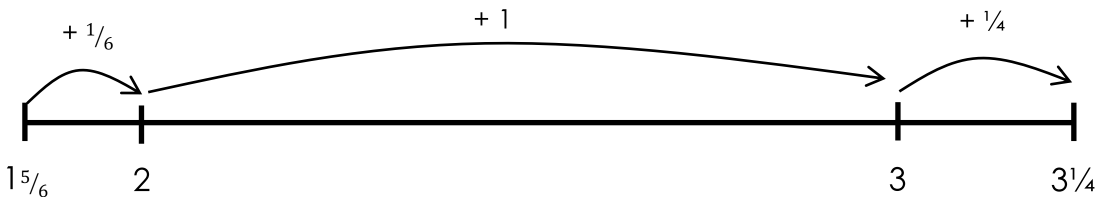
We can make a jump of $\frac{1}{6}$ to get to 2 (Count up $\frac{1}{6}$).




We can make a jump of 1 to get to 3 (Count up 1).



We can make a jump of $\frac{1}{4}$ to get to $3\frac{1}{4}$ (Count up $\frac{1}{4}$).



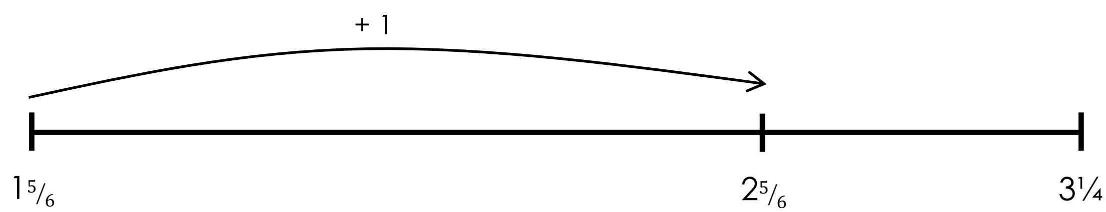

We combine the jumps to get the missing addend.

$$1\frac{5}{6} + \boxed{\frac{1}{6} + 1 + \frac{1}{4}} = 3\frac{1}{4}$$

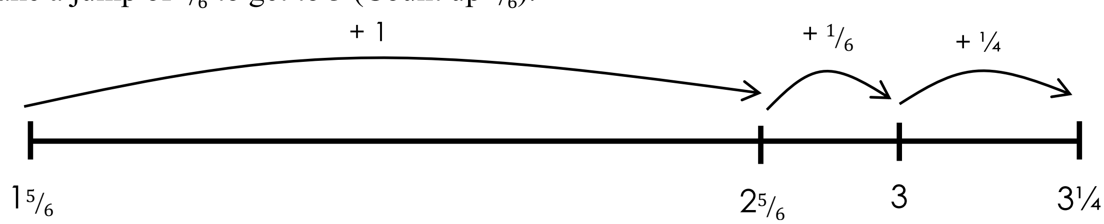
$$1\frac{5}{6} + \boxed{1\frac{5}{12}} = 3\frac{1}{4} \quad \text{so} \quad 3\frac{1}{4} - 1\frac{5}{6} = \boxed{1\frac{5}{12}}$$

<u>Example 2:</u> $1\frac{5}{6}$ + _____ = $3\frac{1}{4}$

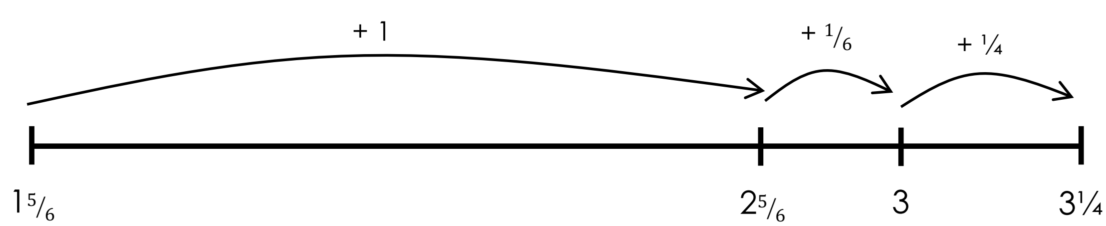
We can make a jump of 1 to get to $2\frac{5}{6}$ (Count up 1).




We can make a jump of $\frac{1}{6}$ to get to 3 (Count up $\frac{1}{6}$).




We can make a jump of $\frac{1}{4}$ to get to $3\frac{1}{4}$ (Count up $\frac{1}{4}$).




We combine the jumps to get the missing addend.

$$1\frac{5}{6} + \mathbf{1 + \frac{1}{6} + \frac{1}{4}} = 3\frac{1}{4}$$

$$1\frac{5}{6} + \boxed{1\frac{5}{12}} = 3\frac{1}{4} \quad \text{so} \quad 3\frac{1}{4} - 1\frac{5}{6} = \boxed{1\frac{5}{12}}$$

# Multiplication & Division
# Models, Strategies, and Drawings

## Multiplication and Division

For many of us, learning multiplication involved memorizing the multiplication facts. Although fact fluency is important, your child is expected to understand all of the ways that we represent multiplication: equal groups, equal rows (array), equal jumps, and area. These models were introduced in grade 3 and are the models your child will see in multiplication story problems. Therefore, we want to make sure that he or she has strong visual images and the language needed to describe multiplication. Language includes: equal groups of, equal rows of, equal jumps of, or “by” (when working with area). For a problem such as 4 x 9, we could read as:

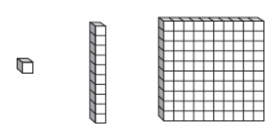
- 4 groups of 9,
- 4 rows of 9,
- 4 jumps of 9, or
- 4 by 9.

In grade 5, we expect your child to be fluent with each of these models and related language as well as multiplicative comparison that was introduced in grade 4. Multiplicative comparison involves the language, “times as many.” For example, Connor has 4 times as many football cards as his younger brother. This builds a foundation for later work with ratios.

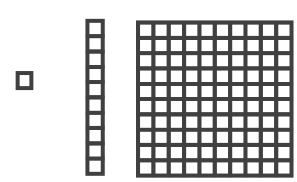
In grade 5, we extend these models from work with 1-digit by up to 4-digits and 2-digit by 2-digit to multi-digit multiplication and multiplying fractions. Many of us learned a single multiplication procedure to use when multiplying. Your child will also learn this procedure in grade 5. However, understanding and using the models mentioned above will move your child from only knowing an arithmetic approach to getting an answer to using algebraic approaches for computing. These approaches will help your child be more successful when multiplying fractions. When your child learns the traditional algorithm, the expectation is that he or she can still use all of the other models that represent algebraic properties.

### Base 10 Materials

**Note:** In grades 2-5 we use Base 10 materials as models for 1s, 10s, and 100s. Your child can easily make a set for home use by cutting 10 by 10, 1 by 10, and 1 by 1 pieces from centimeter grid paper. A sheet with and without suggested cutting lines is included. For Grade 5 cut materials from at least 2 pages. For a greater challenge have your child cut the pieces from an unmarked copy without looking at the marked version. Have an extra copy or 2 available in case your child makes a mistake or if additional pieces are needed for later activities.



The centimeter grid paper (Base 10) materials will look like those to the right.



**Extending the Equal Grouping Model**

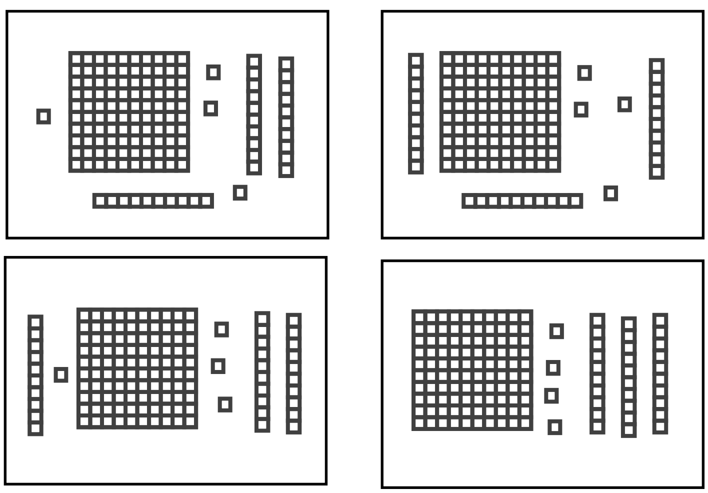
In grade 4 we used Base 10 materials to help your child visualize the Distributive Property. At first your child modeled multiplication problems with base 10 materials. He or she then drew a picture to match the model.

To simplify the drawings, we had your child draw a □ for 100, a | for 10, and a • for 1.

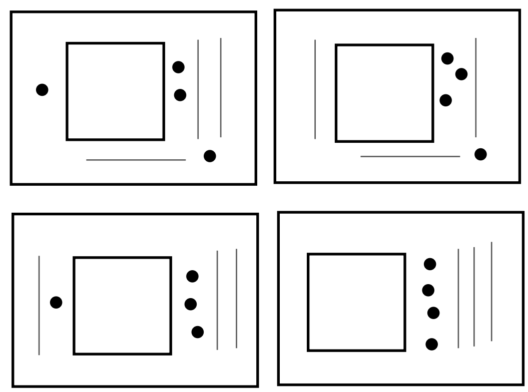
Example 1: 4 x 134

- We first translate this expression into "4 groups of 134." This language helps your child understand what they are to build.
- We have your child build 4 groups of 134 with the materials.
- We then have him or her draw a picture to match.

Materials



Drawing



**Note:** It is important for your child to understand that the materials do not have to be placed in a particular order as long as they have 1 hundred, 3 tens, and 4 ones.

- The important question is, "What did we use to represent (build) 4 groups of 134?"
- We used 4 groups of 100 (4 x 100), 4 groups of 30 (4 x 30), and 4 groups of 4 (4 x 4). This is the Distributive Property.
  To make 134 with the Base 10 materials we thought of it as 100 + 30 + 4 (expanded form).
  So, 4 x 134 = 4 x (100 + 30 + 4) = (4 x 100) + (4 x 30) + (4 x 4)
  4 groups of 134 is the same as 4 groups of 100 put together with 4 groups of 30 put together with 4 groups of 4.
  **Note:** In (4 x 100) + (4 x 30) + (4 x 4) parentheses are not required but are helpful. For this problem your child would be correct if he or she wrote, 4 x 100 + 4 x 30 + 4 x 4.
- You may hear this being referred to as using place value, thinking of 134 as 100 + 30 + 4.

- You may also hear the term “partial products.”
  If we think of 4 x 134 as 4 x 100 + 4 x 30 + 4 x 4, then 4 x 134 = 400 + 120 + 16. 400, 120, and 16 would be the partial products.  You may see this written vertically as,

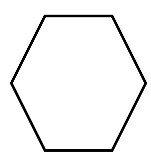
| | | |
|---|---|---|
| 134 | | 134 |
| x 4 | | x 4 |
| 400 | OR | 16 |
| 120 | | 120 |
| +16 | | +400 |
| 536 | | 536 |

  When we use place value or the Distributive Property the order of each partial product will not affect the answer (Commutative Property). When your child is taught the arithmetic procedure in 5th grade that we learned as children, we have them examine a way to collapse the 3 steps (3 partial products).

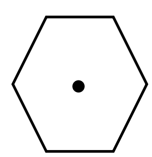
  We show the partial product approach to finding the product and the traditional procedure and have your child identify the connections.

| | |
|---|---|
| 134 | 1 1 |
| x 4 | 1 3 4 |
| 16 | x 4 |
| 120 | 5 3 6 |
| +400 | |
| 536 | |

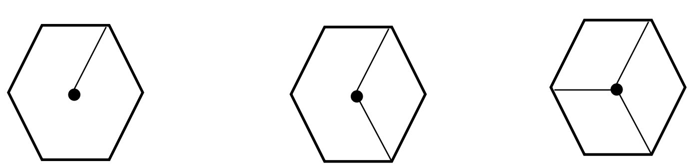
- This approach was extended to a 1-digit times a 4-digit number. Your child learned to draw a cube to represent the 1000 block (With Base 10 materials, ten 100 “flats” are combined to make a 1000). There are many ways to draw a cube. One way is to have your child first draw a hexagon with equal sides.



  Have her or him put a point where the center is.



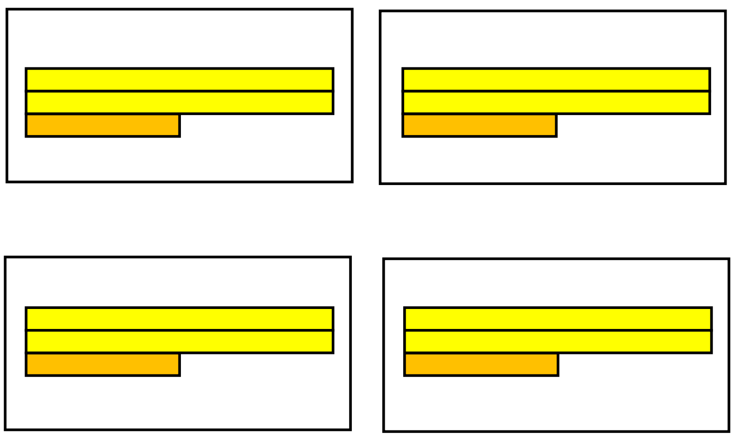
  Your child will then draw a “Y” in the hexagon by drawing a line segment from the point to one vertex (corner). Draw 2 more line segments as shown, skipping a vertex (corner) each time.



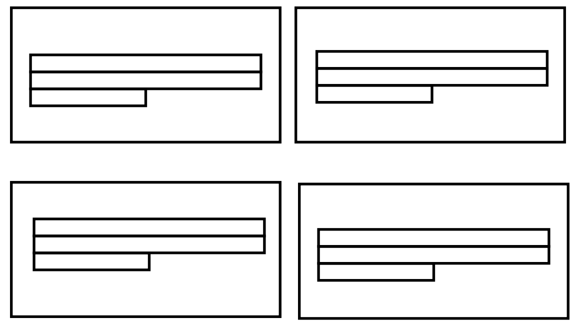
Example 2: In grade 5 it is important for your child to understand and use the Distributive Property for a whole number times a mixed number, 4 x 2½. The equal grouping model for multiplication can be used to build this understanding.

- We first translate this expression into "4 groups of 2½." This language helps your child understand what they are to build.
- We have your child build 4 groups of 2½ with the fraction kit materials. (Your child may need to make extra sets to have enough pieces.)
- We then have him or her draw a picture to match.

Materials



Drawing



- The important question is, "What did we use to represent (build) 4 groups of 2½?"
- We used 4 groups of 2 (4 x 2) and 4 groups of ½ (4 x ½). This is the Distributive Property. To make 2½ with the fraction kit materials we thought of it as 2 + ½ (expanded form).

  $$\text{So, } 4 \times 2\tfrac{1}{2} = 4 \times (2 + \tfrac{1}{2}) = (4 \times 2) + (4 \times \tfrac{1}{2})$$

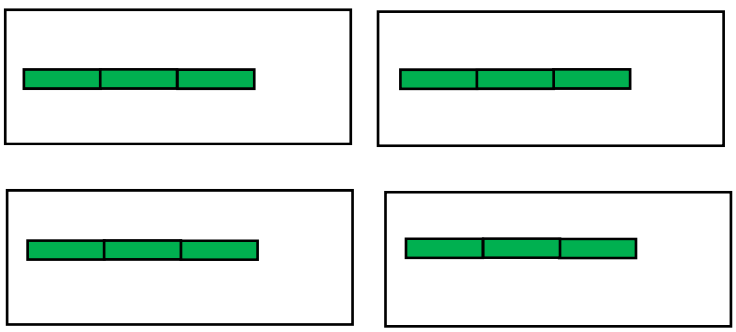
  4 groups of 2½ is the same as 4 groups of 2 put together with 4 groups of ½.
  **Note:** In (4 x 2) + (4 x ½) parentheses are not required but are helpful. For this problem your child would be correct if he or she wrote, 4 x 2 + 4 x ½.

$$\begin{aligned} 4 \times 2\tfrac{1}{2} = 4 \times (2 + \tfrac{1}{2}) &= (4 \times 2) + (4 \times \tfrac{1}{2}) \\ &= 8 + 2 \\ &= 10 \end{aligned}$$

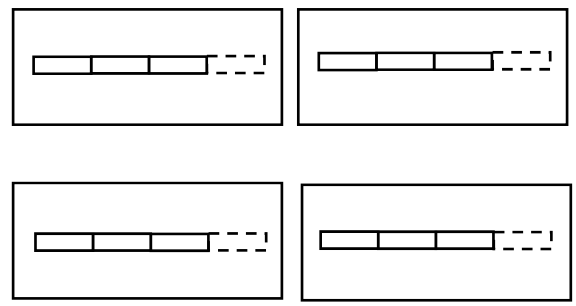
Example 3: Algebraic properties (Associative Property) can also be used for a whole number times a fraction less than 1, 4 x ¾.

- We first translate this expression into "4 groups of ¾." This language helps your child understand what they are to build.
- We have your child build 4 groups of ¾ with the fraction kit materials.
- We then have him or her draw a picture to match. Remember when drawing pictures for fractions less than 1 we used a dashed line to represent the whole and then outline the pieces that make up the fraction (See below).

Materials



Drawing

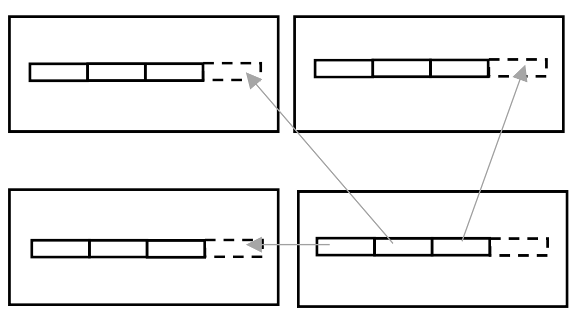


- When we ask, "What did you need to make one of the groups?" we hope your child understands that he or she needed 3, one-fourth pieces for each group (3 x ¼). **Note:** It is important for your child to read ¾ as three-fourths and as 3, one-fourth pieces. The second reading is useful in solving this type of problem.
- So, 4 groups of 3, one-fourth pieces is the same as 4 x (3 x ¼). Instead of associating the 3 with the ¼ we can associate it with the 4,

  (4 x 3) x ¼. 4 x ¾ = (4 x 3) x ¼ = 12 x ¼

  Twelve ¼ pieces can be used to make 3 wholes. These understandings are the foundation for many of the rules we learned later in school.
- Students can also find this answer by noticing that if they take the 4$^{th}$ group of ¾, they can use each of the ¼ pieces to complete the whole for the other 3 groups.

4 x ¾ = 3

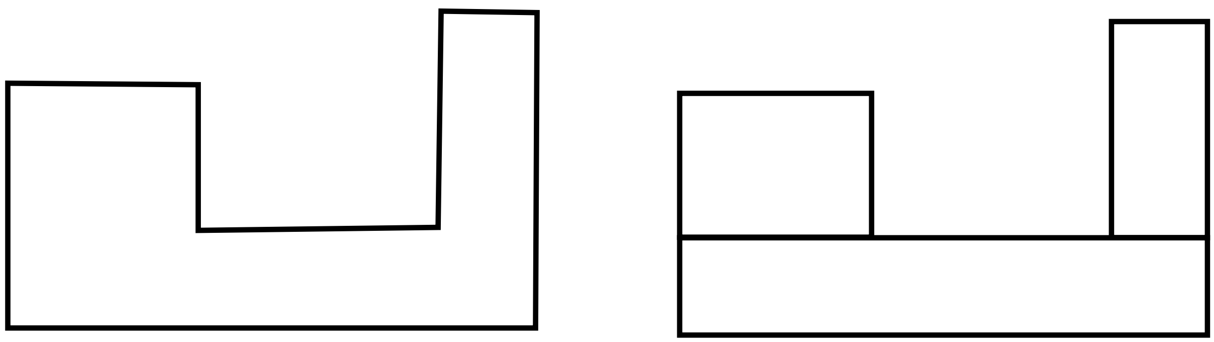


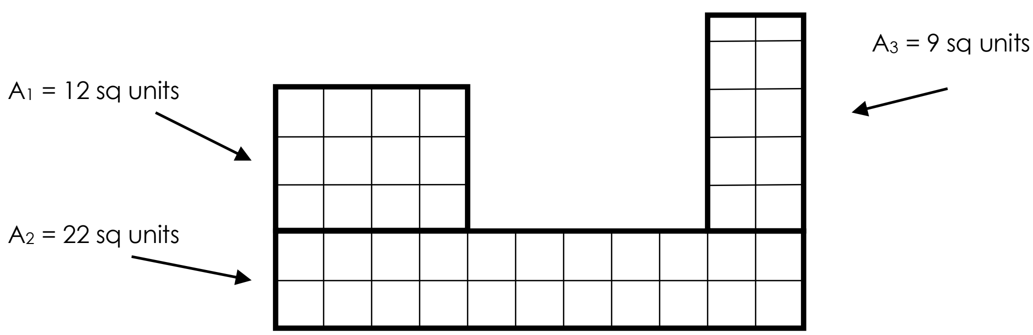
In grade 5 it is important for your child to understand the effects of multiplying by a number less than one versus multiplying by a fraction greater than one.

**Note:** Some children believe that when you multiply you get a larger number. That is not always the case. When you multiply by a number less than one you actually get a "smaller" number.

## Multiplication Using the Area Model

Area models are another powerful visual for developing the Distributive Property and provide the foundation for later work your child will do with algebra tiles and other materials in middle school algebra. In grade 3, your child is introduced to area as a covering in square units. Your child learned that we can divide shapes into smaller pieces, find the area of each of those pieces, and then add them together to get the total area of a shape. This is particularly useful when finding the area of shapes such as the one shown below. This shape can be cut into rectangles as shown on the right. We can then find the area of each rectangle, combine those areas, and obtain the area of the original shape.





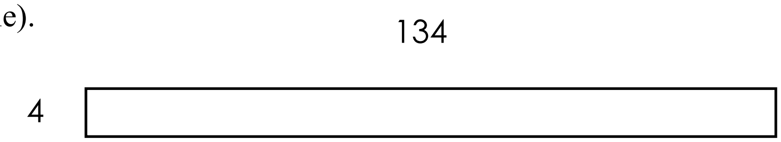

Combining the areas, $A_1$, $A_2$, and $A_3$

$$A_1 + A_2 + A_3 = 12 + 22 + 9 \text{ sq units} = 43 \text{ sq units}$$

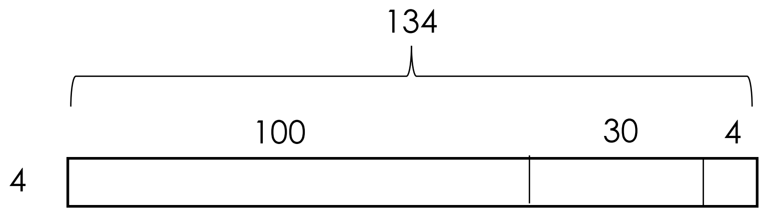
We use this same concept to represent 1-digit by up to 4-digit multiplication or 2-digit by 2-digit multiplication.

Example 1: 4 x 134

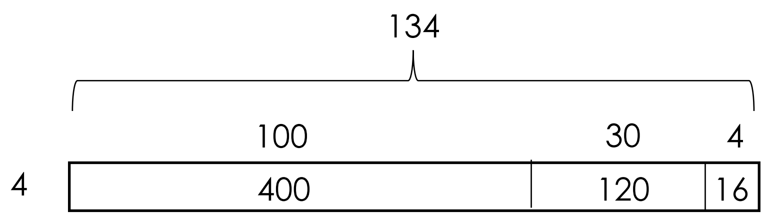
- Since we are using an area model we translate this expression into "4 by 134" (the dimensions of the rectangle).




- To make it easier to find the area of the rectangle we can cut it into smaller sections. An efficient way to cut the rectangle is to think of 134 in expanded form (use place value). We will think of 134 as 100 + 30 + 4. We'll then have 3 rectangles:
    - one with the dimensions 4 by 100,
    - one with the dimensions 4 by 30, and
    - one with the dimensions 4 by 4.



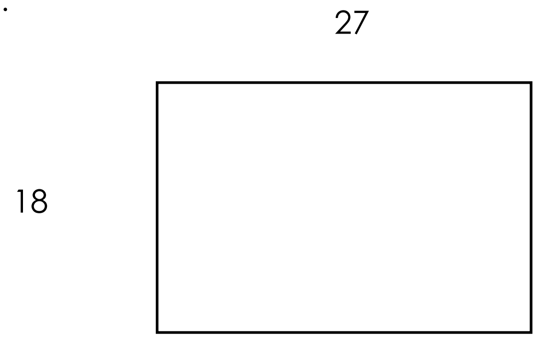

We are thinking of 4 x 134 as 4 x (100 + 30 + 4)
or (4 x 100) + (4 x 30) + (4 x 4).
This is the Distributive Property.




4 x 134 = 4 x (100 + 30 + 4)
= (4 x 100) + (4 x 30) + (4 x 4)
= 400 + 120 + 16
= 536

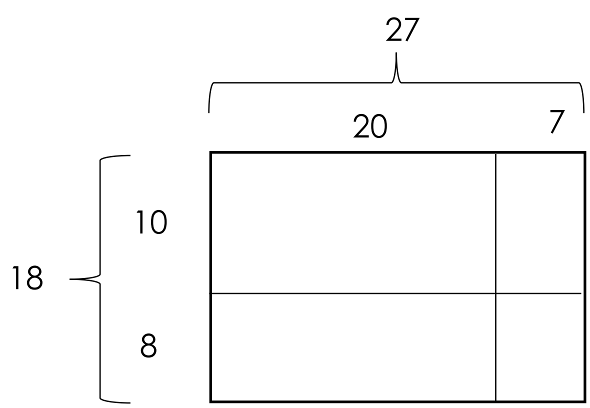
Example 2: 18 x 27

- Since we are using an area model we translate this expression into "18 by 27" (the dimensions of the rectangle).

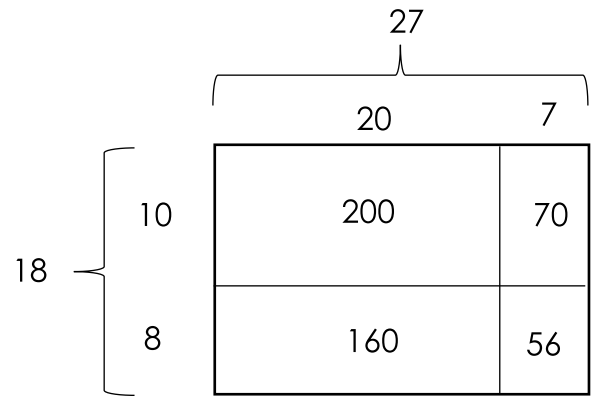



- To make it easier to find the area of the rectangle we can cut it into smaller sections. An efficient way to cut the rectangle is to think of 18 and 27 in expanded form (use place value). We will think of 18 as 10 + 8 and 27 as 20 + 7.

  Notice how we "cut" the rectangle between the 10 and the 8 and between the 20 and 7. We now have 4 rectangles:
  - one with the dimensions 10 by 20,
  - one with the dimensions 10 by 7,
  - one with the dimensions 8 by 20, and
  - one with the dimensions 8 by 7.




We are thinking of 18x 27 as (10 + 8) x (20 + 7)
or (10 x 20) + (10 x 7) + (8 x 20) + (8 x 7)
This is the Distributive Property.



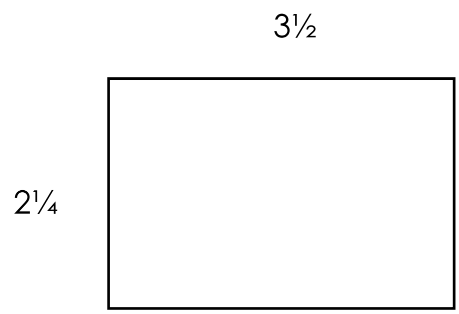

18x 27 = (10 + 8) x (20 + 7)
= (10 x 20) + (10 x 7) + (8 x 20) + (8 x 7)
= 200 + 70 + 160 + 56
= 270 + 216
= 486

Again, the partial products are the areas that are shown within each rectangle, 200, 70, 160, and 56.

In algebra we will call this binomial multiplication. In 4th grade we prepared your child for this later work in algebra by learning this area model representation for multiplication. In 5th grade we ensure that your child is fluent with this strategy and extend its use to mixed number and decimal multiplication.

<u>Example 3:</u> Because we are using the Distributive Property to multiply, we can also use this to multiply mixed numbers. For example, 2¼ x 3½.

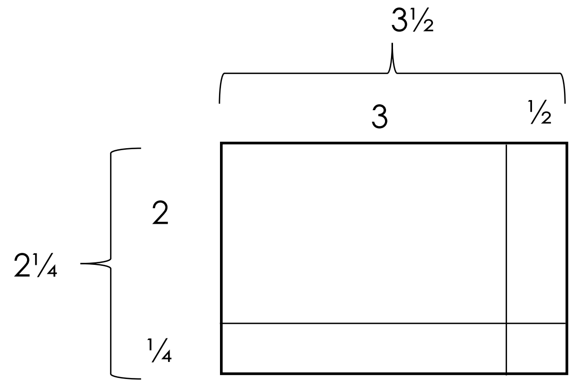
- Since we are using an area model we translate this expression into “2¼ by 3½” (the dimensions of the rectangle).



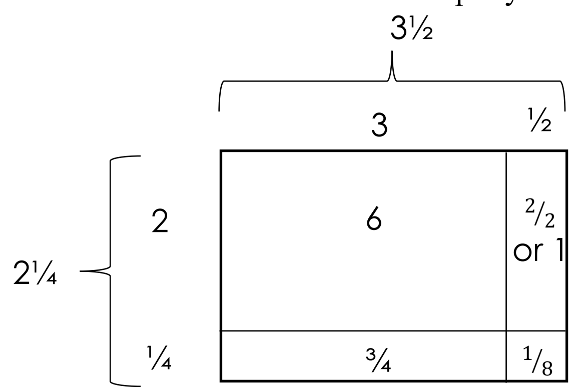

- To make it easier to find the area of the rectangle we can cut it into smaller sections. We know mixed numbers have a whole number piece and a fraction piece. For 2¼ the whole number piece is 2 and the fraction piece is ¼. We can think of 2¼ as 2 + ¼ and 3½ and 3 + ½.

Notice how we "cut" the rectangle between the 2 and the ¼ and between the 3 and ½. We now have 4 rectangles:

- one with the dimensions 2 by 3,
- one with the dimensions 2 by ½,
- one with the dimensions ¼ by 3, and
- one with the dimensions ¼ by ½.



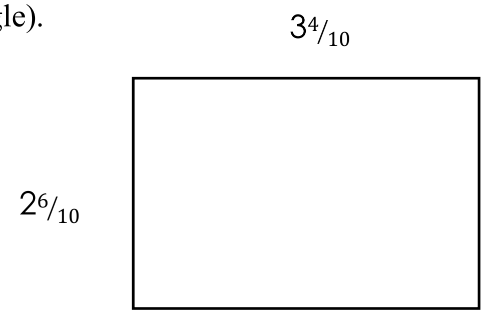

We are thinking of 2¼ x 3½ as (2 + ¼) x (3 + ½
or (2 x 3) + (2 x ½) + (¼ x 3) + (¼ x ½)
This is the Distributive Property.




2¼ x 3½ = (2 + ¼) x (3 + ½)
= (2 x 3) + (2 x ½) + (¼ x 3) + (¼ x ½)
= 6 + 1 + ¾ + $^1/_8$
= 7 + $^7/_8$
= $7^7/_8$

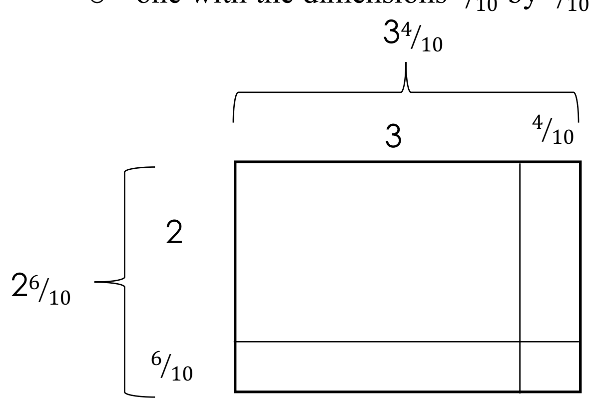
Example 4: Because we are using the Distributive Property to multiply, we can also use this to multiply decimal numbers. For example, $2^6/_{10}$ x $3^4/_{10}$ or 2.6 x 3.4.

- Since we are using an area model we translate this expression into "$2^6/_{10}$ by $3^4/_{10}$" (the dimensions of the rectangle).

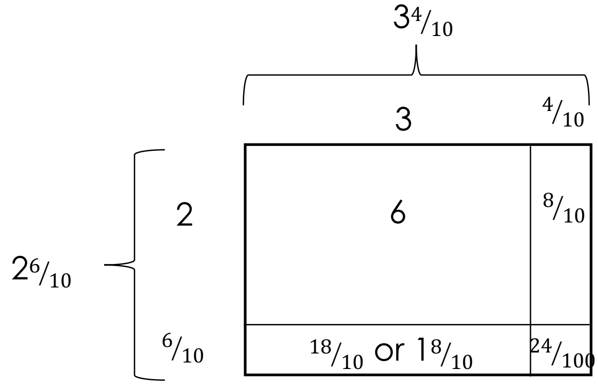



- To make it easier to find the area of the rectangle we can cut it into smaller sections. We know mixed numbers have a whole number piece and a fraction piece. For $2^6/_{10}$ the whole number piece is 2 and the fraction piece is $^6/_{10}$ (or 2 and .6). We can think of $2^6/_{10}$ as 2 + $^6/_{10}$ (or 2 + .6) and $3^4/_{10}$ and 3 + $^4/_{10}$ (3 + .4).

  Notice how we "cut" the rectangle between the 2 and the $^6/_{10}$ and between the 3 and $^4/_{10}$. We now have 4 rectangles:
  - one with the dimensions 2 by 3,
  - one with the dimensions 2 by $^4/_{10}$,
  - one with the dimensions $^6/_{10}$ by 3, and
  - one with the dimensions $^6/_{10}$ by $^4/_{10}$.

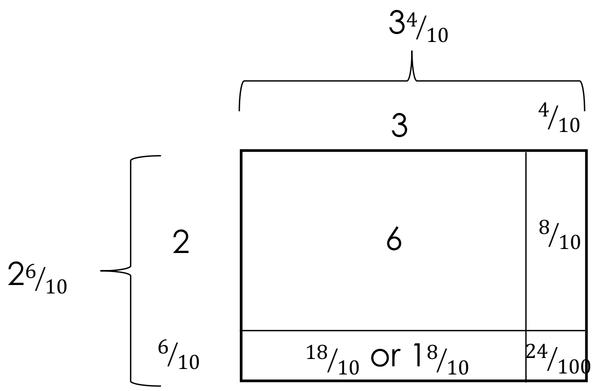


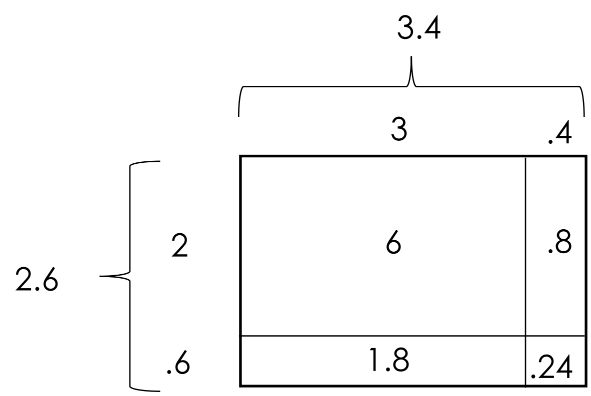

We are thinking of $2^6/_{10}$ x $3^4/_{10}$ as (2 + $^6/_{10}$) x (3 + $^4/_{10}$)
or (2 x 3) + (2 x $^4/_{10}$) + ($^6/_{10}$ x 3) + ($^6/_{10}$ x $^4/_{10}$)
This is the Distributive Property.




$$
\begin{aligned}
2\tfrac{6}{10} \times 3\tfrac{4}{10} &= (2 + \tfrac{6}{10}) \times (3 + \tfrac{4}{10}) \\
&= (2 \times 3) + (2 \times \tfrac{4}{10}) + (\tfrac{6}{10} \times 3) + (\tfrac{6}{10} \times \tfrac{4}{10}) \\
&= 6 + \tfrac{8}{10} + 1\tfrac{8}{10} + \tfrac{24}{100} \\
&= 7 + \tfrac{16}{10} + \tfrac{24}{100} \\
&= 7 + 1\tfrac{6}{10} + \tfrac{24}{100} \\
&= 8\tfrac{84}{100}
\end{aligned}
$$

We could use this information to complete a similar area model for 2.6 x 3.4.



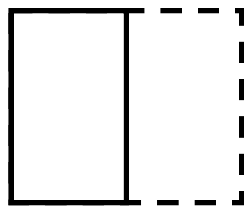




$$
\begin{aligned}
2.6 + 3.4 &= (2 + .6) \times (3 + .4) \\
&= (2 \times 3) + (2 \times .4) + (.6 \times 3) + (.6 \times .4) \\
&= 6 + .8 + 1.8 + .24 \\
&= 8.6 + .24 \\
&= 8.84
\end{aligned}
$$

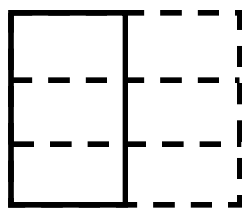
Example 5: In the above fraction multiplication problems, your child needed to multiply 2 fractions less than 1. We can use the area model to represent problems such as $^1/_3$ x $^1/_2$. We can read this as one-third by one-half or one-third of one-half.

- We begin with a rectangular whole represented with a dashed line.

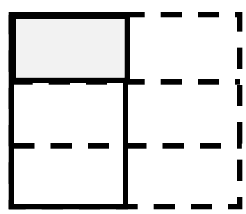
- We cut the whole into 2 equal parts and put a solid line around one-half.



- We now want to cut the halves into 3 equal pieces. Show with a dashed line extended across the whole.

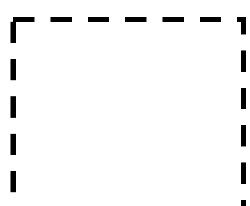


- Shade in a third of the half. We see that $^1/_3$ x $^1/_2$ is $^1/_6$ .

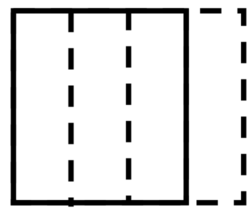


Example 6: $^2/_3$ x $^3/_4$.

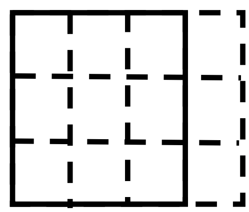
- We begin with a rectangular whole represented with a dashed line.



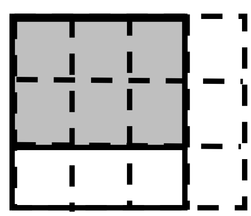
- We cut the whole into 4 equal parts and put a solid line around three-fourths.



- We now want to cut the fourths into 3 equal pieces so that we can find two-thirds of three-fourths. Show with a dashed line extended across the whole.



- Shade in a two-thirds of the three-fourths. We see that $^{2}/_{3}$ x $^{3}/_{4}$ is $^{6}/_{12}$ .

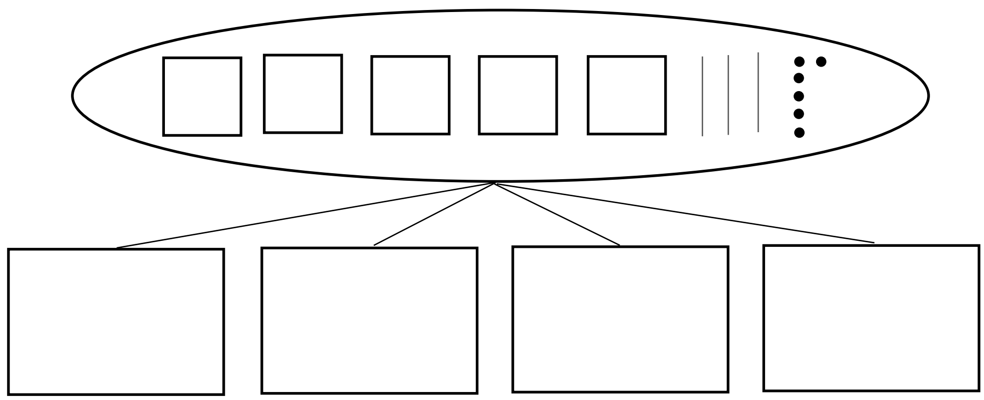


**<u>Division Using the Equal Grouping Model & Partial Quotients</u>**

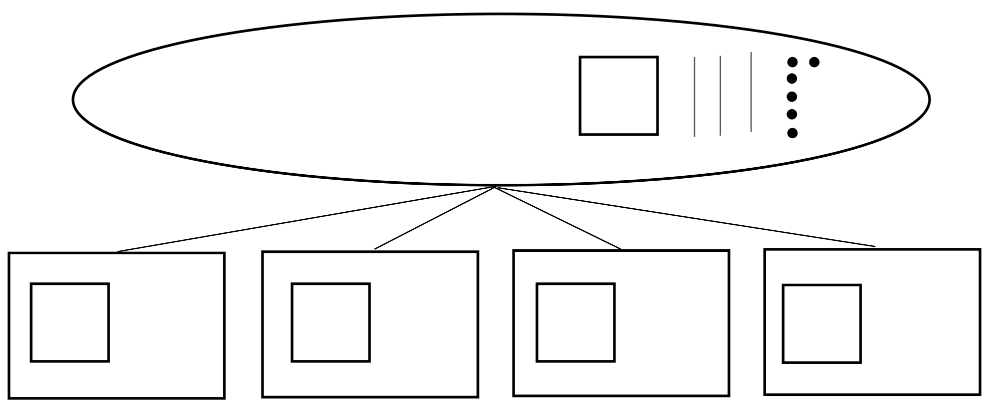
Division is the inverse (opposite) of multiplication. If multiplication is combining equal groups, division is separating into equal groups or pulling off equal groups of a given number. If multiplication is combining equal rows, then division is separating into equal rows or pulling off equal rows of a given number. In grade 3, your child was expected to understand the different representations and language for division. He or she was also expected to use his or her knowledge of multiplication to divide. That is, to find 24 ÷ 6 your child can think, "What times 6 is 24?" (missing factor). In grade 4 we extended these understandings to using partial quotients (similar to partial products) to divide. If using the area model in division, we know one side length and the area. Our goal is to find the missing side length.
Although many of us learned a single procedure (long division) for dividing which your child will learn in Grade 6, your child is expected to understand all of these different ways of thinking of division. These strategies are developed in grades 4 and 5. Many of these strategies are helpful in the work your child will do when dividing fractions. Examples of some of these strategies are shown below.

Example 1: 536 ÷ 4 using an equal grouping model

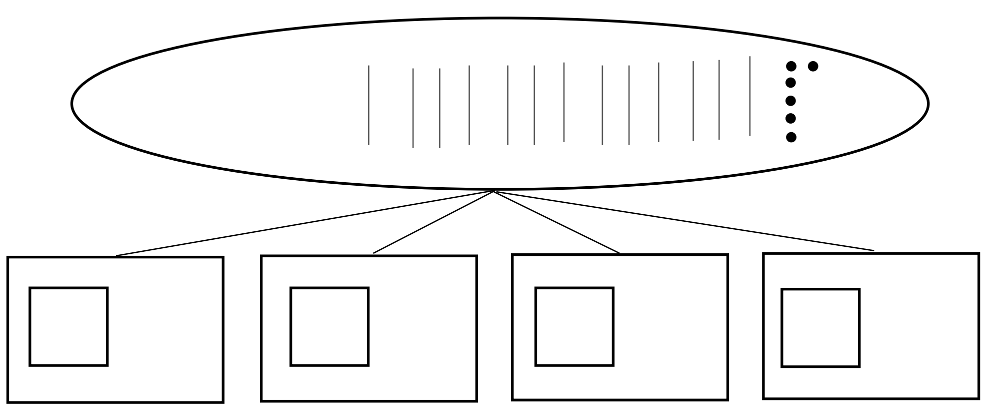
- Begin by translating the division problem into language that helps bring imagery. When dividing by a single digit number it is still efficient to think of division as separating into equal groups. We can translate 536 ÷ 4 as 536 separated into 4 equal groups.



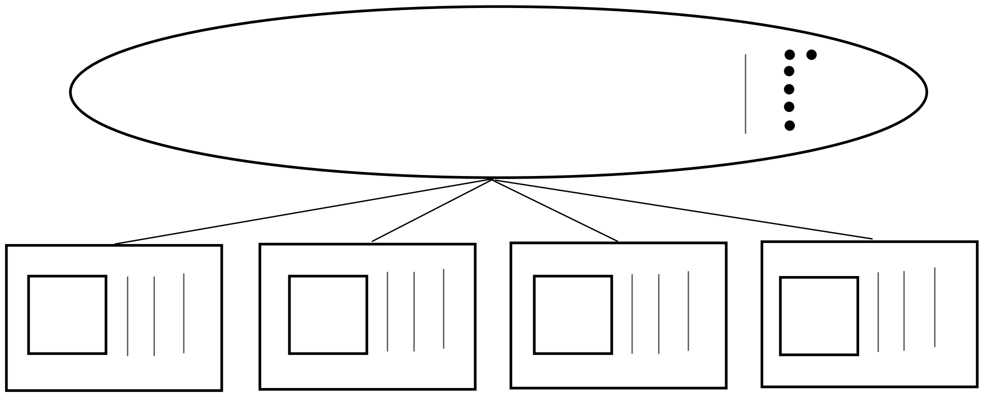
- We can first put 100 in each group.



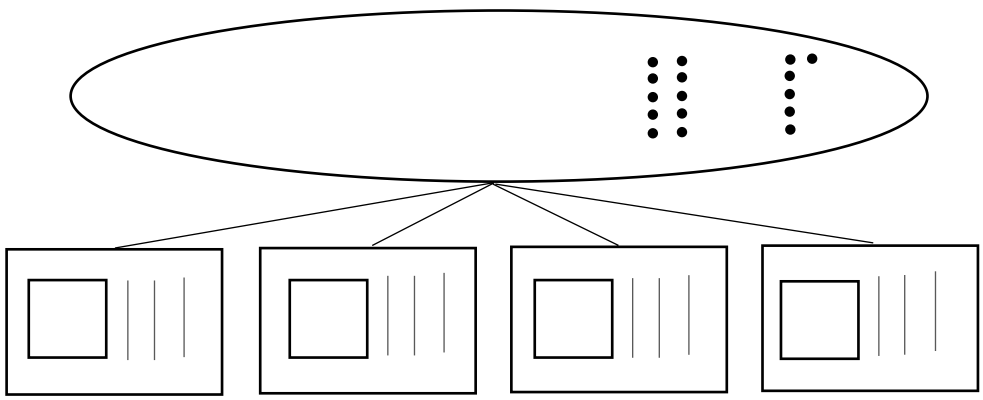
- We know that an equivalent way to think of 1 hundred and 3 tens is 13 tens.



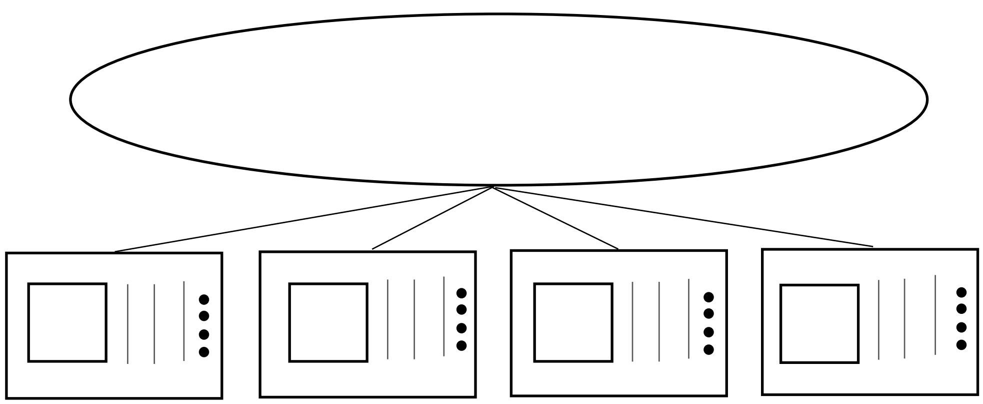
- We can separate the 13 tens into the 4 groups by placing 3 tens in each group.



- We know that an equivalent way of thinking of 1 ten and 6 ones is as 16 ones.



- We can separate the 16 ones into the 4 groups by placing 4 ones in each group.



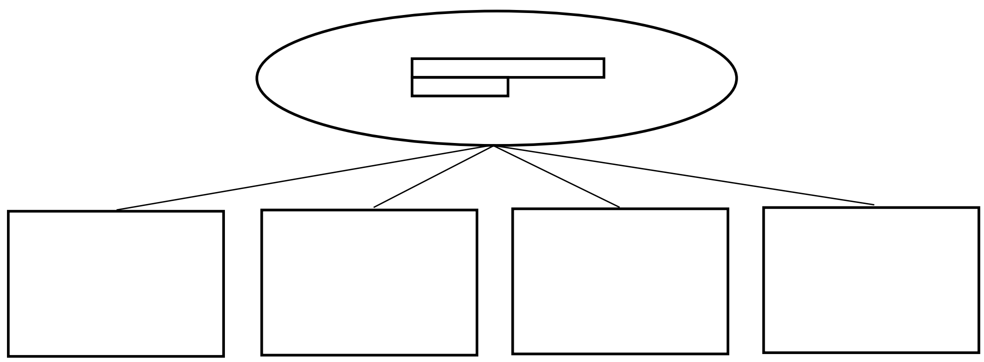
These are the actual steps that are taken when doing the long division procedure, but we begin with understanding how the quantities are being distributed.

- To separate 536 into 4 equal groups we actually thought of 536 as 400 + 120 + 16. Each of these values is a multiple of 4. We first separated 400 into 4 equal groups, 400 ÷ 4. Next we separated the 12 tens (120) into 4 equal groups, 120 ÷ 4. Finally, we separated 16 ones into 4 equal groups, 16 ÷ 4. These are the partial quotients for 536 ÷ 4.

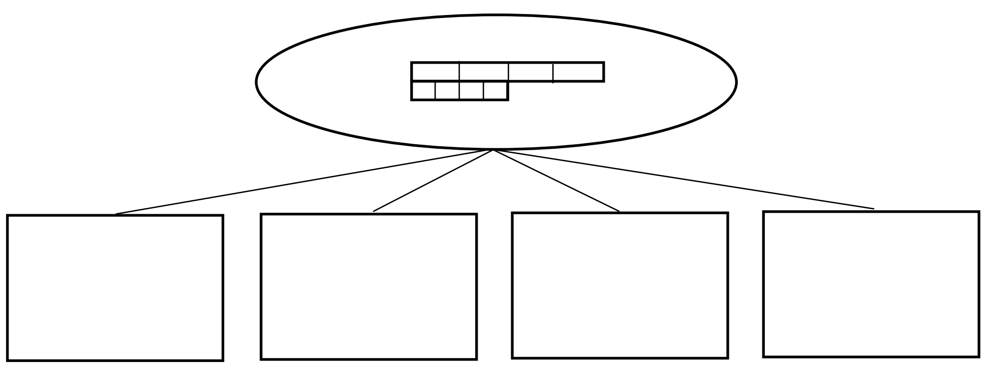
$$\begin{aligned} 536 \div 4 &= (400 \div 4) + (120 \div 4) + (16 \div 4) \\ &= 100 + 30 + 4 \\ &= 134 \end{aligned}$$

<u>Example 2</u>: 1½ ÷ 4 using an equal grouping model

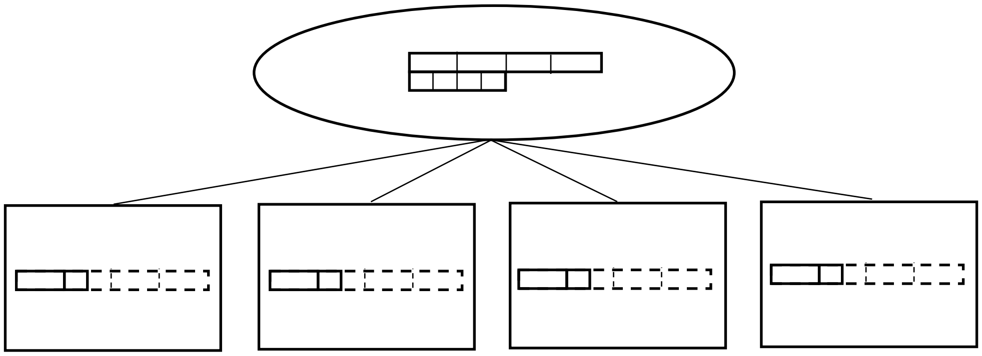
- Begin by translating the division problem into language that helps bring imagery. When dividing by a single digit number it is still efficient to think of division as separating into equal groups. We can translate 1½ ÷ 4 as 1½ separated into 4 equal groups.



- We first cut the whole and the half into 4 equal pieces. When we divide the whole into 4 equal pieces we have 4 fourths. When we divide the half into 4 equal pieces we have 4 eighths.



- We place a $\frac{1}{4}$ piece and a $\frac{1}{8}$ piece in each group. (Remember we draw fractions less than one, that are not part of a mixed number, with a dashed line.) To divide $1\frac{1}{2}$ by 4 we are actually taking a fourth of 1 and a fourth of a half.



$$\begin{aligned} 1\tfrac{1}{2} \div 4 &= (1 \div 4) + (\tfrac{1}{2} \div 4) \\ &= (\tfrac{1}{4} \times 1) + (\tfrac{1}{4} \times \tfrac{1}{2}) \\ &= \tfrac{1}{4} + \tfrac{1}{8} \\ &= \tfrac{3}{8} \end{aligned}$$

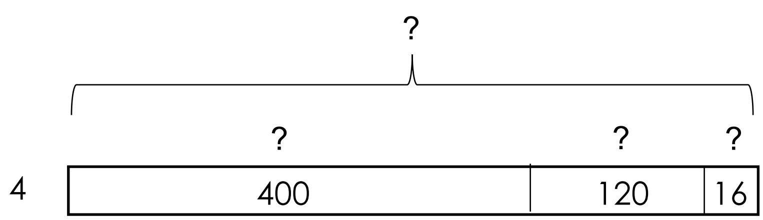
partial quotients

**<u>Division Using the Area Model & Partial Quotients</u>**

<u>Example 1</u>: 536 ÷ 4 using the area model.

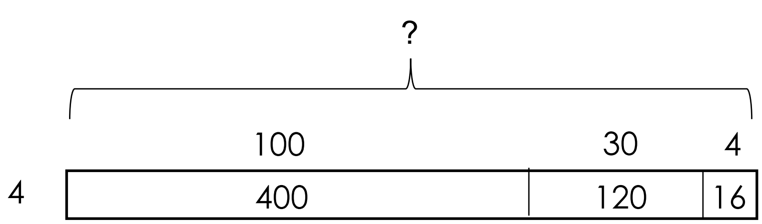
We can also use an area model to find the partial quotients. If we use the area model to find 536 ÷ 4, we think of 536 as the total area and 4 as one of the side lengths. Our goal is to find the other side length.

- We cut the area into multiples of the side length (in this case 4). One possibility is shown below.




- To find the missing values we think, 4 times what will give 400, 4 times what will give 120, and 4 times what will give 16.

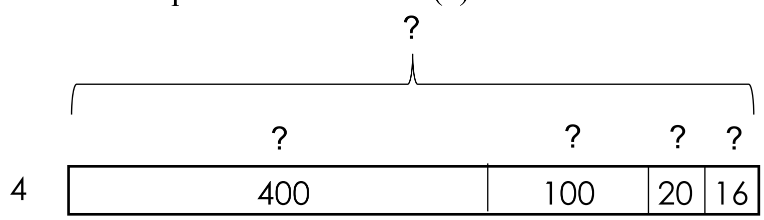
- We know that 4 times 100 is 400, 4 times 30 is 120, and 4 times 4 is 16.



- We add the partial quotients to get the total. So,

536 ÷ 4 is the same as 4 x ___ = 536
4 x ____ = 400
4 x ____ = 120
4 x ____ = 16
So 536 ÷ 4 = 100 + 30 + 4 = 134

**Note:** We can separate the area into any multiple of 4. See below for another example of a way we could separate the area into multiples of 4. Remember, we need the total area to equal 536 and for each partial area to be multiples of the dividend (4).




In this case we would find,

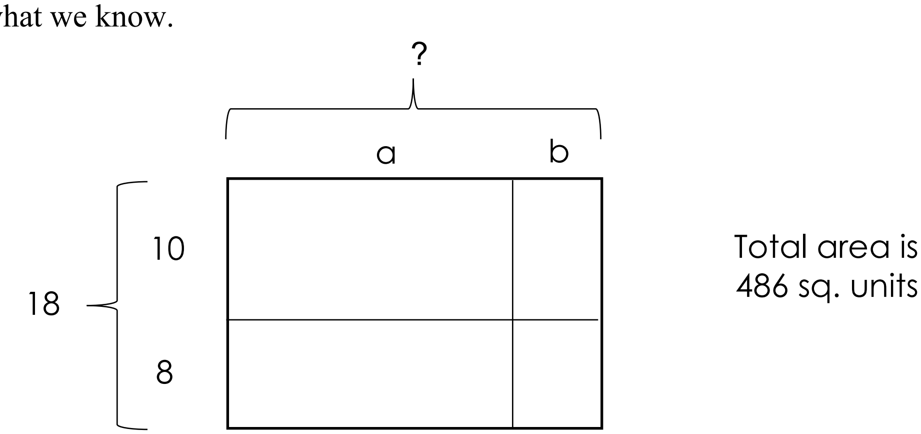
4 x ____ = 400
4 x ____ = 100
4 x ____ = 20
4 x ____ = 16

So 536 ÷ 4 = 100 + 25 + 5 + 4 = 134

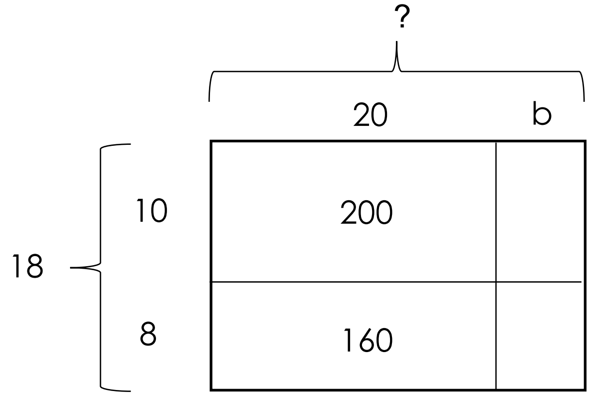
Example 2: 486 ÷ 18 using the area model.

To use the area model to solve 486 ÷ 18, we think of 486 as the total area and 18 as one of the side lengths. We know that the other side length must be a double-digit number. Since 10 times 18 (the first 2-digit number) is 180, we know it cannot be a number less than 10. A single-digit number would be too small. The goal is to determine the areas (partial quotients) that we need to separate 486 into to find the missing 2-digit side length.

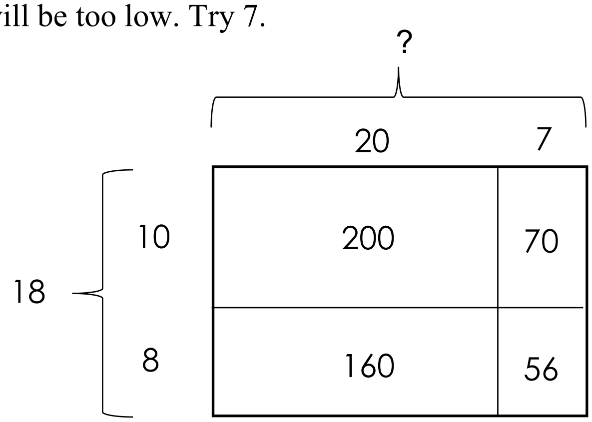
- We begin with what we know.



- Because we have been using expanded form of a number for our dimensions we know the “a” is a multiple of 10. We can try 10, 20, or 30. If we try 30, we will have a 10 by 30 rectangle and an 8 by 30 rectangle. Since 300 + 240 is greater than our total area, 30 will not work. Ten is too low because 10 by 10 + 10 by 8 is 180. We are 306 short of our total area. Let’s try substituting 20 for “a”.




- To determine the value for “b”, your child needs to understand that the ending digit of a multi-digit product is the ending digit obtained when multiplying the numbers in the ones place. In this problem, 486 ends in 6. We know that the number in the bottom right corner must end in 6. That means that “b” must either be 2 or 7 since 2 x 8 = 16 and 7 x 8 = 56. Since we are 126 away from 486, 2 will be too low. Try 7.




$$200 + 70 + 160 + 56 = 486$$

so $486 \div 18 = 27$

**The multiplication and division activities below were introduced in grade 3. These can be used if your child needs additional practice with early models for multiplication.**

## Multiplication Fact Sequence

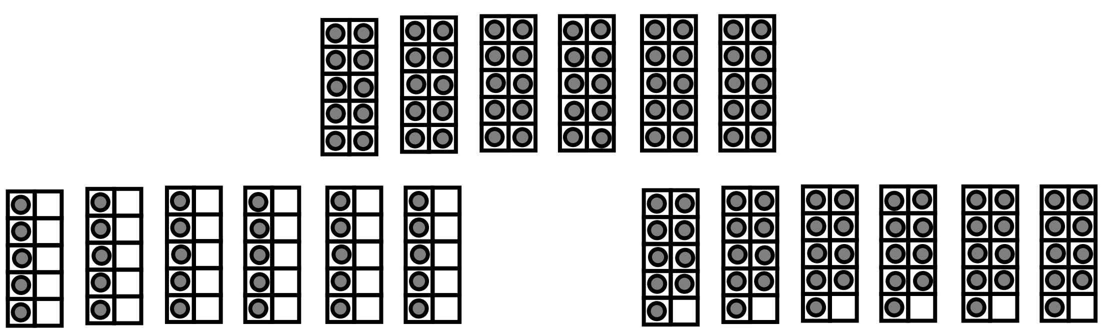
When many of us learned the multiplication facts we began with the 1s and worked our way up to the 12s. We have found that changing the order may help children learn their facts and more importantly learn strategies for working with larger numbers.

Begin with the 0s, 1s, and 10s. The 0s are easy. Your child should know that anything multiplied by 0 will be 0. For fun have your child draw 7 groups of 0, 3 groups of 0, 3 rows of zero. While they are still giggling or looking at you with that look of disbelief, ask them to draw zero groups of 7, zero rows of 5, zero jumps of 9. You get the idea. The 0s are easy.

Do a similar activity with 1s. Have your child draw 1 group of 7 and 7 groups of 1. Although the products are the same, it is important for your child to know that the pictures will be different.

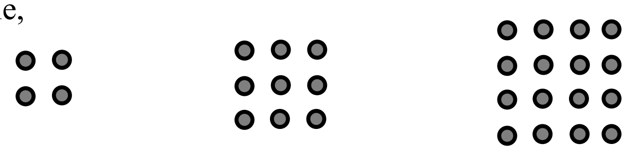

The 10s are one of the next easiest. Your child can picture filled 10 frames or base 10 rods. 5 tens is 50, 8 tens is 80.

Once your child is comfortable with the 10s facts either do the 5s or the 9s. The 5s are half of the 10s facts. For example, if 6 tens is 60, 6 fives is half of that or 30. The 9s are 1 fewer in each group. For example, if 6 tens is 60, 6 nines would be 60 minus 6 or 54.



Once your child is fluent with the 0s, 1s, 10s, 5s, and 9s he or she can practice the doubles. This would have them learn the 2s (doubles—he or she began doubling in grade 1), followed by the 4s (double doubles), followed by the 8s (double, double, double). Repeated doubling is a powerful strategy for mentally multiplying larger numbers by 2, 4, or 8. Repeated halving is used to divide by 2, 4, or 8.

The next multiplication cluster are the square numbers (2 x 2, 3 x 3, 4 x 4, etc.). These are the numbers that make squares. For example,



If your child knows these facts and knows that you can "flip" the known facts to get the same product (Commutative Property: if you know 2 x 3 = 6 then you know 3 x 2 = 6), then your child knows all but 3 facts. We have not listed learning the 3s, 6s, or 7s in this sequence. The only facts that we are missing are 3 x 6, 6 x 3, 3 x 7, 7 x 3, 6 x 7, and 7 x 6. 3 x 6 is double 3 x 3. We are missing 6 x 3 and 6 x 7. If your child knows 3 x 6 is 18 then he or she knows 6 x 3 is 18. 6 x 7 is double 3 x 7. If your child knows 3 x 7 is 21 then 6 x 7 is double that or 42. We are missing 7 x 3 and 7 x 6 but we've already addressed those facts when learning 3 x 7 and 6 x 7. Technically the one fact to memorize is 3 x 7.

## Multiplication and Division Games

There are 3 sets of concentration game cards that we used in grade 3 to help your child develop their understanding and fluency with multiplication and division:

- *Equal Grouping* Cards
- *Division as Separating Concentration* Cards
- *Multiplication 8s, 9s, 10s Facts Concentration* Cards

**You can do Activities 1-4 using any of these card sets. Activity 5 uses the first 2 sets.**

**Activity 1—Find the Match**

- Shuffle the cards well.
- Place cards face up in 4 rows with 6 cards in each row.
- Take turns finding matches.
- Each player must share how he or she knows it is a match before taking the cards.

**Activity 2—Multiplication/Division Concentration**

- Shuffle the cards well.
- Place cards face down in 4 rows with 6 cards in each row.
- Take turns turning over 2 cards and placing face up in the exact same spaces.
- If the cards match, the player must share how he or she knows it is a match before taking the cards. (The defense must make sense.)
- If the cards do not match, the player must share how he or she knows the cards do not match before turning them back over. (The defense must make sense.)
- See who can find the most matches.

**Activity 3—Who has more?**

- Shuffle the cards well.
- Deal the cards so that each player has the same number of cards.
- Each player places their cards in a stack, face down.
- Each player turns over the top card on the stack.
- The player with the greater number states how they know it is greater. If the explanation makes sense, he or she then takes the cards and places on the bottom of the stack.
- See who can capture all of the cards.

**Activity 4—Who has fewer? Who has less?**

- Shuffle the cards well.
- Deal the cards so that each player has the same number of cards.
- Each player places their cards in a stack (face-down).
- Each player turns over the top card on the stack.
- The player with the smaller number states how they know it is smaller. If the explanation makes sense, he or she then takes the cards and places on the bottom of the stack.
- See who can capture all of the cards.

**Activity 5—Multiplication and Division Stories—*Equal Grouping* Cards & *Division as Separating* Cards**

Sample stories are included within these 2 card decks. Use one of the decks at a time. Have your child remove the story cards from one of the decks.

- Shuffle the remaining cards well.
- Place the cards in a stack (face-down).
- Have your child select the top card and tell a story problem to go with the fact or the drawing.
- Complete 4 to 6 stories.
- Repeat for the same deck on a different day or do the same activity for the other deck.

## Comparing Common & Decimal Fractions

Many of us learned to name a fraction such as ¾ as three-fourths or 3 out of 4. It is important for your child to learn to name fractions in 3 ways: three-fourths, 3 out of 4, and 3 one-fourth pieces. In this last way of naming the fraction, your child learns that the numerator, 3, gives us the number of unit fraction pieces (in this case one-fourth). A unit fraction is a fraction with the numerator of 1. This way of reading the fraction will help your child compare fractions. In Grade 3 your child compared fractions with the same denominator. For example,

$$\frac{3}{8} \qquad \frac{5}{8}$$

Your child should think, "Which is bigger 3, one-eighth pieces or 5, one-eighth pieces?" Understanding that you have more one-eighth pieces if you have 5 one-eighth pieces means five-eighths is greater than three-eighths.

Your child compared fractions with the same numerator. For example,

$$\frac{3}{8} \qquad \frac{3}{6}$$

Your child should think, “Which is bigger… 3, one-eighth pieces or 3, one-sixth pieces?” Understanding that since a one-sixth piece is bigger than a one-eighth piece, then 3, one-sixth pieces would be greater.

In grade 4, your child compared fractions with different numerators and denominators. (The denominators were restricted to 2, 3, 4, 5, 6, 8, 10, 12, and 100.) This was done by finding common numerators, common denominators, or comparing to 0, ½, or 1.

**Comparing common and decimal fractions with unlike numerators and denominators**

Example 1: To compare

$$\frac{3}{5} \qquad \frac{6}{8}$$

we can find a common numerator. In this case we can think of $^6/_8$ as ¾ OR we can think of $^3/_5$ as $^6/_{10}$. In both cases we have a common numerator. If we use ¾ for $^6/_8$,

$$\frac{3}{5} \qquad \frac{3}{4}$$

we can think, “Which is bigger… 3, one-fifth pieces or 3, one-fourth pieces?” Since 1/5 is smaller than ¼ we know that 3, one-fifth pieces would be smaller than 3, one-fourth pieces.

If we change $^3/_5$ to its equivalent $^6/_{10}$,

$$\frac{6}{10} \qquad \frac{6}{8}$$

we can think, “Which is bigger…6, one-tenth pieces or 6, one-eighth pieces?” Since $^1/_{10}$ is smaller than $^1/_8$ we know that 6, one-tenth pieces would be smaller than 6, one-eighth pieces.

Example 2: To compare ¾ and $^5/_8$ we can find a common denominator.

$$\frac{3}{4} \qquad \frac{5}{8}$$

In this case we can think of ¾ as $^6/_8$.

$$\frac{6}{8} \qquad \frac{5}{8}$$

We can think, “Which is bigger…6, one-eighth pieces or 5, one-eighth pieces. Since you have more one-eighth pieces with 6, one-eighth pieces, $^6/_8$ must be greater.

Example 3: To compare $^2/_5$ and $^5/_8$ we can compare each fraction to ½.

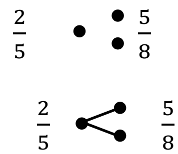
$$\frac{2}{5} \qquad \frac{5}{8}$$

Two-fifths ($^2/_5$) is less than one-half. We know this because 2 is less than half of 5. Another way to think about this is since double 2 is 4 we know that we need to double more than 2 to get to 5.

Five-eighths is more than one-half. We know this because half of 8 is 4. So 5 is more than half of 8. This means $^5/_8$ is more than ½. This means that $^2/_5$ is less than $^5/_8$.

Your child is also expected to use the symbols >, <,  and = correctly. To help with this, have your child put 2 dots next to the larger value and 1 dot next to the smaller number.

Your child then connects the dots.

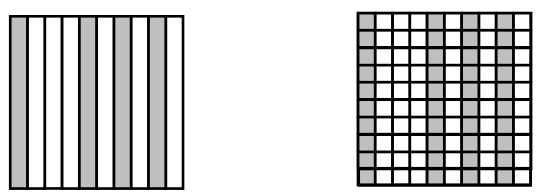



<u>Example 4:</u> To compare 0.4 and 0.37 we use the relationship between tenths and hundredths that was learned in the previous section.

Remember that we can write 0.4 and 0.37 (.4 and .37) as common fractions.

$$\frac{4}{10} \qquad \frac{37}{100}$$

In a prior section we examined the link between a fraction such as $\frac{4}{10}$ and $\frac{40}{100}$. Your child colored in four-tenths in the first whole (Remember any 4, one-tenth pieces can be colored). Your child then pretended to slide the image on the left over the image on the right to show it was equivalent to forty-hundredths.



So,

$$\frac{4}{10} \text{ is equivalent to } \frac{40}{100}$$

To compare $\frac{4}{10}$ and $\frac{37}{100}$, we think of $\frac{4}{10}$ as $\frac{40}{100}$. We are now comparing $\frac{40}{100}$ and $\frac{37}{100}$, fractions with like denominators. Since $\frac{40}{100}$ is greater than $\frac{37}{100}$, $\frac{4}{10}$ is greater than $\frac{37}{100}$. Therefore, 0.4 > 0.37 or .4 > .37 . (**Note:** Your child does not have to write all of this out each time but this is the thought process or understanding we hope he or she will use.)