Grade 5 Mathematics
Key Concepts and Skills
A Parents' Guide

Below are the key mathematics concepts that we hope your child will master by the end of Grade 5.

Grade 5 is the year that we solidify your child's number sense in relation to working with fractions and decimals. We hope they:

- have strong visual images for whole numbers, fractions, and decimals,
- understand place value and the relationships between the places with whole numbers and decimals,
- can easily compare whole numbers, fractions, and decimals,
- can represent whole numbers, fractions, and decimals with objects, drawings, number words, and symbols,
- understand and use open number lines to represent whole numbers, fractions, and decimals,
- notice and describe numeric and computational patterns,
- can create fractional units,
- can represent fractions on open number lines, and
- can compare fractions with like numerators or like denominators.

Grade 5 is the year that your child extends his or her use of a variety of addition and subtraction strategies and representations to add and subtract fractions with unlike denominators. Multiplication strategies and representations are extended to 2-digit by 2-digit multiplication, fraction multiplication, and decimal multiplication. Your child continues to develop fluency with a variety of division strategies including the use of partial quotient division. Your child should be very flexible in the strategies used to solve whole number, fraction, and decimal problems. She or he should be able to solve a problem in more than one way.

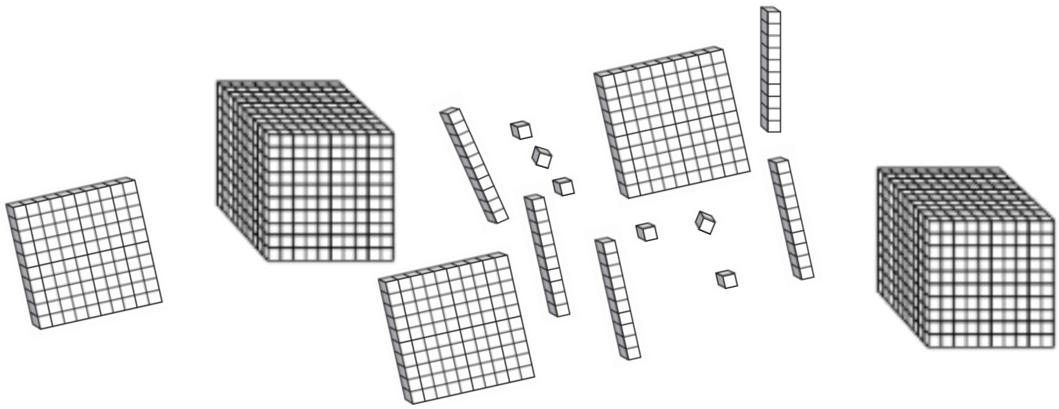
Grade 5 is also the year that your child is introduced to volume, solidifies his or her understanding of the relationships between measurement units to convert between units, and is able to create a hierarchy of 2D shape categories to classify shapes.

Number Concepts

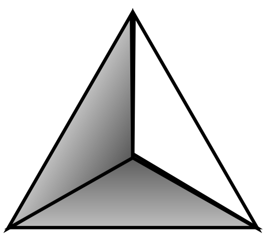
Subitize Numbers

Subitize may be an unfamiliar term but many of us do this without knowing its name. For example, when you play a game involving dice, do you recognize a 6 without counting the pips? That is subitizing, recognizing and naming a quantity without counting the individual objects.

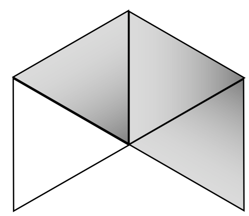
1. Your child should be able to subitize Base 10 materials, fraction representations, and decimal representations in any arrangement and turned in a variety of ways. For example,



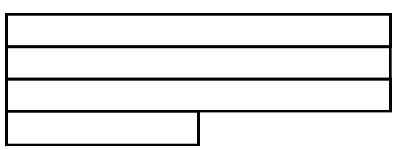
2356



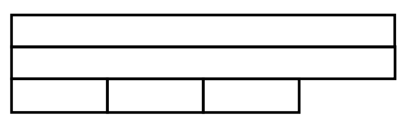
$\frac{2}{3}$



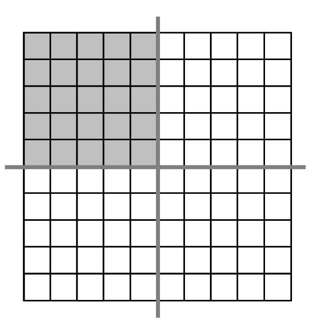
$\frac{3}{4}$



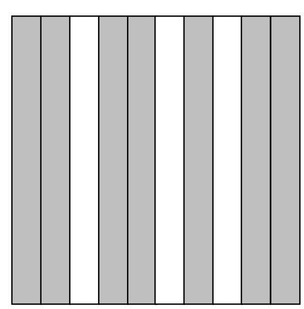
$3\frac{1}{2}$



$2\frac{3}{4}$



$\frac{25}{100}$ or 0.25



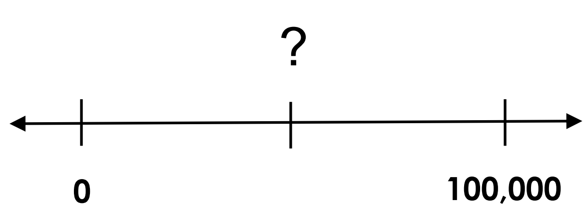
$\frac{7}{10}$ or 0.7

Place Value

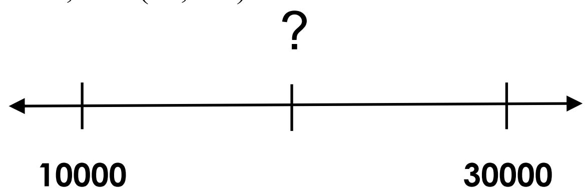
1. Your child should be able to correctly read and write multi-digit numbers. He or she should understand that our number system is divided into periods (units, thousands, millions, billions, trillions, etc.). Each period contains 3 places (unit, tens, hundreds). For example, the units period contains ones, tens, hundreds. Within the thousands period we have thousands, ten thousands, hundred thousands. Within the millions period we have millions, ten millions, hundred millions. This pattern continues within each period. For ease of reading large numbers we can separate the periods with commas (e.g., 132,834,256; one hundred thirty-two million, 8 hundred thirty-four thousand, two hundred fifty-six). However, commas are not required. 132834256 would also be correct.
2. Your child should be able to read and write decimals to thousandths using base 10 numerals and number words (e.g., 3.24, three and twenty-four hundredths). He or she understands and uses the word "and" for the decimal point when reading decimal numbers. Your child understands that the periods are similar across the decimal point beginning with the ones place. That is 3.24 represents the ones, tenths, and hundredths place. We then start the thousandths period (thousandths, ten thousandths, hundred thousandths). This pattern continues.
3. Your child should be able to write multi-digit whole numbers in expanded form (e.g., 3056 = 3000 + 50 + 6 OR 3 x 1000 + 0 x 100 + 5 x 10 + 6 x 1; 999 = 900 + 90 + 9 OR 9 x 100 + 9 x 10 + 9 x 1; 19023 = 10,000 + 9,000 + 20 + 3 OR 1 x 10000 + 9 x 1000 + 0 x 100 + 2 x 10 + 3 x 1).
4. Your child should be able to write decimal numbers to thousandths using expanded form (e.g., $4.259 = 4 \times 1 + 2 \times \frac{1}{10} + 5 \times \frac{1}{100} + 9 \times \frac{1}{1000}$ OR 4 x 1 + 2 x .1 + 5 x .01 + 9 x .01).
5. Your child should understand and explain that a digit in one place is 10 times greater than the same digit in a place to its right and $\frac{1}{10}$ as much as the place to its immediate left. For example, a 5 in the ones place is worth 5. A 5 in the tens place is worth 50, 10 times as much as 5 OR 5 is $\frac{1}{10}$ as much as 50. A 5 in the hundreds place is worth 500, 10 times as much as 50 and 100 times as much as 5 OR 50 is $\frac{1}{10}$ as much as 500, 5 is $\frac{1}{100}$ as much as 500. A 5 in the tenths place is worth 0.5 (five-tenths), $\frac{1}{10}$ as much as 5.
6. Your child should understand and use exponents to represent powers of 10. For example, 1000 is the same as 10 x 10 x 10 (3 factors of 10) or 10^3 (read as 10 cubed, 10 to the third power, or 10 to the third). 100 is the same as 10 x 10 (2 factors of 10) or 10^2 (read as 10 squared, 10 to the second power, or 10 to the second). 10 is the same as 1 factor of 10 or 10^1 (read as 10 to the first power or 10 to the first). 1 is the same as 0 factors of 10 or 10^0 (read as 10 to the zero).
7. Your child should understand and describe the patterns in the number of zeros when multiplying by powers of 10. For example, 10 x 10 x 10 = 10^3 = 1000. 1000 or 10^3 has 3 zeroes. 10 x 10 x 10 x 10 = 10,000 = 10^4. 10,000 or 10^4 has 4 zeroes.

Open Number Lines

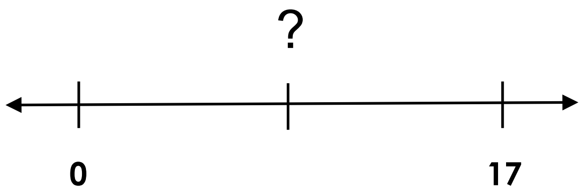
1. Your child should be able to identify points on an "open" number line. An open number line is a number line on which only some of the hash marks are given. We use benchmarks such as halfway points to find the values on the number line. For example, in the number line below, the endpoints 0 and 100000 are given. Your child is asked to find the halfway point, 50000.

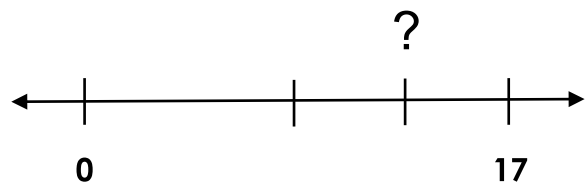


Your child may be asked to solve problems with “0” as one of the endpoints. He or she may also be asked to find the missing values when the starting number is a number other than 0. For example, in the problem below, your child is asked to find the value that is halfway between 10,000 and 30,000. (20,000)



We often think that larger numbers increase the level of difficulty. The problems above are simpler than the problems below.

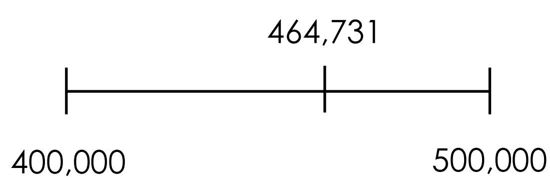




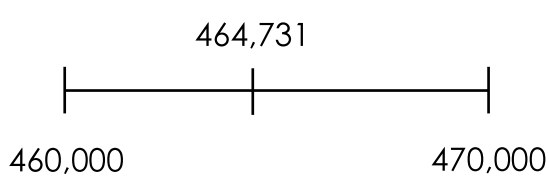
Comparing Quantities

1. Given 2 numbers, your child should be able to tell which is larger, smaller, or the same. He or she should be able to prove using base 10 materials, drawings, and showing on an open number line.
2. Your child should be able to correctly use the symbols >, <, =.
3. When asked to round to the nearest 10, 100, 1000, 10000, 100000, etc., your child should be able to name the 10s, 100s, 1000s, 10000s, 100000s, etc. that the number is between. For example, 464,731 is between 400,000 and 500,000 (hundred thousands it is between). It is between 460,000 and 470,000 (ten thousands it is between). It is between 464,000 and 465,000 (thousands it is between). It is between 464,700 and 464,800 (hundreds it is between). It is between 464,730 and 464,740 (tens it is between). Your child can use these understandings to explain and defend the result of rounding to the given place.

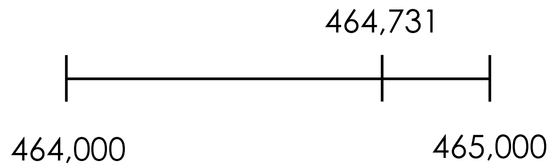
   On an open number line—the hundred-thousands that 464,731 is between:



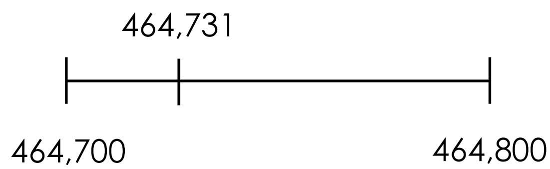
   On an open number line—the ten-thousands that 464,731 is between:



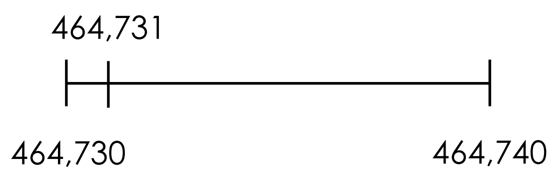
On an open number line—the thousands that 464,731 is between:



On an open number line—the hundreds that 464,731 is between:



On an open number line—the tens that 464,731 is between:



Rounding to the nearest hundred-thousand, 464,731 rounds to 500,000.

Rounding to the nearest ten-thousand, 464,731 rounds to 460,000.

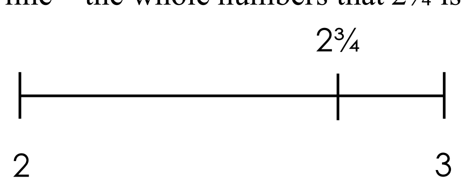
Rounding to the nearest thousand, 464,731 rounds to 465,000.

Rounding to the nearest hundred, 464,731 rounds to 464,700.

Rounding to the nearest ten, 464,731 rounds to 464,730.

4. Your child should be able to name the whole numbers that a fraction is between and use to round to the nearest whole number. For example, 2¾ is between 2 and 3. Your child can use this understanding to explain and defend the result of rounding to the given place.

   On an open number line—the whole numbers that 2¾ is between:



   Rounding to the nearest whole number, 2¾ rounds to 3.

5. Your child understands that when rounding a number at the midpoint, we round up. For example, if we are rounding 65 to the nearest ten we know that 65 is the midpoint between 60 and 70. Therefore we round up to 70. If we are rounding 250 to the nearest hundred we know that 250 is the midpoint between 200 and 300. Therefore, we round up to 300. If we are rounding 2½ or 2.5 to the nearest whole number we know that 2½ or 2.5 is the midpoint between 2 and 3. Therefore, we round up to 3.

Expression

1. Your child should be comfortable with the language associated with each operation (addition, subtraction, multiplication, division) and use symbols to represent.

   Example 1:
   Three times, the sum of five and seven would be represented symbolically as 3 x (5 + 7).

   Example 2:
   The sum of three times five and seven would be represented symbolically as 3 x 5 + 7.

   Example 3:
   Four groups of, eight remove 2 would be represented symbolically as 4 x (8 – 2).

   Example 4:
   Eight more than a number is thirty-two would be represented symbolically as t + 8 = 32.

   Example 5:
   Thirty-six separated into four equal groups is two fewer than a number would be represented symbolically as 36 ÷ 4 = t – 2.

Addition	Subtraction
put together join combine and more than sum plus	remove take away fewer difference ____ fewer than ____ is ____ ____ more than ____ is ____ minus
Multiplication	**Division**
groups of rows of jumps of by times times as many as product	separated into equal groups of separated into equal rows of separated into equal jumps of how many groups of ___ are in ____ how many rows of ____ are in ____ how many jumps of ___ are in ____ ___ by ____ is ____ (Thinking of area, we know the area and one side length. We need to find the missing side length). quotient

2. Your child should be able to “translate” symbols into everyday language. He or she should understand that there are many ways to translate the same symbols. (**Note:** The translation examples given below are not exhaustive. There are additional ways to translate the symbols.)

   Example 1: 2t + 5 could be translated as “5 more than 2 times a number” OR “double a number put together with 5” OR “2 times a number plus 5”.

   Example 2: 23 – 2 x 3 could be translated as “23 remove, 2 groups of 3” OR “23 take away, 2 times 3” OR “How many more than 2 groups of 3 is 23?”

   Example 3: 35 ÷ 7 could be translated as “35 separated into 7 equal groups” OR “35 separated into 7 equal rows” OR “How many sevens are in 35?” OR “7 groups of what number is 35” OR “7 times what number is 35”.

3. Your child should understand and use the conventional order for simplifying expressions:
   - Expressions within parentheses are simplified first.
   - Exponents are then simplified.
   - Multiplication and division are simplified next working left to right.
   - Addition and subtraction are simplified working left to right.

Patterns

1. Your child should be able to continue two relational number patterns given two rules. For example, if given the chart below, your child should be able to complete the next 3 rows.

1	2
2	4
3	6
4	8
5	10

   Your child should be able to describe the rule for each number pattern. For example, he or she may say that the rule for column 1 is “plus 1” or “count by ones.” He or she may say that the rule for column 2 is “even numbers” or “count by twos.”

2. Your child should be able to identify and describe the relationship between two relational number patterns. For example, if given the chart below, your child should be able to describe the relationship between the two columns.

1	6
2	7
3	8
4	9
5	10

She or he may say the 2nd column is the 1st column plus 5 (n, n + 5).

Your child should be able to identify the missing values in a chart without finding every term in between. For example,

1	6
2	7
3	8
4	9
5	10
. . .	
10	?
. . .	
50	?
. . .	
?	100

3. Your child should understand and describe the effect of multiplying a whole number by a power of 10. For example, he or she should look at several examples such as those shown below and then describe the pattern.

Example 1:

5 x 10 = 50	5 x 10^1 = 50
5 x 100 = 500	5 x 10^2 = 500
5 x 1000 = 5000	5 x 10^3 = 5000
5 x 10000 = 50000	5 x 10^4 = 50000

Example 2:

25 x 10 = 250	25 x 10^1 = 250
25 x 100 = 2500	25 x 10^2 = 2500
25 x 1000 = 25000	25 x 10^3 = 25000
25 x 10000 = 250000	25 x 10^4 = 250000

Example 3:

250 x 10 = 2500	250 x 10^1 = 2500
250 x 100 = 25000	250 x 10^2 = 25000
250 x 1000 = 250000	250 x 10^3 = 250000
250 x 10000 = 2500000	250 x 10^4 = 2500000

Your child should notice that the number of zeroes added to the end of the factor matches the exponent (power of 10). For example, $5 \times 10^4 = 50000$, 50000 has 4 zeroes following the 5. For $250 \times 10^2 = 25000$, there are 2 zeroes following the 250.

Note: It is important that your child notices and comes to this conclusion instead of being taught this pattern as a trick. In this way we help your child engage in algebraic thinking.

4. Your child should understand and describe the effect of dividing a whole number by a power of 10. For example, he or she should look at several examples such as those shown below and then describe the pattern.

Example 1:

$25000 \div 10 = 2500$	$25000 \div 10^1 = 2500$
$25000 \div 100 = 250$	$25000 \div 10^2 = 250$
$25000 \div 1000 = 25$	$25000 \div 10^3 = 25$
$25000 \div 10000 = 2.5$	$25000 \div 10^4 = 2.5$
$25000 \div 100000 = .25$	$25000 \div 10^5 = .25$

Example 2:

$50000 \div 10 = 5000$	$50000 \div 10^1 = 5000$
$50000 \div 100 = 500$	$50000 \div 10^2 = 500$
$50000 \div 1000 = 50$	$50000 \div 10^3 = 50$
$50000 \div 10000 = 5$	$50000 \div 10^4 = 5$
$50000 \div 100000 = .5$	$50000 \div 10^5 = .5$
$50000 \div 1000000 = .05$	$50000 \div 10^6 = .05$

Your child should notice that the number of zeroes "removed" from the end of the factor matches the exponent (power of 10). He or she may also notice that the decimal point "moves" to the left the number of places that matches the exponent (power of 10). For example, $50000 \div 10^4 = 5$, 5 has the 4 zeroes removed from the end of 50000. Another way to think of this is that for $50000 \div 10^4 = 5$, the decimal point that is after 50000 has been moved 4 places to the left resulting in 5.

Note: It is important that your child notices and comes to this conclusion instead of being taught this pattern as a trick. In this way we help your child engage in algebraic thinking.

5. Your child should understand and describe the effect of multiplying a decimal number by a power of 10. For example, she or he should look at several examples such as those shown below and then describe the pattern.

Example 1:

$5.126 \times 10 = 51.26$	$5.126 \times 10^1 = 51.26$
$5.126 \times 100 = 512.6$	$5.126 \times 10^2 = 512.6$
$5.126 \times 1000 = 5126$	$5.126 \times 10^3 = 5126$
$5.126 \times 10000 = 51260$	$5.126 \times 10^4 = 51260$

Example 2:

$2.537 \times 10 = 25.37$ $\quad$ $2.537 \times 10^1 = 25.37$
$2.537 \times 100 = 253.7$ $\quad$ $2.537 \times 10^2 = 253.7$
$2.537 \times 1000 = 2537$ $\quad$ $2.537 \times 10^3 = 2537$
$2.537 \times 10000 = 25370$ $\quad$ $2.537 \times 10^4 = 25370$

Your child should notice that the decimal point "moves" to the right the number of places that matches the exponent (power of 10). For example, $2.537 \times 10^2 = 253.7$, the decimal point in 2.537 has been moved 2 places to the right resulting in 253.7.

Note: It is important that your child notices and comes to this conclusion instead of being taught this pattern as a trick. In this way we help your child engage in algebraic thinking.

6. Your child should understand and describe the effect of dividing a decimal number by a power of 10. For example, he or she should look at several examples such as those shown below and then describe the pattern.

Example 1:

$2503.5 \div 10 = 250.35$ $\quad$ $2503.5 \div 10^1 = 250.35$
$2503.5 \div 100 = 25.035$ $\quad$ $2503.5 \div 10^2 = 25.035$
$2503.5 \div 1000 = 2.5035$ $\quad$ $2503.5 \div 10^3 = 2.5035$
$2503.5 \div 10000 = 0.25035$ $\quad$ $2503.5 \div 10^4 = 0.25035$

Example 2:

$256.34 \div 10 = 25.634$ $\quad$ $256.34 \div 10^1 = 25.634$
$256.34 \div 100 = 2.5634$ $\quad$ $256.34 \div 10^2 = 2.5634$
$256.34 \div 1000 = 0.25634$ $\quad$ $256.34 \div 10^3 = 0.25634$
$256.34 \div 10000 = 0.025634$ $\quad$ $256.34 \div 10^4 = 0.025634$

Your child should notice that the decimal point "moves" to the left the number of places that matches the exponent (power of 10). For example, $256.34 \div 10^2 = 2.5634$, the decimal point in 256.34 has been moved 2 places to the left resulting in 2.5634.

Note: It is important that your child notices and comes to this conclusion instead of being taught this pattern as a trick. In this way we help your child engage in algebraic thinking.

Fractions

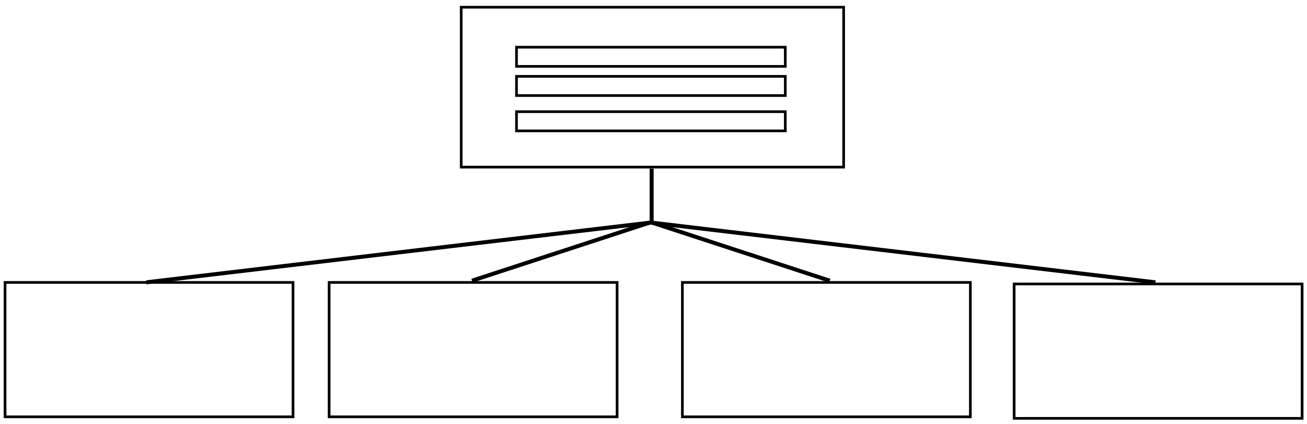
Your child should be able to cut wholes into 2, 4, 8, 3, 6, 12, 5, 10 and 100 equal pieces. Models for cutting a whole into 10 and 100 equal pieces provides a link to decimals. These concepts were developed in grade 4. In grade 5 she or he will extend these understandings to work with any denominator.

Creating Fractional Units

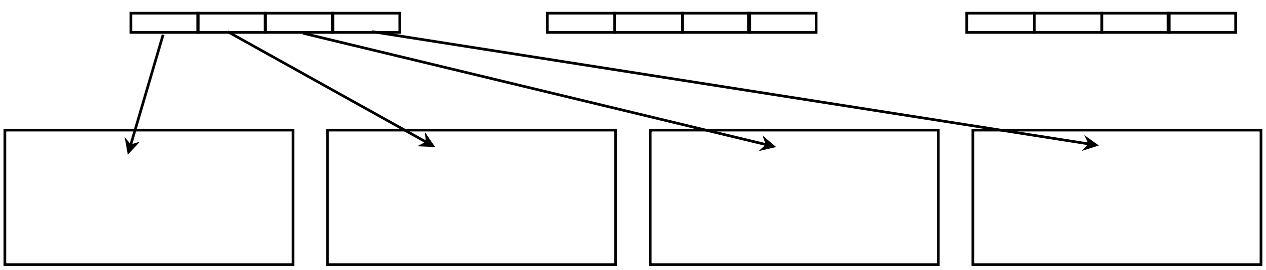
1. Your child should understand the connection between division and fractions. For example, 3 ÷ 4 is the same as ¾. She or he should be able to represent with fraction materials, drawings, and equations.

   <u>Example:</u> 3 ÷ 4

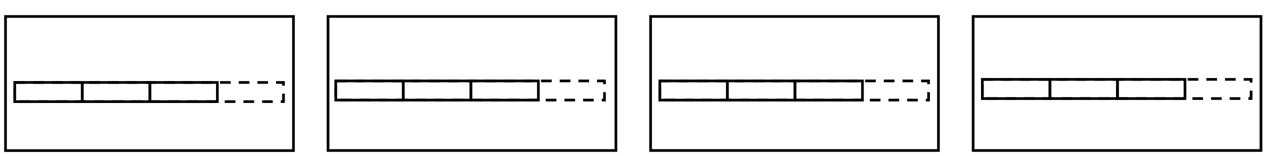
   We can think of 3 ÷ 4 as 3 separated into 4 equal groups.



   We can separate each whole into 4 equal pieces. Then place one of each piece into each group.



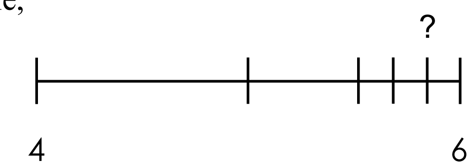
   The results are shown below. 3 separated into 4 equal groups is ¾.



Naming Fractional Units

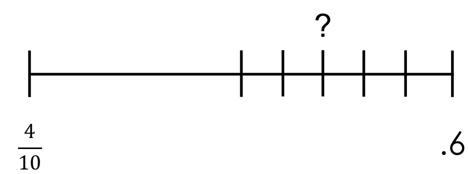
2. Many of us learned to name a fraction such as ¾ as "three-fourths" or "3 out of 4." It is important for your child to learn to name fractions in 3 ways: three-fourths, 3 out of 4, and 3 one-fourth pieces. In this last way of naming the fraction, your child learns that the numerator, 3, gives the number of unit fraction pieces (in this case one-fourth). A unit fraction is a fraction with the numerator of 1. This way of reading the fraction will help your child compare fractions and compute with fractions.

3. Your child should be able to label fractions represented on a number line between any two endpoints. For example,

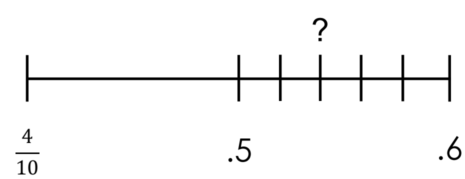


Your child may first notice that there is a halfway point between 4 and 6. That halfway point is 5. There is also a halfway point between 5 and 6. That halfway point is 5½. The distance between 5½ and 6 is divided into 3 equal sections. If you divide the distance between 5 and 5½ into 3 equal sections, then the distance between 5 and 6 would be divided into 6 equal sections. The missing value would be $5\frac{5}{6}$.

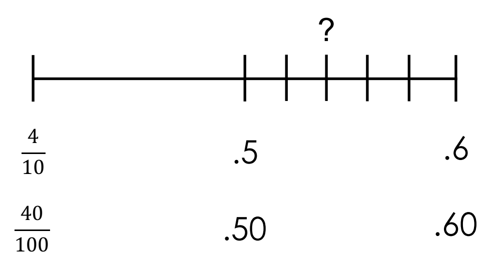
4. Your child can label decimal fractions represented on a number line between any two endpoints (the endpoints may be given as common fractions or as decimal fractions). For example,



Your child may first notice that there is a halfway point between four-tenths (.4) and six-tenths (.6). That halfway point is five-tenths or .5.



Your child should also understand that these points can be labeled in hundredths. For example,



Your child may predict that the hash marks are two-hundredths apart. He or she could then count by two-hundredths (.02) to check, she or he would get from fifty-hundredths (.50) to sixty-hundredths (.60). Counting by two-hundredths from .5 (or .50) she or he would find that the missing value is 0.54 (.54).

Comparing Fractional Units

5. Your child should be able to compare fractions with different numerators and denominators. This can be done by finding common numerators, common denominators, or comparing to 0, ½, or 1. (**Note:** When comparing fractions, we assume that the fractions are from the same-sized wholes.)

   Example 1: To compare

   $$\frac{3}{5} \qquad \frac{6}{8}$$

   we can find a common numerator. In this case we can think of $^6/_8$ as $^3/_4$ OR we can think of $^3/_5$ as $^6/_{10}$. In both cases we have a common numerator. If we use $^3/_4$ for $^6/_8$,

   $$\frac{3}{5} \qquad \frac{3}{4}$$

   we can think, "Which is bigger… 3, one-fifth pieces or 3, one-fourth pieces?" Since $^1/_5$ is smaller than $^1/_4$ we know that 3, one-fifth pieces would be smaller than 3, one-fourth pieces.

   If we change $^3/_5$ to its equivalent $^6/_{10}$

   $$\frac{6}{10} \qquad \frac{6}{8}$$

   we can think, "Which is bigger…6, one-tenth pieces or 6, one-eighth pieces?" Since $^1/_{10}$ is smaller than $^1/_8$ we know that 6, one-tenth pieces would be smaller than 6, one-eighth pieces.

   Example 2: To compare $^3/_4$ and $^5/_8$ we can find a common denominator.

   $$\frac{3}{4} \qquad \frac{5}{8}$$

   In this case we can think of ¾ as $^6/_8$.

   $$\frac{6}{8} \qquad \frac{5}{8}$$

   We can think, "Which is bigger…6, one-eighth pieces or 5, one-eighth pieces. Since you have more one-eighth pieces with 6, one-eighth pieces, $^6/_8$ must be greater.

   Example 3: To compare $^2/_5$ and $^5/_8$ we can compare each fraction to $^1/_2$.

   $$\frac{2}{5} \qquad \frac{5}{8}$$

   Two-fifths ($^2/_5$) is less than one-half. We know this because 2 is less than half of 5. Another way to think about this is since double 2 is 4 we know that we need to double more than 2 to get to 5.

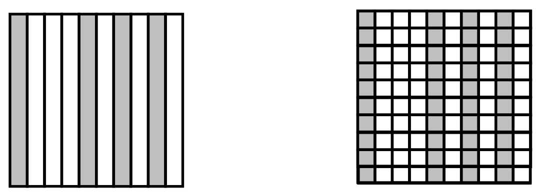
   Five-eighths is more than one-half. We know this because half of 8 is 4. So 5 is more than half of 8. This means $^5/_8$ is more than $^1/_2$. Therefore, $^2/_5$ is less than $^5/_8$.

6. Your child should understand that on a number line, the number to the right is larger. She or he can use this understanding to compare fractions on a number line.

7. Your child is also expected to use the symbols >, <, and = correctly to show the relationship between the two fraction or decimal values.

Comparing Decimal Fractions

8. Your child should understand that comparing .3 and .5 is the same as comparing $\frac{3}{10}$ and $\frac{5}{10}$ (like denominators). 3, one-tenth pieces is fewer than 5, one-tenth pieces. Likewise, comparing .03 and .05 is the same as comparing $\frac{3}{100}$ and $\frac{5}{100}$ (like denominators). 3, one-hundredth pieces is less than 5, one-hundredth pieces.
9. Your child should understand that comparing .3 and .03 is the same as comparing $\frac{3}{10}$ and $\frac{3}{100}$ (like numerators). Since one-tenth is larger than one-hundredth, 3, one-tenth pieces would be greater than 3, one-hundredth pieces.
10. Your child should understand that comparing 0.4 and 0.37 is the same as comparing $\frac{4}{10}$ and $\frac{37}{100}$. To compare your child uses his or her understanding that $\frac{4}{10}$ is equivalent to $\frac{40}{100}$. To compare 0.4 and 0.37 he or she can compare 0.40 and 0.37 (like denominators). For example,



$\frac{4}{10}$ is equivalent to $\frac{40}{100}$

Since $\frac{40}{100}$ is greater than $\frac{37}{100}$, $\frac{4}{10}$ is greater than $\frac{37}{100}$. Therefore, 0.4 > 0.37 or .4 > .37 .

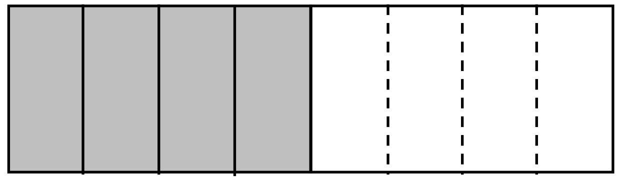
Equivalent Fractions

11. Your child should understand that we can divide the same region into smaller equal-sized pieces. For example, we can begin with ½.

We can cut the halves into 2 equal pieces. We see that 2, one-fourth pieces is equivalent to one-half.

$\frac{1}{2} = \frac{2}{4}$

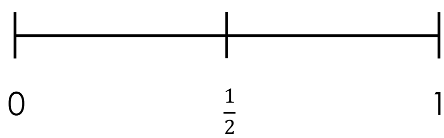
We can cut the fourths into 2 equal pieces. We see that 4, one-eighth pieces is equivalent to one-half.



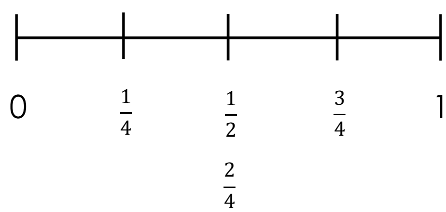
$$\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$$

Your child should understand how to create equivalent fractions using fraction pieces or drawings. He or she should notice and explain the relationship between $\frac{1}{2}$ and $\frac{4}{8}$ ($\frac{4}{8}$ can be obtained by multiplying the numerator and denominator of $\frac{1}{2}$ by the common factor 4). Your child should be able to use this technique when computing with fractions.

12. Your child understands that we can name the same location on a number line in more than one way. For example, suppose we cut the interval from 0 to 1 into 2 equal sections.

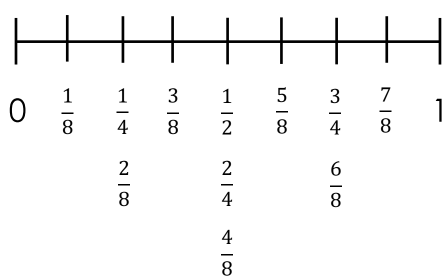


We then cut the interval from 0 to 1 into 4 equal sections.



$\frac{1}{2}$ and $\frac{2}{4}$ name the same location on the number line so $\frac{1}{2}$ is equivalent to $\frac{2}{4}$.

Suppose we repeat and cut the interval from 0 to 1 into 8 equal sections.



$\frac{1}{4}$ and $\frac{2}{8}$ name the same location on the number line so $\frac{1}{4}$ is equivalent to $\frac{2}{8}$.

$\frac{1}{2}$, $\frac{2}{4}$, and $\frac{4}{8}$ name the same location on the number line so they are equivalent.

$\frac{3}{4}$ and $\frac{6}{8}$ name the same location on the number line so $\frac{3}{4}$ is equivalent to $\frac{6}{8}$.

Your child should understand that equivalent fractions on a number line name the same location. She or he should be able to represent equivalent fractions on a number line and defend that the fractions are equivalent.

Addition & Subtraction

Word Problems

1. Your child should be able to solve word problems involving the addition of fractions using a variety of strategies including drawings, properties, and equations.
2. Your child should be able to solve word problems involving the subtraction of fractions using a variety of strategies including drawings, properties, and equations.
3. Your child should be able to estimate mentally the answers to addition and subtraction of fractions word problems.
4. Your child should be able to determine the reasonableness of answers to fraction addition and subtraction word problems.

Addition Strategies

5. Your child should understand that when you add fractions, you combine like sized units. For example, halves with halves, fourths with fourths, eighths with eighths, etc. He or she should understand that it is possible to create equivalent fractions if needed. For example, we cannot combine $^1/_2$ and $^3/_4$ until we rewrite using like sized units (equivalent forms). In this case we can rewrite $^1/_2$ in its equivalent form, $^2/_4$. Now that both fractions are written using like sized units (fourths), we can combine them. Two, one-fourth pieces ($^2/_4$) combined with three, one-fourth pieces ($^3/_4$) is the same as five, one-fourth pieces ($^5/_4$).

$$^1/_2 + {}^3/_4 = {}^2/_4 + {}^3/_4$$
$$= {}^5/_4 \text{ or } 1^1/_4$$

Five, one-fourth pieces can be used to make 1 whole with $^1/_4$ left over ($1^1/_4$).

6. Your child should understand that when you add decimals, you combine like places. For example, tens with tens, ones with ones, tenths with tenths, hundredths with hundredths, and so on. She or he should understand that we can use equivalent forms for a number to write the result in simplest form (10 tenths is equivalent to 1 whole; 10 hundredths is equivalent to 1 tenth).

Bridge to a Whole (Associative Property)

The bridge to a whole strategy is an extension of the bridge to 10 strategy. The bridge to 10 strategy is a powerful addition strategy for your child to know. It was introduced to your child in grade 1. He or she learned that another way to think of 9 + 4 is as 10 + 3 or 8 + 6 as 10 + 4. In grade 2 we used this strategy to solve problems such as 38 + 16 knowing that it is the same as 40 + 14. In grade 3, your child learned that this strategy is really the Associative Property as he or she used it to solve problems such as 538 + 197, linking it to 535 + 200. For example, to solve 538 + 197 we thought of 538 as 535 + 3. Instead of associating the 3 with the 535 we associate it with 197,

$$538 + 197 = (535 + 3) + 197 = 535 + (3 + 197).$$
$$\text{So,}\quad 538 + 197 = 535 + 200 = 735$$

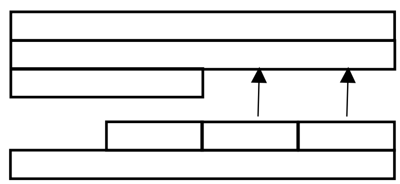
In grade 4 your child extended this idea to adding fractions. In grade 5 your child extends this idea to adding decimals.

$$\begin{aligned} 5.38 + 1.97 &= (5.35 + 0.03) + 1.97 \\ &= 5.35 + (.03 + 1.97) \\ &= 5.35 + 2 \\ &= 7.35 \end{aligned}$$

7. Your child should be able to use the Bridge to a Whole (Associative Property) to add fractions with unlike denominators. An important component of using this strategy with fractions is the drawing of pictures.

   Example 1: $2\frac{1}{2} + 1\frac{3}{4}$. I could draw a picture like the one below.

   But if we rotate the image for $1\frac{3}{4}$ and place it under $2\frac{1}{2}$ it is much easier to see how two of the fourths can slide into the empty space of $2\frac{1}{2}$ to complete the whole (bridge to a whole).



   Note: It doesn't matter which of the $\frac{1}{4}$ pieces we slide to complete the whole.

   We are calling it Bridge to a Whole but it is the Associative Property. To find $2\frac{1}{2} + 1\frac{3}{4}$ we think of $1\frac{3}{4}$ as $\frac{2}{4} + 1\frac{1}{4}$. So $2\frac{1}{2} + 1\frac{3}{4} = 2\frac{1}{2} + (\frac{2}{4} + 1\frac{1}{4})$. Instead of associating the $\frac{2}{4}$ with $1\frac{1}{4}$, we associate it with $2\frac{1}{2}$.

$$2\frac{1}{2} + (\frac{2}{4} + 1\frac{1}{4}) = (2\frac{1}{2} + \frac{2}{4}) + 1\frac{1}{4}$$

   We can rewrite $2\frac{1}{2} + \frac{2}{4}$ using common denominators. Your child should understand that we can rewrite $2\frac{1}{2}$ using fourths or we can rewrite $\frac{2}{4}$ using halves.

$$(2\frac{1}{2} + \frac{2}{4}) + 1\frac{1}{4} = (2\frac{2}{4} + \frac{2}{4}) + 1\frac{1}{4} \quad \text{OR} \quad (2\frac{1}{2} + \frac{2}{4}) + 1\frac{1}{4} = (2\frac{1}{2} + \frac{1}{2}) + 1\frac{1}{4}$$

In either case the result is $3 + 1\frac{1}{4}$ or $4\frac{1}{4}$

$$2\frac{1}{2} + 1\frac{3}{4} = 4\frac{1}{4}$$

8. Your child should be able to use the Bridge to a Whole (Associative Property) to add decimals (decimal fractions).

   Example 1: 5.7 + 3.6

   For this problem we can either bridge to the whole using 5.7 or we can bridge to the whole using 3.6. If we use 5.7, we know that an additional 3-tenths is needed to make the whole. Thus, we think of 3.6 as .3 + 3.3.

$$\begin{aligned} 5.7 + 3.6 &= 5.7 + (.3 + 3.3) \\ &= (5.7 + .3) + 3.3 \\ &= 6 + 3.3 \\ &= 9.3 \end{aligned}$$

If we use 3.6, we know that an additional 4-tenths is needed to make the whole. Thus, we think of 5.7 as 5.3 + .4.

$$\begin{aligned} 5.7 + 3.6 &= (5.3 + .4) + 3.6 \\ &= 5.3 + (.4 + 3.6) \\ &= 5.3 + 4 \\ &= 9.3 \end{aligned}$$

<u>Example 2:</u> 6.89 + 3.97

For this problem we can either bridge to the whole using 6.89 or we can bridge to the whole using 3.97. If we use 6.89, we know that an additional 11-hundredths is needed to make the whole. Thus, we think of 3.97 as .11 + 3.86.

$$\begin{aligned} 6.89 + 3.97 &= 6.89 + (.11 + 3.86) \\ &= (6.89 + .11) + 3.86 \\ &= 7 + 3.86 \\ &= 10.86 \end{aligned}$$

If we use 3.97, we know that an additional 3-hundredths is needed to make the whole. Thus, we think of 6.89 as 6.86 + .03.

$$\begin{aligned} 6.89 + 3.97 &= (6.86 + .03) + 3.97 \\ &= 6.86 + (.03 + 3.97) \\ &= 6.86 + 4 \\ &= 10.86 \end{aligned}$$

<u>Example 3:</u> 8.75 + 2.5

For this problem we can either bridge to the whole using 8.75 or we can bridge to the whole using 2.5. If we use 8.75, we know that an additional 25-hundredths is needed to make the whole. Thus, we think of 2.5 (or 2.50) as .25 + 2.25.

$$\begin{aligned} 8.75 + 2.5 &= 8.75 + (.25 + 2.25) \\ &= (8.75 + .25) + 2.25 \\ &= 9 + 2.25 \\ &= 11.25 \end{aligned}$$

If we use 2.5, we know that an additional 5-tenths or 50-hundredths is needed to make the whole. Thus, we think of 8.75 as 8.25 + .50.

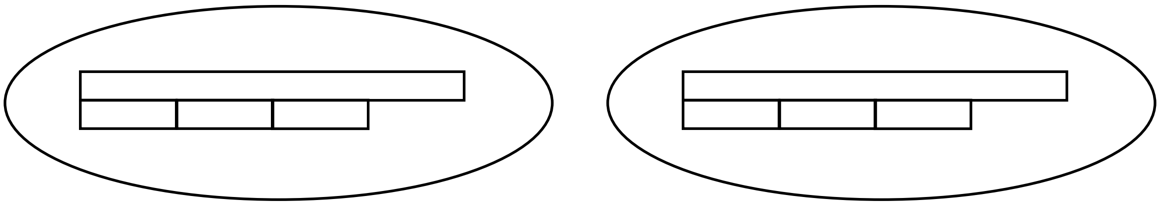
$$\begin{aligned} 8.75 + 2.5 &= (8.25 + .50) + 2.5 \\ &= 8.25 + (.50 + 2.5) \\ &= 8.25 + 3 \\ &= 11.25 \end{aligned}$$

Doubling Strategies

Doubles and near doubles (doubles plus 1, doubles minus 1, doubles plus 2, doubles minus 2) are important addition strategies for your child to know. In grade 1, your child learned to double the numbers 1 to 9 (e.g., double 8 is 16, double 6 is 12). In grade 2 we used these strategies for doubling 2-digit numbers. In grade 3 we used these strategies to quickly solve problems such as 199 + 199. Your child can think of this problem as double 200 minus 2, 56 + 56 as double 50 plus double 6 or 112. In grade 4 we use this same strategy to solve 2 x 199 or 2 x 56. We also used the doubling strategy to double fractions less than one and mixed numbers. In grade 5 we use these strategies to double decimal numbers.

9. Your child is expected to be able to double mixed numbers and record using pictures, an addition equation, and a multiplication equation.

   Example 1: Double $1\frac{3}{4}$.



   Double $1\frac{3}{4}$ can be written as, $1\frac{3}{4} + 1\frac{3}{4}$ or $2 \times 1\frac{3}{4}$. We can describe what we have as 2 groups of 1, or double 1, put together with 2 groups of $\frac{3}{4}$, or double $\frac{3}{4}$.

   $$\text{So } 2 \times 1\tfrac{3}{4} = 2 \times 1 + 2 \times \tfrac{3}{4}.$$

   This is the Distributive Property and is linked to a whole number times a mixed number when doing fraction multiplication.

10. Your child is expected to be able to use doubling and near doubles strategies to double decimal numbers.

    Example 1: 1.99 + 1.99

    To solve we can think of 1.99 + 1.99 as double 2 minus .02. So,

    $$\begin{aligned} 1.99 + 1.99 &= 2 + 2 - .02 \\ &= 4 - .02 \\ &= 3.98 \end{aligned}$$

    Note: This is similar to the strategy we use to solve 199 + 199 or 2 x 199. We think of it as double 200 minus 2.

    Example 2: Double 3.14 OR 3.14 + 3.14 OR 2 x 3.14

    To solve we can think of 3.14 as 3 + .1 + .04.

    To double 3.14 we can double 3, double .1, and double .04.

    $$\begin{aligned} 2 \times 3.14 &= 2 \times (3 + .1 + .04) \\ &= 2 \times 3 + 2 \times .1 + 2 \times .04 \\ &= 6 + .2 + .08 \\ &= 6.28 \end{aligned}$$

    We can also think of 3.14 as 3 + .14

    To double 3.14 we can double 3 and double 0.14.

    $$\begin{aligned} 2 \times 3.14 &= 2 \times (3 + .14) \\ &= 2 \times 3 + 2 \times .14 \\ &= 6 + .28 \\ &= 6.28 \end{aligned}$$

Subtraction Strategies

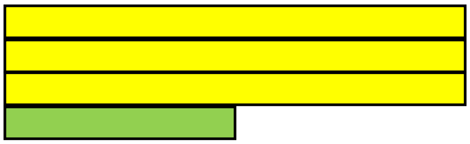
1. Your child should be able to subtract fractions with unlike denominators using a variety of strategies (e.g., round and adjust, decompose a whole, connect to addition, count up, find the missing part). He or she should be able to prove that they correctly used the strategy with fraction strips, drawings, and equations.

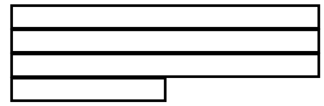
Round and Adjust Strategy

Example: 3½ – 1¾

To illustrate the strategy, we begin with showing 3½ using fraction kit materials and with drawings.

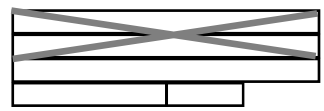
Materials | Drawings





To remove 1¾, we remove 2 and then give back ¼. We rounded 1¾ to 2 and then needed to adjust (give ¼ back) because we took away one-fourth too many. The illustrations below show the changes in our materials and drawing after using this strategy.

Materials | Drawings



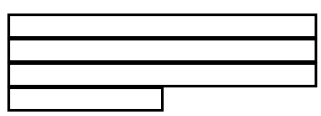
I removed 2 wholes and gave back ¼. (Note: It doesn't matter which of the wholes are removed or crossed out.)

$$\begin{aligned} 3\tfrac{1}{2} - 1\tfrac{3}{4} &= 3\tfrac{1}{2} - 2 + \tfrac{1}{4} \\ &= 1\tfrac{1}{2} + \tfrac{1}{4} \\ &= 1\tfrac{3}{4} \end{aligned}$$

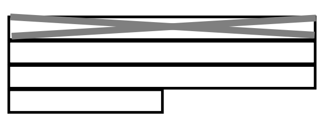
Decompose a Whole

Example: 3½ – 1¾

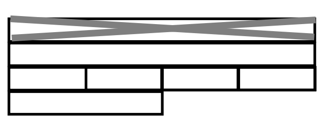
To illustrate the strategy, we begin by drawing 3½.



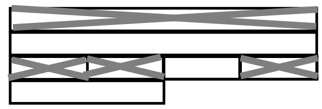
We think of 1¾ as 1 and ¾ (1 + ¾). First remove 1.



We don't have ¾ to remove so we "cut" a whole into fourths (Note: It doesn't matter which whole we cut into fourths).



We “remove” 3, one-fourth pieces and we are left with the answer (Note: It doesn’t matter which of the one-fourth pieces we remove).



$$3\tfrac{1}{2} - 1\tfrac{3}{4} = 3\tfrac{1}{2} - 1 - \tfrac{3}{4} = 1\tfrac{3}{4}$$

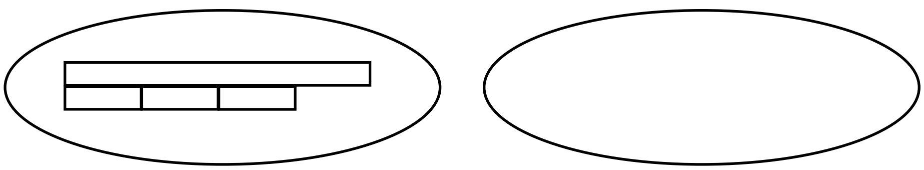
<u>Connect to Addition (Missing Addend)</u>

Using this strategy we think of problems such as $3\frac{1}{2} - 1\frac{3}{4}$ as how many more than $1\frac{3}{4}$ is $3\frac{1}{2}$ or how many do we need to add to $1\frac{3}{4}$ to get to $3\frac{1}{2}$.

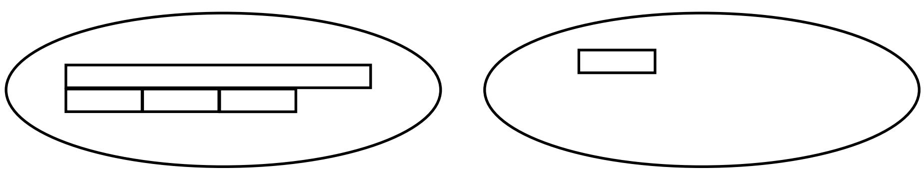
$$1\tfrac{3}{4} + \_\_\_ = 3\tfrac{1}{2}$$

<u>Example:</u> $1\frac{3}{4} + \_\_\_\_ = 3\frac{1}{2}$

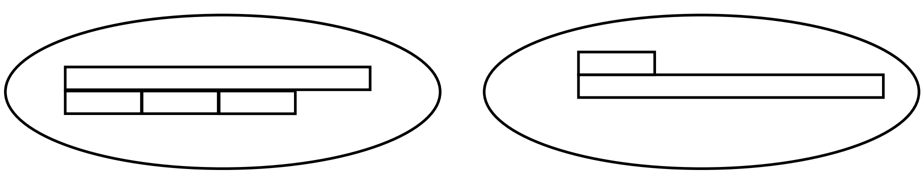
To illustrate the strategy, we begin with drawing the part you know ($1\frac{3}{4}$) in one section (the loop on the left). We will draw the pieces for the missing part in the other section.



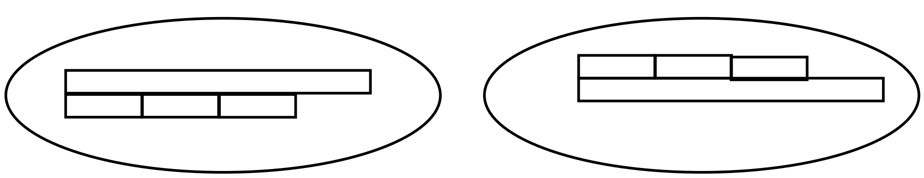
We add fourths and 1s until we reach the total ($3\frac{1}{2}$). For example, if we add $\frac{1}{4}$, we have a total of 2. (**Note:** I don’t have to begin by adding fourths. I could add a 1 first until I get close to the goal number.)



We add a whole and we have a total of 3.



We then add two more fourths (or $\frac{1}{2}$) and we have a total of $3\frac{1}{2}$.



The missing part is 1¾.

1¾ + [1¾] = 3½ so 3½ – 1¾ = [1¾]

<u>Counting Up on an Open Number Line</u>

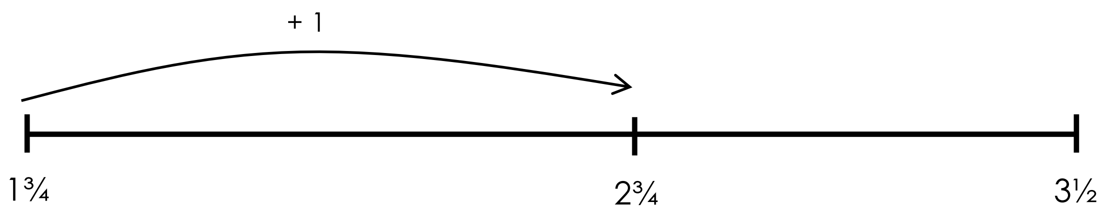
Using this strategy we think of problems such as 3½ – 1¾ as how many more than 1¾ is 3½ or how many do we need to add to 1¾ to get to 3½. We use an open number line to find the missing addend.

1¾ + ___ = 3½

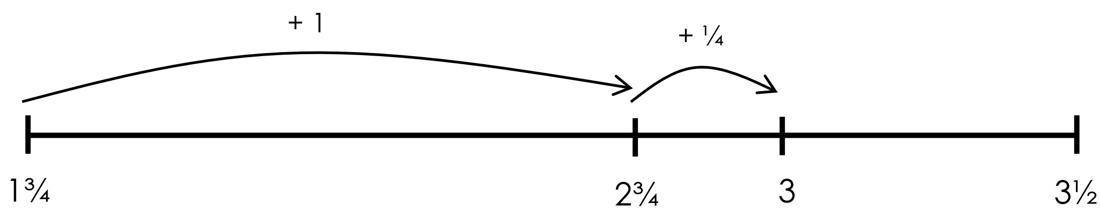
<u>Example:</u> 1¾ + ____ = 3½

To illustrate the strategy, we begin with an open number. 1¾ is the left endpoint. 3½ is the right endpoint.

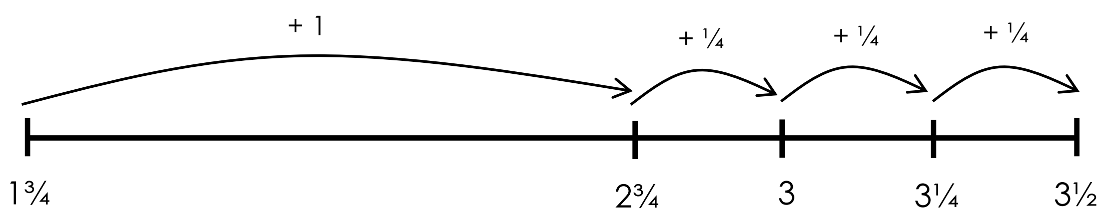
We can make a jump of 1 to get to 2¾ (Count up 1).



We can make a jump of ¼ to get to 3 (Count up ¼).



We can make two jumps of ¼ to get to 3½ (Count up ¼).



We combine the jumps to get the missing addend.

1¾ + **<u>1 + ¼ + ¼ + ¼</u>** = 3½

1¾ + [1¾] = 3½ so 3½ – 1¾ = [1¾]

2. Your child should be able to subtract decimals using a variety of strategies (e.g., round and adjust, decompose a whole, connect to addition, count up, find the missing part). He or she should be able to prove that they correctly used the strategy with drawings and equations.

Round and Adjust Strategy

Example: 8.4 – 3.8

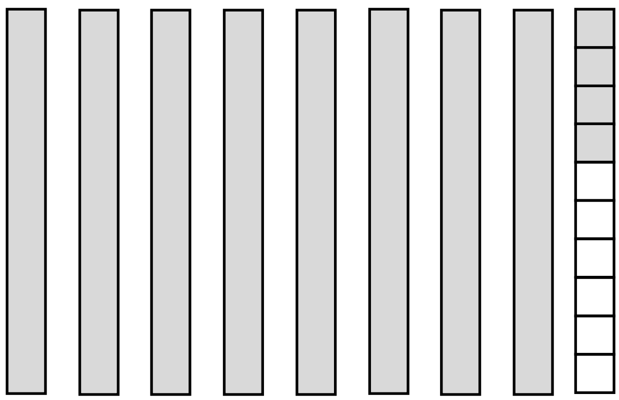
To remove 3.8, we remove 4 and then give back 2-tenths (0.2). We rounded 3.8 to 4 and then needed to adjust (give .2 back) because we took away two-tenths too many.

$$\begin{aligned} 8.4 - 3.8 &= 8.4 - 4 + .2 \\ &= 4.4 + .2 \\ &= 4.6 \end{aligned}$$

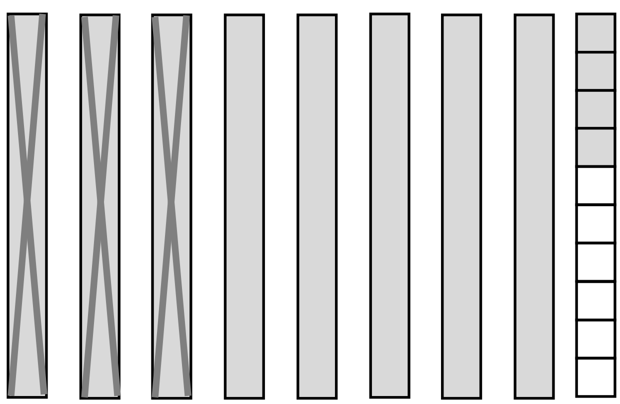
Decompose a Whole

Example: 8.4 – 3.8

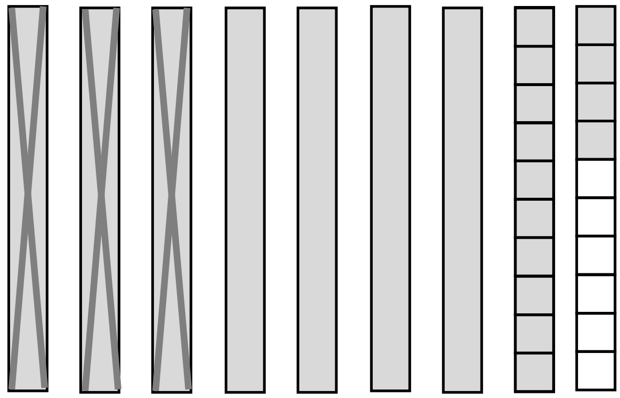
To illustrate the strategy, we begin by drawing 8.4.



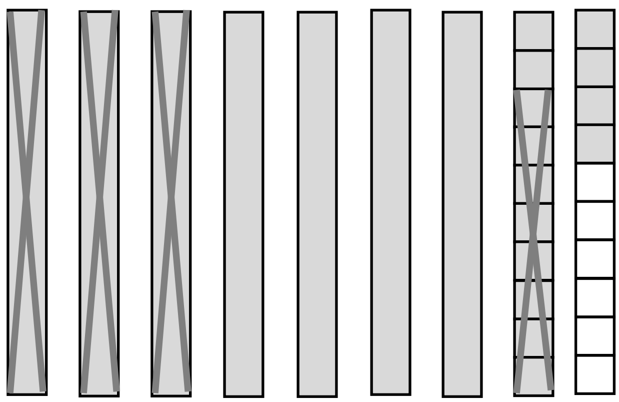
We think of 3.8 as 3 and .8 (3 + .8). First remove 3.



We don't have .8 to remove so we "cut" a whole into tenths (Note: It doesn't matter which whole we cut into tenths).



We can think of ten-tenths as two-tenths put together with eight-tenths. We "remove" 8, one-tenth pieces and we are left with the answer (Note: It doesn't matter which of the one-tenth pieces we remove).



$$\begin{aligned} 8.4 - 3.8 &= 8.4 - 3 - .8 \\ &= 5.4 - .8 \\ &= 4.4 + (.2 + .8) - .8 \\ &= (4.4 + .2) + .8 - .8 \\ &= 4.6 \end{aligned}$$

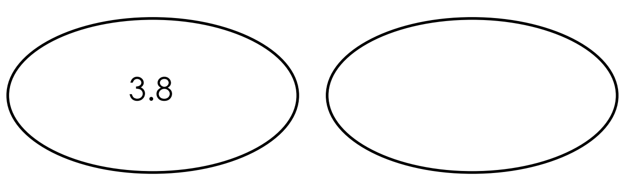
<u>Connect to Addition (Missing Addend)</u>

Using this strategy we think of problems such as 8.4 – 3.8 as how many more than 3.8 is 8.4 or how many do we need to add to 3.8 to get to 8.4.

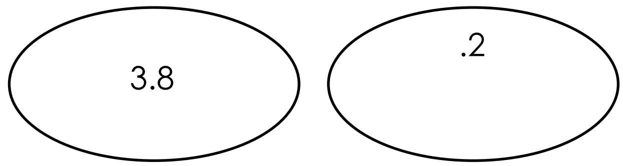
3.8 + ___ = 8.4

<u>Example 1:</u> 3.8 + ____ = 8.4

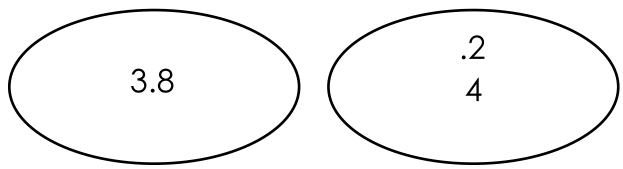
To illustrate the strategy, we begin with writing the part we know (3.8) in one section (the loop on the left). We will write the missing parts in the other section.



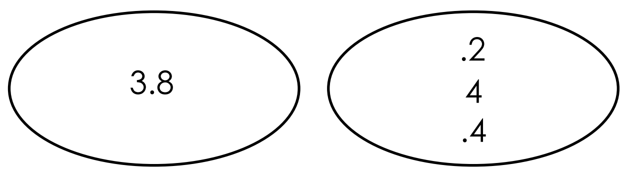
We add tenths and 1s until we reach the total (8.4). For example, if we add .2, we have a total of 4. (**Note:** I don't have to begin by adding tenths. I could add 1s first until I get close to the goal number.)



We add 4 and we have a total of 8.



We then add four more tenths (or .4) and we have a total of 8.4.

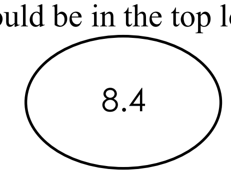


The missing part is 4.6.

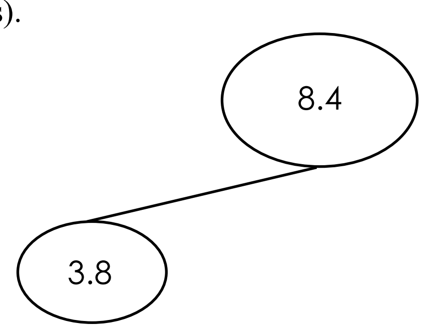
3.8 + [4.6] = 8.4 so 8.4 – 3.8 = [4.6]

Example 2: 3.8 + ____ = 8.4 using a number bond

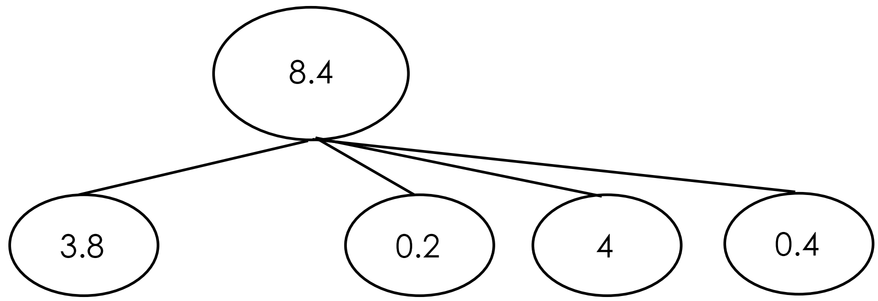
Another representation your child is expected to understand and use is a number bond. A number bond is similar to the image above except each number would be separated. For example, the goal number would be in the top loop. In this case it would be 8.4.



There would then be branches coming off of the top loop that shows the parts into which we can separate 8.4. We already know one of the parts. Our goal is to determine the missing part(s).



We can be strategic in determining the missing parts as we were above. We find a part that takes us to the nearest whole number. In this case it would be 0.2. We then find the part that gets us close to the goal number. Four will get us to 8. We then find the part that gets us to the goal number. 0.4.



<u>Counting Up on an Open Number Line</u>

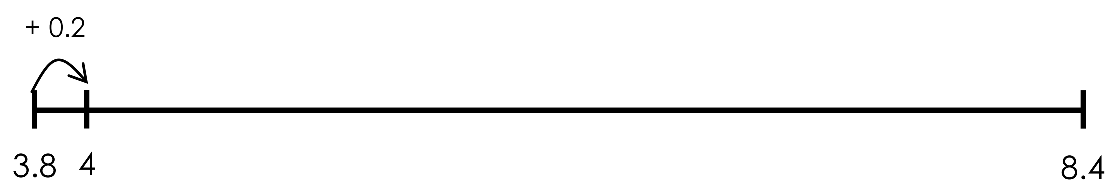
Using this strategy we think of problems such as 8.4 – 3.8 as how many more than 3.8 is 8.4 or how many do we need to add to 3.8 to get to 8.4. We use an open number line to find the missing addend.

3.8 + ___ = 8.4

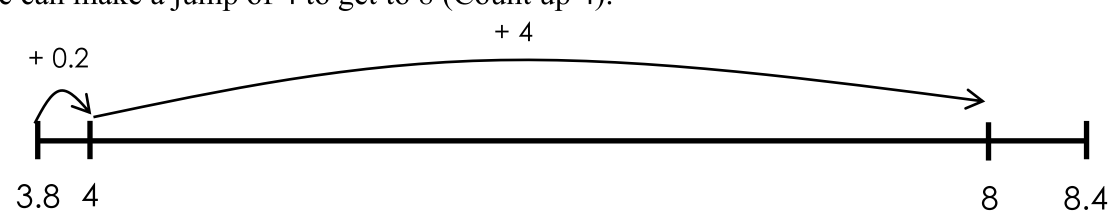
<u>Example:</u> 3.8 + ____ = 8.4

To illustrate the strategy, we begin with an open number. 3.8 is the left endpoint. 8.4 is the right endpoint.

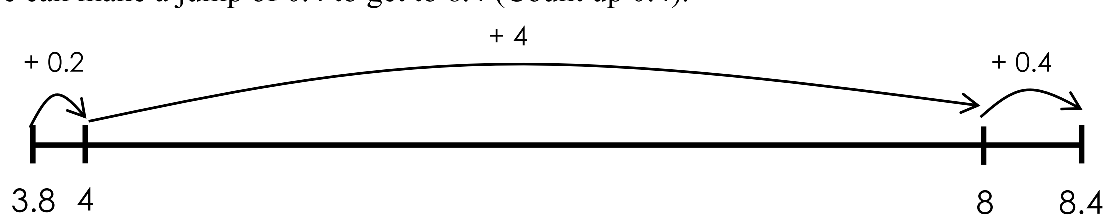
We can make a jump of 0.2 to get to 4 (Count up 0.2).



We can make a jump of 4 to get to 8 (Count up 4).



We can make a jump of 0.4 to get to 8.4 (Count up 0.4).



We combine the jumps to get the missing addend.

$$3.8 + \underline{\mathbf{\ 0.2 + 4 + 0.4\ }} = 8.4$$

$$3.8 + \boxed{4.6} = 8.4 \quad \text{so} \quad 8.4 - 3.8 = \boxed{4.6}$$

Multiplication and Division

Multiplication Concepts

1. Your child should understand the effect of multiplying one factor (number) by another (multiplication as scaling). He or she should understand that multiplication does not always result in a larger number. That is, if a factor is multiplied by a number greater than one, the answer will be larger than the starting factor. However, if a factor is multiplied by a number less than one, the answer will be less than the starting factor.

   Example 1: 4 x 8 (4 groups of 8) results in 32. This is 4 times as many as 8.

   Example 2: ½ x 8 results in 4. This is half of 8, less than the starting factor.

   After examining many examples of multiplying by a number greater than one, equal to one, and less than 1, your child should be able to describe the effects of multiplication.

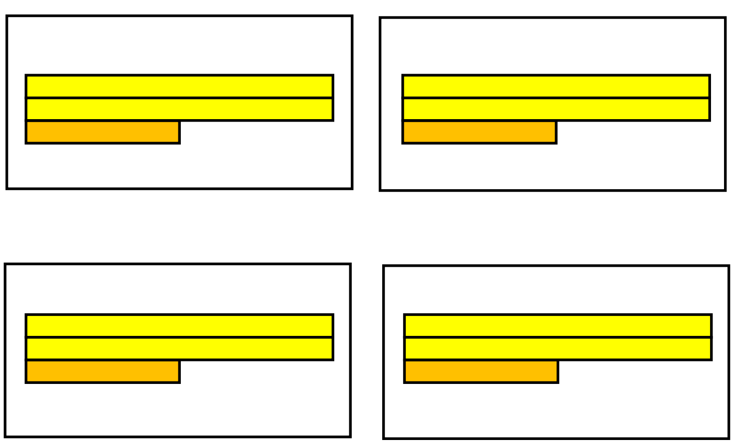
Equal Grouping Model

2. Your child should understand and use the equal grouping model to multiply a fraction by a whole number. He or she should understand its relationship to the Distributive Property.

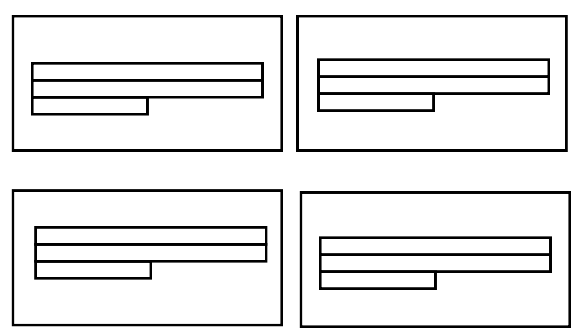
   Example 1: 4 x 2½

4 x 2½
4 groups of 2½

Materials



Drawing



To represent 4 groups of 2½ we used 4 groups of 2 (4 x 2) and 4 groups of ½ (4 x ½). This is the Distributive Property.

To make $2\frac{1}{2}$ we think of it as $2 + \frac{1}{2}$ (expanded form).

$$4 \times 2\tfrac{1}{2} = 4 \times (2 + \tfrac{1}{2}) = (4 \times 2) + (4 \times \tfrac{1}{2})$$

4 groups of $2\frac{1}{2}$ is the same as 4 groups of 2 put together with 4 groups of $\frac{1}{2}$.

Note: In $(4 \times 2) + (4 \times \frac{1}{2})$ parentheses are not required but are helpful. For this problem your child would be correct if she or he wrote, $4 \times 2 + 4 \times \frac{1}{2}$.

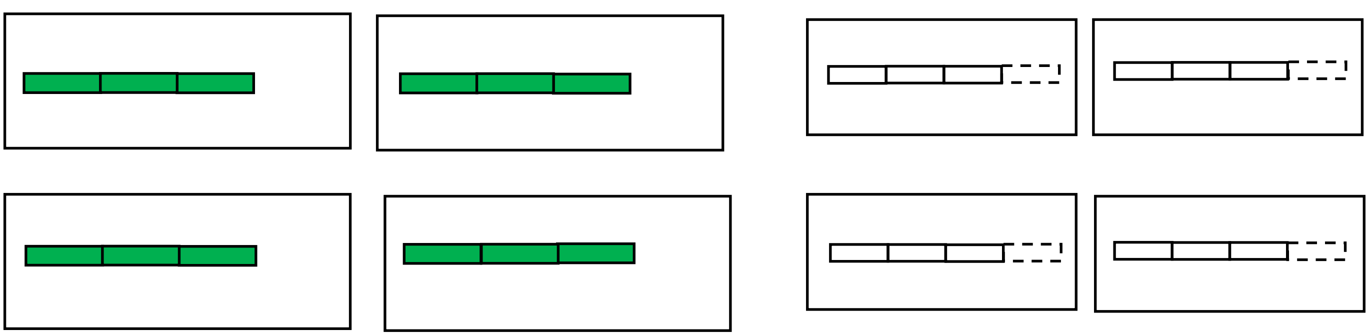
$$\begin{aligned} 4 \times 2\tfrac{1}{2} = 4 \times (2 + \tfrac{1}{2}) &= (4 \times 2) + (4 \times \tfrac{1}{2}) \\ &= 8 + 2 \\ &= 10 \end{aligned}$$

3. Your child should understand and use an equal grouping model representation of the Associative Property to find the product of a whole number times a fraction less than one.

Example 1: $4 \times \frac{3}{4}$.

4 x ¾
4 groups of ¾

Materials | Drawing



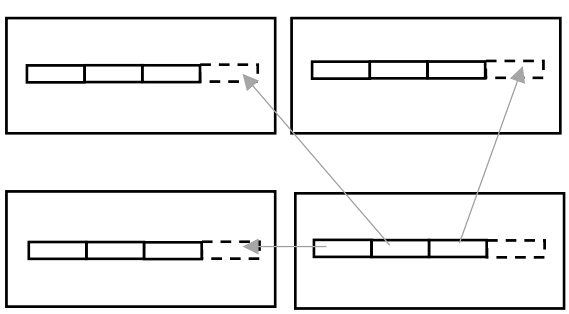
3, one-fourth pieces is needed for each group $(3 \times \frac{1}{4})$.
4 groups of 3, one-fourth pieces is the same as $4 \times (3 \times \frac{1}{4})$.
Instead of associating the 3 with the $\frac{1}{4}$ we can associate it with the 4, $(4 \times 3) \times \frac{1}{4}$.

$$4 \times \tfrac{3}{4} = (4 \times 3) \times \tfrac{1}{4} = 12 \times \tfrac{1}{4}.$$

12, one-fourth pieces can be used to make 3 wholes.

Students can also find this answer by noticing that if they take the 4th group of $\frac{3}{4}$, they can use each of the $\frac{1}{4}$ pieces to complete the whole for the other 3 groups.

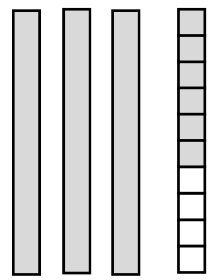
$$4 \times \tfrac{3}{4} = 3$$



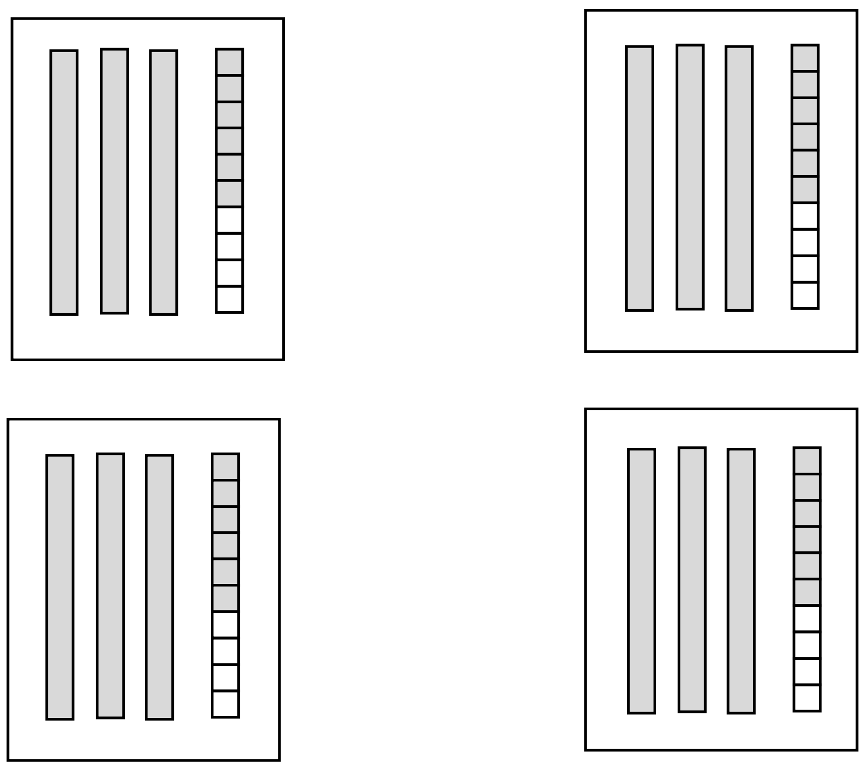
4. Your child should understand and use the equal grouping model to multiply a whole number times a decimal number. He or she should understand its relationship to the Distributive Property.

   <u>Example 1:</u> 4 x 3.6 (4 groups of 3.6)

   We think of 3.6 as 3 + .6.

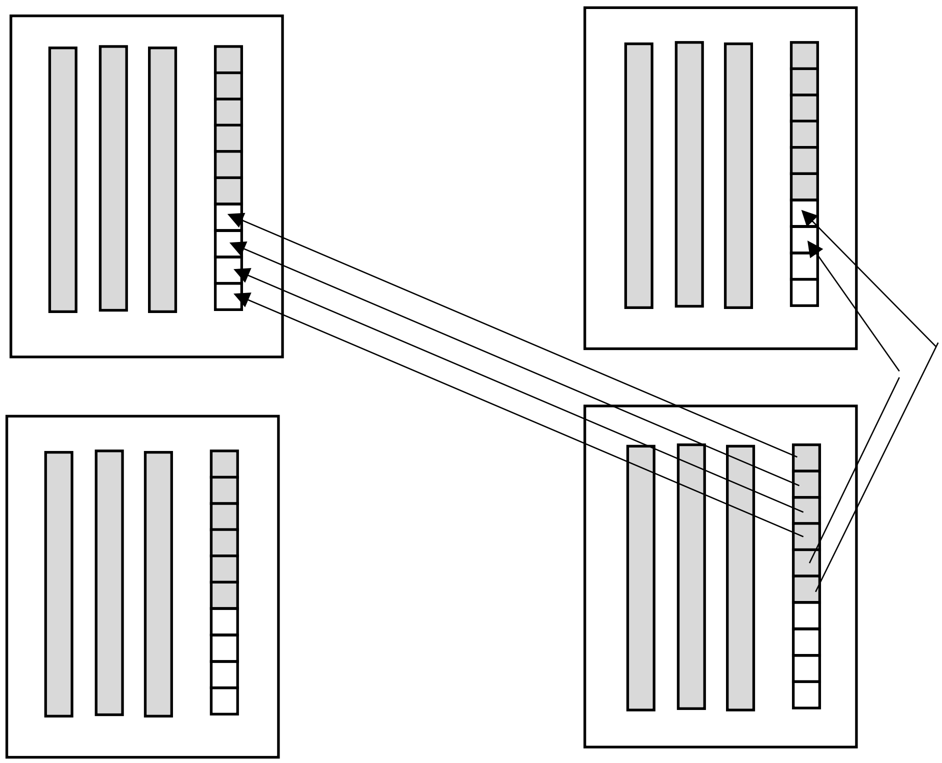


   Four groups of 3.6 would consist of 4 groups of 3 (4 x 3) put together with 4 groups of 0.6 (4 x .6).

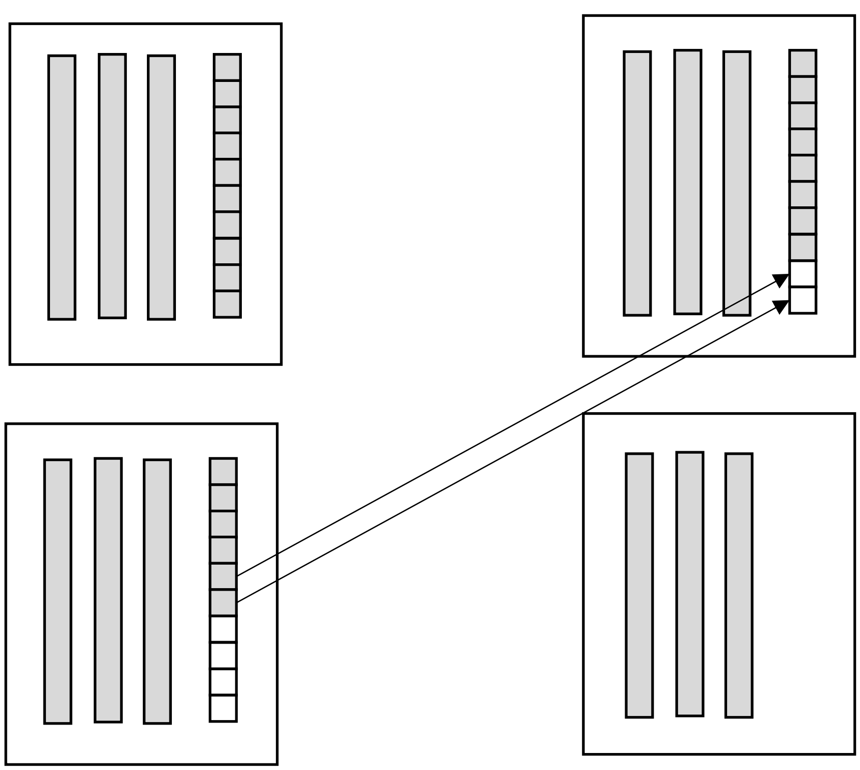


$$4 \times 3.6 = 4 \times 3 + 4 \times .6$$
(This is the Distributive Property)
$$= 12 + 2.4$$
(12 and 2.4 would also be called partial products)
$$= 14.4$$

   If your child has difficulty with 4 x .6 = 2.4 he or she can use the picture to help him or her think about how the wholes can be filled. For example, 4 of the one-tenth pieces in the last group can be used to complete the whole in group 1. The remaining 2, one-tenth pieces can be used to help fill the whole in group 2.



Two more one-tenth pieces would be needed from group 3 to finish filling the whole in group 2, leaving 4 one-tenth pieces in that group.



Your child may also need additional experience with the effects of multiplying by powers of 10 and dividing by powers of 10. To multiply 4 x .6 we can use a known fact, 4 x 6 to get 24. We then take a tenth of the answer since .6 was the factor instead of 6.

Since 4 x 6 = 24
4 x .6 = 2.4

OR

4 x .6 = 4 x 6 x .1
= 24 x .1
= 2.4

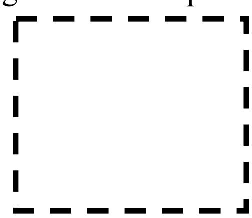
Area Model

1. Your child should use their understanding of the area model for multiplication to multiply a fraction by a fraction.

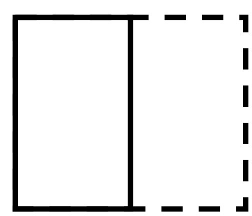
   Example 1: $^1/_3$ x $^1/_2$

   We can use the area model to represent problems such as $^1/_3$ x $^1/_2$. We can read this as one-third by one-half or one-third of one-half.

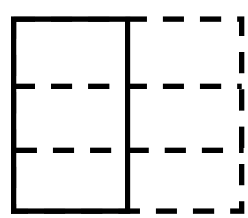
   - We begin with a rectangular whole represented with a dashed line.



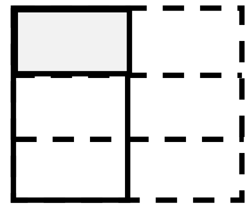
   - We cut the whole into 2 equal parts and put a solid line around one-half.



   - We now want to cut the halves into 3 equal pieces. Show with a dashed line extended across the whole.



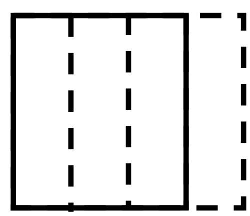
- Shade in a third of the half. We see that $^1/_3$ x $^1/_2$ is $^1/_6$.



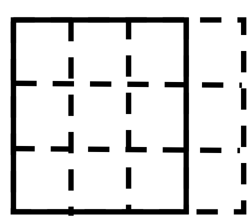
<u>Example 2:</u> $^2/_3$ x $^3/_4$.

- We begin with a rectangular whole represented with a dashed line.

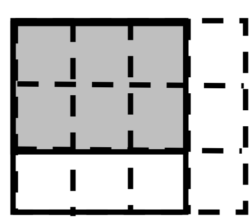
- We cut the whole into 4 equal parts and put a solid line around three-fourths.



- We now want to cut the fourths into 3 equal pieces so that we can find two-thirds of three-fourths. Show with a dashed line extended across the whole.



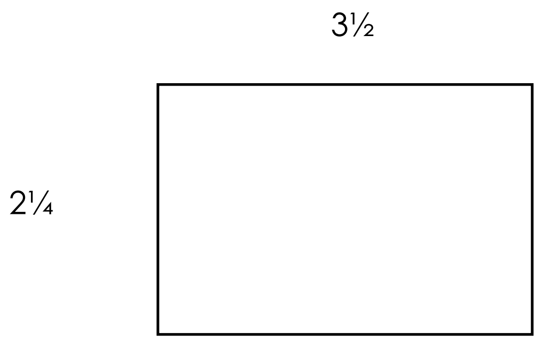
- Shade in two-thirds of the three-fourths. We see that $^2/_3$ x $^3/_4$ is $^6/_{12}$.



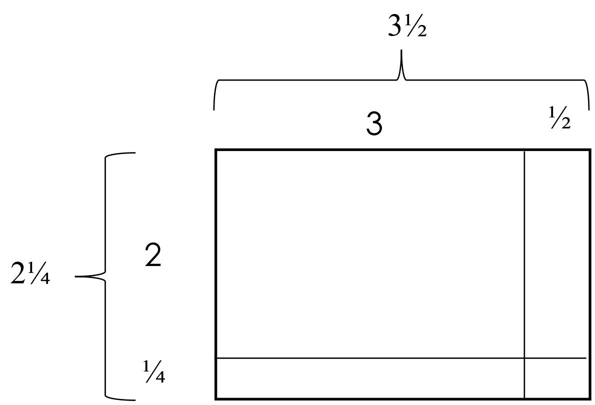
2. Your child should be able to represent mixed number multiplication using the area model. She or he should understand its relationship to the Distributive Property.

   Example 2: 2¼ x 3½

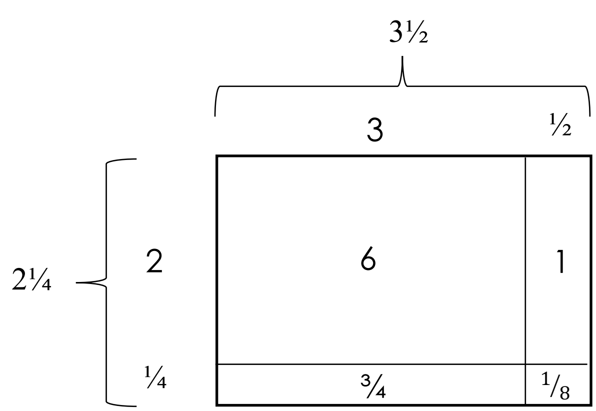
   2¼ x 3½
   2¼ by 3½



Cut the area into smaller sections using expanded form. Thinking of 2¼ as 2 + ¼ and 3½ as 3 + ½ we'll have 4 rectangles: one with the dimensions 2 by 3, one with the dimensions 2 by ½, one with the dimensions ¼ by 3, and one with the dimensions ¼ by ½.



2¼ x 3½ = (2 + ¼) x (3 + ½)
OR (2 x 3) + (2 x ½) + (¼ x 3) + (¼ x ½)
This is the Distributive Property.



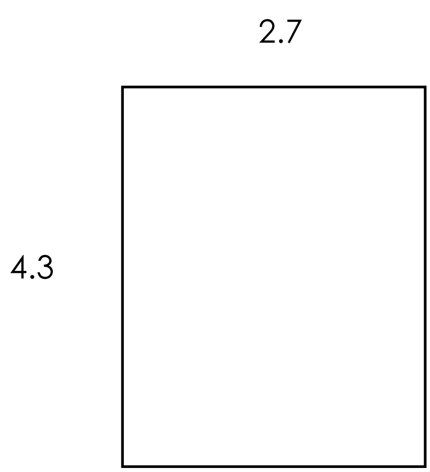
$$2\tfrac{1}{4} \times 3\tfrac{1}{2} = (2 + \tfrac{1}{4}) \times (3 + \tfrac{1}{2})$$
$$= (2 \times 3) + (2 \times \tfrac{1}{2}) + (\tfrac{1}{4} \times 3) + (\tfrac{1}{4} \times \tfrac{1}{2})$$
$$= 6 + 1 + \tfrac{3}{4} + \tfrac{1}{8}$$
$$= 7 + \tfrac{6}{8} + \tfrac{1}{8}$$
$$= 7 + \tfrac{7}{8}$$
$$= 7\tfrac{7}{8}$$

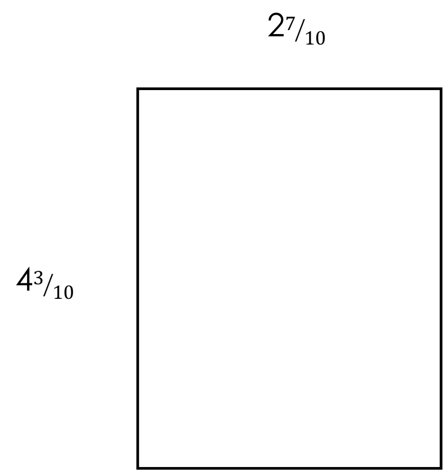
3. Your child should be able to represent decimal multiplication using the area model. She or he should understand its relationship to the Distributive Property.

Example1: 4.3 x 2.7

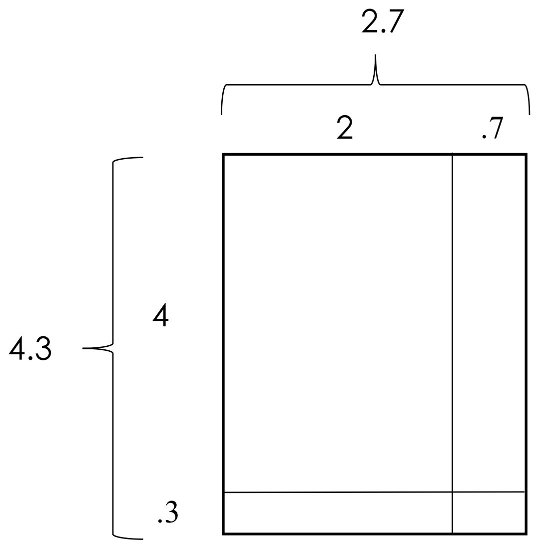
The representations used for mixed number multiplication can be used for decimal multiplication. Some students may find that it is easier to use the common fraction representations for four and three-tenths and two and seven-tenths and then translate the end result back to decimal fraction representations. For example,

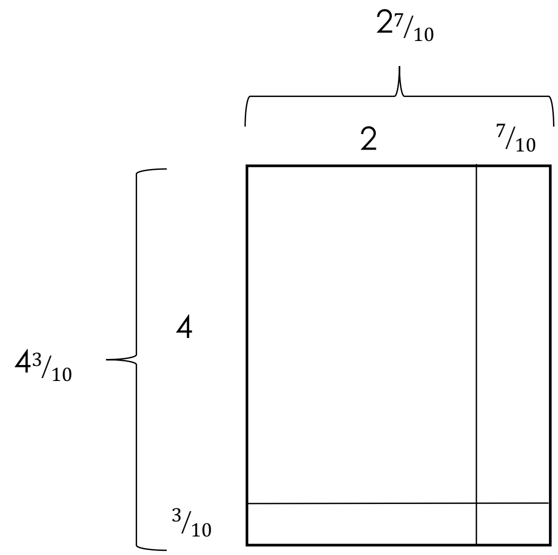
Decimal Fraction	Common Fraction
4.3 x 2.7 4.3 by 2.7	$4\frac{3}{10} \times 2\frac{7}{10}$ $4\frac{3}{10}$ by $2\frac{7}{10}$





Cut the area into smaller sections using expanded form. Thinking of 4.3 as 4 + .3 and 2.7 as 2 + .7 we'll have 4 rectangles: one with the dimensions 4 by 2, one with the dimensions 4 by .7, one with the dimensions .3 by 2, and one with the dimensions .3 by .7.

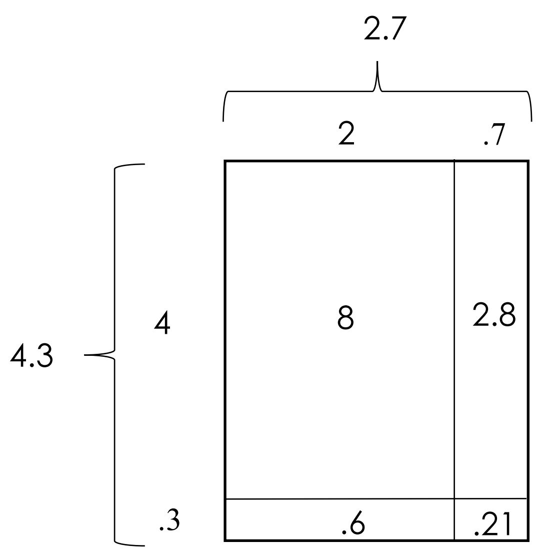


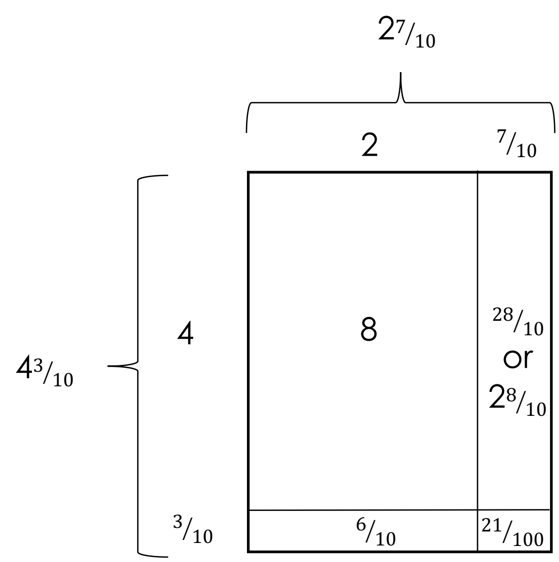


4.3 x 2.7 = (4 + .3) x (2 + .7)
OR (4 x 2) + (4 x .7) + (.3 x 2) + (.3 x .7)
This is the Distributive Property.

$4^{3}/_{10}$ x $2^{7}/_{10}$ = (4 + $^{3}/_{10}$) x (2 + $^{7}/_{10}$)
OR (4 x 2) + (4 x $^{7}/_{10}$) + ($^{3}/_{10}$ x 2) + ($^{3}/_{10}$ x $^{7}/_{10}$)
This is the Distributive Property.

Your child may find that it is easier to complete the rectangle on the right and use those values to fill in the rectangle on the left.



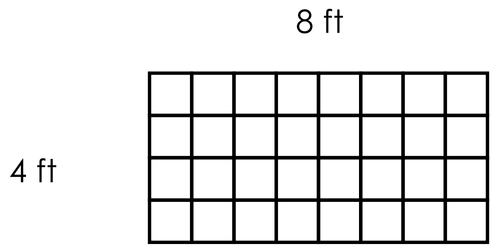


4.3 x 2.7 = (4 + .3) x (2 + .7)
= (4 x 2) + (4 x .7) + (.3 x 2) + (.3 x .7)
= 8 + 2.8 + .6 + .21
= 8 + 3.4 + .21
= 8 + 3.61
= 11.61

$4^{3}/_{10}$ x $2^{7}/_{10}$ = (4 + $^{3}/_{10}$) x (2 + $^{7}/_{10}$)
= (4 x 2) + (4 x $^{7}/_{10}$) + ($^{3}/_{10}$ x 2) + ($^{3}/_{10}$ x $^{7}/_{10}$)
= 8 + $2^{8}/_{10}$ + $^{6}/_{10}$ + $^{21}/_{100}$
= 8 + $3^{4}/_{10}$ + $^{21}/_{100}$
= 8 + $3^{61}/_{100}$
= $11^{61}/_{100}$

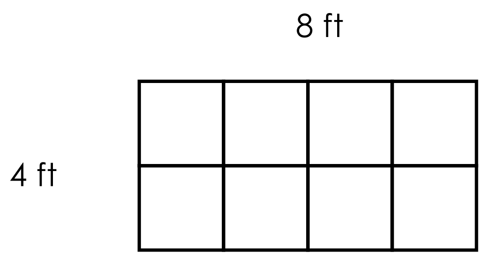
4. Your child should understand that an area can be tiled with square tiles that have a dimension other than 1. He or she should understand that the area of each tile is determined by the side lengths. The area of the entire shape can be found by finding the sum of the areas of the tiles.

   <u>Example 1:</u> We could tile a 4 ft by 8 ft area with 1 ft by 1 ft tiles.
   Each 1 ft by 1 ft tile would have an area of 1 square foot.



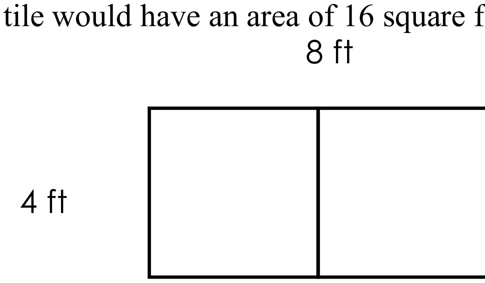
   The area of this shape would be 32 sq. ft.

   <u>Example 2:</u> We could tile the 4 ft by 8 ft area with 2 ft by 2 ft tiles.
   Each 2 ft by 2 ft tile would have an area of 4 square feet.



   The area is still 32 sq. ft. Instead of thirty-two 1 sq. ft. tiles, we used eight, 4 sq. ft. tiles.

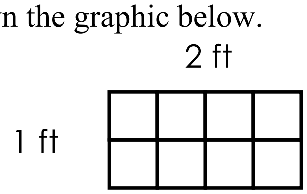
   <u>Example 3:</u> We could also tile the 4 ft by 8 ft area with 4 ft by 4 ft tiles.
   Each 4 ft by 4 ft tile would have an area of 16 square feet.



   The area is again 32 sq ft. This time we used two 16 sq. ft. tiles.

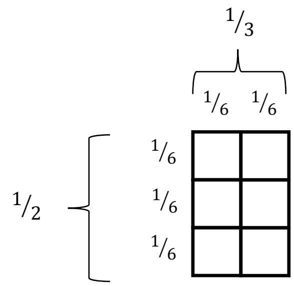
   <u>Example 4:</u> Your child should understand that we can tile an area with a variety of different sized tiles. The area of each of the tiles is determined by the tile's side length. Sometimes the side lengths are fractional units.

   Suppose we were shown the graphic below.



The tiles used to tile the 1 ft by 2 ft region were ½ ft by ½ ft each. The area for a ½ ft by ½ ft tile is ¼ square foot. If we add eight quarter square ft tiles together we get the expected 2 square feet.

Example 5: Your child is expected to be able to extend the above strategies to tiling regions with fractional side lengths. Suppose we have a region that is $^1/_2$ by $^1/_3$. Your child would need to find a square tile that can be used to tile this region (using whole tiles). The tile needed can be determined by finding the common denominator (common unit fraction piece) for $^1/_2$ and $^1/_3$, $^1/_6$.



The area of each of the $^1/_6$ by $^1/_6$ tiles is $^1/_{36}$ square units.
Six, $^1/_{36}$ square units is $^6/_{36}$ or $^1/_6$ square units.

Division

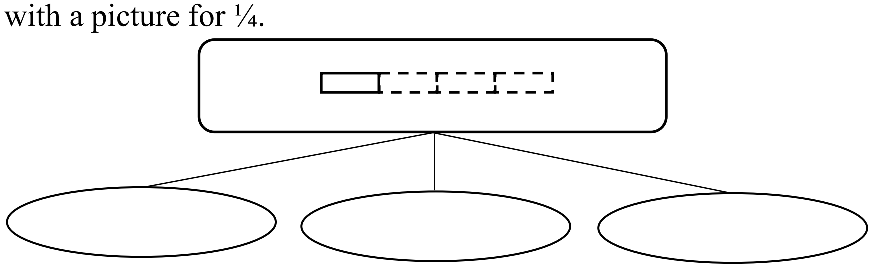
1. Your child should understand that multiplication and division are inverse operations. That is, your child should understand that:
   - if multiplication is combining equal groups, division is separating into equal groups or pulling off equal groups of a given number.
   - if multiplication is combining equal rows, then division is separating into equal rows or pulling off equal rows of a given number.
   - if multiplication is combining equal jumps, then division is separating into equal jumps.
   - if multiplication is finding the area when given the dimensions of a rectangle, then division is finding an unknown side length when the area and one side length is known.

2. Your child should be able to divide a unit fraction by a whole number using a drawing. She or he should be able to connect the division of unit fractions by a whole number to multiplication (e.g., 1/3 ÷ 4 = 1/12 is the same as 4 x 1/12 = 1/3).

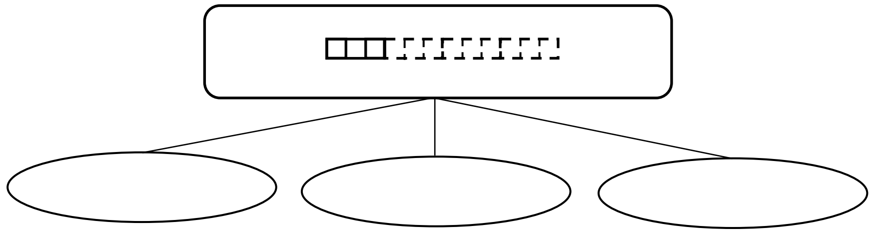
Example: ¼ ÷ 3

¼ ÷ 3
¼ separated into 3 equal groups

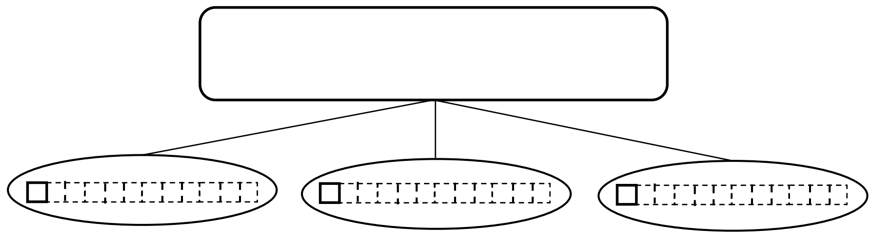
We begin with a picture for ¼.



We separate the ¼ into 3 equal pieces (**Note**: to make it easier to determine the size of the new pieces, we divide each of the fourths into 3 equal pieces using dashed lines.)



We place one piece (1/12) in each group to show the result.



$^1/_4 \div 3 = {}^1/_{12}$

OR

$3 \times \_\_\_ = {}^1/_4$

We can also relate this equation to $^1/_4 \times \_\_\_ = {}^1/_{12}$

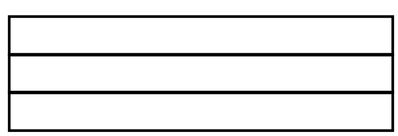
In this last example we begin to connect division to multiplying by the reciprocal. For our example, the reciprocal of 3 (or $^3/_1$) is $^1/_3$.

3. Your child should be able to divide a whole number by unit fractions and represent with fraction materials, drawings, and equations.

<u>Example:</u> $3 \div {}^1/_4$

3 ÷ ¼
How many fourths are in 3?

We can begin by drawing 3 wholes using the fraction strip representation that your child used in grades 3 and 4.



We then cut each whole into fourths.

We see that each whole is made up of 4, one-fourth pieces. There is a total of 12, one-fourth pieces in 3 wholes.

$3 \div {}^1/_4$ 12

Division using the equal grouping model and partial quotients

1. Your child should be able to divide up to 4-digit numbers by 1-digit or 2-digit numbers using the equal grouping model and partial quotients (Distributive Property).

   Example 1: 5336 ÷ 4 using an equal grouping model

   - Begin by translating the division problem into language that helps bring imagery. When dividing by a single digit number it is still efficient to think of division as separating into equal groups. We can translate 5336 ÷ 4 as 5336 separated into 4 equal groups.

5336 ÷ 4 5336 separated into 4 equal groups

   - We can first put 1000 in each group.

1000	1000	1000	1000

   - We've used 4000 of 5336. We have 1336 remaining (5336 – 4000).
   - We know that an equivalent way to think of 1 thousand, 3 hundreds, 3 tens, and 6 ones is as 13 hundreds, 3 tens, and 6 ones.
   - We can put 3 hundreds in each group.

1000 300	1000 300	1000 300	1000 300

   - We've used 1200 of the remaining 1336. We have 136 remaining (1336 – 1200).
   - We know that an equivalent way to think of 1 hundred, 3 tens, and 6 ones is as 13 tens and 6 ones.
   - We can put 3 tens in each group.

1000 300 30	1000 300 30	1000 300 30	1000 300 30

   - We've used 120 of the remaining 136. We have 16 remaining (136 – 120).
   - We know that an equivalent way to think of 1 ten and 6 ones is as 16 ones.
   - We can put 4 ones in each group.

1000 300 30 4	1000 300 30 4	1000 300 30 4	1000 300 30 4

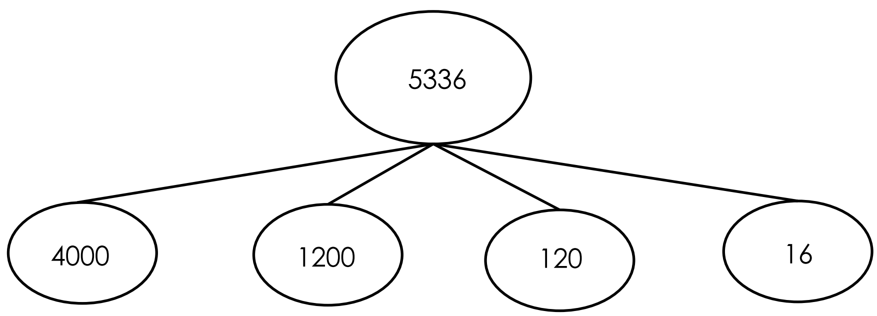
- To find 5336 ÷ 4 we thought of 5336 as 4000 + 1200 + 120 + 16. Notice that each of the addends are multiples of 4 (the divisor).

  So, 5336 ÷ 4 = (4000 ÷ 4) + (1200 ÷ 4) + (120 ÷ 4) + (16 ÷ 4)
  = 1000 + 300 + 30 + 4
  = 1334

  4000 ÷ 4, 1200 ÷ 4, 120 ÷ 4, 16 ÷ 4 are partial quotients.

<u>Example 2</u>: 5336 ÷ 4 using an equal grouping model and a number bond representation.

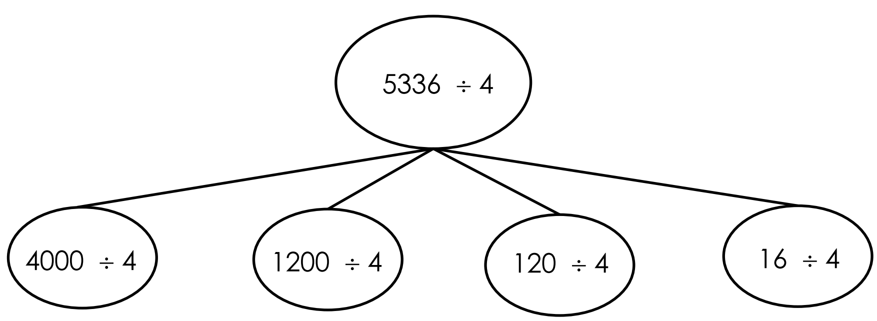
- We can use a number bond representation to show ways to separate 5336 (the dividend) into multiples of 4 (the divisor). A number bond is a part-part-total or total - part-part representation (there can be more than 2 parts). In the example below, the total is given in the top bubble. Parts that make up the total are given in the 4 bubbles below. Although there are many ways to separate 5336 into parts, we are requiring the parts to be multiples of 4 since we are dividing by 4.



  Notice that 4000 + 1200 + 120 + 16 is equal to 5336. Each of those numbers are also multiples of 4.

- We can then use this representation to show the partial quotients.

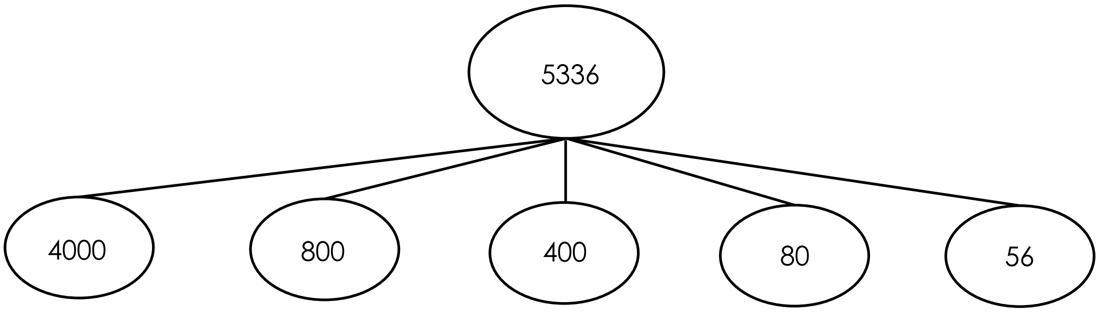
5336 ÷ 4
5336 separated into 4 equal groups



5336 ÷ 4 = (4000 ÷ 4) + (1200 ÷ 4) + (120 ÷ 4) + (16 ÷ 4)
= 1000 + 300 + 30 + 4
= 1334

Example 3: 5336 ÷ 4 using an equal grouping model and a number bond representation.

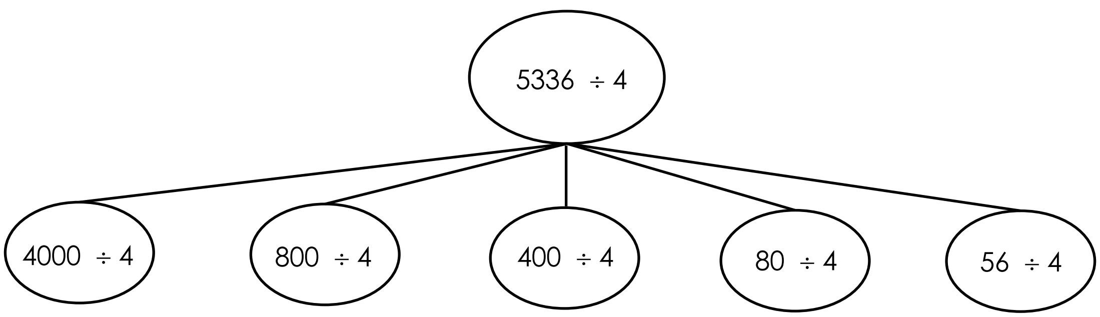
- There are many ways to separate 5336 into multiples of 4. One other way is shown below.



Notice that 4000 + 800 + 400 + 80 + 56 is equal to 5336 and each of those numbers are multiples of 4.

- We can then use this representation to show the partial quotients.

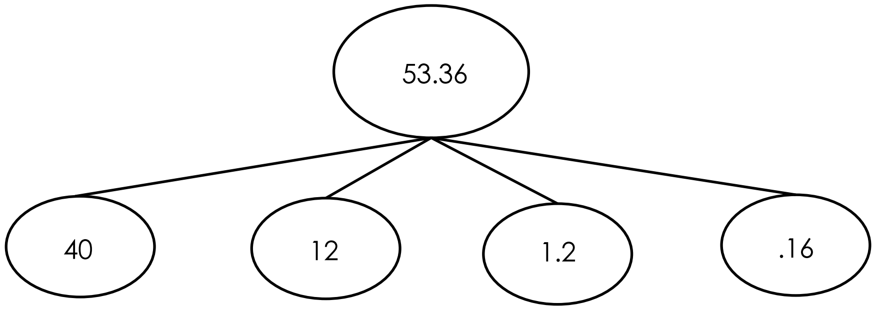
5336 ÷ 4
5336 separated into 4 equal groups



$$\begin{aligned} 5336 \div 4 &= (4000 \div 4) + (800 \div 4) + (400 \div 4) + (80 \div 4) + (56 \div 4) \\ &= 1000 + 200 + 100 + 20 + 14 \\ &= 1334 \end{aligned}$$

Example 4: 53.36 ÷ 4 using an equal grouping model and a number bond representation.

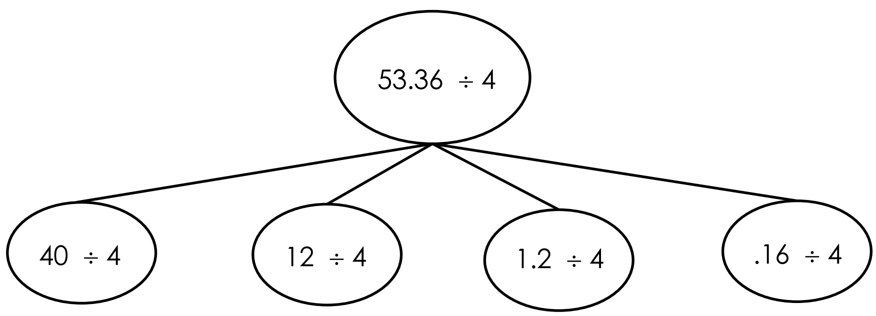
- The approach used above can also be used with a decimal dividend (53.36).



Notice that 40 + 12 + 1.2 + .16 is equal to 53.36 and each of the numbers is a multiple of 4. One and two-tenths (1.2) or 12-tenths is a multiple of 4. Four groups of 3-tenths (.3) is the same as 12-tenths or 1.2. Sixteen hundredths (.16) is also a multiple of 4. Four groups of four hundredths (.04) is the same as 16-hundredths (.16)

- We can then use this representation to show the partial quotients.

53.36 ÷ 4
53.36 separated into 4 equal groups



$$
\begin{aligned}
53.36 \div 4 &= (40 \div 4) + (12 \div 4) + (1.2 \div 4) + (.16 \div 4) \\
&= 10 + 3 + .3 + .04 \\
&= 13.34
\end{aligned}
$$

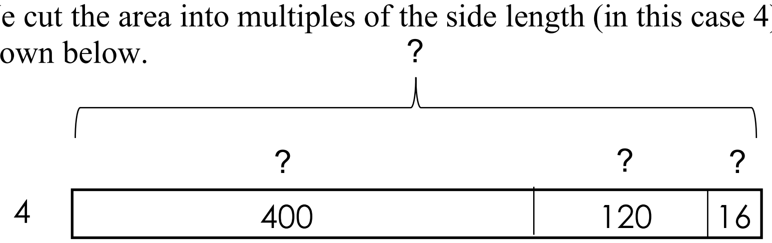
Division Using the Area Model & Partial Quotients

2. Your child should be able to represent division using the area model, dividing up to a 4-digit number by a 1-digit number or 2-digit number. She or he should understand the relationship to the Distributive Property.

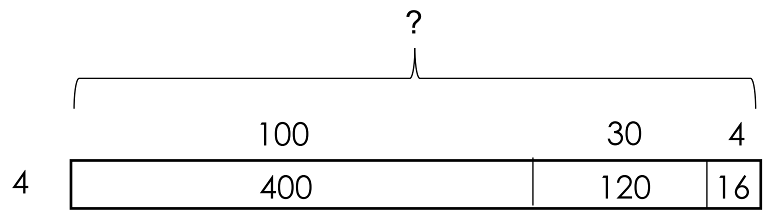
<u>Example 1</u>: 536 ÷ 4 using the area model.

We can also use an area model to find the partial quotients. If we use the area model to find 536 ÷ 4, we think of 536 as the total area and 4 as one of the side lengths. Our goal is to find the other side length.

- We cut the area into multiples of the side length (in this case 4). One possibility is shown below.

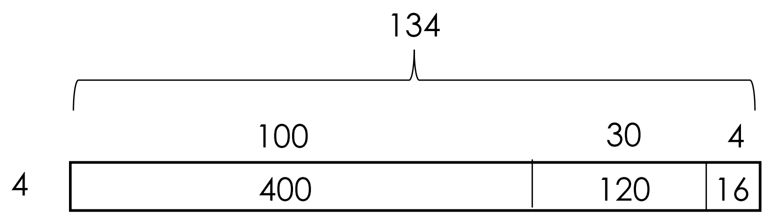


- To find the missing values we think, 4 times what number will give 400, 4 times what number will give 120, and 4 times what number will give 16.
- We know that 4 times 100 is 400, 4 times 30 is 120, and 4 times 4 is 16.

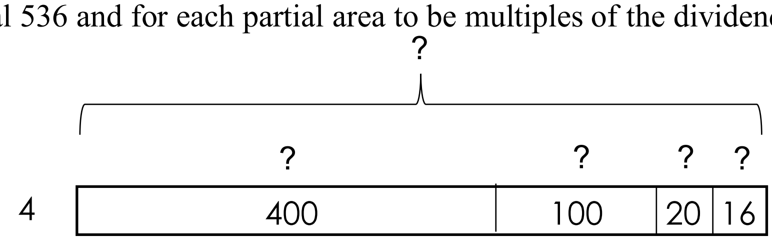


- We add the partial quotients to get the total. So,

536 ÷ 4 is the same as 4 x ___ = 536
4 x ____ = 400
4 x ____ = 120
4 x ____ = 16
So 536 ÷ 4 = 100 + 30 + 4 = 134



Note: We can separate the area into any multiple of 4. See below for another example of a way we could separate the area into multiples of 4. Remember, we need the total area to equal 536 and for each partial area to be multiples of the dividend (4).



In this case we would find,

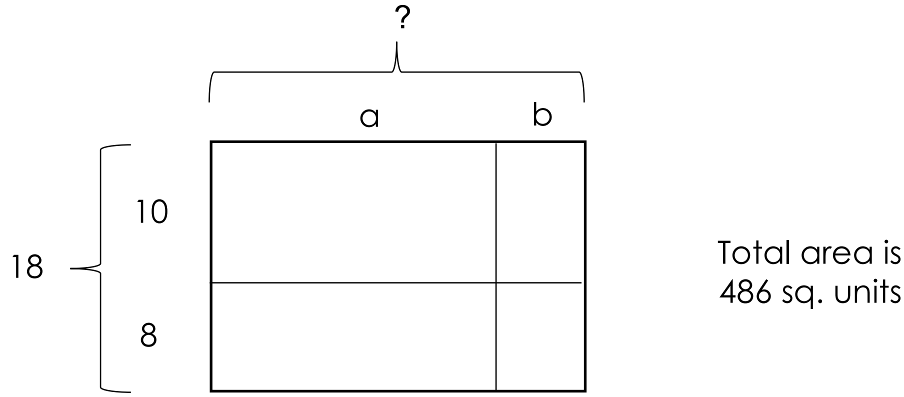
4 x ____ = 400
4 x ____ = 100
4 x ____ = 20
4 x ____ = 16

So 536 ÷ 4 = 100 + 25 + 5 + 4 = 134

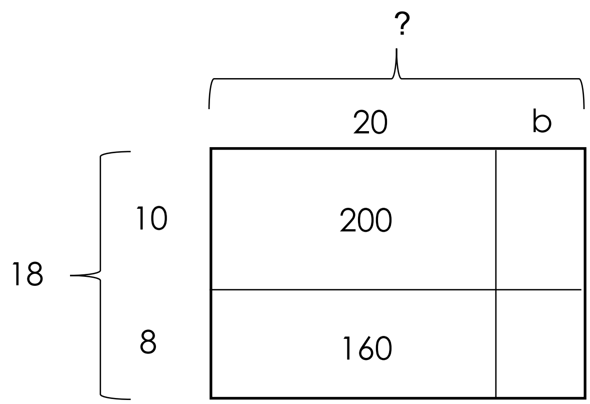
<u>Example 2</u>: 486 ÷ 18 using the area model.

To use the area model to solve 486 ÷ 18, we think of 486 as the total area and 18 as one of the side lengths. We know that the other side length must be a double-digit number. Since 10 times 18 (the first 2-digit number) is 180, we know it cannot be a number less than 10. A single-digit number would be too small. The goal is to determine the areas (partial quotients) that we need to separate 486 into to find the missing 2-digit side length.

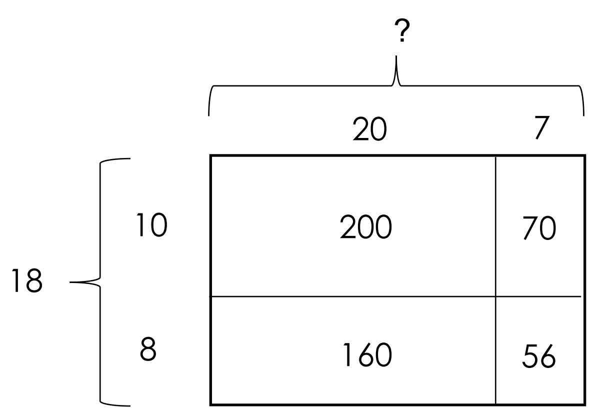
- We begin with what we know.



- Because we have been using expanded form of a number for our dimensions we know that “a” is a multiple of 10. We can try 10, 20, or 30. If we try 30, we will have a 10 by 30 rectangle and an 8 by 30 rectangle. Since 300 + 240 is greater than our total area, 30 will not work. Ten is too low because 10 by 10 combined with 10 by 8 is 180. We are 306 short of our total area. Let’s try substituting 20 for “a”.



- To determine the value for “b”, your child needs to understand that the ending digit of a multi-digit product is the ending digit obtained when multiplying the numbers in the ones place. In this problem, 486 ends in 6. We know that the number in the bottom right corner must end in 6. That means that “b” must either be 2 or 7 since 2 x 8 = 16 and 7 x 8 = 56. Since we are 126 away from 486, 2 will be too low. Try 7.



200 + 70 + 160 + 56 = 486
so 486 ÷ 18 = 27

Word Problems

1. Your child should be able to represent and solve multi-step word problems involving the multiplication of fractions and mixed numbers.
2. Your child should be able to represent and solve multi-step word problems involving the division of a unit fraction by a whole number and a whole number by a unit fraction.
3. Your child should be able to create story problems for division of a unit fraction by a whole number and a whole number by a unit fraction.

Measurement Concepts

Time

1. Your child should be able to represent and solve multi-step word problems involving time. This includes representing times on a number line to find elapsed time.

Money

2. Your child should be able to represent and solve multi-step word problems involving money. She or he should be able to correctly write money values using $ and ¢ symbol and should be able to identify and describe errors made in recording money values.

   Note: Remember that when using the ¢ symbol we do not use a decimal point. Think of ¢ as representing the value of a penny. We have 50¢ but not **.**50¢. The latter is the equivalent of half a penny. That is not part of US currency. We can also show 50 cents as a part of a dollar. A dollar is the equivalent of 100 cents. 50 cents is the equivalent of fifty hundredths of a dollar, or $**.**50. So to record 50 cents we can write it as 50¢ or $**.**50. We would write 25 cents as either 25¢ or $**.**25, 10 cents as 10¢ or $**.**10, 5 cents as 5¢ or $**.**05, and 1 cent as 1¢ or $**.**01 (one-hundredth of a dollar).

Measurement Conversion

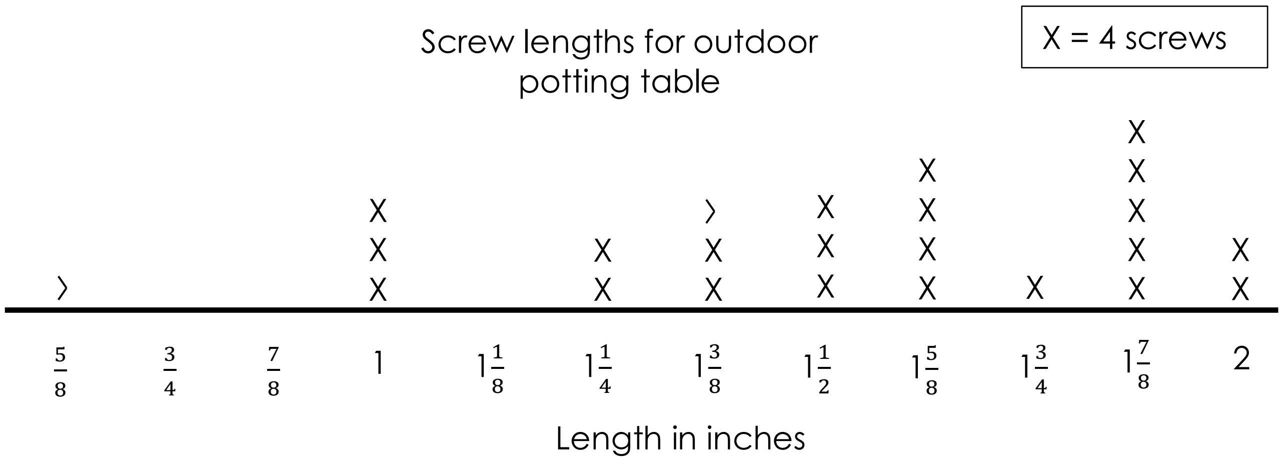
3. Your child should understand the relationship between units of measure and use when solving word problems that require measurement conversion. For example, when given a measure in gallons, your child should also be able to write it in pints, quarts, cups, etc.
4. Your child should be able to represent and solve multi-step word problems that require conversion of measurement. This includes problems involving simple fractions and decimals and giving answers in a smaller unit of measure when given a larger unit (e.g., giving the answer in inches when the problem is given in feet).

Area & Perimeter

5. Your child should understand the difference between area (number of square units in a shape) and perimeter (number of linear units to "outline" the shape) and use to solve multi-step word problems.

Represent and Interpret Data

6. Your child should be able to measure a collection of items to the nearest eighth of an inch and create a line plot to show the data. For example,

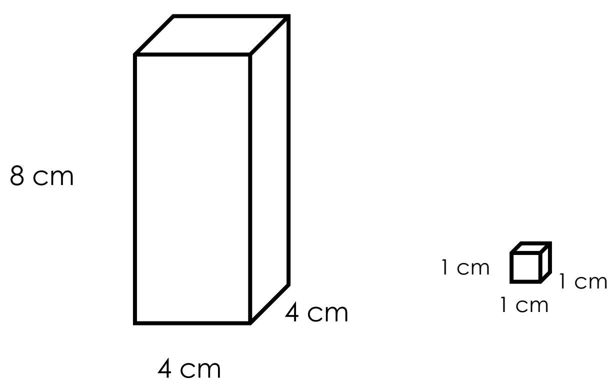


7. Your child should be able to answer questions and solve problems using the data in a line plot.

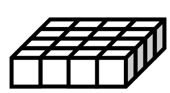
Volume

8. Your child should understand that volume is a measure of cubic units.
9. Your child should understand that the volume of a right rectangular prism with whole-number edge (side) lengths can be found by packing the shape with unit cubes. A unit cube is a cube in which each edge is 1 unit long (e.g., inch, ft, cm, yd, etc.). The volume of the unit cube is 1 cubic unit (e.g., 1 cubic ft or 1 ft^3).

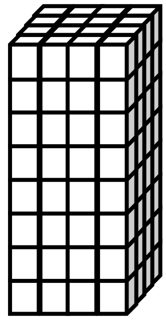
   <u>Example:</u> We can find the volume of the shape below by packing it with unit cubes.



   If we fill this right rectangular prism with the unit cubes, the bottom layer will contain 16 cubes (4 by 4).



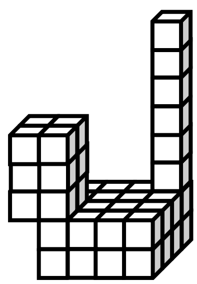
To fill the prism, we will need 8 layers of the 16 cubes (8 x 16 or 8 x 4 x 4)



10. Your child should understand that the volume of right rectangular prisms can be found by multiplying the edge lengths as shown in the above example.
11. Your child should understand that to find the volume of "odd shaped" solid figures that are made up of non-overlapping rectangular prisms we can find the volume of each rectangular prism. We then add the volumes of each rectangular prism to get the total volume.

<u>Example:</u>

Your child may notice that the shape below is made of 3 rectangular prisms. The base prism is 4 by 4 by 2. Its volume is 32 cubic units. The prism on the front left corner is 2 by 2 by 3. Its volume is 12 cubic units. The prism on the back right corner is 1 by 1 by 6. Its volume is 6 cubic units.



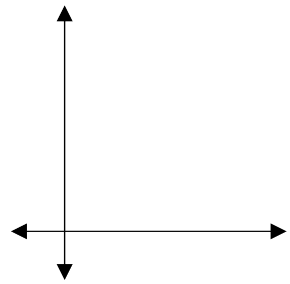
By combining the volumes of each of the 3 prisms (32 + 12 + 6) we get the volume of the entire shape.

32 + 12 + 6 = 50 cubic units

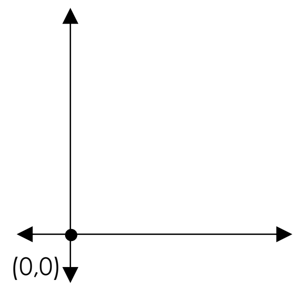
Geometry

Coordinate Grids

1. Your child should understand that a coordinate grid is made up of a pair of perpendicular number lines. He or she should know that we call these number lines "axes". (**Note:** In grade 5 we focus on the positive ends of the number lines).



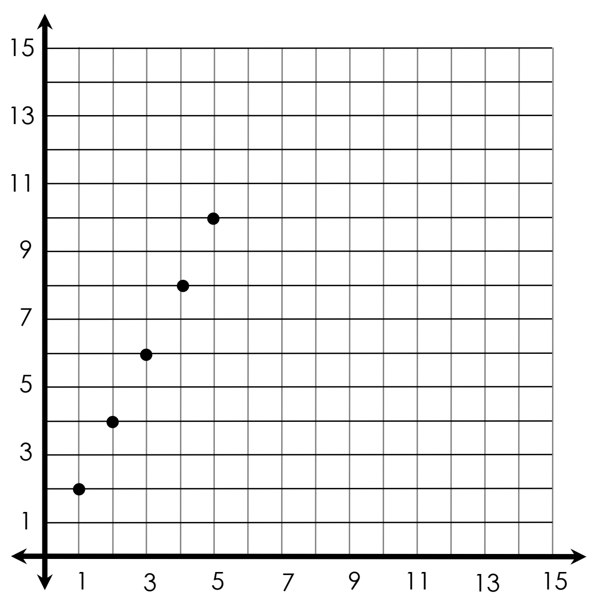
2. Your child should understand that the origin is the point of intersection of the perpendicular number lines that make up a coordinate grid. We label the origin with the point (0,0).



3. Your child should understand that points are named on the coordinate grid using 2 values, the horizontal location and the vertical location. For example (2,3) would be the point that is 2 to the right and 3 "up". The point (2,3) is called the coordinate or ordered pair.
4. Your child should be able to graph the relationship shown on two number patterns on a coordinate grid.

<u>Example:</u> If given the two patterns below, your child should be able to graph the points (1, 2), (2, 4), (3, 6), (4, 8), and (5, 10) on a coordinate plane.

1	2
2	4
3	6
4	8
5	10



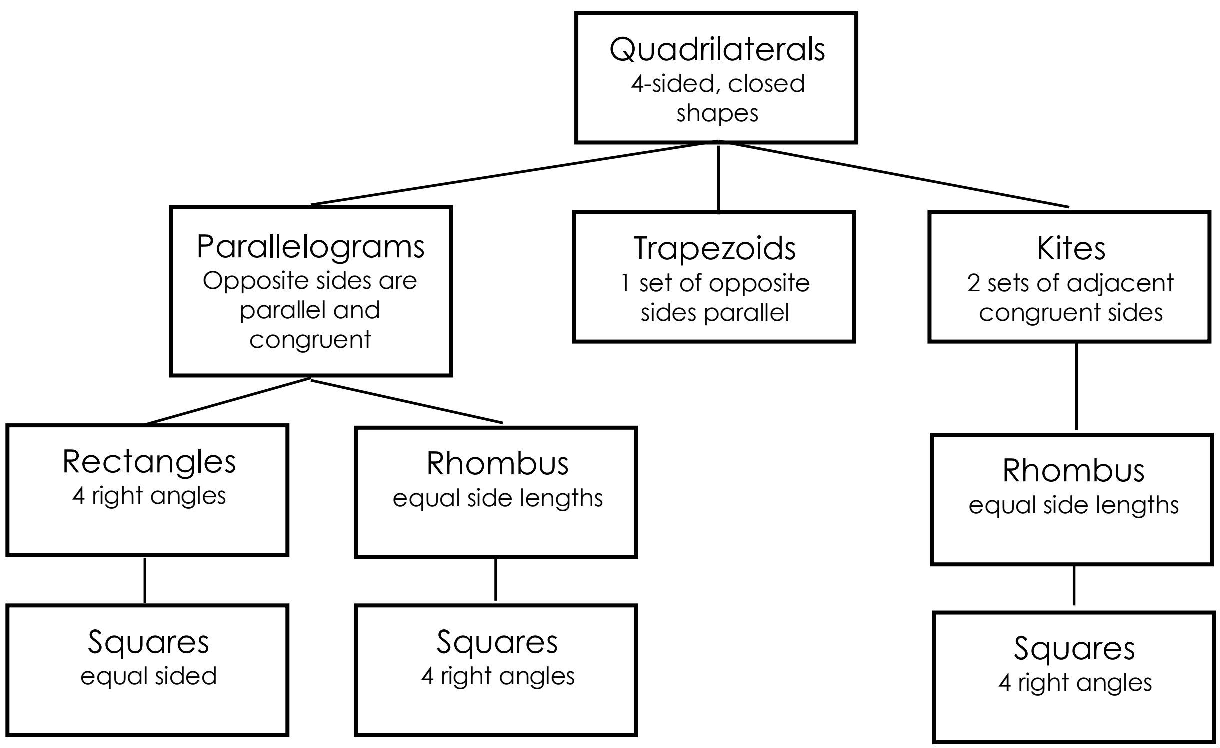
She or he should be able to describe patterns they notice and use to predict where other points may go. (e.g., These points form a line. A point is 2 up and 1 to the right from the previous point. This observation is the foundation for slope that they will do in later grades).

5. Your child should be able to represent and solve word problems by graphing points on the coordinate plane (**Note:** grade 5 problems are limited to quadrant 1—the positive ends of the number lines).

2-Dimensional Geometry

6. Your child should understand that 2D shapes can be classified according to their attributes. These classifications form a hierarchy based on the properties of the shapes. He or she should understand and describe the ways in which a single shape can be sorted into more than one group. according to a hierarchy based on properties.

   Example: 4-sided closed shapes can be sorted into the following categories. In the categories below, there are some 4-sided shapes that will fit the subcategories of parallelogram, trapezoid, kite and others that will remain in the category, quadrilateral. (**Note**: This is not an exhaustive diagram).



7. Your child should understand that attributes of a category of shapes belong to all of the sub-categories.

   Example: A square has the characteristics of a rectangle, the characteristics of a parallelogram, and the characteristics of a quadrilateral. A square has equal sides, right angles, opposite sides are parallel and congruent, and it has 4 sides. Similarly, a square is a rhombus with right angles. It has opposite sides that are parallel and congruent. So, a square could fit in the following categories: quadrilateral, parallelograms, rectangles, rhombus, and kites.