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An EWMA monitoring scheme with a single auxiliary variable for industrial processes

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ABSTRACT

When using control charts to monitor manufacturing processes, Shewhart control chart is known to be useful for detecting transient shifts, while the EWMA and CUSUM charts are useful for detecting persistent shifts. The efficiency of EWMA chart in monitoring location parameter can be improved by using an auxiliary variable that is closely related to the variable of interest. In this paper, an EWMA-type scheme using ratio estimator is developed to further increase the effectiveness of the classical EWMA chart in monitoring the location parameter. The proposed procedure outperforms the classical EWMA and even the mixed EWMA-CUSUM chart, especially when there is a strong positive relationship between the variable of interest and the auxiliary variable. Finally, a real data set is used to show the implementation procedures of the proposed chart.

1. Introduction

Statistical process control (SPC) is a data-driven statistical method for monitoring and controlling a process in manufacturing industry. The Shewhart control chart (Shewhart, 1924) is an example of the memoryless-type control chart. The two most commonly used memory type control charts are the exponentially weighted moving average (EWMA) control chart and the cumulative sum (CUSUM) control chart, proposed by Page (1954) and Roberts (1959), respectively. The memory control charts utilize previous information with present information to yield a better result for detecting small to moderate shifts, unlike Shewhart-type charts that use only the current information. Research works related to the Shewhart-type charts and CUSUM-type charts can be found in Hawkins and Olwell (1998), Mukherjee and Sen (2015), Li, Mukherjee, Su, and Xie (2016), Chong, Mukherjee, and Khoo (2017), Lombard, Hawkins, and Potgieter (2017), and Sanusi, Riaz, Abbas, and Abujiya (2017). Different EWMA-type schemes have also been proposed in the literature. Noorossana, Fathizadan, and Nayebpour (2016) investigated the joint effect of non-normality and parameter estimation on EWMA chart. Also, Tamirat and Wang (2016) introduced an acceptance sampling plan scheme based on an EWMA statistic. In the case of unknown parameters, EWMA median control chart with estimated parameters was introduced to monitor the location parameter of a normal process (Castagliola, Maravelakis, & Figueiredo, 2016). Also, Zhou, Shu, and Jiang (2016) suggested a one-sided EWMA scheme with varying sample sizes for monitoring rare events. Furthermore, EWMA control chart found early applications in economics (Muth, 1960) and in inventory control and forecasting (Dushman, Lafferty, & Brown, 1962). For more works on the improvement of EWMA chart, interested readers can see Liu, Xie, Goh, and Chan (2007), Sheu, Tai, Hsieh, and Lin (2009), Teh, Khoo, and Wu (2011), Xie, Xie, and Goh (2011), Nishimura, Matsuura, and Suzuki (2015), and Zwetsloot, Schoonhoven, and Does (2016).

The traditional EWMA chart monitors the process mean (say $\overline{Y}$) of a process distribution. The $\overline{Y}$ is computed using the famous simple random sampling (SRS) approach. However, in the presence of an auxiliary variable (X) that is closely related to the study variable (Y), $\overline{Y}$ can be estimated more efficiently. Consequently, Cochran (1940) used the advantage of auxiliary information to propose a ratio mean estimator ($\overline{y}_r$) for estimating the population mean of Y. Choudhury and Singh (2012) noted that $\overline{y}_r$ is most effective when there is a positive linear relationship (which passes through the origin) between Y and X and the mean square error (MSE) of Y is proportional to X. Murthy (1964) suggested the use of $\overline{y}_r$ when $\rho_{YX} C_Y / C_X > 0.5$, where ρ_{YX}, C_Y, and C_X are, respectively, the correlation coefficient between Y and X, the coefficient of variation of the study variable (Y), and the coefficient of variation of X. Also, Adebola, Adegoke, and Sanusi (2015) introduced an efficient estimator, with cum-dual ratio estimator as intercept, for estimating the population mean.

Riaz (2008a) popularized the idea of using an auxiliary variable at

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the stage of estimating the plotting statistic of a monitoring chart. He presented a Shewhart-type scheme based on a regression-type estimator for detecting shifts in process variability and also showed the superiority of the scheme over the traditional Shewhart control chart. Also, a Shewhart-type control chart was suggested by Riaz (2008b) for monitoring process location. The chart is based on the regression mean estimator and it was shown that it outperforms the Shewhart's X - bar chart and the cause-selecting charts. The work was later extended to the EWMA set-up for detecting small to moderate shifts in the process mean (Abbas, Riaz, & Does, 2014). It was revealed that the chart outperforms other existing univariate and bivariate charts Also, some efficient estimators with an auxiliary variable are used to improve the performance of the combined Shewhart-CUSUM chart for detecting both small and large shifts in the location parameter of a process (Sanusi, Riaz, & Abbas, 2017).

In this article, an auxiliary variable is introduced at the estimation stage to monitor the location parameter of a process distribution. The proposed chart, denoted as MrEWMA, is an EWMA-type control chart based on the ratio mean estimator. This motivation further enhances the sensitivity of the control chart, especially in detecting shifts of small magnitudes. The average run length (ARL) approach is used to evaluate the performance of the chart. Also, other performance measures such as the standard deviation of the run length (SDRL), the extra quadratic loss (EQL), the relative average run length (RARL), and the performance comparison index (PCI) are considered. The management perspective of the proposed scheme is also briefly discussed.

One of the three activities for the successful execution of an efficient management of a process is quality control. This requirement ensures that products are up to standard through continuous improvement. The proposed MrEWMA scheme would further help to continosly improve the performance of a product, which will lead to a long-term reward for industries. Also, the early detection of shifts would avoid mass inspection in controlling quality, since a good quality is achieved by preventing defective items, instead of inspecting the items for bad products. Moreover, industry with a modern method of improving products quality, and can demonstrate process capability and control, has an edge over other competitors. These are in agreement with the Deming philosophy in improving management strategies (Montgomery, 2009).

The rest of this article is arranged as follows: The statistical preliminaries of the proposed scheme; the structural framework of the classical EWMA chart including its plotting statistic, control limits, and ARL; and the design of the proposed chart are presented in Section 2. The performance evaluations and the major findings of the proposed MrEWMA chart are provided in Section 3. The comparison of the MrEWMA control chart with its existing counterparts is given in Section 4. A real-life illustrative example is given in Section 5. Finally, the summary and conclusion of the findings are provided in Section 6.

2. The proposed monitoring scheme

The motivation of this work is to enhance the sensitivity of EWMA control chart in detecting shifts in the location parameter of a control process. This is achieved by introducing an auxiliary variable, in the form of a ratio estimator, to the charting scheme. In the next subsections, the mathematical preliminaries of the proposed scheme are explained, followed by a brief description of the classical EWMA, and then, the construction of the proposed MrEWMA chart.

2.1. Statistical preliminaries

Let X represents an auxiliary variable which is positively correlated with the study variable (Y) of a control process. The extra information provided by the auxiliary variable can increase the efficiency in estimating the population mean ($\overline{Y}$). One of the ways of integrating the extra information is to use the ratio mean estimator defined as:

$$\overline{y}_r = (\overline{y}/\overline{x})\overline{X}, \tag{2.1}$$

The estimator is biased, that is, $E(\overline{y}_r) = \overline{Y} + B(\overline{y}_r)$, where $E(\overline{y}_r)$ is the expectation of the ratio mean estimator ($\overline{y}_r$) and $B(\overline{y}_r)$ is the bias of $\overline{y}_r$. The $B(\overline{y}_r)$ is defined as:

$$B(\overline{y}_r) = \overline{Y}(C_X^2 - \rho_{YX} C_Y C_X)/n, \tag{2.2}$$

For the Case-U, $B(\overline{y}_r)$ can be estimated as:

$$\widehat{B}(\overline{y}_r) = \overline{y}(c_X^2 - r_{YX} c_Y c_X)/n, \tag{2.3}$$

where $C_X = \sigma_X/\overline{X}$, $C_X = \sigma_Y/\overline{Y}$, $c_X = s_X/\overline{x}$, and $c_Y = s_Y/\overline{y}$.

Furthermore, the mean squared error of $\overline{y}_r$ is given as

$$MSE(\overline{y}_r) = \overline{Y}^2(C_Y^2 + C_X^2 - 2\rho_{YX} C_Y C_X)/n, \tag{2.4}$$

which can be estimated as (Singh & Mangat, 1996)

$$mse(\overline{y}_r) = \overline{y}^2(c_y^2 + c_x^2 - 2r_{yx} c_y c_x)/n. \tag{2.5}$$

Note that sampling with replacement and approximation up to the first order are assumed. Also, the choice of ρ_{YX} that gives an efficient estimation is determined by the condition $\rho_{YX} > C_X/2C_Y$ (Singh & Mangat, 1996). More often, $C_X \approx C_Y$, which implies that $C_X/C_Y \approx 1$. Consequently, we have $\rho_{YX} > 0.5$. As a result, in the presence of an auxiliary variable, it is more efficient to apply the ratio mean estimator instead of the usual SRS mean estimator if $\rho_{YX} > 0.5$ (Murthy, 1964). Murthy (1964) noted that in the case of large samples and when both Y and X are both positive or negative, the product estimator, the usual mean estimator, and the ratio estimator should be used when $-1 \leqslant \rho_{YX} < -C_X/2C_Y$, $-C_X/2C_Y \leqslant \rho_{YX} \leqslant C_X/2C_Y$, and $C_X/2C_Y < \rho_{YX} \leqslant 1$, respectively. If either Y or X is negative, the conditions for the product and ratio estimators will be reversed. In Fig. 1, the broken horizontal line is the standard error (SE) of the usual SRS mean, while the curve is the SE of the ratio estimator which varies with different ρ_{YX} values. The two lines meet at a point $C_X/2C_Y$. It is shown that the SE of the ratio estimator is less than the SE of the usual SRS mean when $C_X/2C_Y < \rho_{YX} \leqslant 1$. In this research, it is assumed that both Y and X are greater than zero. Hence, only positive values of ρ_{YX} (ρ_{YX} = 0.25, 0.50, 0.75, and 0.95) are considered. Furthermore, the usual SRS estimator may be better than the ratio estimator when near proportionality between Y and X does not exist (Cochran, 1940). This happens when the regression line of Y on X passes through a point on Y-axis that is not close to the origin. Before constructing the proposed control chart, the classical EWMA will be introduced briefly.

2.2. The classical EWMA chart

The classical EWMA control chart is a memory-type chart that detects persistent shifts in a control process. It is frequently used for monitoring single observations, though it can also be used for monitoring process mean of rational subgroups of size $n > 1$ (Montgomery, 2009). Consequently, the procedures for constructing an EWMA chart

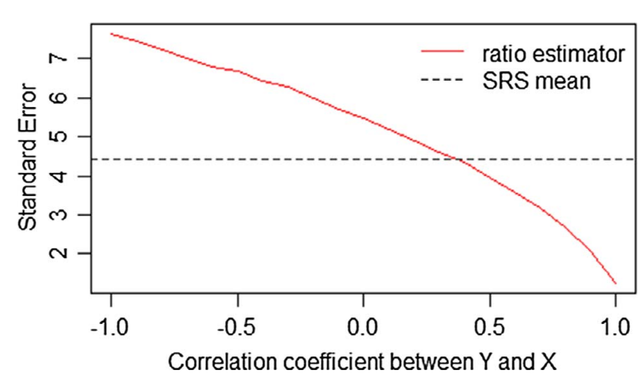

Fig. 1. Estimated SE of the ratio estimator vs. SE of the usual SRS mean estimator.

with rational subgroup is explained. Assume that the study variable (Y) of a control process is from a normal distribution with population mean $\overline{Y}$ and population standard deviation σ_Y, that is, $Y \sim N(\overline{Y},\sigma_Y^2)$. For the case of monitoring the process mean $\overline{y} = \sum_{i=1}^{n} Y_i/n$, where $\overline{y} \sim N(\overline{Y},\sigma_Y^2/n)$, the plotting statistic of the classical EWMA scheme for the tth rational subgroup is given as

$$W_t = \lambda\overline{y}_t + (1-\lambda)W_{t-1}, \quad t = 1,2,... \tag{2.6}$$

where λ is the smoothing parameter such that $0 < \lambda \leqslant 1$. The structure becomes a Shewhart X -bar chart whenever $\lambda = 1$. Also, the sensitivity of the chart to small shifts is inversely related to the smoothing parameter. Since the EWMA-type charts aim to detect small shifts, $\lambda = 0.1$ or 0.2 is often recommended in the literature. The quantity W_{t-1} denotes the past value, while W_0 is the initial value. If the process parameters are known (often referred to as the Case-K), W_0 is taken to be the target or in-control mean (say $\overline{Y}_0$). For an in-control process, the mean and the variance of the EWMA statistic (Roberts, 1959) are, respectively,

$$E(W_t) = \overline{Y}_0 \quad \text{and} \quad V(W_t) = (\sigma_Y^2/n)(\lambda/(2-\lambda))(1-\lambda)^{2t}. \tag{2.7}$$

For the case of unknown process parameters (often referred to as the Case-U), the parameters $\overline{Y}_0$ and σ_Y^2 are estimated from preliminary samples.

The center line (CL), the upper control limit (UCL), and the lower control limit (LCL) of the EWMA chart at time t can be defined as follows:

$$LCL_t = \overline{Y}_0 - L(\sigma_Y/\sqrt{n})\sqrt{(\lambda/(2-\lambda))(1-(1-\lambda)^{2t})}, \tag{2.8}$$

$$CL = \overline{Y}_0, \tag{2.9}$$

$$UCL_t = \overline{Y}_0 + L(\sigma_Y/\sqrt{n})\sqrt{(\lambda/(2-\lambda))(1-(1-\lambda)^{2t})}. \tag{2.10}$$

The LCL_t and UCL_t have subscript t because the chart is time-varying. Asymptotically, the limits become constant as time increases. Hence, the asymptotic limits are given in Eqs. (2.11) and (2.12). The constant L regulates the width of the control limits. Its value is selected to fix a particular in-control ARL (ARL_0).

$$LCL = \overline{Y}_0 - L(\sigma_Y/\sqrt{n})\sqrt{(\lambda/(2-\lambda))}, \tag{2.11}$$

$$UCL = \overline{Y}_0 + L(\sigma_Y/\sqrt{n})\sqrt{(\lambda/(2-\lambda))}. \tag{2.12}$$

Table 1 gives the ARL values of the classical EWMA control chart with time-varying limits (Steiner, 1999). In the Table, δ *is* the amount of shift defined as $\delta = |\overline{Y}_1 - \overline{Y}_0|/(\sigma_Y/\sqrt{n})$, where $\overline{Y}_0$ and $\overline{Y}_1$ are the in-control and the out-of-control mean, respectively.

Many authors have proposed different modifications of the memory-type control charts that give smaller out-of-control ARL values for a particular ARL_0. Some of the modifications are the use of auxiliary variable and the introduction of first initial response (Lucas & Crosier, 1982; Rhoads, Montgomery, & Mastrangelo, 1996; Sanusi et al., 2017), among others.

2.3. Design of the proposed MrEWMA control chart

In this study, the values of Y and X are gotten in paired form for each sample observation, and the population mean and variance of X are assumed to be known. More so, Y and X are assumed to follow a bivariate normal distribution, that is, $(Y,X) \sim N_2(\overline{Y},\overline{X},\sigma_Y,\sigma_X,\rho_{YX})$ where N_2 represents the bivariate normal distribution. Based on the ratio estimator in Eq. (2.1), the plotting statistic of the proposed MrEWMA chart is given as:

$$W_t = \lambda\overline{y}_{rt} + (1-\lambda)W_{t-1}, \quad t = 1,2,... \tag{2.13}$$

Based on the statistical properties of $\overline{y}_r$, the control limits for MrEWMA chart are defined as:

$$LCL_t = \overline{Y}_0 - L\sqrt{(MSE(\overline{y}_r))(\lambda/(2-\lambda))(1-(1-\lambda)^{2t})} \tag{2.14}$$

$$CL = \overline{Y}_0 \tag{2.15}$$

$$UCL_t = \overline{Y}_0 + L\sqrt{(MSE(\overline{y}_r))(\lambda/(2-\lambda))(1-(1-\lambda)^{2t})} \tag{2.16}$$

Eqs. (2.14)–(2.16) represent the time-varying control limits. For constant limits, they can be written as:

$$LCL = \overline{Y}_0 - L\sqrt{(MSE(\overline{y}_r))(\lambda/(2-\lambda))} \tag{2.17}$$

$$UCL = \overline{Y}_0 + L\sqrt{(MSE(\overline{y}_r))(\lambda/(2-\lambda))}. \tag{2.18}$$

More argument about the efficiency of the proposed chart over the classical EWMA chart is given in Appendix A, where it is proved that the MSE of the plotting statistic of MrEWMA is less than that of the plotting statistic of the classical EWMA when $C_X/2C_Y < \rho_{YX} \leqslant 1$.

3. Performance measures

Following Steiner (1999), a comprehensive assessment of the proposed MrEWMA chart in terms of ARL and SDRL is performed with the aid of *R* software (R Core Team, 2016). ARL is the average number of samples plotted until the first out-of-control signal (Montgomery, 2009), while SDRL is the standard deviation of the number of samples plotted until the first out-of-control signal. ARL_0 is the ARL when a process is in-control, while ARL_1 is the ARL when a process is out of control. It is expected that ARL_0 has a large value while ARL_1 has a small value (Abbasi, Riaz, & Miller, 2012; Ahmad, Abbasi, Riaz, & Abbas, 2014). Knoth (2007) gave an accurate ARL calculation of EWMA chart for the simultaneous monitoring of normal mean and variance. To show that the sensitivity of EWMA-type chart decreases with increase in λ, small and large values of λ ($\lambda = 0.05$, 0.25, and 0.50) are considered, and the corresponding values of L that fix ARL_0 to be 500 are searched through a Monte Carlo search algorithm. This is to ensure a fair comparison of the MrEWMA chart with other existing charts of same ARL_0. Also, the run length properties of the proposed chart, such as average, standard deviation and median, are investigated for different shift sizes, say $\delta = 0$, 0.25, 0.50, 0.75, 1, 1.50, 2, 2.50, 3, 4, and 5 (Table 2). The results are computed using the time-varying limits.

Some authors have raised concern about the use of ARL for comparing different charts because it is based on the effectiveness of charts at a specific shift point (Wu, Yang, Jiang, & Khoo, 2008). Consequently, we further measure the efficiency of the proposed chart using performance measures that give the performance of a chart over all the domain of shifts considered. The performance measures are:

- Extra Quadratic Loss (EQL): This measures the effectiveness of a chart over all the domain of the process shifts. It is the weighted average ARL over the entire shifts of a charting structure. It makes use of the square of shift (δ^2) as weight.

Table 1
ARL values for the classical EWMA chart with $ARL_0 = 500$.

	δ	0	0.25	0.5	0.75	1	1.5	2	2.5	3	4	5
$\lambda = 0.05$	L = 2.639	500	77.75	23.71	11.87	7.31	3.77	2.43	1.77	1.41	1.09	1.01
$\lambda = 0.25$	L = 3.001	500	169.5	47.38	19.32	10.41	4.78	2.94	2.09	1.62	1.16	1.02
$\lambda = 0.50$	L = 3.072	500	254.7	88.47	35.59	17.18	6.27	3.39	2.26	1.70	1.18	1.03

Table 2
ARL values of the proposed MrEWMA chart with $ARL_0 = 500$ and $n = 10$.

	δ	0	0.25	0.5	0.75	1	1.5	2	2.5	3	4	5
$\rho_{XY} = 0.25$	ARL	501.50	87.45	26.72	13.54	8.24	4.25	2.72	1.99	1.56	1.16	1.03
$\lambda = 0.05$	SDRL	514.01	79.10	20.81	9.58	5.50	2.57	1.50	1	0.70	0.38	0.16
L = 2.647	Median	343	65	22	12	7	4	2	2	1	1	1
$\rho_{XY} = 0.25$	ARL	498.11	173.02	53.20	22.47	12.09	5.44	3.33	2.35	1.80	1.27	1.06
$\lambda = 0.25$	SDRL	499.58	168.72	49.90	19.28	9.23	3.42	1.82	1.18	0.83	0.48	0.23
L = 3.015	Median	346	122	38	17	10	5	3	2	2	1	1
$\rho_{XY} = 0.25$	ARL	500.56	238.07	92.45	40.20	20.11	7.42	3.92	2.61	1.92	1.30	1.07
$\lambda = 0.50$	SDRL	490.95	236.11	91.81	38.41	18.37	5.82	2.61	1.46	0.96	0.51	0.26
L = 3.093	Median	351	166	65	28	14	6	3	2	2	1	1
$\rho_{XY} = 0.50$	ARL	501.21	64.41	19.50	9.92	6.03	3.16	2.08	1.56	1.26	1.04	1
$\lambda = 0.05$	SDRL	519.15	56.48	14.48	6.78	3.83	1.80	1.07	0.70	0.48	0.19	0.04
L = 2.649	Median	339	49	16	8	5	3	2	1	1	1	1
$\rho_{XY} = 0.50$	ARL	499.65	137.90	36.19	15.06	8.26	3.95	2.48	1.79	1.41	1.08	1.01
$\lambda = 0.25$	SDRL	500.26	135.99	32.85	11.92	5.68	2.25	1.24	0.82	0.58	0.27	0.08
L = 3.0105	Median	346	97	26	12	7	4	2	2	1	1	1
$\rho_{XY} = 0.50$	ARL	500.23	208.35	66.64	26.10	12.85	4.91	2.79	1.90	1.47	1.09	1.01
$\lambda = 0.50$	SDRL	496.65	206.79	64.80	24.59	11.07	3.40	1.60	0.94	0.63	0.29	0.09
L = 3.0889	Median	347	145	47	19	9	4	2	2	1	1	1
$\rho_{XY} = 0.75$	ARL	499.80	38.56	11.79	5.99	3.76	2.03	1.40	1.13	1.03	1	1
$\lambda = 0.05$	SDRL	508.88	31.48	8.20	3.78	2.19	1.02	0.59	0.35	0.17	0.02	0
L = 2.647	Median	344	31	10	5	3	2	1	1	1	1	1
$\rho_{XY} = 0.75$	ARL	495.57	85.40	19.22	8.21	4.73	2.42	1.60	1.23	1.06	1	1
$\lambda = 0.25$	SDRL	497.11	82.34	15.87	5.67	2.78	1.19	0.70	0.44	0.25	0.04	0
L = 3.01	Median	343	60	15	7	4	2	1	1	1	1	1
$\rho_{XY} = 0.75$	ARL	502.20	152.35	35.97	12.75	6.25	2.69	1.69	1.26	1.07	1	1
$\lambda = 0.50$	SDRL	504.94	150.34	34.34	10.96	4.63	1.49	0.78	0.48	0.27	0.04	0.01
L = 3.085	Median	349	106	25.5	9	5	2	2	1	1	1	1
$\rho_{XY} = 0.95$	ARL	500.15	15.73	4.86	2.57	1.71	1.10	1	1	1	1	1
$\lambda = 0.05$	SDRL	512.42	11.06	2.91	1.36	0.79	0.31	0.07	0	0	0	0
L = 2.647	Median	346	13.5	4	2	2	1	1	1	1	1	1
$\rho_{XY} = 0.95$	ARL	498.81	29.07	6.49	3.14	2.01	1.19	1.01	1	1	1	1
$\lambda = 0.25$	SDRL	495.81	25.35	4.13	1.63	0.93	0.41	0.11	0.01	0	0	0
L = 3.013	Median	349	22	6	3	2	1	1	1	1	1	1
$\rho_{XY} = 0.95$	ARL	504.56	63.63	9.85	3.76	2.17	1.22	1.01	1	1	1	1
$\lambda = 0.50$	SDRL	501.93	61.70	7.92	2.33	1.11	0.44	0.12	0.02	0	0	0
L = 3.099	Median	355	45	8	3	2	1	1	1	1	1	1

Let Π be the loss subject to an out-of-control state,

$$\Pi(\delta) = f(\delta)M(\delta) \tag{3.1}$$

where f = EQL per unit = (quadratic loss with respect to δ) − (quadratic loss when process is in-control).M = number of units produced when the process is in an out-of-control state.

$$f(\delta) = Q_c\{\sigma_0^2 + (\mu-\mu_0)^2\} - Q_c\sigma_0^2 = Q_c\sigma_0^2\delta^2 \tag{3.2}$$

where Q_c = constant cost for an individual process$M(\delta) = b \times j \times \{ARL(\delta)-0.5\}$, b = rate of production, and j = regularly spaced time interval. $M(\delta)$ assumes that the random time of a mean shift follows a uniform distribution (Reynolds, Amin, & Arnold, 1990). Considering all shifts in the process domain,

$$\begin{aligned} EQL &= \frac{1}{\delta_{\max}-\delta_{\min}}\int_{\delta_{\min}}^{\delta_{\max}} \Pi(\delta)d\delta \\ &= b \times j \times Q_c \times \sigma_0^2\{(\delta_{\max}-\delta_{\min})^{-1}\int_{\delta_{\min}}^{\delta_{\max}} \delta^2 ARL(\delta)d\delta \\ &\quad -(\delta_{\max}^2 + \delta_{\max}\delta_{\min} + \delta_{\min}^2)/6\} \end{aligned} \tag{3.3}$$

Without loss of generalization and for simplicity purpose, $b \times j \times Q_c \times \sigma_0^2$ and $(\delta_{\max}^2 + \delta_{\max}\delta_{\min} + \delta_{\min}^2)/6$ are constants and can be eliminated from Eq. (3.3) since they do not affect the performance comparison and the optimization solution (Wu, Jiao, Yang, Liu, & Wang, 2009). Hence, we have:

$$EQL = (\delta_{\max}-\delta_{\min})^{-1}\int_{\delta_{\min}}^{\delta_{\max}} \delta^2 ARL(\delta)d\delta \tag{3.4}$$

Note that a chart with a high EQL value is said to be inferior to a chart with a low EQL value.

- Relative Average Run Length (RARL): It uses ARL to determine the global performance of a chart with respect to a benchmark chart. The benchmark chart has a RARL value of one. Apparently, a chart with RARL greater than one is said to be inferior to the benchmark chart (Wu et al., 2009). The classical EWMA chart is used as the benchmark chart in this research.

$$RARL = \frac{1}{\delta_{\max}-\delta_{\min}}\int_{\delta_{\min}}^{\delta_{\max}} \frac{ARL(\delta)}{ARL_{bmk}(\delta)}d\delta \tag{3.5}$$

where $ARL_{bmk}(\delta)$ is the ARL of the benchmark chart when there is a shift of size δ.

Both EQL and RARL can be calculated using numerical integration.

- Performance Comparison Index (PCI): This is the ratio of the average extra quadratic loss (AEQL) of a particular chart to the AEQL of a benchmark chart. Ou, Wu, and Tsung (2012) used the best chart as the benchmark. For consistency, the classical EWMA is used as the benchmark in our case. A chart of PCI equals to one performs equally as the benchmark chart, while a chart with PCI lesser (greater) than one performs better (worse) than the benchmark chart. The mathematical expression of the PCI is given as:

$$PCI = AEQL/AEQL_{bmk}$$

But $AEQL = (\delta_{\max}-\delta_{\min})^{-1}\int_{\delta_{\min}}^{\delta_{\max}}\delta^2 ATS(\delta)d\delta$ and $ATS = ARL \times h$, where ATS is the average time to signal and h is a regularly spaced time interval. If $h = 1$, $ATS = ARL$ and $AEQL = EQL$. Hence,

$$PCI = EQL/EQL_{bmk} \tag{3.6}$$

3.1. Step-by-step algorithm of the MrEWMA

The illustrative procedures of the proposed chart for the Case-K are listed.

1. Determine the in-control parameters ($\overline{Y}$,$\overline{X}$,σ_Y,σ_X,ρ_{YX}) and the appropriate constants λ and L.
2. Construct the control limits using Eqs. (2.14), (2.16) for the time-varying limits, or Eqs. (2.17) and (2.18) for the constant limits.
3. Obtain the tth test sample (x_{ti},y_{ti}) of size n in the course of monitoring, where $t = 1,2,...$.
4. Compute the ratio mean estimate $\overline{y}_{rt}$ using Eq. (2.1).
5. Compute the plotting statistic $W_t = \lambda\overline{y}_{rt} + (1-\lambda)W_{t-1}$.
6. Plot W_t against the control limits.
7. If W_t falls within the control limits, the process is declared to be in-control, then the process goes back to Step 3 for the monitoring of the next test sample. Alternatively, if W_t falls outside the control limits, the process is said to have shifted to an out-of-control state. Consequently, the monitoring process terminates and the cause of the shift in the process mean is investigated.

For the Case-U, the in-control parameters can be determined from a phase-1 analysis.

3.2. Major findings of the proposed MrEWMA chart

The findings of the proposed MrEWMA chart are summarized as follow:

(1) The introduction of the auxiliary variable (in the form of ratio estimator) to the plotting statistic of the classical EWMA control chart improves the performance of the chart for values of $\rho_{XY} \geqslant 0.50$ (Table 2). Also, for $\rho_{XY} = 0.25$, the ARL_1 values of the MrEWMA chart are slightly greater than ARL_1 values of the classical EWMA chart. Hence, it is less effective in monitoring the location parameter of a control process. In addition, for $\rho_{XY} \geqslant 0.50$, the MrEWMA chart performs better than the classical EWMA chart. In fact, the performance of the proposed chart increases as ρ_{XY} increases. This is evident from the lower ARL_1 values of the MrEWMA chart (Fig. 2).

(2) For different sensitivity parameter values (that is, $\lambda = 0.05$, 0.25, and 0.50), it is shown that the MrEWMA chart is more sensitive to small and moderate shifts than the classical EWMA chart (Table 2).

(3) The MrEWMA chart is ARL unbiased for all values of λ, L and ρ_{XY}, i.e., ARL_0 is always greater than ARL_1 for any choice of δ (Table 2). Discussion on biased ARL charts can be found in Knoth and Morais (2015).

(4) The out-of-control ARL and SDRL values decrease quickly for all δ, and the decrease is more significant for $\rho_{XY} = 0.95$ (Table 2).

(5) When $\delta = 0$, there is no significant difference between the ARL and the SDRL values. This is in agreement with the geometric distribution property of run length. Also, when $\delta > 0$, the ARL and the SDRL values are directly proportional but indirectly proportional to the shift (δ) in the location parameter (Table 2).

(6) The chart detects very large shifts at the first out-of-control sample. Thus, the out-of-control ARL and SDRL values tend to 1 and 0, respectively, as the shift increases (Table 2).

(7) The ARL values are always greater than the median of the run length (except for some sampling error). This implies that the run length distribution is skewed to the right.

(8) The EQL, RARL, and PCI, which explain the overall performance of a chart over all range of shifts, show that the classical EWMA is inferior to the proposed chart for $\rho_{XY} \geqslant 0.50$ (see Table 3). The classical EWMA chart is used as the benchmark chart. A chart with lower EQL, RARL, and PCI values than the benchmark chart is said to be superior to the benchmark chart. It is shown in Fig. 3 that the proposed chart possesses all these attributes when $\rho_{XY} \geqslant 0.50$. These further show the supremacy of the proposed chart over the classical EWMA chart when the correlation between the study variable and the auxiliary variable is not weak.

4. Comparisons with other location monitoring charts

A general comparison of the proposed scheme with the classical EWMA, MxEWMA, and the Mixed EWMA-CUSUM charts is provided in this section, by presenting a one by one comparison of MrEWMA chart with its existing counterparts.

4.1. MrEWMA vs. classical EWMA

Section 2 broadly explains the classical EWMA chart and Table 1 shows the ARL of the classical EWMA chart with time-varying limits. For all the values of λ considered, it is observed that the classical EWMA chart outperform the proposed chart when there is a weak correlation between the study variable and the auxiliary variable. Contrarily, when $\rho_{XY} \geqslant 0.50$, the proposed MrEWMA chart outperforms the classical EWMA chart for all the values of the process mean shift. This is as a result of the lower ARL_1 values of the MrEWMA chart for all the λ and δ considered. In fact, the supremacy of the proposed chart is more significant when $\rho_{XY} = 0.95$ (Table 1 vs. Table 2). This is expected since the proposed chart will perform best for a very high positive correlation coefficient between the study variable and the auxiliary variable.

Furthermore, it is worthy to note that a special case of the MrEWMA chart is the classical EWMA chart when $\rho_{XY} = 0$. Also, the L values of the MrEWMA chart are slightly different from that of the classical EWMA chart because the values are adjusted for the bias in the ratio estimator.

4.2. MrEWMA vs. MxEWMA

The MxEWMA proposed by Abbas et al. (2014) uses auxiliary information in the form of regression estimator to improve the classical EWMA chart. It slightly outperforms the proposed chart. However, the MrEWMA chart competes closely for higher ρ_{XY} (Table 2 vs. Table 4). The supremacy of the M_XEWMA chart is not surprising because the regression estimator has higher efficiency (or lower MSE) than the ratio estimator. Nevertheless, the ratio estimator is often preferred to the regression estimator because of the simplicity of its structure and its merits when the estimates of total (of the study variable) or the ratio of totals (of the study variable to the auxiliary variable) are required (Särndal, Swensson, & Wretman, 2003).

4.3. MrEWMA vs. mixed EWMA-CUSUM (MEC)

The concept of the mixed EWMA-CUSUM chart (denoted as MEC hereafter) was proposed to further increase the sensitivity of the EWMA and CUSUM charts to small shifts. The design structure of the scheme can be found in Abbas, Riaz, and Does (2013). When $\lambda = 0.25$, the MEC chart outperforms the MrEWMA chart in detecting small shifts ($\delta \leqslant 1$) in process mean when the correlation between X and Y is not strong (that is, $\rho_{XY} \leqslant 0.50$), but inferior for detecting large shifts ($\delta > 1$) in the

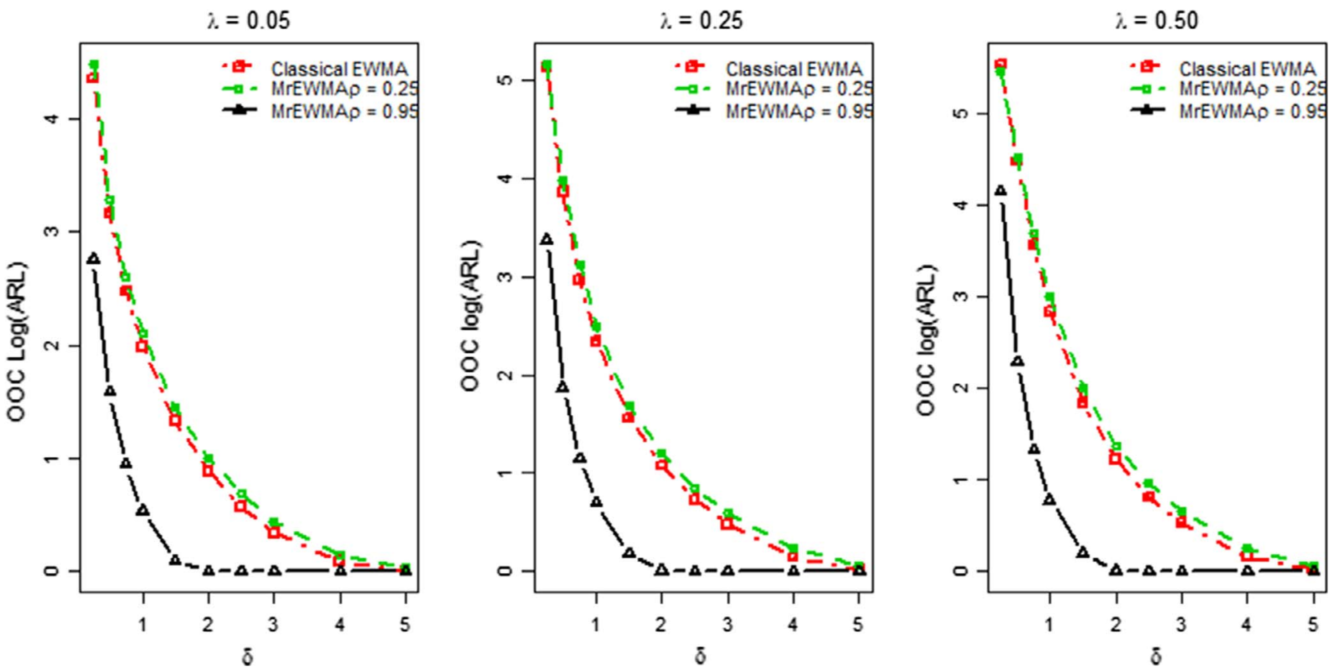

Fig. 2. Out-of-control ARL curve of the classical EWMA chart and the MrEWMA chart (for ρ_{XY} = 0.25 and 0.95).

Table 3
Performance comparison of the proposed MrEWMA chart with the classical EWMA chart as the benchmark chart.

	λ	Classical EWMA	MrEWMA			
			$\rho_{XY} = 0.25$	$\rho_{XY} = 0.5$	$\rho_{XY} = 0.75$	$\rho_{XY} = 0.95$
EQL	0.05	9.69	10.67	8.62	6.73	5.52
	0.25	12.60	13.97	10.76	7.74	5.75
	0.5	16.48	18.14	13.51	9.16	6.16
RARL	0.05	1	1.11	0.87	0.63	0.46
	0.25	1	1.11	0.85	0.59	0.41
	0.5	1	1.10	0.83	0.56	0.39
PCI	0.05	1	1.10	0.89	0.69	0.57
	0.25	1	1.11	0.85	0.61	0.46
	0.5	1	1.10	0.82	0.56	0.37

location parameter of a process. However, the proposed MrEWMA chart outperforms the MEC chart for all shifts when $\rho_{XY} \geqslant 0.50$ (Fig. 4(a)).

When $\lambda = 0.25$, the MEC only performs better than the MrEWMA chart in detecting shifts of $\delta \leqslant 1.50$, $\delta \leqslant 1$, and $\delta \leqslant 0.50$ when $\rho_{XY} = 0.25$, $\rho_{XY} = 0.50$, and $\rho_{XY} = 0.75$, respectively, but inferior to the MrEMWA chart in detecting all sizes of shift when $\rho_{XY} = 0.95$ (Fig. 4(b)). Hence, it can generally be inferred that the MEC chart only performs better than the proposed MrEWMA chart in detecting small shifts when the correlation between X and Y is not strong.

5. Illustrative example

In this section, we provide an illustrative example to show the implementation of our proposed chart using a practical example. For this purpose, we have considered the dataset based on the nonisothermal continuous stirred tank chemical reactor model (namely CSTR process) originally proposed by Marlin (1995) and has been widely used as a benchmark in fault detection and diagnosis (Yoon & MacGregor, 2001; Shi, Lv, Fei, & Liang, 2013). The CSTR process comprises of nine process variables, among which we have considered Outlet temperature as the study variable (Y) and cooling water temperature as the auxiliary variable (X). Details of other variables may be seen in Yoon and MacGregor (2001) and Shi et al. (2013).

The said data set originally contains 1024 values that have been collected on sampling interval of half minute. The first 512 data points which are in-control are used to estimate the population parameters. These estimates come out to be $\bar{y} = 368.23$, $\bar{x} = 365.02$, $s_y = 0.4671$, $s_x = 0.5439$, and $r_{yx} = 0.71$. Considering these estimates as the known parameters, we have generated 15 paired observations of size $n = 5$ from a bivariate normal distribution and disturbance is introduced to the mean of the study variable after the sixth observation, which makes the mean to be shifted to a new value ($\mu_1 = 368.70$). The inspiration of generating dataset in such manner is taken from Singh and Mangat (1996).We have applied the classical EWMA (with $\lambda = 0.03$ and $L = 2.483$) and the MrEWMA (with $\lambda = 0.03$ and $L = 2.4841$) to the generated datasets. Fig. 5 shows that the classical EWMA detects the shift after the 12th sample, while the proposed chart detects the shift earlier at the 9th sample (Fig. 6). These results are in agreement with the results of the comparison of the proposed chart with the classical EWMA chart in Section 4.

6. Summary and conclusion

Control charts are used in monitoring process to detect any special cause variations. Primarily, they are classified into memoryless control charts and memory control charts. Memoryless control charts aimed at detecting large shifts in a process parameter, while memory control charts have design structures that are efficient in detecting small to moderate shifts in a process parameter. We proposed an EWMA-type chart that uses an auxiliary variable in the form of ratio estimator. The auxiliary variable must be strongly positively related to the study variable for the proposed MrEWMA chart to perform better than its existing counterparts. Except for the case of weak correlation value, it is shown that the proposed MrEWMA chart outperforms the classical EWMA chart in detecting small to moderate shifts in the location parameter of a control process. It gives the best performance when the correlation is 0.95. The proposed MrEWMA chart also performs better than the mixed EWMA-CUSUM chart when $\rho_{XY} \geqslant 0.50$. Even when there is a weak correlation between Y and X, the proposed chart outperforms the MEC chart in detecting large shifts in process mean. In addition, the proposed chart performs closely with the EWMA-type chart that estimates the process mean using the regression estimator.

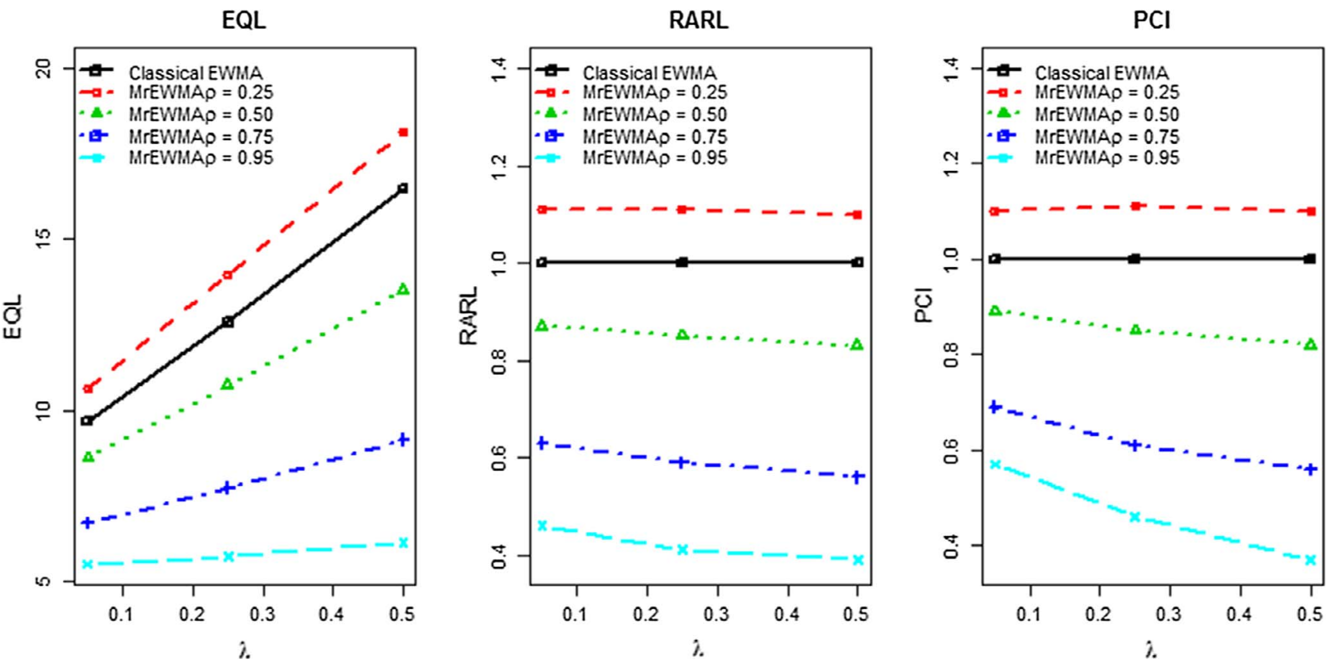

Fig. 3. EQL, RARL, and PCI of the classical EWMA chart and the MrEWMA chart.

Table 4
ARL values of the MxEWMA chart with ARL_0 = 500.

ρ_{XY}	(λ, L)	δ										
		0	0.25	0.5	0.75	1	1.5	2	2.5	3	4	5
0.25	(0.05, 2.639)	500.54	73.80	22.43	11.25	6.93	3.58	2.31	1.70	1.36	1.07	1.01
	(0.25, 3.001)	499.80	161.97	44.18	18	9.77	4.50	2.80	1.99	1.55	1.13	1.01
	(0.50, 3.072)	499.95	245.96	83.01	32.90	15.83	5.83	3.18	2.15	1.62	1.15	1.02
0.50	(0.05, 2.639)	499.56	61.10	18.54	9.33	5.76	3.01	1.97	1.48	1.22	1.03	1
	(0.25, 3.001)	499.69	135.77	34.58	14.11	7.80	3.72	2.35	1.70	1.35	1.05	1
	(0.50, 3.072)	500.79	216.10	65.40	24.80	11.91	4.56	2.59	1.80	1.39	1.06	1
0.75	(0.05, 2.639)	499.59	38.46	11.71	5.93	3.72	2.03	1.40	1.13	1.03	1	1
	(0.25, 3.001)	500.41	84.13	18.97	8.08	4.72	2.41	1.59	1.23	1.06	1	1
	(0.50, 3.072)	499.29	146.89	34.90	12.46	6.15	2.68	1.68	1.26	1.07	1	1
0.95	(0.05, 2.639)	499.34	10.61	3.40	1.87	1.31	1.02	1	1	1	1	1
	(0.25, 3.001)	500.23	16.70	4.26	2.21	1.48	1.03	1	1	1	1	1
	(0.50, 3.072)	500.01	30.23	5.39	2.42	1.55	1.04	1	1	1	1	1

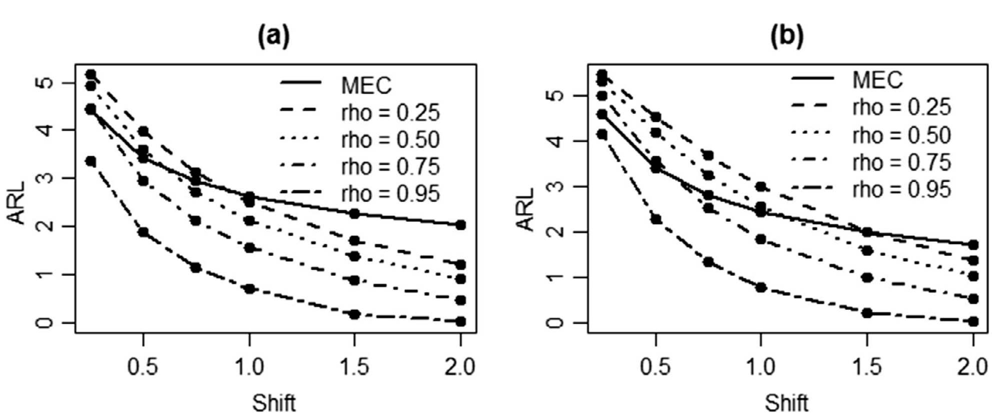

Fig. 4. Out-of-control ARL curve of the mixed EWMA-CUSUM (MEC) chart and the MrEWMA chart (for ρ_{XY} = 0.5, 0.75, and 0.95) when (a) λ = 0.25 and (b) λ = 0.50.

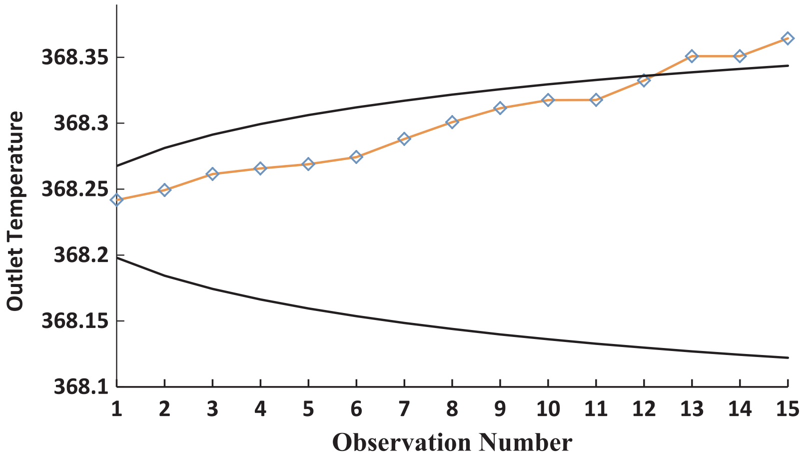

Fig. 5. Classical EWMA chart for the CSTR dataset.

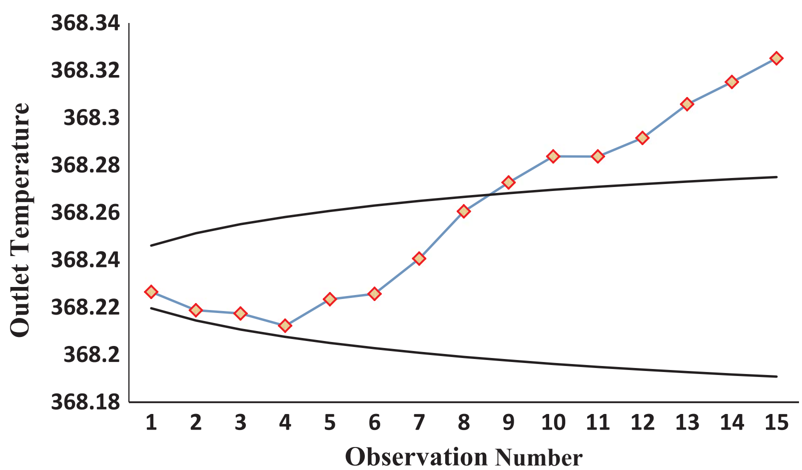

Fig. 6. MrEWMA chart for the CSTR dataset.

Furthermore, the performance of the proposed chart is assessed in terms of ARL, EQL, RARL, and PCI for different positive values of the correlation coefficient between the study variable and the auxiliary variable. At last, the proposed chat is applied to a real dataset and it detects the disturbance in the data earlier than the classical EWMA.

In future, the runs rule schemes of the proposed chart may also be studied. The scope of the study may be extended to the multivariate EWMA control chart using multivariate auxiliary information. Also, the study may be extended to monitor shifts in processes variability using ARL and the overall performance measures.

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Appendix A

Theorem. *The MrEWMA control chart is more efficient than the classical EWMA control chart when* $C_X/2C_Y < \rho_{YX} \leqslant 1$.

Proof. MSE(Plotting statistic of MrEWMA) $<$ MSE(Plotting statistic of the classical EWMA)

$$\Rightarrow (MSE(\bar{y}_r))(\lambda/(2-\lambda))(1-(1-\lambda)^{2t}) < (MSE(\bar{y}))(\lambda/(2-\lambda))(1-\lambda)^{2t}$$

$$\bar{Y}^2(C_Y^2 + C_X^2 - 2\rho_{YX}C_YC_X)/n < (S_Y^2/n)$$
$$\bar{Y}^2(C_Y^2 + C_X^2 - 2\rho_{YX}C_YC_X)/n < C_Y^2\bar{Y}^2(1/n)$$
$$\bar{Y}^2(C_X^2 - 2\rho_{YX}C_YC_X)/n < 0$$
$$C_X^2 < 2\rho_{YX}C_YC_X$$
$$\rho_{YX} > C_X/2C_Y$$

But $0 \leqslant \rho_{YX} \leqslant 1$.

Then, $C_X/2C_Y < \rho_{YX} \leqslant 1$.

Generally, if $C_Y \approx C_X$, we say, the MrEWMA control chart is more efficient than the classical EWMA control chart when $0.5 < \rho_{YX} \leqslant 1$. □

A.1. Symbols and acronyms

Symbols and acronyms	Meaning
Y	The variable of interest
X	The auxiliary variable
$\overline{X}$ and $\overline{Y}$	The population means of X and Y
$\overline{x}$ and $\overline{y}$	The sample means of X and Y
$\overline{Y}_0$ and $\overline{Y}_1$	The in-control (or target) mean and the out-of-control mean of Y
$\overline{y}_r$	The ratio estimator of population mean $\overline{Y}$
E(), $B()$ and $MSE()$	Expectation, bias and Mean Squared Error
σ_X and σ_Y	The population standard deviations of X and Y
s_x and s_y	The sample standard deviations of X and Y
σ_r	The population standard deviation of ratio estimator
N	The sample size
ρ_{YX} and r_{YX}	The population and sample correlation values between X and Y
C_X and C_Y	The population coefficient of variations of X and Y
c_x and c_y	The sample coefficient of variations of X and Y
λ	The smoothing parameter of EWMA chart
L	Controls the width of the control limits of EWMA chart
δ	The shift in process mean
MrEWMA	The proposed EWMA-type chart
ARL and SDRL	The average run length and standard deviation of run length
EQL	The extra quadratic loss
RARL	The relative average run length
PCI	The performance comparison index.
ATS	Average time to signal
AEQL	Average extra quadratic loss
SRS	Simple random sampling
SE	Standard error
Case-K, Case-U	Known case and unknown case

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