WHY WE USE THIS EQUATION

The quadratic-to-“slightly cubic” step is the reason for the modern Spirule problem

The equation in The First Root-Locus Problem was chosen for a historical reason. It deliberately reconstructs the conceptual move Evans and his colleagues remembered from 1948: begin with a familiar quadratic servo, add one small neglected lag, and watch the characteristic equation become “slightly cubic.” The added pole looks minor at first, but it changes the stability conclusion and creates exactly the sort of root-motion problem for which root locus is useful.

Use a simple negative-unity-feedback position servo:

1. Begin with the quadratic servo

$$L_2(s) = K / [s(s + 1)]$$

Its closed-loop characteristic equation follows from $1 + L_2(s) = 0$:

$$s^2 + s + K = 0$$

At K = 0 the roots start at 0 and −1. As gain increases they meet and become a complex-conjugate pair, but their real part remains negative. In this idealized second-order model there is no finite positive gain at which instability begins.

2. Add the small amplifier lag

Now include a 0.25-second amplifier time constant. Its factor 1/(1 + s/4) adds a third pole at s = −4:

$$L_3(s) = K / [s(s + 1)(1 + s/4)]$$

The characteristic equation becomes

$$s(s + 1)(1 + s/4) + K = 0$$
$$s^3 + 5s^2 + 4s + 4K = 0$$

This is the remembered teaching move. Jeff Schmidt later recalled Evans starting with a simple quadratic, adding another term, and calling the result “slightly cubic” when the added term only slightly bent the locus.

3. Why our handout uses s(s + 1)(s + 4)

Absorb the factor of 4 into the gain by defining $\bar{K} = 4K$. Then

$$s^3 + 5s^2 + 4s + \bar{K} = 0$$

which comes directly from

$$L(s) = \bar{K} / [s(s + 1)(s + 4)]$$

That is the exact equation used in The First Root-Locus Problem. The numbers are modern and intentionally clean, but the structure was chosen to recreate the quadratic-to-cubic episode—not as an arbitrary textbook example.

The extra pole at −4 initially seems remote from the dominant poles at 0 and −1. But the cubic behaves differently at higher gain. Two branches eventually head toward the right half-plane, so they must cross the imaginary axis.

4. The “small” lag changes the engineering answer

$$\text{Set } s = j\omega \text{ in } s^3 + 5s^2 + 4s + \bar{K} = 0$$
$$4\omega - \omega^3 = 0 \rightarrow \omega = 2 \text{ rad/s}$$
$$\bar{K} - 5\omega^2 = 0 \rightarrow \bar{K} = 20$$

The contrast is the point: the quadratic approximation suggests stability for every positive gain; the slightly cubic model has a finite stability ceiling. A physically small lag has produced a qualitatively different design conclusion.

5. Why this leads naturally to root locus

The real design problem is not merely to solve one cubic. As gain changes, every closed-loop root moves. Repeatedly expanding and factoring a new polynomial conceals that motion.

Root locus makes the motion itself visible. Evans’s 1950 paper defined the method as a graphical way to determine the roots of the control-system differential equation as a parameter changes. The open-loop poles give the starting points; the feedback equation supplies the angle and magnitude conditions; the locus shows when the roots cross into instability.

That is why this equation is useful for the Spirule. It lets the student encounter the same conceptual surprise: a term that looks secondary can alter the entire stability picture. The Spirule then becomes a tool for seeing why.

Historical note

The numerical values here are a modern reconstruction, not a claim to reproduce Evans’s exact September 1948 classroom numbers. The historical basis is Jeff Schmidt’s recollection of Evans teaching from a quadratic, adding another term, and calling the result “slightly cubic.” The technical basis is Walter R. Evans, “Control System Synthesis by Root Locus Method,” AIEE Transactions 69(1), 66–69 (1950).