The First Root-Locus Problem

Evans credited a student's "slightly cubic" question as the spark. It was the first root locus problem.

An invitation, not a lesson plan.
Page 1 is meant to stand alone. Use it as a short demonstration, a pair exercise, or a loaner challenge. The sequence is intentionally: predict > reason > construct > verify.

Three low-friction classroom uses

8–10 minute demonstration	Put the three poles on the board. Ask K = 0 and K → ∞ first. Ask where a stability boundary must be crossed. Then let the Spirule locate it. Show MATLAB last.
15–20 minute pair exercise	One student works the Spirule; the other sketches and records predictions. They must agree where to search before calculating, then compare their picture with rlocus.
Loaner challenge	Hand the Spirule and Page 1 to one curious student: "Bring it back with a hand sketch, a predicted stability limit, and a MATLAB plot that tests your reasoning."

Answer key and reasoning

1. Characteristic equation. From 1 + L(s) = 0:

$$\mathbf{s(s+1)(s+4) + K = 0 \;\rightarrow\; s^3 + 5s^2 + 4s + K = 0}$$

2. K = 0. The roots begin at 0, −1 and −4.

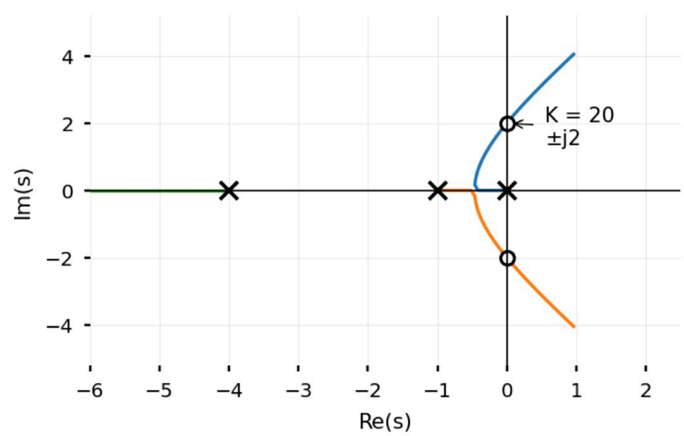
3. K very large. The dominant balance is $s^3 + K \approx 0$, giving far-field directions 60°, 180° and 300°. Two branches therefore head toward the right half-plane.

4. Real-axis clue. The 180° angle condition gives locus segments on (−∞, −4) and (−1, 0). This is a consequence of L(s) = −1, not a separate rule that must be memorized.

5. Where to test. Because the complex branches ultimately reach the right half-plane, test the imaginary axis. Set s = jω:

$$\mathbf{(K - 5\omega^2) + j(4\omega - \omega^3) = 0}$$

For a nonzero crossing, ω = 2 rad/s and then K = 20.



6. Spirule check. At s = j2, the pole angles are 90°, 63.4° and 26.6°, totaling 180°. The pole distances are 2, √5 and √20; their product is 20.

7. Exact boundary. At K = 20:

$$\mathbf{s^3 + 5s^2 + 4s + 20 = (s+5)(s^2+4)}$$

Poles: −5 and ±j2. Stable range for this model: 0 < K < 20.

Questions that deepen understanding

- What did you know before doing arithmetic?
- Why was the imaginary axis the right place to test?
- What did the Spirule make visible or convenient?
- What did MATLAB verify rather than discover?
- If the third pole moved farther left, what would you expect to happen to the allowable gain?

The connection to *Into Stability*

Chapter 3: Understanding Beats Memorization.
Glasgow's advice: "Take extreme cases."
Evans later described learning as self-study and problem solving, beginning with simple examples and working upward; he compared the teacher's role to that of a coach getting students into the game.

Sources and publication note

Historical/educational context: Gregory W. Evans, Into Stability, Second Edition, Chapters 3 and 7. • Technical source: Walter R. Evans, "Control System Synthesis by Root Locus Method," Transactions of the AIEE 69(1), 66–69 (1950), IEEE Xplore document 5060121. • Software verification: MathWorks Control System Toolbox, rlocus documentation.

Publication note: This 2026 numerical example is intentionally simple and is inspired by the documented 1948 "slightly cubic" classroom episode. It does not claim to reproduce Evans's exact September 1948 numerical example.