The First Root-Locus Problem

A modern reconstruction inspired by Walter R. Evans's 1948 "slightly cubic" servo problem

Evans's way of problem solving: *start with simple examples and work my way up*. Also, take "extreme cases":—a problem solving "trick" Walter Evans learned from his favorite professor, Roy Glasgow, and cited in his September 14, 1948 servomechanism lecture notes.
Predict first. Reason from the feedback equation. Use the Spirule to test the geometry. Ask MATLAB last.

Historical starting point

In the summer of 1948, Evans's servomechanism class had already studied a quadratic servo when a student asked what happened if a small amplifier lag made it "slightly cubic." That question helped lead to the root-locus method. The numerical model below is a modern teaching reconstruction, not a claim to reproduce the exact 1948 numbers.

THE SYSTEM	DESIGN QUESTION
$L(s) = K / [\, s(s + 1)(s + 4)\,]$ Characteristic condition: $1 + L(s) = 0$	As K increases, when does the closed-loop system first reach the boundary of stability?

1. Start with one extreme: K = 0. Where are the closed-loop roots? Mark the three starting points.

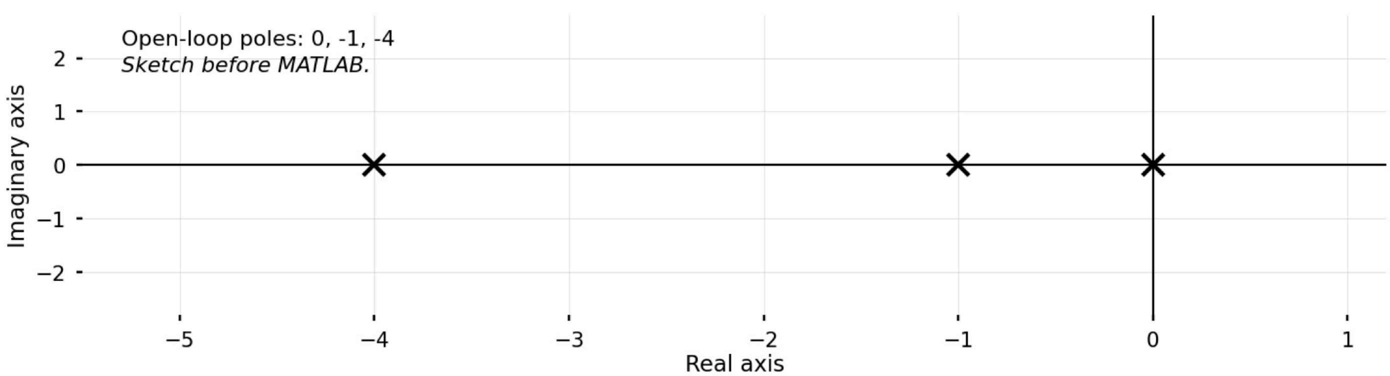
2. Take the other extreme: K → ∞. There are no finite zeros. Far from the origin, $s^3 + K \approx 0$. What three directions does that suggest? What does it tell you before detailed calculation?

3. Derive the two root-locus tests. From $1 + L(s) = 0$, a point p is on the locus only when $L(p) = -1$: its angle is 180° and its magnitude is 1. Use the angle idea on the real axis to get a rough map.

4. Decide where to look. If two branches ultimately head toward the right half-plane, continuity says they must cross the imaginary axis. Let $p = j\omega$. Move a trial point up that axis and use the Spirule to find where the total pole angle is 180°.

5. Let the Spirule find the gain. At a point on the locus, $|L(p)| = 1$, so $K = |p|\,|p + 1|\,|p + 4|$.
Record: $\omega \approx$ _____ $K \approx$ _____

6. Predict before MATLAB. Sketch the locus and state the positive range of K you expect to be stable. Commit to the prediction before you ask software to draw the curve.



COMMIT YOUR PREDICTION BEFORE SOFTWARE

Imaginary-axis crossing: $\omega \approx$ _____ $K \approx$ _____

Predicted stable range: ______________________

NOW VERIFY — DO NOT DISCOVER — WITH MATLAB

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s = tf('s');    G = 1/(s*(s+1)*(s+4));
rlocus(G); grid on
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The computer should confirm a picture you already understand — including the finite ceiling on stable gain.

Historical basis: Into Stability, Second Edition, Chapters 3 and 7. Technical basis: W. R. Evans, "Control System Synthesis by Root Locus Method," AIEE Transactions 69(1), 66–69 (1950).

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