WHERE THE CHARACTERISTIC EQUATION COMES FROM

Companion explanation for The First Root-Locus Problem

The simplified servo used in the problem has the open-loop transfer function

$$L(s) = K \,/\, [s(s + 1)(s + 4)]$$

The system uses negative unity feedback. For such a feedback loop, the closed-loop transfer function is

$$T(s) = L(s) \,/\, [1 + L(s)]$$

The closed-loop poles—the values that determine whether the system settles, oscillates, or becomes unstable—occur where the denominator of T(s) is zero. Therefore,

$$1 + L(s) = 0$$

Substituting the open-loop model gives

$$1 + K \,/\, [s(s + 1)(s + 4)] = 0$$

Multiplying through by s(s + 1)(s + 4),

$$s(s + 1)(s + 4) + K = 0$$

$$s^3 + 5s^2 + 4s + K = 0$$

This is the characteristic equation. For every value of K, its three roots are the three closed-loop poles. Root locus asks: Where do those roots move as K changes?

Why this leads directly to the root-locus test

Instead of repeatedly solving the cubic for different values of K, return to the characteristic condition and rearrange it:

$$K \,/\, [s(s + 1)(s + 4)] = -1$$

So any point s = p on the root locus must make L(p) equal to −1. That gives two conditions:

- **Angle:** L(p) must point in the negative-real direction—an angle of 180°.
- **Magnitude:** |L(p)| must equal 1.

These are not arbitrary rules to memorize. They fall directly out of the feedback characteristic equation—and they are precisely the two quantities the Spirule was designed to help evaluate.

MATLAB connection. The rlocus command displays the trajectories of the closed-loop poles as feedback gain varies. In this exercise, MATLAB comes last: it verifies the picture already developed from the characteristic equation and the Spirule.